$B_{PCM} = \frac{R_b}{2} = \frac{n2B}{2} = nB$

$SNR_{PCM,dB} = \alpha + 6n$

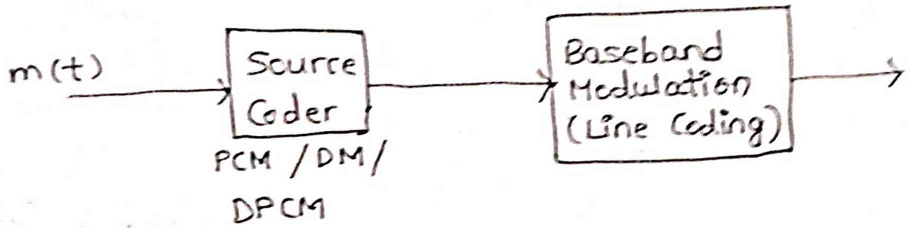
$SNR_{DPCM} = G\ SNR_{PCC}$

$SNR_{DPCM,dB} = 20\log_{10}\left(\frac{m_p}{m_d}\right) + \alpha + 6n.$

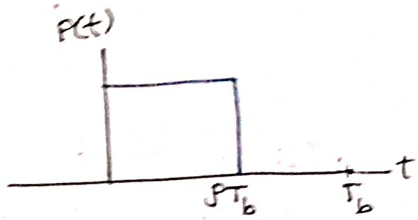
$m_d \Rightarrow d[k] = m[k] - \hat{m}_q[k]$

RZ: Return to Zero

NRZ: Not return to zero.

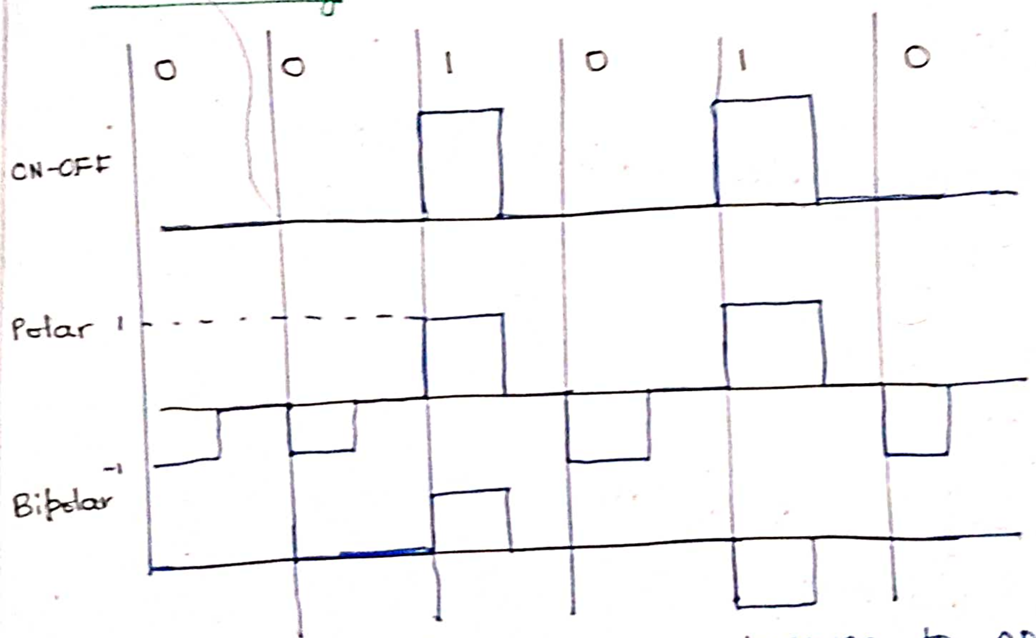


Line Coding : RZ Scheme (Return to Zero)



→ Fractional

~~Pulse Coded signal~~



nature of seq matching error

* Polar ~~needs~~ mo scheme is more immune to noise and distance b/n + and - is more than +, 0.

Hence, polar scheme is more immune to noise and more power efficient compared to ON-OFF.

* We can count the number of ones.

Hence, Bipolar has error detection capabilities.

$$SNR_{ON\text{-}OFF} = \frac{A'^2}{2N_0} = K \qquad SNR_{Polar} = \frac{A^2}{N_0} = K$$

$A' > A$ i.e. $A' = \sqrt{2}\,A$

'K' is suppose the desired SNR. So, to equate the SNR for both schemes, ON-OFF will require amplitude and hence more power

ISI control
PSD of pulsing
more power

PLL
CLK/carrier recovery

Favourable requirements for Line Encoding.

(i) Power efficient

(ii) BW efficient

(iii) Favourable PSD (Power Spectral Density) of pulses to avoid ISI (Inter-Symbolic Interference)

(iv) Error ~~computation~~ correct / detection capability

(v) Favourable for Clock recovery

Calculating Power Spectral Density (PSD) of Pulses

Pulse Coded Signal

PSD ∝ FT Deterministic component of a Line Coding scheme

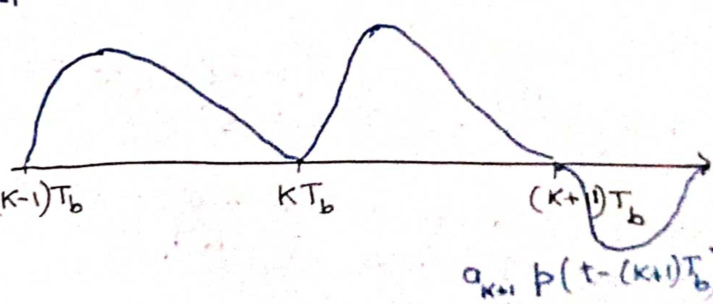
ON-OFF $\quad a_K = \begin{cases} 1, & \text{for '1'} \\ 0, & \text{for '0'} \end{cases}$

Polar $\quad a_K = \begin{cases} +1, & \text{for '1'} \\ -1, & \text{for '0'} \end{cases}$

Bipolar

Y(t): Line Coding Signal

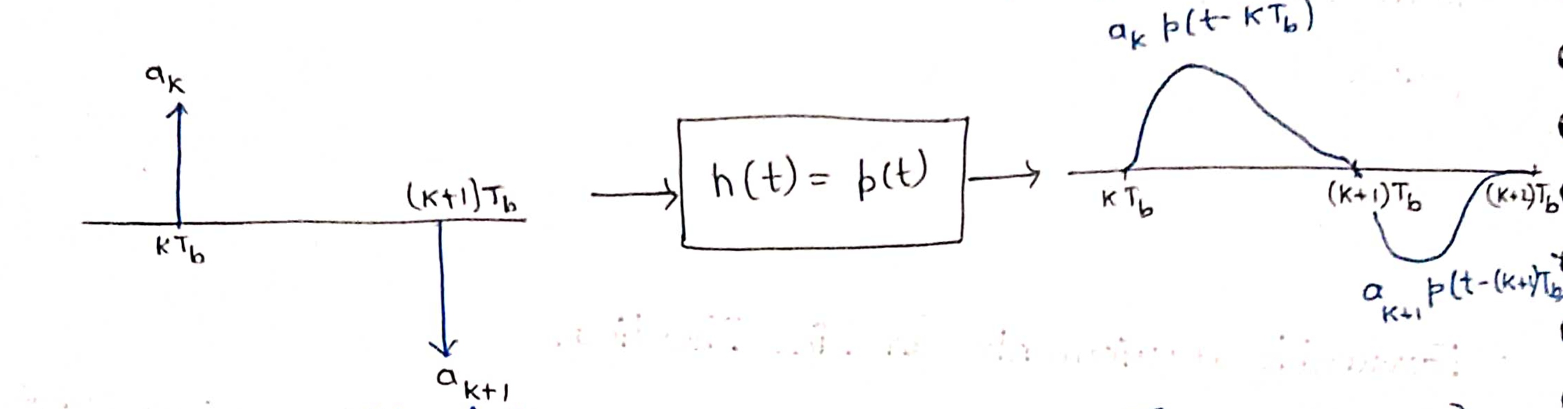
$$Y(t) = \sum_{K=-\infty}^{\infty} \underbrace{a_K}_{\text{Random sequence}} \underbrace{p(t - KT_b)}_{\text{Deterministic}}$$



Representing $Y(t)$ as an output of a filter will help us calculate its PSD directly acc. to PSD of input. using formula:

$$S_Y(f) = |P(f)|^2 \, S_X(f)$$

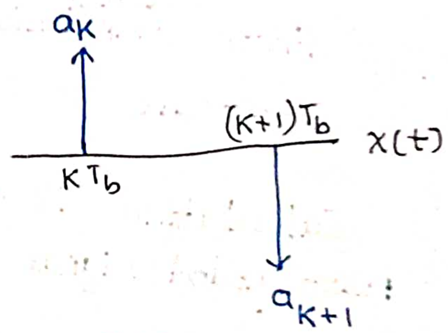
This helps for decoupling dependency of $Y(t)$ from a_K and $p(t-KT_b)$.



$x(t)$: impulse at multiples of T_b.

$$Y(t) = \sum_K a_K \, p(t - KT_b)$$

$x(t)$: original signal.



Finding PSD of input.

→ $S_x(f) = FT\{R_x(\tau)\}$

→ $R_x(\tau) = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} x(t)\, x(t-\tau)\, dt$

→ $R_x(\tau) = \lim_{\varepsilon\to 0} R_{\hat{x}}(\tau)$

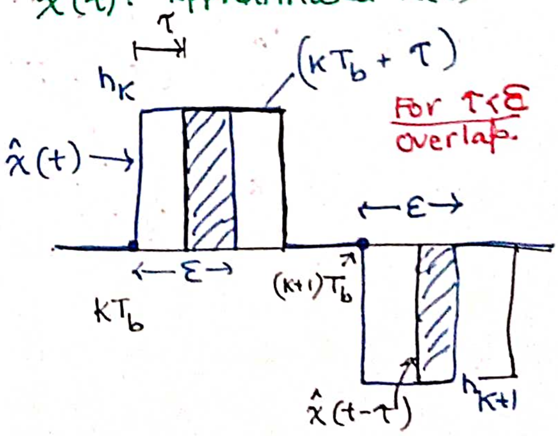
We will calculate above integration with $x(t) = \hat{x}(t)$ using cases below:

$\hat{x}(t) \xrightarrow{\varepsilon\to 0} x(t)$

$\hat{x}(t)$: Approximated $x(t)$

CASE I

For $\tau < \varepsilon \Rightarrow \lim_{T\to\infty} \frac{1}{T} \sum_K h_K^2 (\varepsilon - \tau)$



→ Impulses can be seen as limiting case for rectangular pulses.

$h_K \, \varepsilon = a_K$ as $\varepsilon \to 0$

→ $R_{\hat{x}}(\tau) = \lim_{T\to\infty} \frac{1}{T} \sum \frac{a_K^2}{\varepsilon^2} (\varepsilon - \tau)$

$h_K = \frac{a_K}{\varepsilon}$ as $\varepsilon \to 0$

→ Let N be the number of pulses transmitted in time T. Then, $T = N T_b$.

$\rightarrow R_{\hat{x}}(\tau) = \underbrace{\left(\lim_{N\to\infty} \frac{1}{N} \sum_i a_k^2\right)}_{R_0} \frac{1}{T_b\,\varepsilon}\left(1 - \frac{\tau}{\varepsilon}\right)$

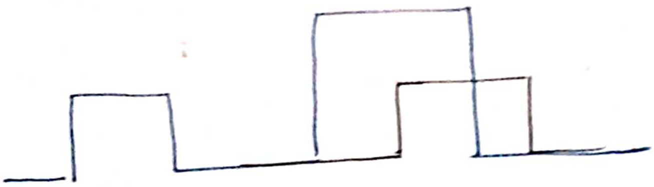
$\rightarrow R_{\hat{x}}(\tau) = \frac{R_0}{T_b\,\varepsilon}\left(1 - \frac{|\tau|}{\varepsilon}\right)$ for $|\tau| < \varepsilon$

CASE 2

$\rightarrow R_{\hat{x}}(\tau) = 0$ for $\varepsilon < \tau \leq T_b - \varepsilon$

As there would be no overlap.

$\rightarrow R_{\hat{x}}(\tau)$ for $T_b - \varepsilon < \tau < T_b + \varepsilon$ CASE 3



$$R_{\hat{x}}(\tau) = \lim_{T\to\infty} \frac{1}{T} \sum_i h_k h_{k+1} (\varepsilon - \tau)$$

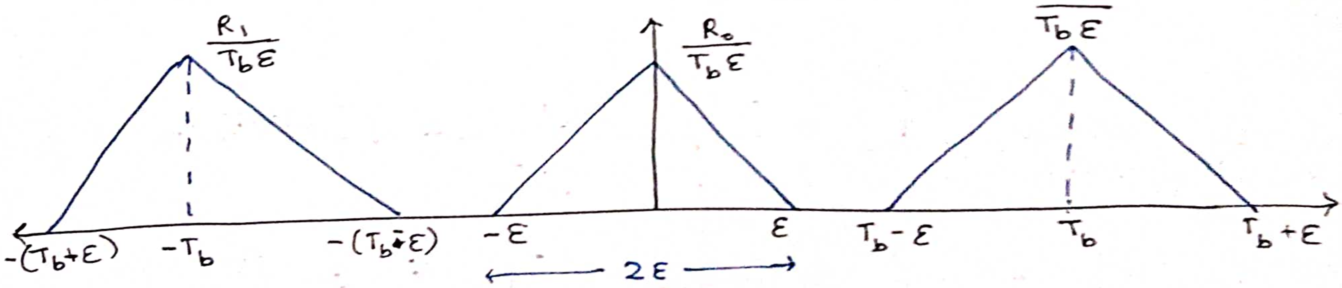
$$= \frac{R_1}{T_b\,\varepsilon}\left(1 - \frac{|\tau|}{\varepsilon}\right)$$

where, $R_1 = \lim_{N\to\infty} \frac{1}{N} \sum_i a_k a_{k+1} = \widetilde{a_k a_{k+1}}$

$\rightarrow$ For $nT_b - \varepsilon < \tau < nT_b + \varepsilon$

$$R_{\hat{x}}(\tau) = \frac{R_n}{T_b\,\varepsilon}\left(1 - \frac{|\tau|}{\varepsilon}\right)$$

$\rightarrow$



$$\rightarrow R_X(\tau) = \lim_{\epsilon \to 0} R_{\hat{X}}(\tau)$$

$$R_X(\tau) = \frac{1}{T_b} \sum_n R_n \, \delta(t - nT_b)$$

$$S_X(f) = \frac{1}{T_b} \sum_n R_n \exp(-j 2\pi f n T_b)$$

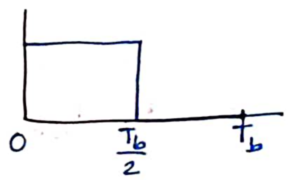
$$S_Y(f) = \frac{|P(f)|^2}{T_b} \sum_{n=-\infty}^{\infty} R_n \exp(-j 2\pi f n T_b)$$

$$= \frac{|P(f)|^2}{T_b} \left[\sum_{n=1}^{\infty} 2 R_n \cos(2\pi f n T_b) + R_0 \right]$$

$$\rightarrow R_0 = \widetilde{a_K^2} = 1^2 \cdot \frac{1}{2} + 0^2 \cdot \frac{1}{2} = \frac{1}{2}$$

$$R_n = \widetilde{a_K \, a_{K+n}} = 1^2 \cdot \frac{1}{4} + 0^2 \, \frac{3}{4} = \frac{1}{4}$$

$\rightarrow$ For $p(t) =$



$$P(f) = \frac{T_b}{2} \operatorname{sinc}\left(\pi \frac{T_b}{2} f\right)$$

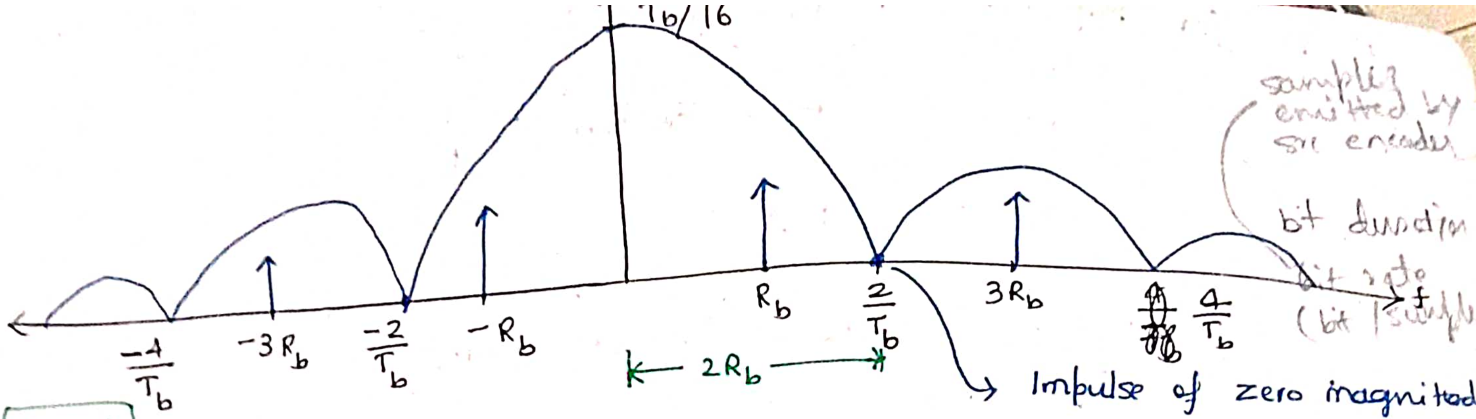
$$|P(f)|^2 = \frac{T_b^2}{4} \operatorname{sinc}^2\left(\pi \frac{T_b}{2} f\right)$$

$$\rightarrow S_Y^{\text{ON-OFF}}(f) = \frac{T_b^2}{4T_b} \operatorname{sinc}^2\left(\pi \frac{T_b}{2} f\right) \left[\frac{1}{2} + \sum_{n \neq 0} \frac{1}{4} \exp(j 2\pi f n T_b) \right]$$

$$= \frac{T_b}{16} \operatorname{sinc}^2\left(\pi \frac{T_b}{2} f\right) \left[1 + \sum_{n=-\infty}^{\infty} \exp(j 2\pi n f T_b) \right]$$

$$= \frac{T_b}{16} \operatorname{sinc}^2\left(\frac{\pi}{2} T_b f\right) \left[1 + \frac{1}{T_b} \sum \delta\left(f - \frac{n}{T_b}\right) \right]$$

Fourier series represent of $\sum \delta\left(f - \frac{n}{T_b}\right)$ is $\sum \exp(j 2\pi n f T_b)$.



→ Impulse of zero magnitude

sampling emitted by src encoder

bit duration

bit rate (bit / sample)

$\boxed{R_b = \frac{1}{T_b}}$

So, $S_y(f)$ consists of both continuous (sync) and discrete (impulse) part.

null-to-null BW

Ideally, BW should be infinite, but we look at null to null BW.

$B_T = \frac{BW}{2}$ (min BW required channel BW.)

We consider BW to ~~consi~~ have main lobe in sinc function.

$BW_{ON\text{-}OFF} = 2R_b = 4B_T$ if pulse width is $T_b/2$.

$BW_{ON\text{-}OFF} = R_b = 2B_T$ if pulse width is T_b if For full T_b

where, $B_T = \frac{R_b}{2}$. B_T : Minimum channel B.W.

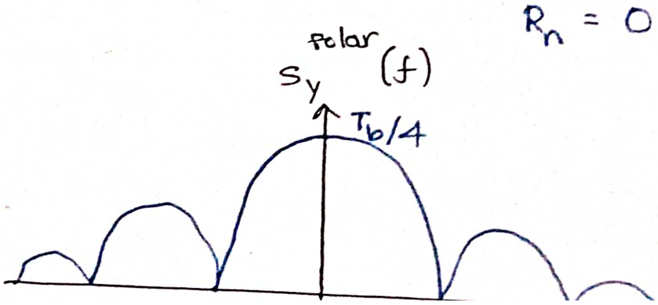
We need to increase pulse width to make it more closer to B_T.

PSD for Polar

More power eff. ∵ sinc amplitude is less

Polar ⇒ $a_k = \begin{cases} +1, & \text{for '1'} \\ -1, & \text{for '0'} \end{cases}$ ⇒ $R_n = \overline{a_k\, a_{k+n}}$ ⇒ $R_0 = 1$, $R_n = 0$

$$S_y^{polar}(f) = \frac{T_b}{4}\,\text{sinc}^2\left(\pi \frac{T_b}{2} f\right)$$



BW is same as ON-OFF

PSD for Bipolar

$$a_k = \begin{cases} \pm 1 & \text{for '1'} \\ 0 & \text{for '0'} \end{cases}$$

$$R_0 = \widetilde{a_k^2} = (\pm 1)^2 \frac{1}{2} = \frac{1}{2}$$

$$R_1 = \widetilde{a_k a_{k+1}} = \frac{-1}{4}$$

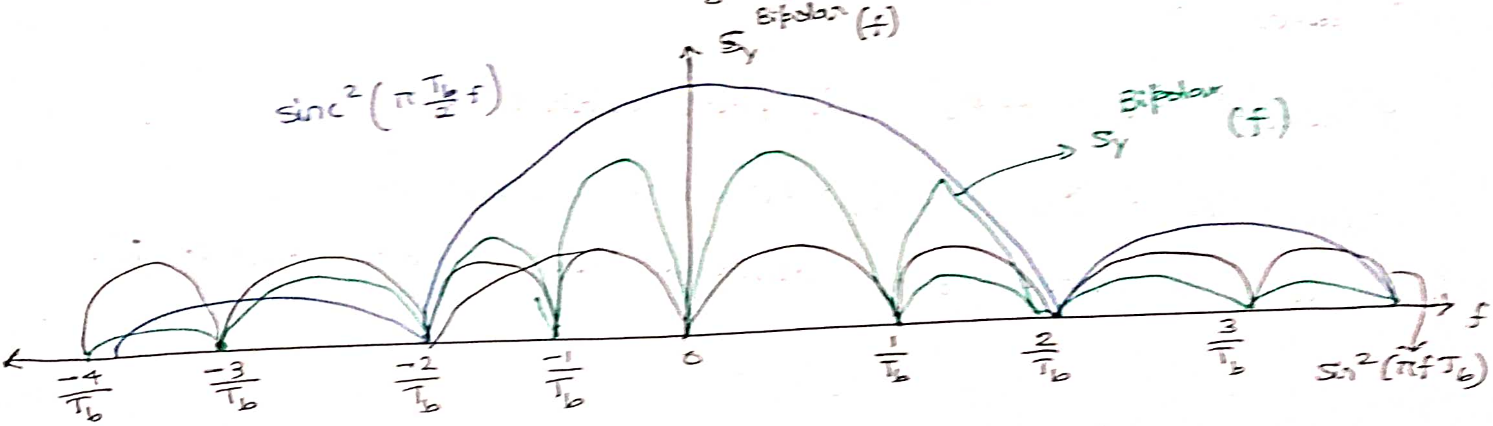
$$R_n = 0$$

$$S_y(f) = \frac{|P(f)|^2}{T_b} \left[\frac{1}{2} - \frac{1}{2} \cos(2\pi f T_b) \right]$$

$$= \frac{|P(f)|^2}{2T_b} \left[1 - \cos(2\pi f T_b) \right]$$

$$= \frac{T_b}{4} \underbrace{\text{sinc}^2\left(\pi \frac{T_b}{2} f\right)}_{\to \pm \frac{n2}{T_b}} \underbrace{\sin^2(\pi f T_b)}_{\to \pm \frac{n}{T_b}}$$

Values at which these terms become zero.



BW = R_b

Q1. To show periodic rectangular waveform with period T_p can be used as carrier to generate DSB-SC signal.

Sol^n:

→ $s(t)$ is a periodic signal with period T_p.. Let $f_p = \frac{1}{T_p}$

$$s(t) = \sum_{n} x(t - nT_p)$$

$$= a_0 + \sum_{n=1}^{\infty} a_n \cos(2\pi n f_p t) + \sum_{n=1}^{\infty} b_n \sin(2\pi n f_p t)$$

(Fourier Series Representation)

→ As $s(t)$ is an odd signal, $a_n = 0 \quad \forall n \geq 1.$

→ $$s(t) = a_0 + \sum_{n=1}^{\infty} b_n \sin(2\pi n f_p t)$$

→ $$a_0 = \frac{1}{T_p} \int_{\langle T_p \rangle} s(t)\, dt = \frac{1}{T_p}\left[\int_0^{T_p/2} 1\, dt + \int_{T_p/2}^{T_p} (-1)\, dt \right] = 0$$

→ $$b_n = \frac{2}{T_p}\left[\int_0^{T_p} s(t) \sin(2\pi n f_p t)\, dt \right]$$

$$= \frac{2}{T_p}\left[\int_0^{T_p/2} \sin(2\pi n f_p t)\, dt + \int_{T_p/2}^{T_p} -\sin(2\pi n f_p t)\, dt \right]$$

$$= \frac{2}{T_p}\left[\frac{\cos(2\pi n f_p t)\Big|_{T_p/2}^{0}}{2\pi n f_p} + \frac{\cos(2\pi n f_p t)\Big|_{T_p/2}^{T_p}}{2\pi n f_p} \right]$$

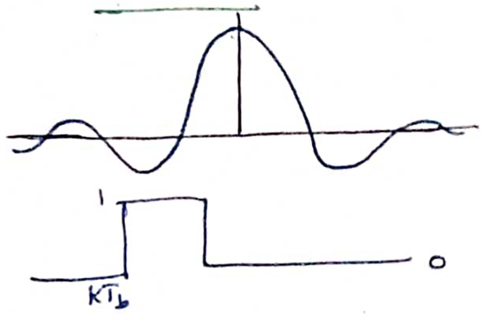
$$= \frac{2}{T_p}\left[\frac{1 - \cos(\pi n f_p T_p) + \cos(2\pi n f_p T_p) - \cos(\pi n f_p T_p)}{2\pi n f_p} \right]$$

$$= \frac{2}{T_p}\left[\frac{2 - 2\cos(n\pi)}{2\pi n f_p} \right] = \frac{2}{n\pi}\left[1 - \cos n\pi \right]$$

$$= \begin{cases} 0 & , \text{ if 'n' is odd} \\ 4/n\pi & , \text{ if 'n' is even} \end{cases}$$

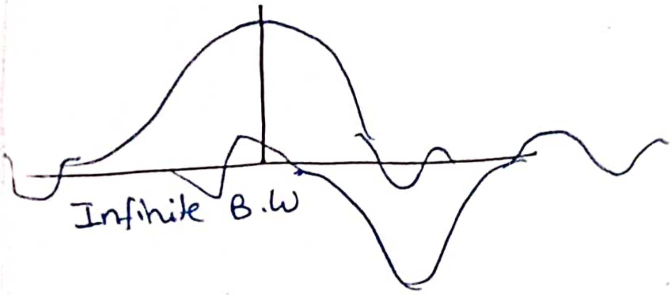
ISI

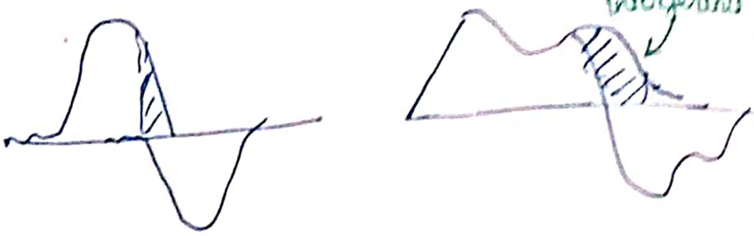
30 March, 2024



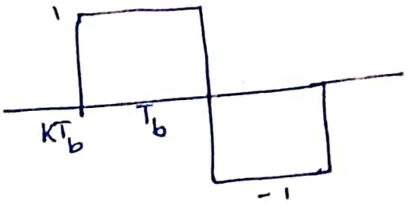
$\rightarrow$ channel $B.W = 2R_b$

B.W limited Channel.





ISI: Inter symbolic interference



Bandwidth limited pulse $\rightarrow$ Pulse overlapping in time $\rightarrow$ ISI before even folding to channel

Time limited pulse $\rightarrow$ Infinite B.W $\rightarrow$ Output of B.W limited channel causing plus spreading in time $\rightarrow$ ISI at output.

Avoiding ISI

$$\rightarrow \boxed{P(t) = \begin{cases} 1 & \text{at } t = 0 \\ 0 & \text{at } t = \pm nT_b \end{cases}}$$

Infinite time duration pulse

B.W limited pulse

$P(t)$ can take any value at 't' other than $\pm nT_b \cup \{0\}$.

$$\rightarrow y(t) = \sum_l a_K \, p(t - KT_b)$$

$$= a_l \, P(t - lT_b) + \sum_{K \neq l} a_K \, P(t - KT_b)$$

$$\rightarrow y(lT_b) = a_l$$

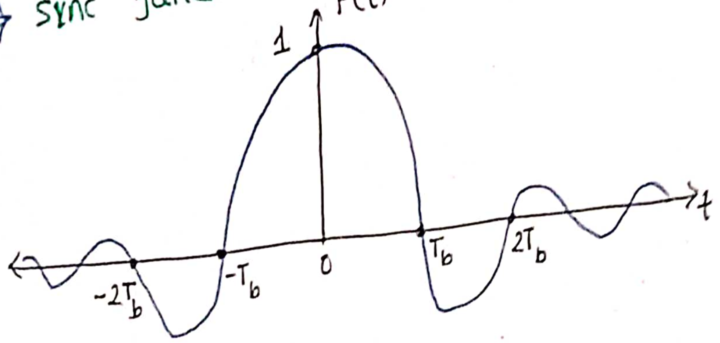
$\rightarrow B_T = \frac{R_b}{2}$

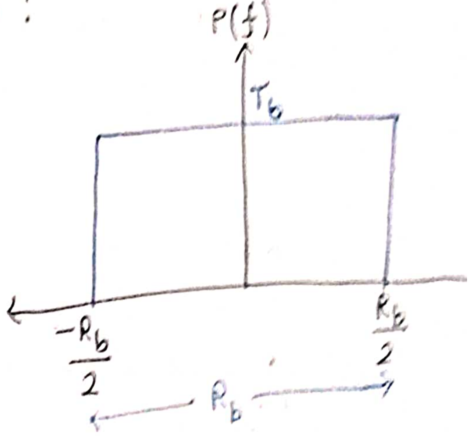
→ Min Channel B.W to transmit bits sampled at rate R_b.

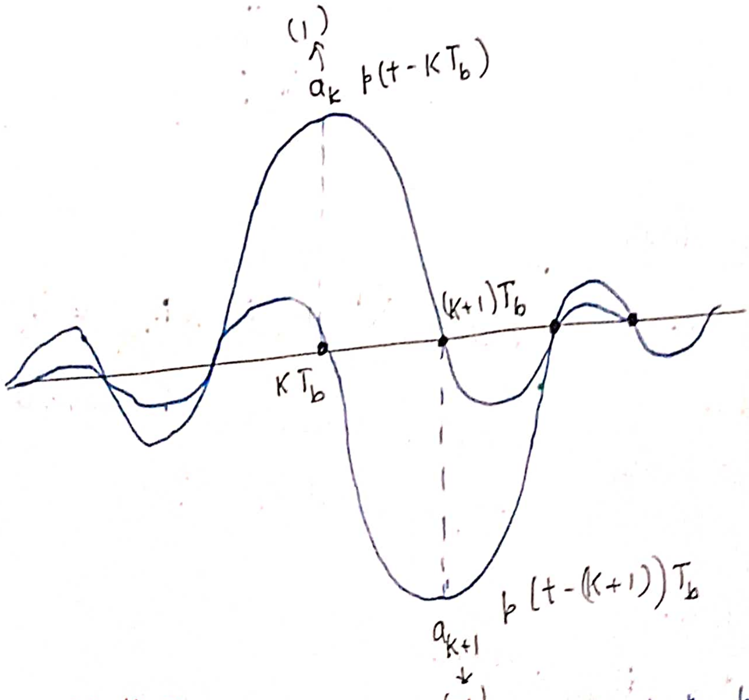
→ So, $\boxed{BW \text{ of } p(t) \leq B(t)}$.

As now pulse is associated with each bit now, p(t) should also have the same constraint of B.W so that it is transmitted effectively.

→ sinc function have both the property!



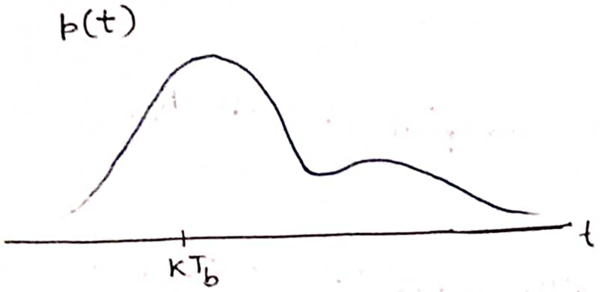


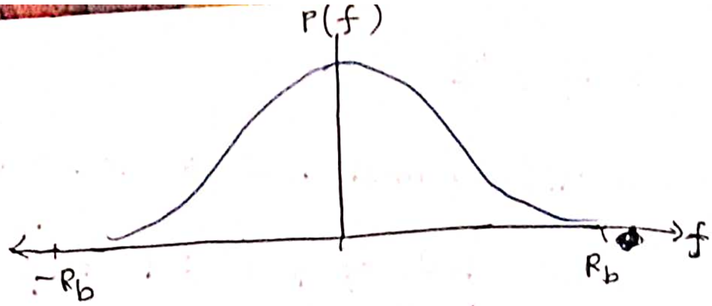


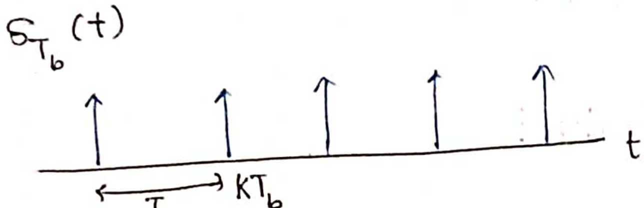
Limitations

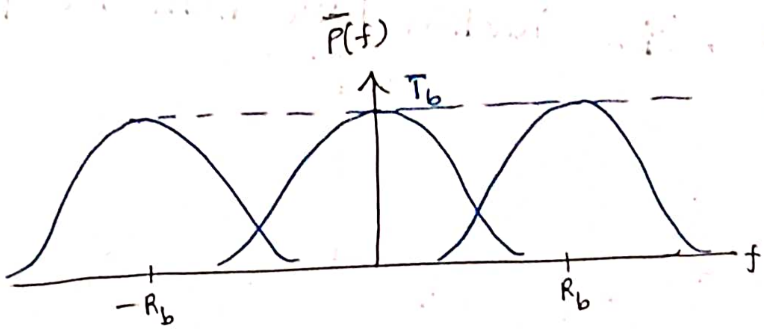
① Little bit error in sampling can lead to high levels of ISI.

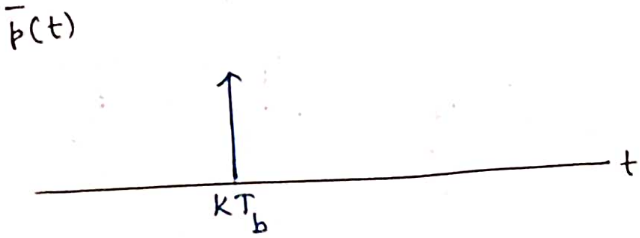
$\because \operatorname{sinc} x = \frac{\sin x}{x} \qquad \sum \frac{1}{x} \rightarrow$ large value





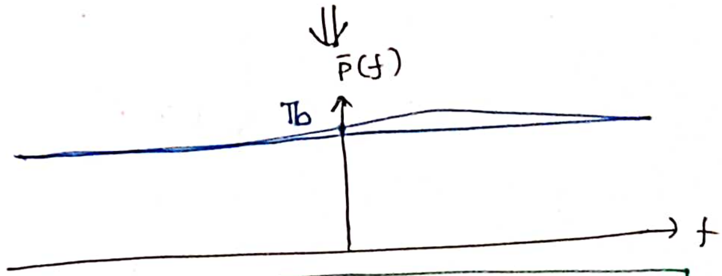






$\bar{p}(t) = p(t) \times \delta_{T_b}(t)$

$\Downarrow$



If we choose $\bar{P}(f) = T_b$, then we would get a constant amplitude of T_b which would lead to $\bar{p}(t) = \delta(KT_b)$ which we want.

$$\boxed{\bar{P}(f) = \sum P(f - KR_b) = T_b}$$

Sampling would lead to summation of $P(f)$ shifted by KR_b.

$$\bar{p}(t) = \begin{cases} 1 & , \text{ for } t = KT_b \\ 0 & , \text{ for } t = \pm KT_b \end{cases} , \; n \neq K.$$

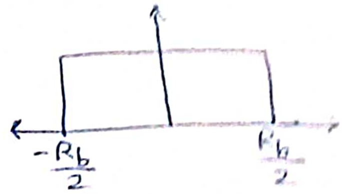
Over any band (frequency) only two pulse are overlapping.

$$T_b = P(f + 0.5R_b) + P(f - 0.5R_b)$$

$m(t) = \sum a_k p(t - kT_b)$

1) $p(t) = \begin{cases} 1 & t = 0 \\ 0 & t = \pm n T_b \end{cases}$

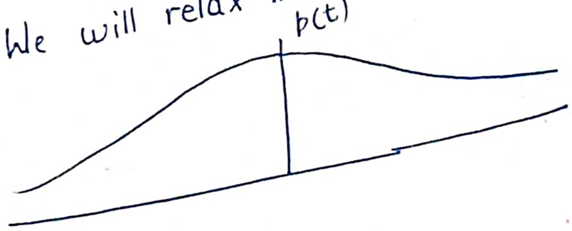
2) $B \leq \frac{R_b}{2}$ $\qquad P(f) = \Pi\left(\frac{f}{R_b}\right)$



$p(t) = R_b \operatorname{sinc}(\pi R_b t)$

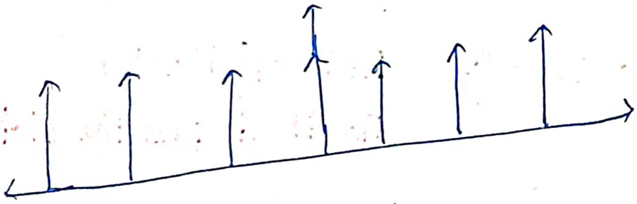
We will relax the second criteria.

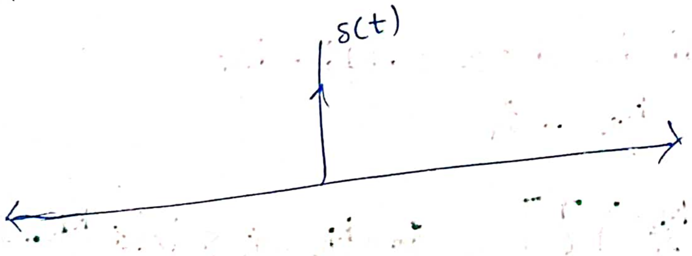
$\overline{p(t)} = p(t)\, \delta_{T_b}(t) = \delta(t)$



$R_b \sum P(f - K R_b) = 1$

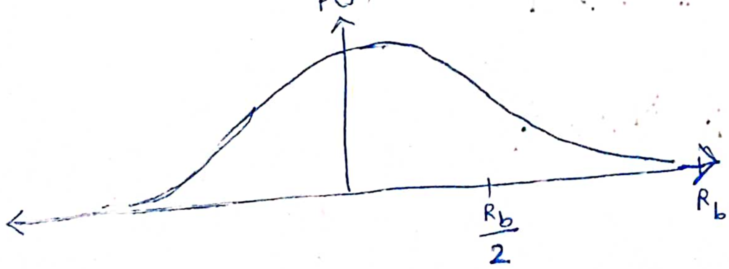
$\boxed{\sum P(f - K R_b) = T_b}$





$P\left(f + \frac{R_b}{2}\right) + P\left(f - \frac{R_b}{2}\right) = T_b$

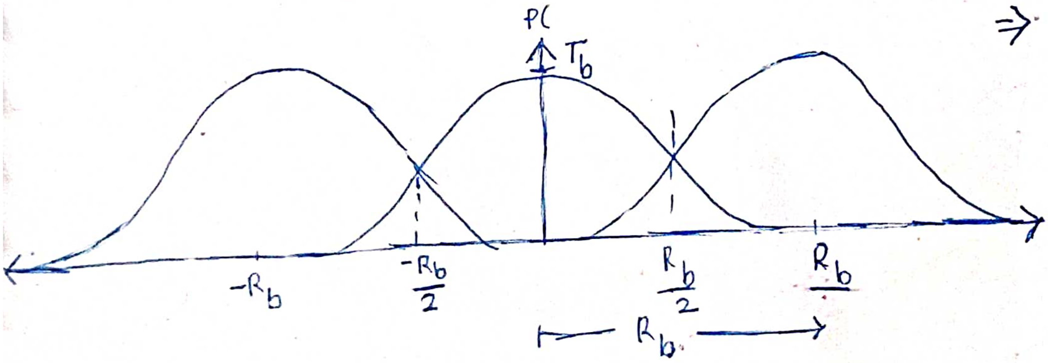
$\left|P\left(f + \frac{R_b}{2}\right)\right| + \left|P\left(-f + \frac{R_b}{2}\right)\right| = T$

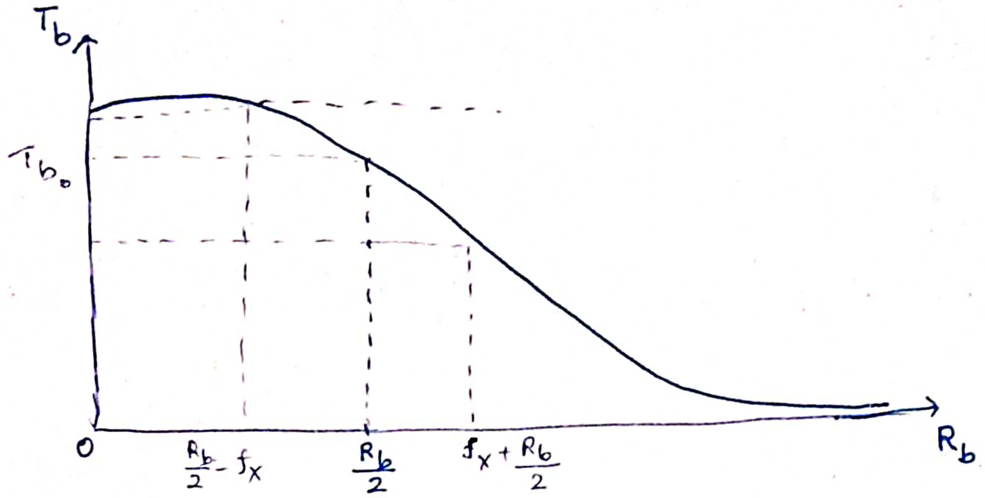


$p(t) \Rightarrow$ Real

$P(f) \Rightarrow P(f) = P^*(-f)$

$\Rightarrow |P(f)| = |P(-f)|$





$$\left|P\left(\frac{R_b}{2}\right)\right| = \frac{1}{2}|P(0)|$$

Let say sinc = 0 at $B.W = f_x + \frac{R_b}{2}$

B.W increased from $\frac{R_b}{2}$ to $f_x + \frac{R_b}{2}$.

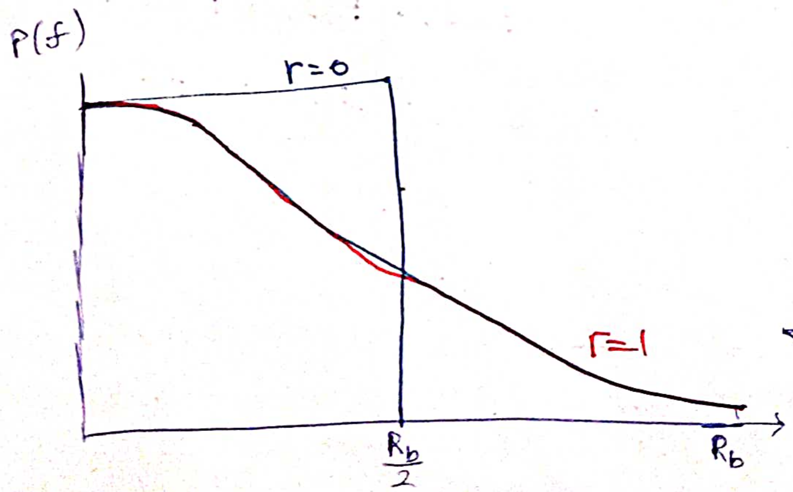
$B.W. = \frac{R_b}{2}(1+r)$ where, r : Increased fraction.

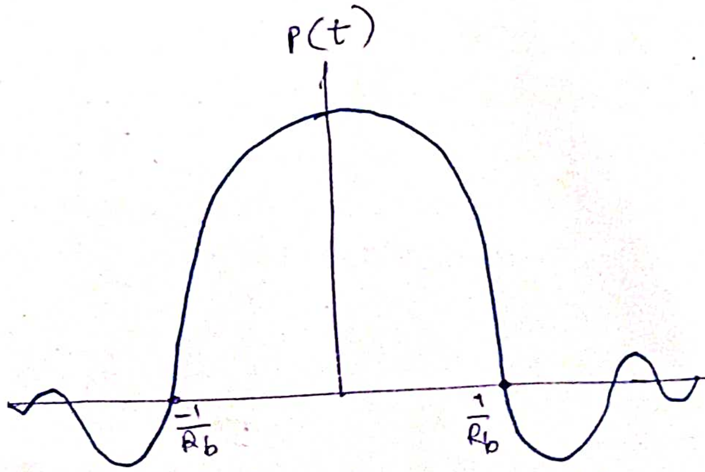
$\boxed{r = 2 f_x T_b}$ $0 < r < 1$, $0 < f_x < \frac{R_b}{2}$ Roll off Factor

As it decides the slope

The family of pulses which satisfies above criteria is:

$$P(f) = \begin{cases} 1 & \text{for } |f| \leq \frac{R_b}{2} - f_x \\ \frac{1}{2}\left[1 - \sin\left\{\pi\left(\frac{f - \frac{R_b}{2}}{2f_x}\right)\right\}\right] & \text{for } \frac{R_b}{2} - f_x \leq |f| \leq \frac{R_b}{2} + f_x \\ 0 & \text{for } |f| \geq \frac{R_b}{2} + f_x \end{cases}$$





r = 0   r = 1   r = 0.5

$f_x = 0$ $f_x = \frac{R_b}{2}$ $f_x = R_b/4$

For $r = 1$ $\qquad\qquad 0 < |f| \leq R_b$

$$P(f) = \frac{1}{2}\left[1 - \sin\left(\pi f R_b - \frac{\pi}{2}\right)\right]$$

$$= \frac{1}{2}\left[1 + \cos\left(\frac{\pi f}{R_b}\right)\right]\Pi\left(\frac{f}{R_b}\right)$$

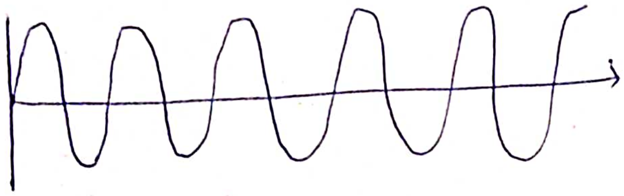
$$= \cos^2\left(\frac{\pi f}{2R_b}\right)\Pi\left(\frac{f}{R_b}\right)$$

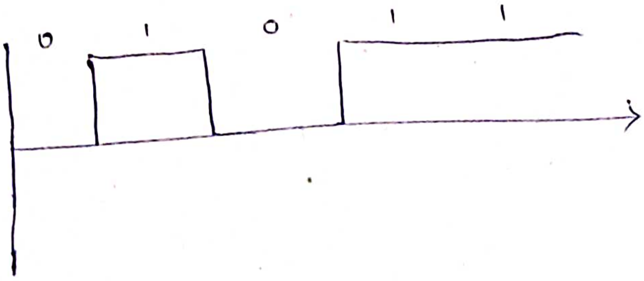
$$\boxed{p(t) = \frac{R_b \cos(\pi t R_b)}{1 - \pi R_b^2 f^2} \; \frac{\sin(\pi t R_b)}{\pi t R_b}}$$

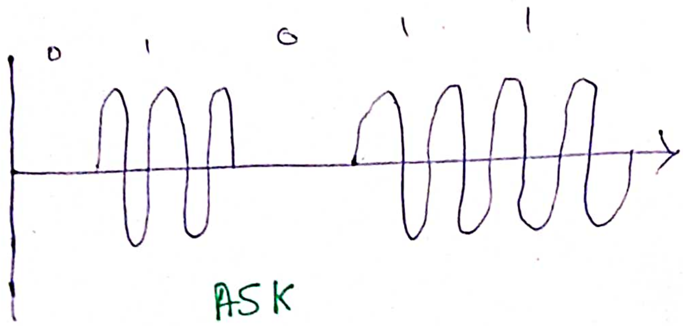
Raised Cosine Pulse.

Baseband Modulation

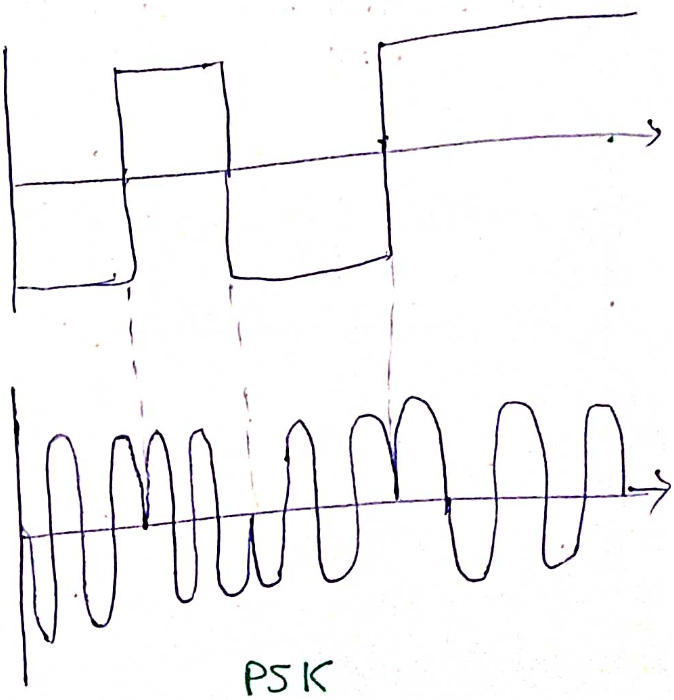
$$m(t) = \sum a_K \, p(t - K T_b)$$







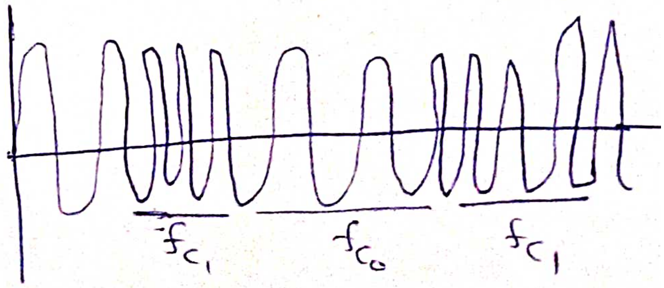
ASK



PSK

$$a_{K_{PSK/ON\text{-}OFF}} = \begin{cases} 1 & \text{for '1'} \\ 0 & \text{for '0'} \end{cases}$$

$$\phi_{ASK}(t) = \begin{cases} \cos(2\pi f_c t) & \text{for '1'} \\ 0 & \text{for '0'} \end{cases}$$



FSK

$$\phi_{PSK} = \begin{cases} \cos(2\pi f_c t) & \text{for '1'} \\ -\cos(2\pi f_c t) & \text{for '0'} \end{cases} \quad 0 \le t \le T_b$$

$$\phi_{FSK} = \begin{cases} \cos(2\pi f_{c_1} t) & \text{for '1'} \\ \cos(2\pi f_{c_0} t) & \text{for '0'} \end{cases} \quad 0 \le t \le T_b$$

$$m(t) = \sum_1 a_K \, p(t - K T_b) \qquad \begin{cases} \text{ON-OFF} & \text{- ASK} \\ \text{Polar} & \text{- PSK} \end{cases}$$

$$\phi(t) = m(t) \cos(2\pi f_c t)$$

$$\psi(f) = \frac{M(f + f_c) + M(f - f_c)}{2}$$

$$\phi(f) = \lim_{T \to \infty} \frac{|\psi_T(f)|^2}{T}$$

$$\boxed{\phi(f) = \frac{1}{4} S_M(f + f_c) + \frac{1}{4} S_M(f - f_c)}$$