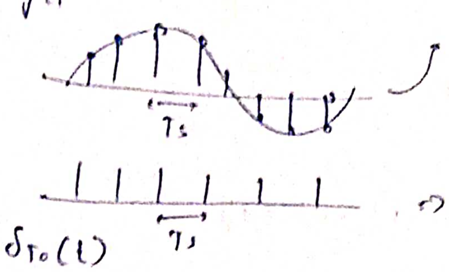
A2D conversion



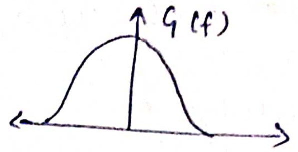
$\bar{g}(t) = g(t) \cdot \delta_{Ts}(t)$

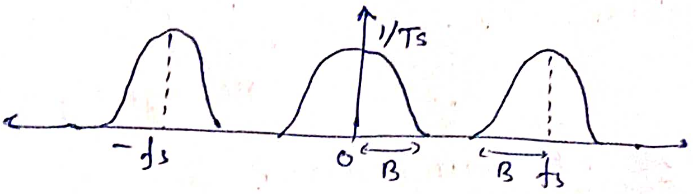
$\delta_{Ts}(t) = \sum_{n=-\infty}^{\infty} \delta(t - nTs)$

$\Rightarrow \bar{g}(t) = \sum_{n=-\infty}^{\infty} g(nTs) \cdot \delta(t - nTs)$

taking F.T. $\overline{G(f)} = \frac{1}{Ts} \sum_{n=-\infty}^{\infty} G(f - nfs)$

Let $G(f) = FT(g(t))$





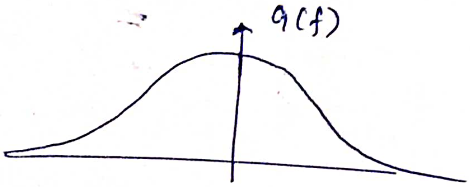
The lobes donot overlap only when $f_s \geq 2B$ OR $\frac{1}{Ts} \geq 2B$

OR $\boxed{Ts \leq \frac{1}{2B}}$

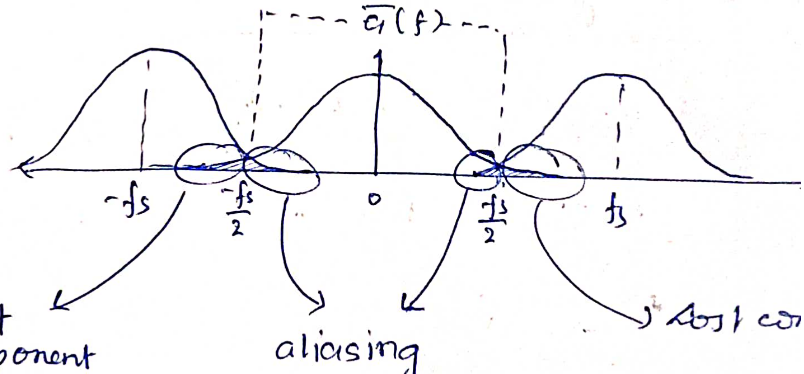
Note that when $fs = 2B$, $fs \Rightarrow$ Niquyst Rate

but if we choose $fs = 2B$ then we would require ideal LPF to reconstruct $g(t)$ back.

* But in actual signals of finite time, B.W is ∞

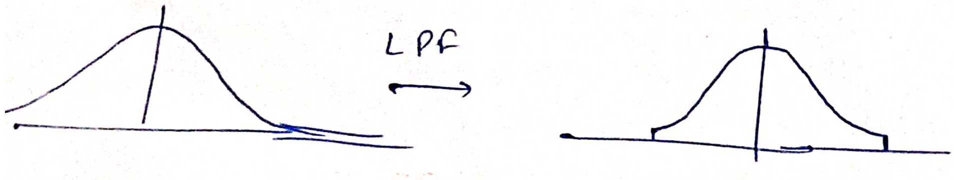


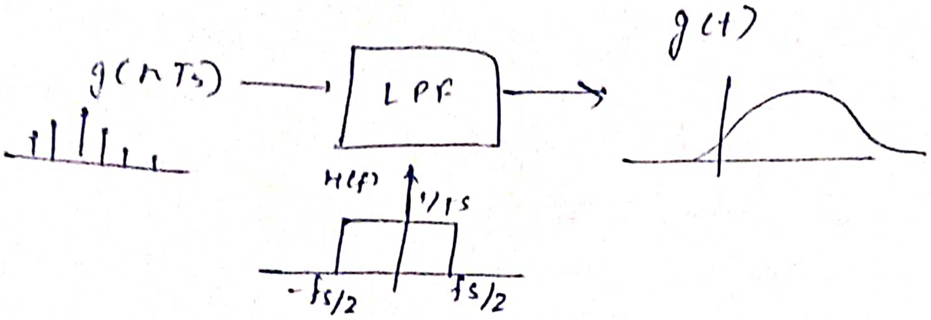
$\Rightarrow$



(distortion in this region). 2- folded spectrum.

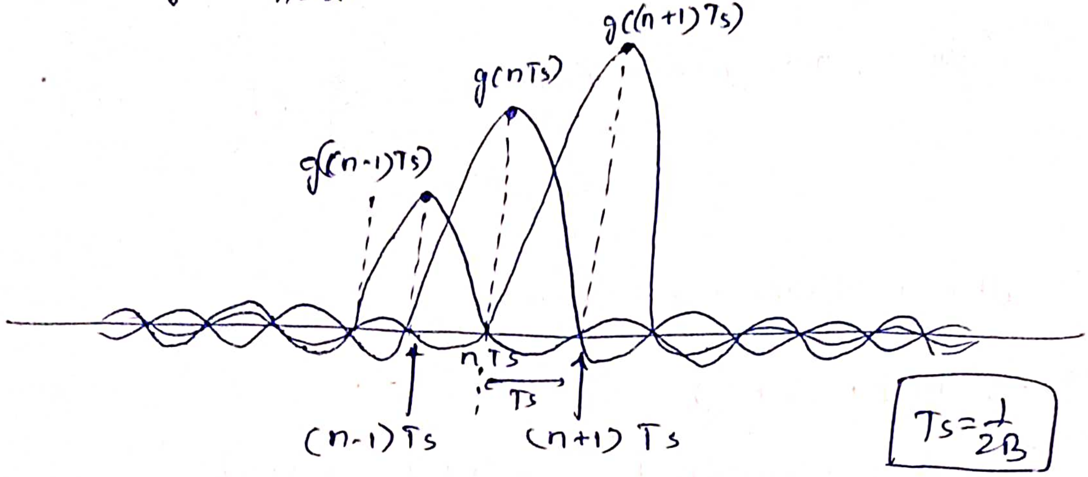
We can not recover the lost component but we can remove aliasing by initially passing $g(t)$ through LPF to make it band limited.





for $t = nT_s$, output $= g(nT_s) \cdot \text{sinc}(\pi f_s (t - nT_s))$

$$\Rightarrow \quad g(t) = \sum_{n=-\infty}^{\infty} g(nT_s) \cdot \text{sinc}(\pi f_s (t - nT_s)).$$

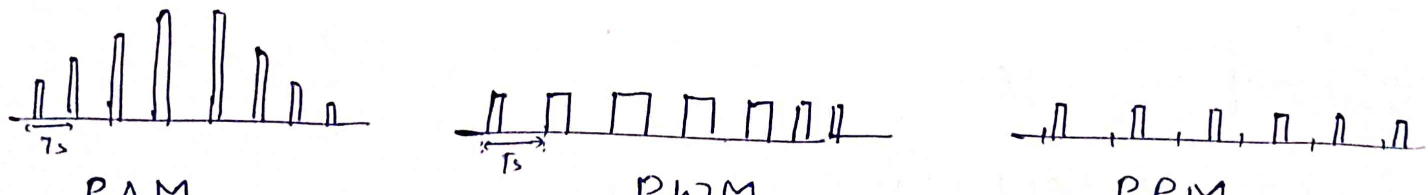


$\boxed{T_s = \frac{1}{2B}}$

at $\boxed{t = k \cdot T_s \quad k \in \mathbb{Z}}$,

the reconstructed signal = original signal.

for the signal

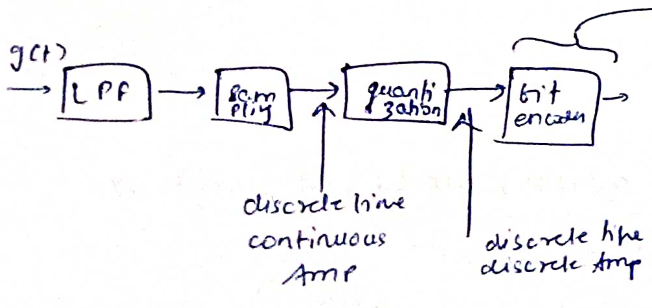


PAM — Pulse Amplitude Mod

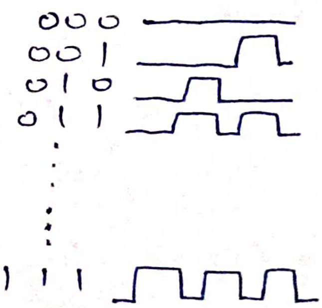
PWM — Pulse width mod.

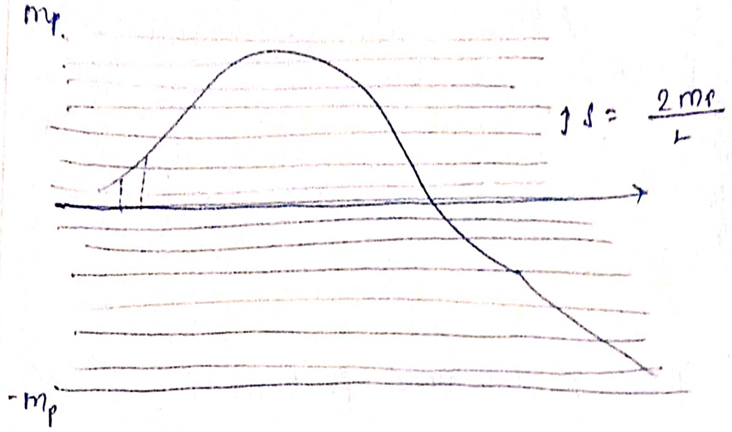
PPM — Pulse Position modulation

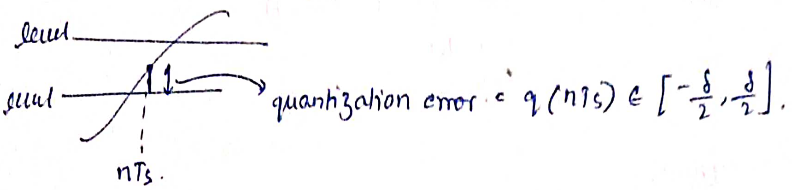
PCM
(Pulse Coded Modulation).



eg. of 3 bits or 8-ary $\Rightarrow$







Thus, after quantization, we have

$$\hat{g}(nT_s) = g(nT_s) + q(nT_s)$$

When this $\hat{g}(nT_s)$ is recovered, $\hat{g}(nT_s) \longrightarrow \boxed{LPF} \longrightarrow \hat{g}(t)$.

$$= \underbrace{g(t)}_{\text{Power} = S_o} + \underbrace{q(t)}_{\text{Power} = N_o}$$

$$\boxed{SNR = \frac{S_o}{N_o}} = \frac{(\widetilde{m(t)})^2}{(\widetilde{q(t)^2})}$$

$$q(nT_s) \longrightarrow \boxed{LPF} \longrightarrow q(t).$$

$$q(t) = \sum_{n=-\infty}^{\infty} q(nT_s).\text{sinc}(2Bt - n\pi)$$

~~the~~ ~~to~~ $$\tilde{q}^2(t) = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} q^2(t)\, dt.$$

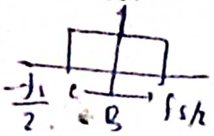
~~Since~~ $$q(t) = \sum_n a_n(t)$$

a_n are orthogonal

$$\Rightarrow \quad q^2(t) = \sum_n a_n^2(t).$$

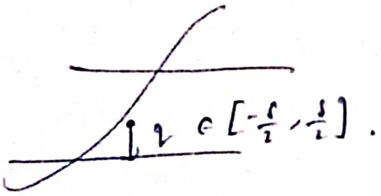
$$\tilde{q}(t)^2 = \sum_n \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} q(nT_s).\text{sinc}(2Bt - n\pi)\, dt$$

$$\widetilde{q(t)^2} = \sum_{n=-\infty}^{\infty} \lim_{T\to\infty} \frac{1}{T} \cdot q(nT_s) \int_{-T/2}^{T/2} \text{sinc}^2(\pi B - n\pi)\, dt$$



$$\widetilde{q(t)} = \sum_{n=-\infty}^{\infty} \lim_{T\to\infty} \frac{1}{T} q^2(nT_s) \cdot \frac{1}{2B}$$

$$= \lim_{T\to\infty} \frac{1}{T2B} \sum_{n=-\infty}^{\infty} q^2(nT_s) = \widetilde{q^2(nT_s)}$$



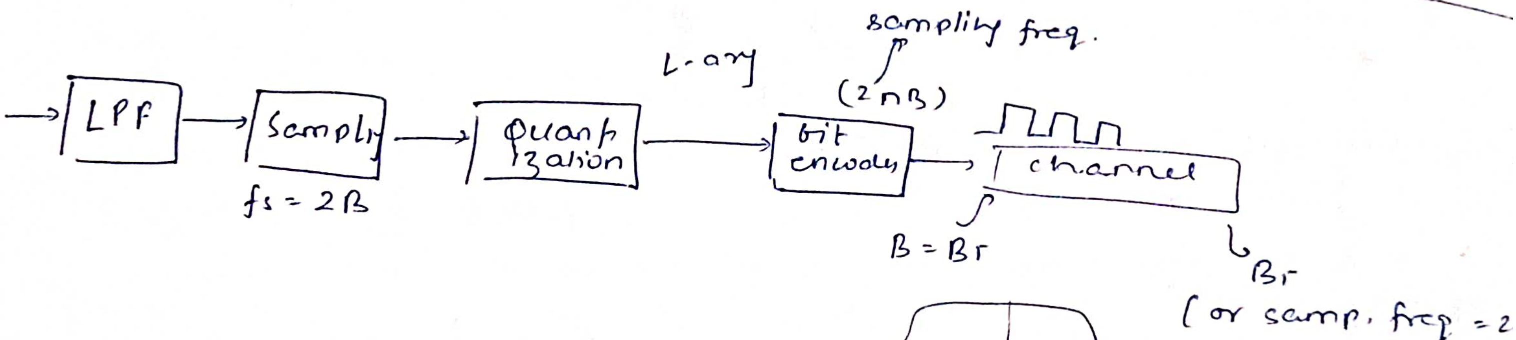
$q \in [-\frac{\delta}{2}, \frac{\delta}{2}]$.

$$\overline{q^2} = \frac{1}{\delta}\int_{-\delta/2}^{\delta/2} q^2 \cdot dq$$ (assuming uniform distribution of q).

$$\overline{q^2} = \frac{\delta^2}{12} = \frac{m_p^2}{3L^2} \Rightarrow \boxed{SNR = \frac{3L^2\, \widetilde{m(t)^2}}{m_p^2}}$$

→ Larger L ⇒ Larger SNR or smaller quant. noise.

P.C.M.



input samples per sec = $2nB$

^ sampling (niquyst) (if B.W of channel is B_T) is $2B_T$

Thus, this niquyst Rate should be atleast greater than/equal to input samples per sec.

$$\Rightarrow \quad 2B_T \geq 2nB \Rightarrow \boxed{B_T \geq nB}$$

$$\boxed{\text{min. Bandwidth Required } B_T = nB}$$

Note: as n increases SNR increases but B_T also increases.

The problem with uniform quantizer

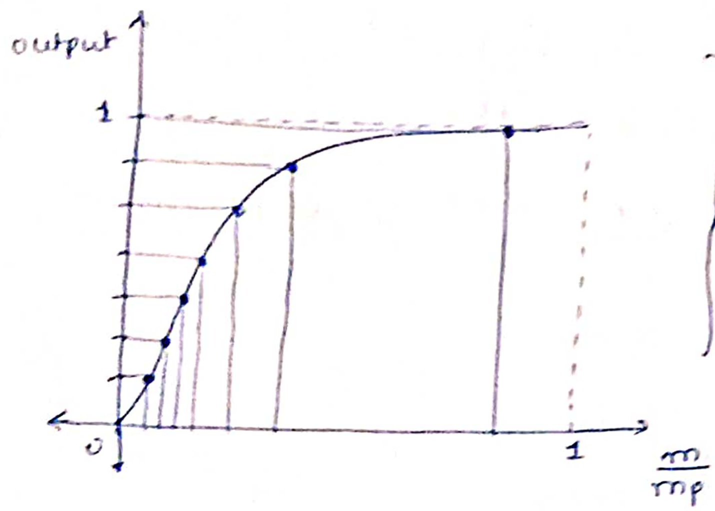
$N_o = \frac{\delta^2}{12} = \frac{m_p^2}{3L}$ is independent of level.

$\frac{S_o}{N_o} = \underbrace{\left[\frac{3L^2}{m_p^2}\right]}_{\text{const}} \overline{m(t)^2}$ } High for larger $\overline{m(t)^2}$, low for smaller $\overline{m(t)^2}$

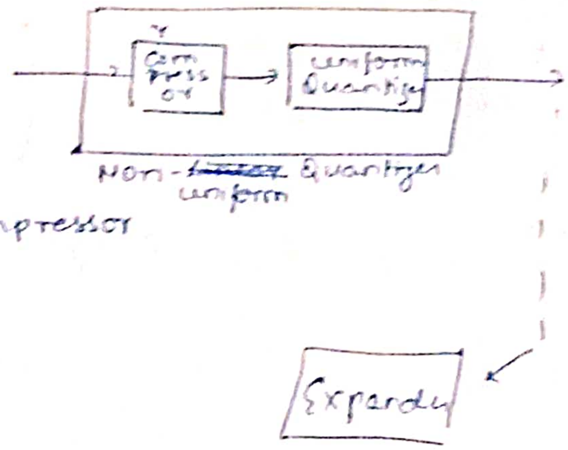
We want: that S_o/N_o is same $\forall$ $\overline{m(t)^2}$, thus N_o must be function of $\overline{m(t)^2}$

Desired:
* small amplitude of $m(t)$ $\Rightarrow$ small N_o $\Rightarrow$ step size small
* large amplitude of $m(t)$ $\Rightarrow$ large N_o $\Rightarrow$ step size large.

~~Say~~ Suppose a block with i/o characteristics



compressor



"It attenuates the higher amplitudes & amplifies the lower amplitudes"

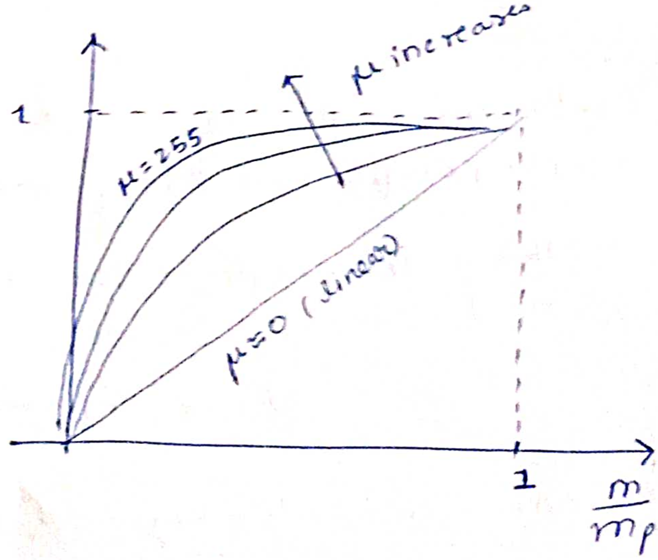
μ law

$$y = \frac{1}{\ln(1+\mu)} \ln\left(1 + \mu \frac{m}{m_p}\right)$$

(apply taylor's expansion for $\mu \to 0$ to get linear expression)

A- law

$$y = \begin{cases} \dfrac{A|x|}{1+\ln(A)} & |x| < \dfrac{1}{A} \\[2ex] \dfrac{1+\ln(A|x|)}{1+\ln(A)} & \dfrac{1}{A} < |x| < 1 \end{cases}$$



SNR for μ-law $\Rightarrow \frac{S_o}{N_o} = \frac{3L^2}{(\ln(1+\mu))^2}$

$$\frac{S_o}{N_o} = \begin{cases} \frac{3L^2 \, \widetilde{m(t)}^2}{m_p^2} & : \text{uniform quantizer} \\ \frac{3L^2}{(\ln(1+\mu))^2} & : \text{non-uniform quantizer} \end{cases}$$

Let $c = \begin{cases} \frac{3\widetilde{m(t)}^2}{m_p^2} & : \text{uniform} \\ \frac{3}{[\ln(1+\mu)]^2} & : \text{non-uniform} \end{cases} \Rightarrow \frac{S_o}{N_o} = c \cdot L^2$

$$SNR = cL^2 = c \cdot 2^{2n}$$

$$SNR_{dB} = 10 \log_{10}(SNR) = 10 \log_{10}(c \cdot 2^{2n})$$

$$SNR_{dB} = 10 \log_{10}(c) + 20n(\log 2).$$

$$\boxed{SNR_{dB} = \alpha + 6n}$$

Eg: if $\alpha = 3, n = 4$. $\quad SNR_{dB} = 3 + 24 = 27\,dB$

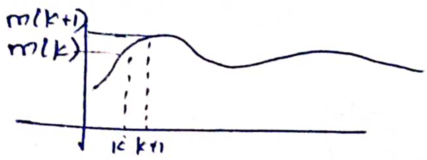
$\alpha = 3, n = 5 \quad SNR_{dB} = 3 + 30 = 33\,dB$

Thus, we can choose n according to the following eqns.

$\boxed{SNR_{dB} = \alpha + 6n}$ and. $\boxed{B_T = nB}$

---

* suppose $m(t)$ is constant, then we can transmit it using 1 sample
* suppose $m(t)$ is not much varying



Let $d[k] = \cancel{m[k+1]} \; m[k] - m[k-1]$

$$\underset{m_d}{\downarrow} \quad << \quad \underset{m_p}{\downarrow}$$

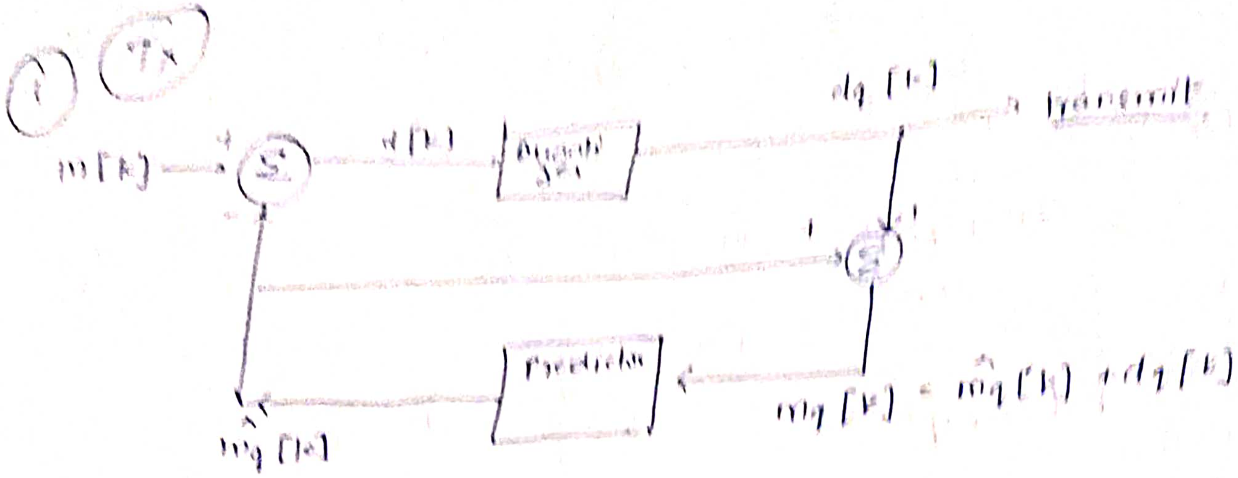
$$\delta' = \frac{2m_d}{L} << \delta = \frac{2m_p}{L}$$

$d[k]$ will be better quantized.

Thus, we have control over m_p part of $\delta = \frac{2m_p}{L}$.

$\Rightarrow$ Reduced δ can be obtained from: $\begin{cases} \text{same } n, \; SNR \uparrow, \; B_T \text{ same} \\ \text{OR} \\ n \downarrow, \; \text{same SNR}, \; B_T \downarrow \end{cases}$

1 Tx



$d[k] = m[k] - \hat{m}_q[k]$

$d_q[k] = d[k] + q[k] \rightarrow$ quantization error.

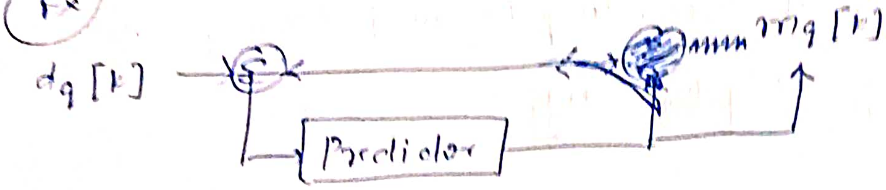
$m_q[k] = \hat{m}_q[k] + d_q[k]$

$= \cancel{\hat{m}_q[k] +}$

$= m[k] - d[k] + d_q[k]$

$m_q[k] = m[k] + q[k] \rightarrow$ same quant. error

2 Rx



$$G_p = \frac{SNR_{DPCM}}{SNR_{PCM}} = \frac{N_{o,PCM}}{N_{o,DPCM}} = \frac{m_p^2}{m_d^2}$$

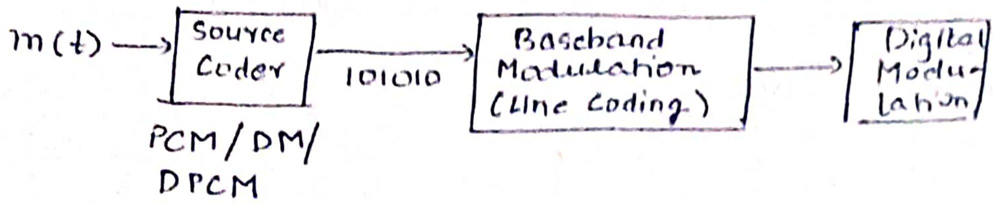
$$SNR^{dB}_{DPCM} = 10\log_{10}(SNR_{DPCM}) = 10\log_{10}\left(\left(\frac{m_p}{m_d}\right)^2\right) + 10\log_{10}(SNR_{PCM})$$

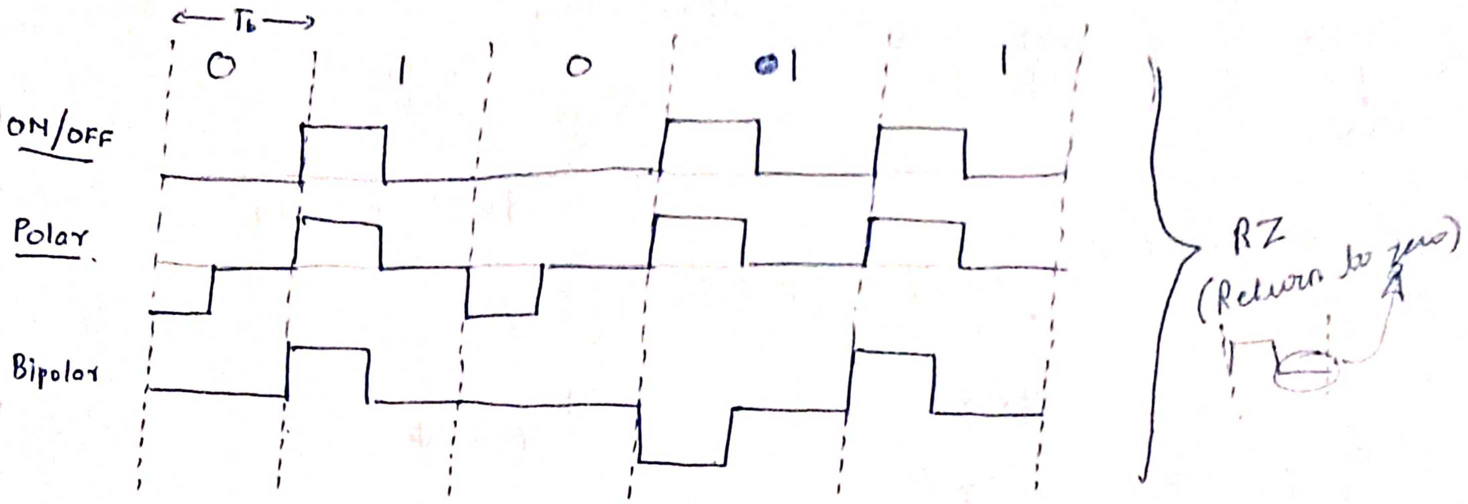
$$SNR^{dB}_{DPCM} = 20\log_{10}\frac{m_p}{m_d} + \alpha + 6n$$

$$\boxed{SNR^{dB}_{DPCM} = 20\log_{10}\left(\frac{m_p}{m_d}\right) + \underbrace{\alpha + 6n}_{SNR^{dB}_{PCM}}}$$

Study Delta Modulation.

Line Coding





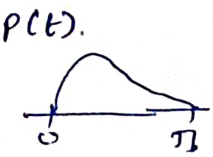
* Polar is more immune to noise than ON/OFF. Thus in order to reach a certain SNR, more power is required in ON/OFF message signal. Thus, Polar is power efficient

* Bipolar keeps track of parity and thus has error detection property.

Requirements for a good Line Coding

1) Power efficient
2) BandWidth efficient.
3) Favourable PSD
4) Error correction/detection capability
5) Clock or carrier Recovery

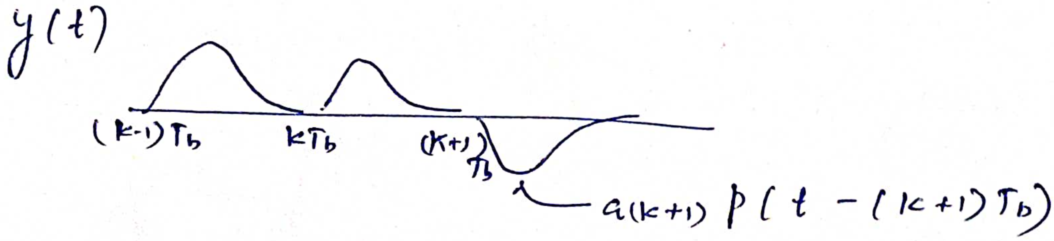
Suppose the pulse shape is this



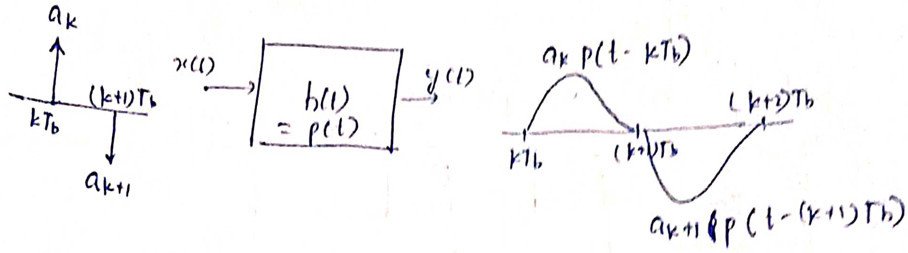
Let

$$a_k = \begin{cases} 0 & \text{for } k^{th} \text{ bit '0'} \\ 1 & \text{for } k^{th} \text{ bit '1'} \end{cases} \text{ for ON-OFF}$$

$$a_k = \begin{cases} +1 & \text{for } k^{th} \text{ bit '1'} \\ -1 & \text{for } k^{th} \text{ bit '0'} \end{cases} \text{ for POLAR.}$$



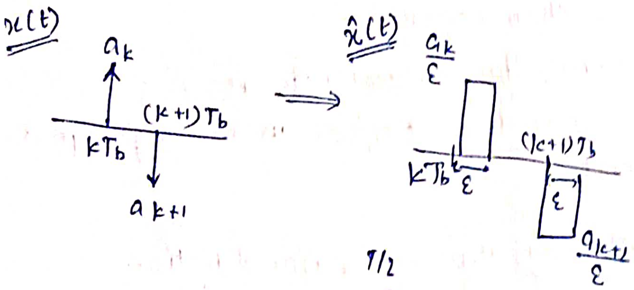
$$\Rightarrow y(t) = \sum_k \underset{\text{Random}}{a_k}\; \underset{\text{deterministic}}{p(t - kT_b)}$$



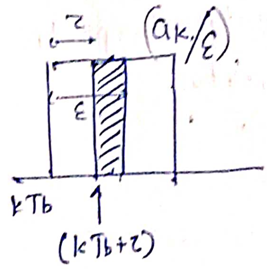
Thus, output PSD $\boxed{S_y(f) = |P(f)|^2 \cdot S_x(f)}$

Output PSD — input PSD

$$S_x(f) = \text{F.T.}(R_x(\tau))$$



$$R_x(\tau) = \lim_{\varepsilon \to 0} R_{\hat{x}}(\tau)$$

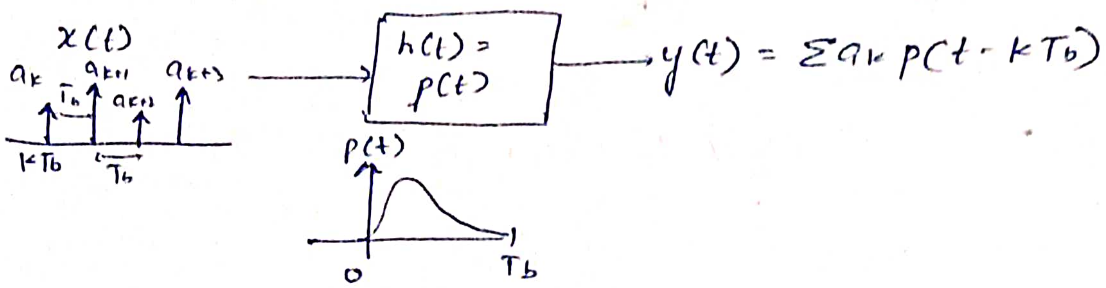


$$\lim_{\varepsilon \to 0} R_{\hat{x}}(\tau) = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} \hat{x}(t)\, \hat{x}(t-\tau)\, dt$$

for $\tau < \varepsilon \Rightarrow \lim_{T\to\infty} \frac{1}{T} \sum h_k (\varepsilon - \tau)$

where $h_k = a_k/\varepsilon$

$$y(t) = \sum a_k\, p(t - kT_b)$$

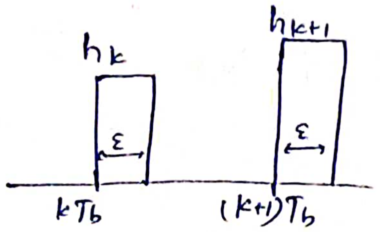


Now $S_y(f) = |Y(f)|^2 = |P(f)|^2 \cdot S_x(f)$.

$\hookrightarrow$ F.T. $\{R_x(\tau)\}$

$R_x(\tau)$ = autocorrelation of $x(t) = \lim_{\epsilon \to 0} R_{\hat{x}}(\tau)$.

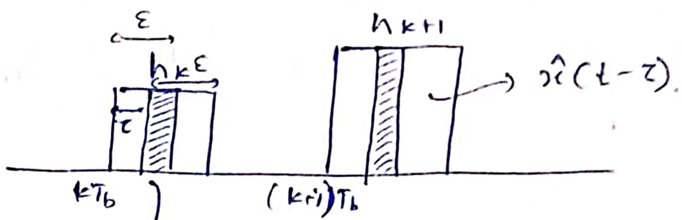
where $\hat{x}(t)$:-



$$R_{\hat{x}}(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} \hat{x}(t)\, \hat{x}(t-\tau)\, dt$$

$$\boxed{h_k = \frac{a_k}{\varepsilon}}$$

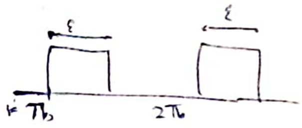
Now for $\tau < \varepsilon$



$$\sum_k h_k^2 (\varepsilon - \tau).$$

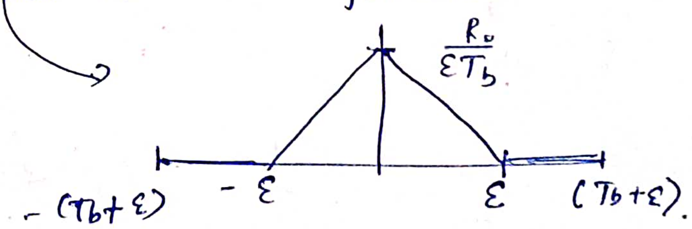
$$\Rightarrow \quad R_{\hat{x}}(\tau) = \lim_{T \to \infty} \frac{1}{T} \sum \frac{a_k^2}{\epsilon^2}(\epsilon - \tau)$$

$$= \underbrace{\left(\lim_{N \to \infty} \frac{1}{N} \sum a_k^2\right)}_{R_0} \frac{1}{T_b\, \epsilon}\left(1 - \frac{\tau}{\epsilon}\right)$$



$$\boxed{R_{\hat{x}}(\tau) = \frac{R_0}{T_b\, \varepsilon}\left(1 - \frac{|\tau|}{\varepsilon}\right) \quad \text{for} \quad |\tau| < \varepsilon}$$

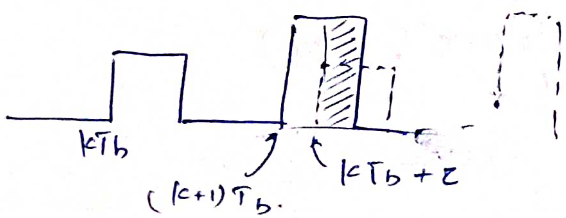
replaced τ by $|\tau|$ since $R_{\hat{x}}$ is even function.



Now if $\varepsilon \le \tau \le T_b - \varepsilon$

$\Longleftrightarrow R_{\hat{x}}(\tau) = 0$

if $T_b - \varepsilon < \tau < T_b + \varepsilon$



$$\Rightarrow \quad R_{\hat{x}}(\tau) = \lim_{T \to \infty} \frac{1}{T} \sum h_k h_{k+1} (\varepsilon - \tau)$$

$$R_{\hat{x}}(\tau) = \frac{R_1}{T_b\, \varepsilon}\left(1 - \frac{|\tau|}{\varepsilon}\right)$$

here $R_1 = \lim_{N \to \infty} \frac{1}{N} \sum a_k a_{k+1} = \overline{a_k a_{k+1}}$

Now area of each triangle is $\frac{R_n}{T_b}$

$\Rightarrow$ $\lim R_x(\tau) = \lim_{\epsilon \to 0} R_x(\tau) = \frac{1}{T_b} \sum_n R_n(\delta(t - nT_b))$

$R_x(\tau) = \frac{1}{T_b} \sum_n R_n(\delta(t - nT_b))$

$FT(R_x(\tau)) = S_x(f) = \frac{1}{T_b} \sum_n R_n \exp(-j2\pi f n T_b)$.

$\Rightarrow$ $S_y(f) = |P(f)|^2 \cdot S_x(f)$

$$\boxed{S_y(f) = \frac{|P(f)|^2}{T_b} \sum_{n=-\infty}^{\infty} R_n \exp(-j2\pi f n T_b)}$$

$$\boxed{S_y(f) = \frac{|P(f)|^2}{T_b} \sum_{n=1}^{\infty} 2R_n \cdot \cos(2\pi f n T_b) + R_0}$$

<u>Now Examples:</u>

* for ON/OFF, let $a_k = \begin{cases} 1 & \text{for "1"} \\ 0 & \text{for "0"} \end{cases}$

$p(t) \rightarrow$

we can find $P(f) \Rightarrow |P(f)|^2$
we only have to find $R_n \; \forall n \in [0, \infty]$ to get $S_y(f)$

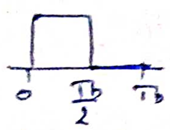
Now, $R_0 = 1^2 \cdot \frac{1}{2} + 0^2 \cdot \frac{1}{2} = \frac{1}{2}$

$\vdots$

$R_n = \overline{a_k a_{k+n}} = \frac{1}{4} \cdot (1)(1) + \frac{3}{4}(0) = \frac{1}{4}$

assuming uniform distribution

let $p(t) \Rightarrow$



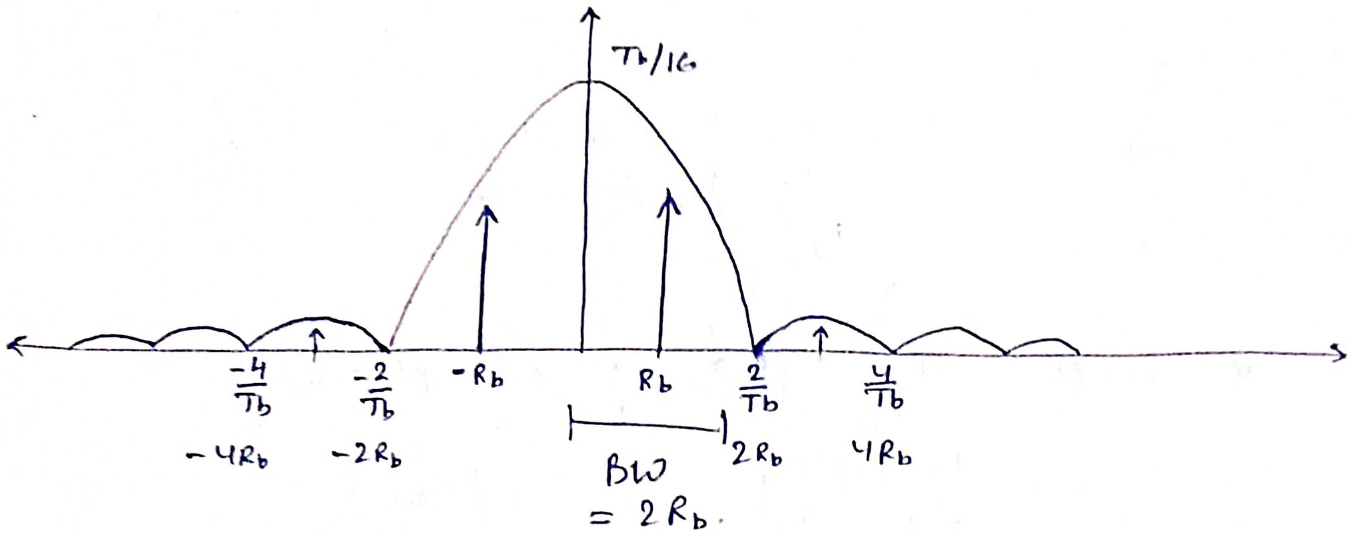
$\Rightarrow |P(f)|^2 = \frac{T_b^2}{4} \operatorname{sinc}^2\left(\frac{\pi T_b f}{2}\right)$

$$S_y^{ON\ OFF}(f) = \frac{T_b}{4}\, sinc^2\left(\frac{\pi T_b f}{2}\right).\left[\frac{1}{2} + \frac{1}{4}\sum_{n\neq 0}^{\infty} \exp(-j2\pi f n T_b)\right]$$

$$= \frac{T_b}{4}\, sinc^2\left(\frac{\pi T_b f}{2}\right)\left[\frac{1}{4} + \frac{1}{4}\sum_{n=-\infty}^{\infty} \exp(-j2\pi f n T_b)\right]$$

$$= \frac{T_b}{16}\, sinc^2\left(\frac{\pi T_b f}{2}\right)\left[1 + \sum_{n=-\infty}^{\infty} \exp(-j2\pi f n T_b)\right]$$

$$\boxed{S_y^{ON\text{-}OFF}(f) = \frac{T_b}{16}\, sinc^2\left(\frac{\pi T_b f}{2}\right)\left[1 + \frac{1}{T_b}\sum_{-\infty}^{\infty} \delta\left(f - \frac{n}{T_b}\right)\right]}$$



But $B_T = R_b/2$

$$\boxed{\begin{array}{l} B_{ON\text{-}OFF} = 2R_b = 4B_T \\ B_{ON\text{-}OFF} = R_b = 2B_T \end{array}}$$

→ $PW = T_b/2$

→ $P.W = T_b$

* find. $S_y(f)$ of Polar.

~~only~~ $P(f)$ ~~changes~~. remains same.

$R_0 = \widetilde{a_k^2} = (1)^2\frac{1}{2} + (-1)^2\frac{1}{2} = 1$

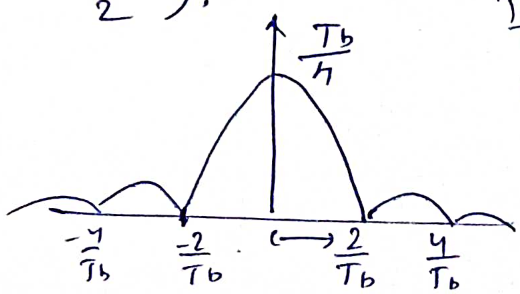
$R_1 = \widetilde{a_k a_{k+1}} = \frac{1}{4}[-1-1+1+1] = 0 = R_n$

$$\Rightarrow.\quad S_y^{POLAR} = \frac{T_b}{4}\, sinc^2\left(\frac{\pi T_b f}{2}\right).\left[1 . e(0)\right]$$

$$S_y^{POLAR}(f) = \frac{T_b}{4}\, sinc^2\left(\frac{\pi T_b f}{2}\right).$$

$\frac{T_b f}{2} = n$

$f = \frac{2n}{T_b}$



$BW_{POLAR} = 2R_b = 2(2B_T) = 4B_T$

Bipolar $\Rightarrow a_k = \begin{cases} \pm 1 & \text{for '1'} \\ 0 & \text{for '0'} \end{cases}$

$$R_0 = \overline{a_k^2} = \frac{1}{2}(0) + \frac{1}{2}\left[\cancel{\tfrac{1}{2}}1^2 + \frac{1}{2}(-1)^2\right] = \frac{1}{2}$$

$$R_n = \overline{a_k a_{k+n}} =$$

$$R_1 = \overline{a_k a_{k+1}} = \frac{1}{4}\cdot[0] + \frac{1}{4}\cdot[0] + \frac{1}{4}[0] + \frac{1}{4}(1(-1)) = \frac{-1}{4}$$

$$\begin{matrix} 0 & 0 \\ 0 & 1 \\ 1 & 0 \\ 1 & 1 \end{matrix}$$

$$R_2 = \overline{a_k a_{k+2}} = 0$$

$$R_n = \overline{a_k a_{k+n}} = 0 \Rightarrow R_n = 0 \ \forall\, n \neq 0, 1, -1$$

$$R_0 = \frac{1}{2} \quad \cancel{\text{for } n=0}$$

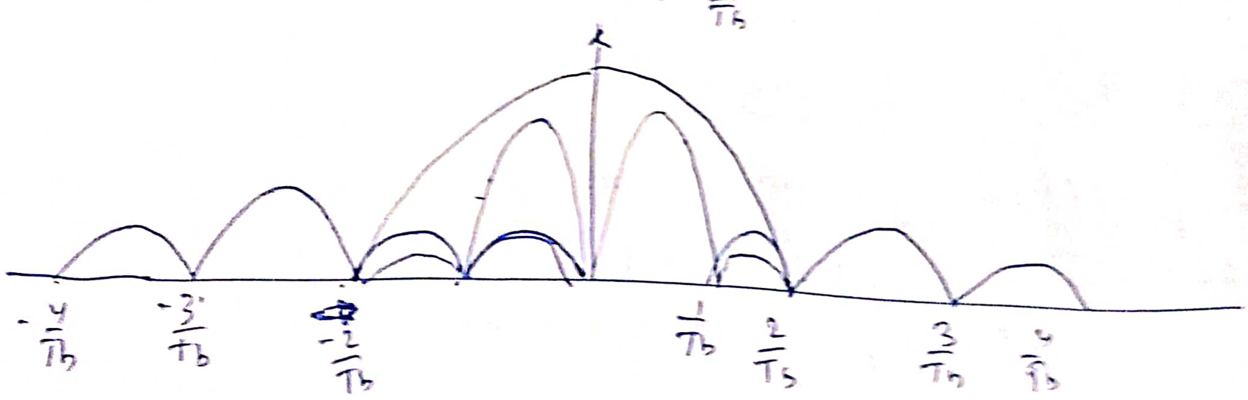
$$R_1 = \frac{-1}{4}$$

$$\Rightarrow S_x(f) = \frac{|P(f)|^2}{T_b}\left[\frac{1}{2} - \frac{2}{4}\cos(2\pi f (1) T_b)\right]$$

$$= \frac{|P(f)|^2}{2T_b}\left(1 - \cos(2\pi f T_b)\right)$$

$$= \frac{T_b}{4}\,\mathrm{sinc}^2\left(\frac{\pi T_b f}{2}\right)\sin^2(\pi f T_b)$$

$$\hookrightarrow = n\frac{2}{T_b} \qquad \hookrightarrow = n/T_b$$



$$\boxed{BW = R_b}$$

	P_s	B_T	Complexity
DSB - SC	$\frac{P_m}{2}$	$2 \cdot B_m$	carrier Recovery req
A.M. $(A + m_p > 0)$ [DSB - FC]	$\frac{A^2}{2} + \frac{P_M}{2}$	$2 \cdot B_m$	Reduced Complexity [Envelope Detection]
S.S.B - SC	$\frac{P_M}{4}$	B_m	Hilbert transform required (complex)
V.S.B	$\frac{P_M}{2} > P_s > \frac{P_M}{4}$	$2B_m > B_T > B_m$	similar of DSBSC

DSBSC
[Normal]

AM (DSBFC)
$[A + m_p > 0]$

SSB - SC
[Hilbert transform]

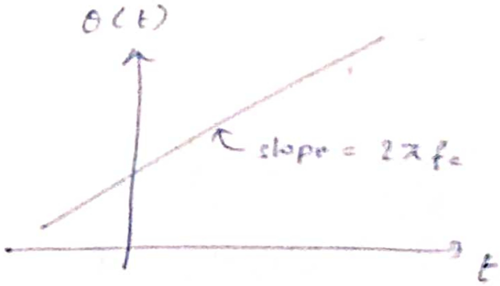
VSB
[High Pass filter]

Frequency Modulation

$$f_i(t) = f_c + k_f \cdot m(t)$$

if $-m_p < m(t) < m_p \Rightarrow f_i(t) = f_c \pm m_p$ (max).

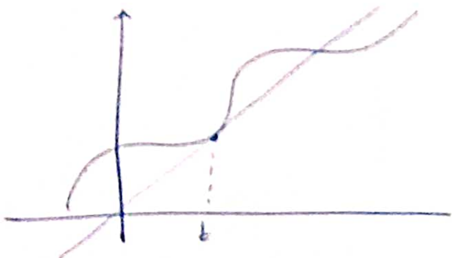
$$A \cdot \cos(\underbrace{2\pi f_c t + \theta_0}_{\theta(t)}) \Rightarrow \frac{d\theta(t)}{dt} = 2\pi f_c$$



Now

$$\boxed{f_i(t) = \frac{1}{2\pi} \frac{d\theta(t)}{dt}}$$

$$\boxed{\theta(t) = 2\pi \int_{-\infty}^{t} f_i(\tau)\, d\tau}$$



* Instantaneous frequency is derative of instantaneous angle

df $\boxed{f = \frac{1}{2\pi}\frac{d\theta}{dt}}$

Phase Modulation (PM)

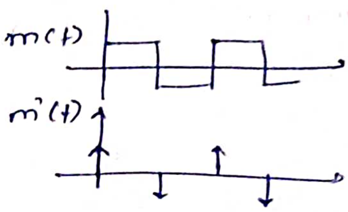
$$\theta(t) = 2\pi f_c t + \cancel{\theta_0}^{\,0} + 2\pi K_P\, m(t)$$

$$\boxed{S_{pm}(t) = A\cos\left(2\pi f_c t + K_P 2\pi\, m(t)\right)}$$

instantaneous frequency $f_i(t)$ is given by

$$f_i(t) = \frac{1}{2\pi}\left[2\pi f_c + 2\pi K_p\, m'(t)\right] = f_c + K_p\, m'(t).$$

$$\boxed{f_i(t) = f_c + k_p\, m'(t)}$$



* Instantaneous frequency is proportional to $m'(t)$

In fm, we have $f_i(t) = f_c + \frac{k_m}{2\pi}\, m(t)$

and ~~$\theta(t) = f_c t$~~

$$\theta(t) = 2\pi\int_{-\infty}^{t} f_i(t)\,d\tau = 2\pi f_c t + K_f \cdot \int_{-\infty}^{t} m(\tau)\,d\tau$$

$$\Rightarrow \boxed{S_{FM}(t) = A\cdot\cos\left(2\pi f_c t + K_f \cdot \int_{-\infty}^{t} m(\tau)\,d\tau\right).}$$

* Instantaneous freq. $\propto$ $m(t)$
* Instantaneous phase $\propto$ $\int m(\tau)\,d\tau$

Suppose we have a derivative block and we pass $m(t)$ through that to get $m'(t) = \dot{m}(t)$

$$m(t) \longrightarrow \boxed{s} \longrightarrow \dot{m}(t). \; (= \dot{m}(t))$$

Now if this $m'(t)$ is sent for F.M.

$$S_{FM}(t) = A \cdot \cos\left(2\pi f_c t + k_f \int_{-\infty}^{t} m'(\tau)\, d\tau\right).$$

$$S_{FM}(t) = A \cos\left(2\pi f_c t + k_f \cdot m(t)\right) \nwarrow \text{PM signal.}$$

* $m(t) \longrightarrow \underset{\text{}}{\boxed{\frac{d}{dt}}}^{\;2\pi \cdot s} \xrightarrow{\dot{m}(t)} \boxed{F.M} \longrightarrow S_{PM}(t).$

* $m(t) \longrightarrow \boxed{\int}^{\;\frac{1}{2\pi s}} \xrightarrow{\frac{1}{2\pi}\int m(t)\, d\tau} \boxed{PM} \longrightarrow S_{FM}(t).$

F.M.: $\theta(t) \propto \int_{-\infty}^{t} m(\tau)\, d\tau = u(t) \circledast m(t)$

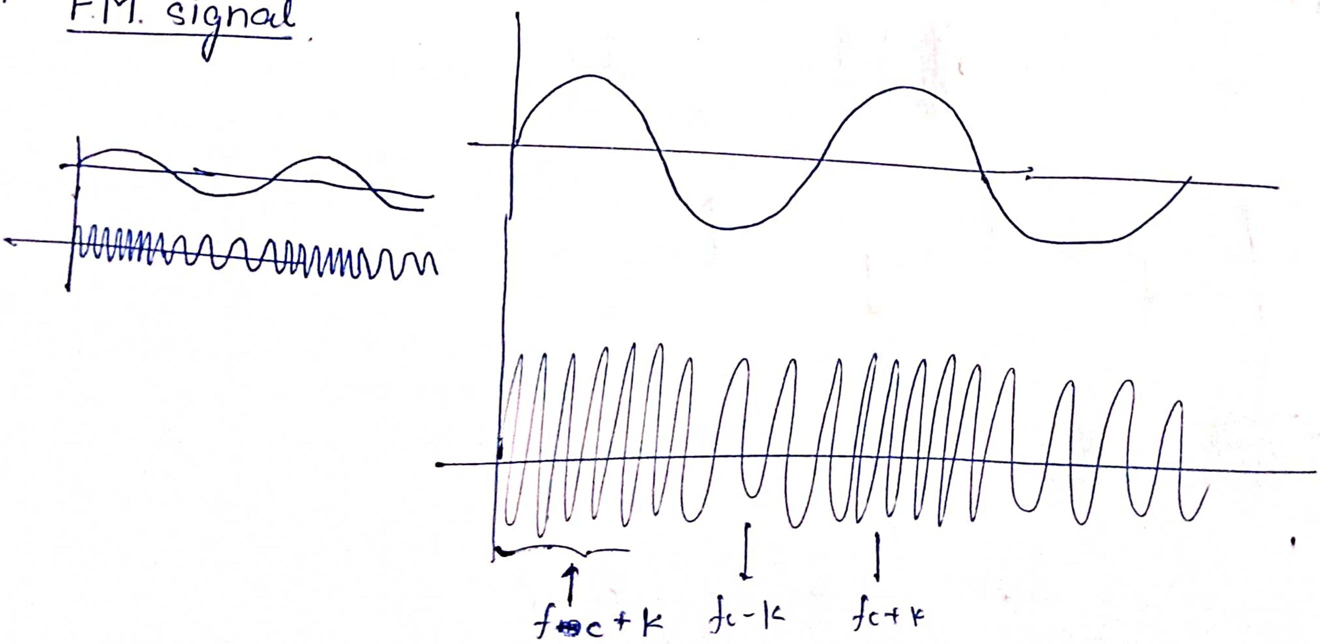
P.M.: $\theta(t) \propto m(t) = \delta(t) \circledast m(t).$

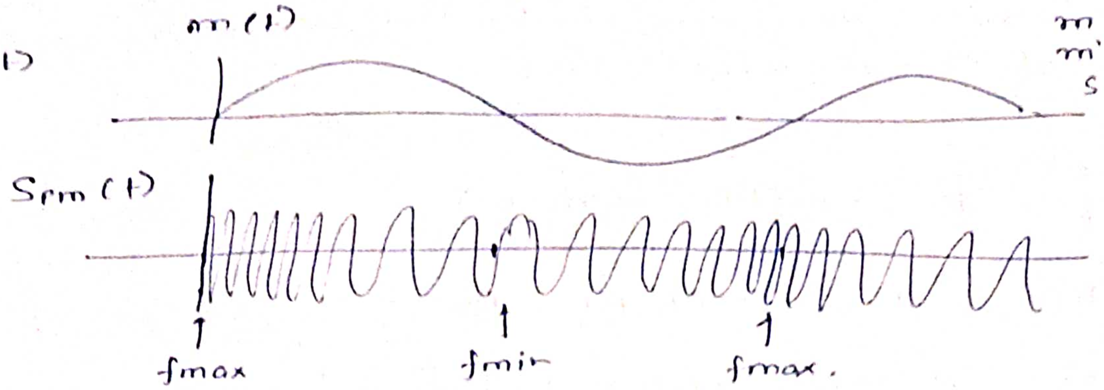
In general, we can have any $h(t)$

$$m(t) \longrightarrow \boxed{h(t)} \longrightarrow \theta(t) = m(t) \circledast h(t)$$

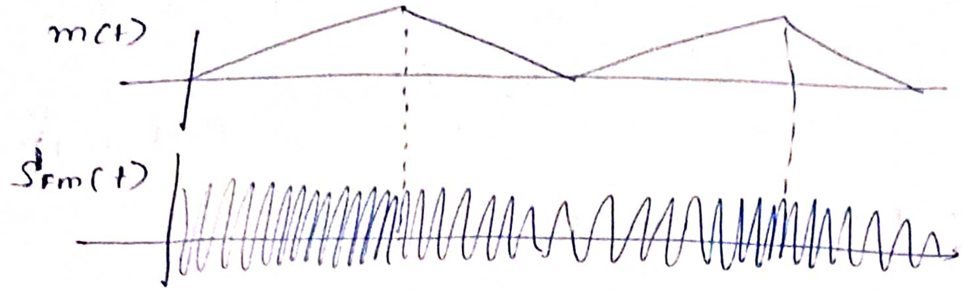
for F.M. $h(t) = k_f \cdot u(t)$
P.M. $h(t) = k_p \, \delta(t)$

* F.M. signal.



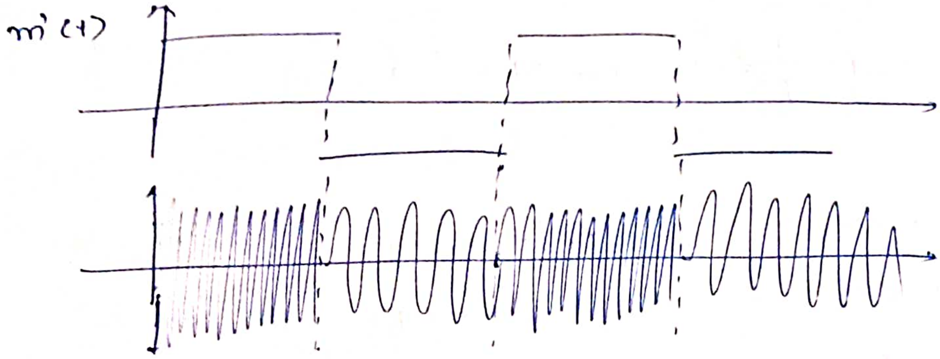
* 

$m(t) \rightarrow$ smooth
$m'(t) \rightarrow$ smooth.
S_{pm} & S_{pm}
not differential

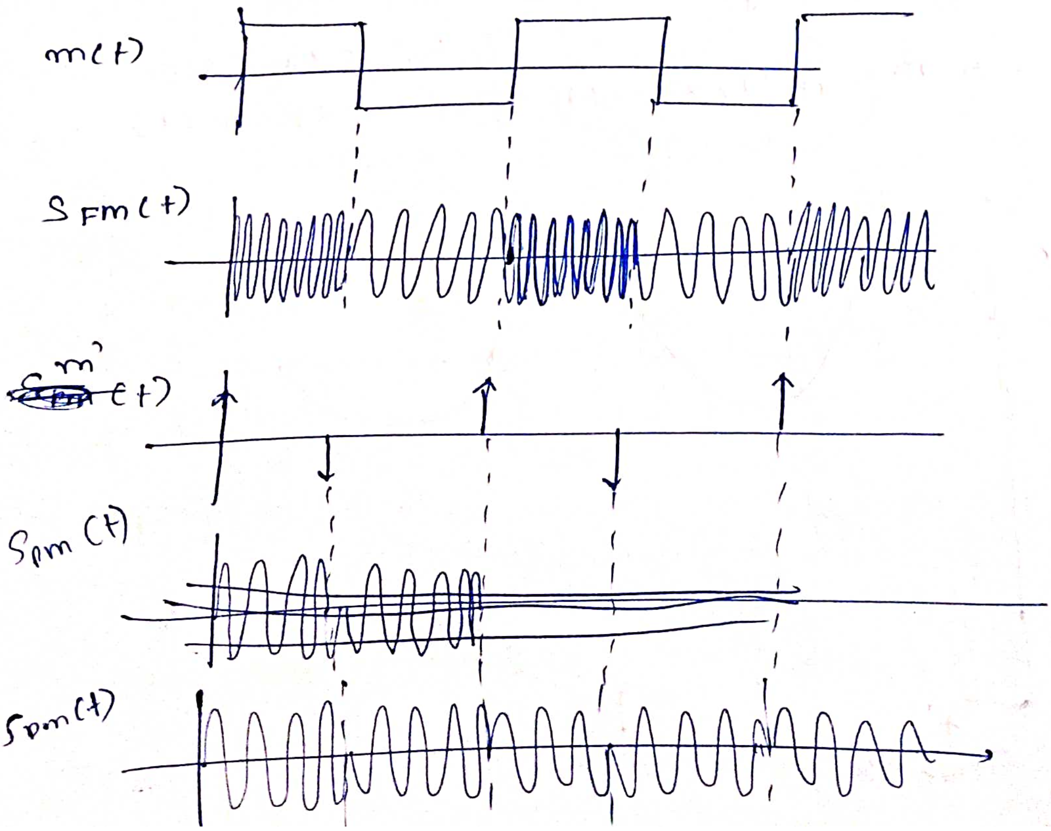
* 

$m(t) \leftarrow$ smooth.

$\boxed{\text{Sfm and Spm are differentiable}}$



$m'(t) \leftarrow$ not smooth

* 

For FM/PM $\boxed{P_c = \frac{A^2}{2}}$ $\boxed{P_M = P_{FM} = \frac{A^2}{2}}$

Now for Bandwidth:

$$S_{FM}(t) = A\cdot\cos\left(2\pi f_c t + k_f \int_{-\infty}^{t} m(\tau)d\tau\right).$$

Let $a(t) = \int_{-\infty}^{t} m(\tau)d\tau$

$$\Rightarrow S_{FM}(t) = A\cos(2\pi f_c t + k_f a(t))$$

$$= \phi_{FM}(t) = \text{Re}\{\hat{\phi}_{FM}(t)\} \quad (\text{say}).$$

$$\hat{\phi}_{FM}(t) = A\cdot e^{j2\pi f_c t}\cdot e^{jk_f a(t)}$$

$$= A\cdot e^{j2\pi f_c t}\left[1 + jka(t) + \frac{[jk(a(t))]^2}{2!} + \frac{(jka(t))^3}{3!} + \ldots\right]$$

$$\Rightarrow \cos(2\pi f_c t) + j\sin(2\pi f_c t)$$

$$\text{Re}(\hat{\phi}_{FM}(t)) = A\left[\cos(2\pi f_c t) - \underbrace{k_f a(t)\sin(2\pi f_c t)}_{2B} - \underbrace{\frac{(k_f a(t))^2}{2!}\cos(2\pi f_c t)}_{4B} - \ldots\right.$$

$$= \phi_{FM}(t).$$

Ideally B.W. of $\phi_{FM}(t)$ would be infinite but $\frac{(k_f a(t))^n}{n!}$ decreases with increasing n.

$\Rightarrow$ The choice of $k_f.a(t)$ decides how many terms are to be included.

Case I $\quad |k_p.m_p| \ll 1$

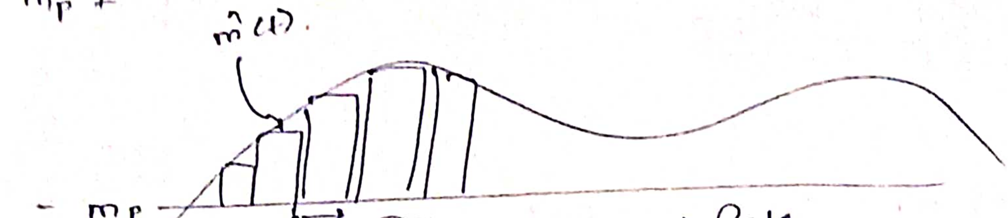
$$\Rightarrow \boxed{\phi_{FM}(t) = A\cos(2\pi f_c t) - A.k_f a(t).\sin(2\pi f_c t)}$$

$\hookrightarrow$ AM signal (but Amp. is const.) $\quad \rightarrow$ out of phase with carrier.

$\boxed{B_{FM} = 2.B_M} \Rightarrow$ N.B.F.M.

(Narrow Band F.M.)

CASE 2 | General case



$\left(\frac{1}{2B}\right)$ for Niquyst Rate

$$f_i(t_k) = f_c + \frac{k_f}{2\pi}\hat{m}(t_k).$$

Note: we know a rect. pulse. $\Pi\left(\frac{t}{T}\right) = \begin{cases} 1 & \text{for } |t| < T \\ 0 & \text{otherwise} \end{cases}$

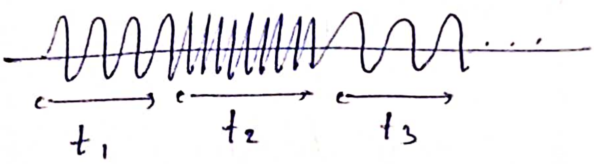
has its FT as $\tau \operatorname{sinc}\left(\frac{f\tau}{2}\right)$.

$$\pi(t \cdot 2B)\cos(2\pi f_c t + k_f \cdot \hat{m}(t))$$

$$\Updownarrow$$

$$\frac{1}{2}\operatorname{sinc}\left(\frac{2\pi f \pm (2\pi f_c + k_f \hat{m}(t_k))}{4B}\right)$$

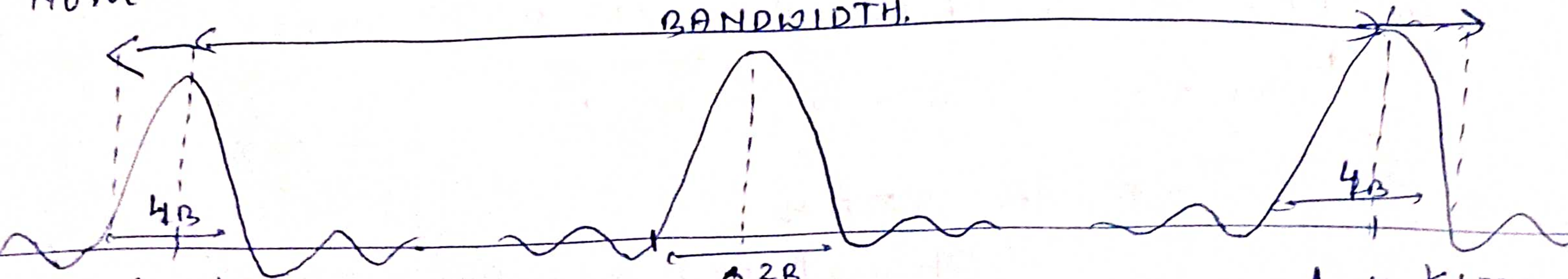
Thus, in the kth block, there is only 1 freq. given by value at $\hat{m}(t_k)$. $f_i(t_k) = f_c + \frac{k_f}{2\pi}\hat{m}(t_k)$



where $f(t_k)$ depends upon $\hat{m}(t_k)$.

each of these kth block will have sinc spectrum centered around $f_i(t_k) = f_c + \frac{k_f}{2\pi}\hat{m}(t_k)$.

Now $\hat{m}(t_k) \in [-m_p, m_p]$.



$f_c - \frac{k_f m_p}{2\pi}$ $\left[f_c + \frac{k_f}{2\pi}\hat{m}(t_k)\right]$ $f_c + \frac{k_f m_p}{2\pi}$

Leftmost spectrum

Rightmost Spectr

$$B.W = \frac{2k_f}{2\pi}m_p + 4B$$

Max. deviation in frequency. $f_i(t) = f_c + \frac{k_f}{2\pi} m(t)$

$$\Delta f = \frac{\left[\frac{k_f}{2\pi}(2m_p)\right]}{2} - \frac{\left[\frac{k_f \cdot m_p}{\pi}\right]}{2} = \frac{k_f m_p}{2\pi}$$

$$\boxed{\Delta f = \frac{k_f \cdot m_p}{2\pi}} \Rightarrow \boxed{B_{\hat{\phi}} = 2\Delta f + 4B}$$

frequency deviation (but we have overestimated B. (B.W. of staircase) as B.W. of $m(t)$ < B.W of $\hat{m}(t)$).

actual bandwidth $B_\phi \in [2\Delta f, 2\Delta f + 4B]$.

<u>Now</u>

if $\boxed{\Delta f \simeq 0} \Rightarrow (k_p \cdot m_p << 1) \Rightarrow$ Narrow Band signal

$$\boxed{B_\phi = 2B}$$

but $B_{\hat{\phi}} = 2\cancel{\Delta f}^{0} + 4B = 4B$

$\Rightarrow$ we can correct the over estimation

Let $B_{\hat{\phi}} = 2\Delta f + 4B + c$

correction term

at $\Delta f = 0$, $B_\phi = 0 \Rightarrow c = -2B$

$$\Rightarrow \boxed{B_\phi = 2\Delta f + 2B}$$

modulation index

$$\boxed{\beta = \frac{\Delta f}{B}} \Rightarrow \boxed{\hat{B}_\phi = 2\beta B + 2B}$$

$$B_\phi = 2B(\beta + 1)$$

Carson's Rule

► Bandwidth for m(t) = sinusoid (tone modulation)

Now let $m(t) = \alpha \cdot \cos(2\pi f_m t)$ (tone modulation).

$$\Rightarrow \quad a(t) = \alpha \int_{-\infty}^{t} m(\tau)\, d\tau = \frac{\alpha}{2\pi f_m} \sin(2\pi f_m t)$$

$$\phi_{FM}(t) = A \cdot \cos\left(2\pi f_c t + \frac{\alpha k_f}{2\pi f_m} \sin(2\pi f_m t)\right)$$

$$\phi_{FM}(t) = \mathrm{Re}\Big\{ \underbrace{A \cdot e^{j\left(2\pi f_c t + \frac{\alpha k_f}{2\pi f_m} \sin(2\pi f_m t)\right)}}_{\hat{\phi}_{FM}(t)} \Big\}.$$

$$\Rightarrow \quad \hat{\phi}_{FM}(t) = A \cdot e^{j 2\pi f_c t} \cdot \underbrace{e^{j\beta \sin(2\pi f_m t)}}$$

$$\boxed{\beta = \frac{\Delta f}{f_m} = \frac{\alpha k_f}{2\pi f_m}}$$

$$\left. e^{j\beta \sin(2\pi f_m t)} = \sum_{n=-\infty}^{\infty} D_n e^{j 2\pi f_m n t} \right\} \text{ F.S. Representation.}$$

$$D_n = \frac{\langle e^{j\beta \sin(2\pi f_m t)}, e^{j 2\pi f_m n t} \rangle}{\langle e^{j 2\pi f_m n t}, e^{j 2\pi f_m n t} \rangle} =$$

$$D_n = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} e^{j\beta \sin(2\pi f_m t)} \cdot e^{-j 2\pi n f_m t}\, dt$$

Now let $2\pi f_m t = x \Rightarrow dt = dx / 2\pi f_m$

$$t \in [-T_0/2, T_0/2) \Rightarrow x \in \left[2\pi f_m \frac{T_0}{2}, 2\pi f_m \frac{T_0}{2}\right].$$

$$\Rightarrow x \in [-\pi, \pi]$$

$$\Rightarrow \quad \left. D_n = \frac{1}{2\pi \cancel{f_m} \cancel{T_0}} \int_{-\pi}^{\pi} e^{j(\beta \sin x - n x)}\, dx \right\} \text{ nth order Bessel function of 1st kind}$$

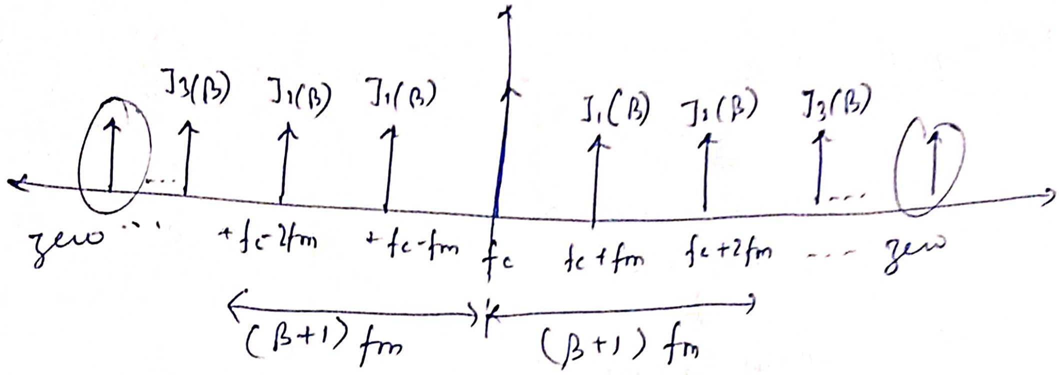
$$D_n = J_n(\beta)$$

$$\Rightarrow \quad e^{j\beta \sin(2\pi f_m t)} = \sum_{n=-\infty}^{\infty} J_n(\beta)\, e^{j 2\pi n f_m t}$$

$$\Rightarrow \hat{\phi}_{FM} = A \cdot \sum_{n=-\infty}^{\infty} J_n(\beta)\, e^{j(2\pi n f_m t + 2\pi f_c t)}$$

$$\Rightarrow \boxed{\phi_{FM} = \sum_{n=-\infty}^{\infty} A \cdot J_n(\beta) \cdot \cos(2\pi f_c t + 2\pi n f_m t)}$$

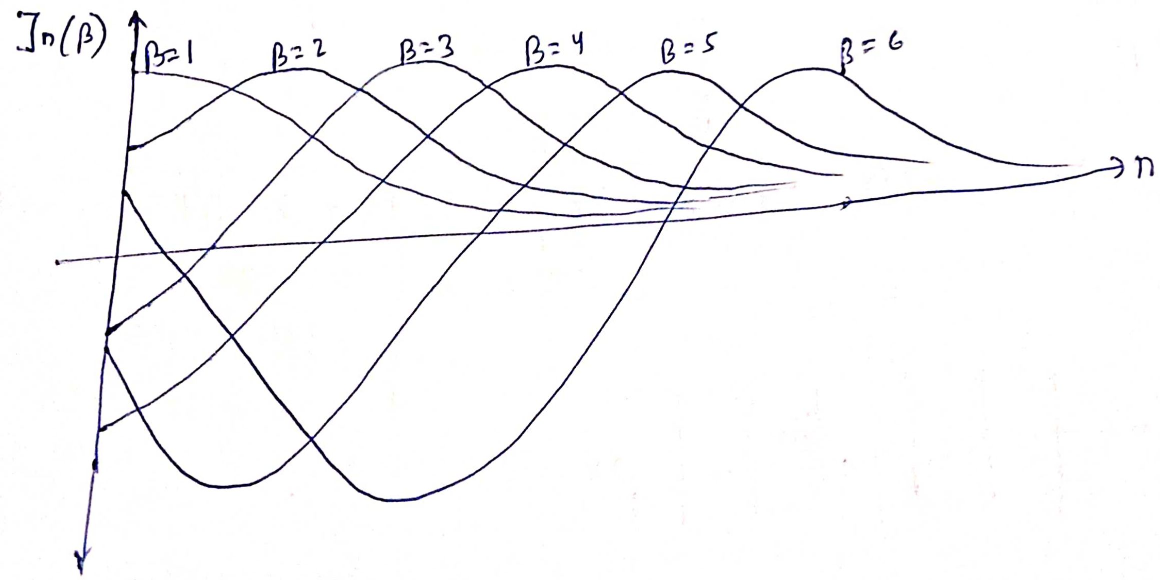
Drawing frequency spectrum :



for $n \geq \beta + 1$; $J_n(\beta) \approx 0$.

$\Rightarrow$ Bandwidth = $2 f_m (\beta + 1)$

Bessel function of 1st kind



We had for NBFM, $k_f \cdot m_p << 1 \Rightarrow 2\pi \Delta f << 1$.

and. NBFM signal = DSBSC + carrier

$$\varphi_{NBFM}(t) = A\cos(2\pi f_c t) - A \cdot k_f a(t) \sin(2\pi f_c t)$$

has Amplitude variation
but in FM, Amplitude = const.

$\Rightarrow$ $\boxed{\text{NBFM signal} = \text{DSBSC} + \text{carrier} + \text{Amp. var removed}}$

$\Downarrow$

frequency deviation conversion

$\Downarrow$

NBFM for any Δf.

Amp. variation Removal

$v_i(t) \longrightarrow$ [Hard Limiter] $\longrightarrow V_o(t)$.

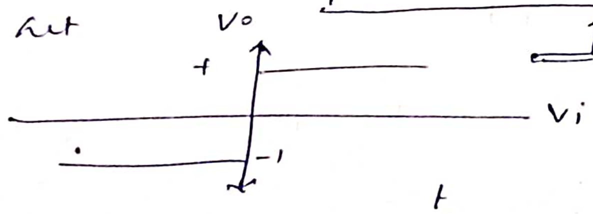
$A(t)\cos(\theta(t))$

but $A(t) >> 0$ always

Ckt

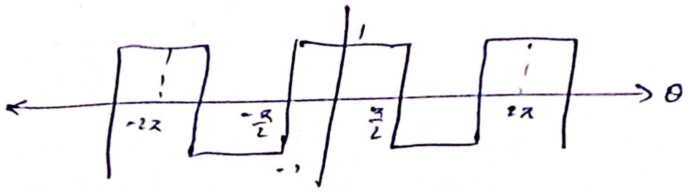
$$\boxed{V_o = \begin{cases} +1 & \text{if } \cos\theta > 0 \\ -1 & \text{if } \cos\theta < 0 \end{cases}}$$

Ckt



Now $\theta(t) = 2\pi f_c t + k_f \int_{-\infty}^{t} m(\tau)\,d\tau = 2\pi f_c t + k_f a(t)$.

$V_o(\theta)$



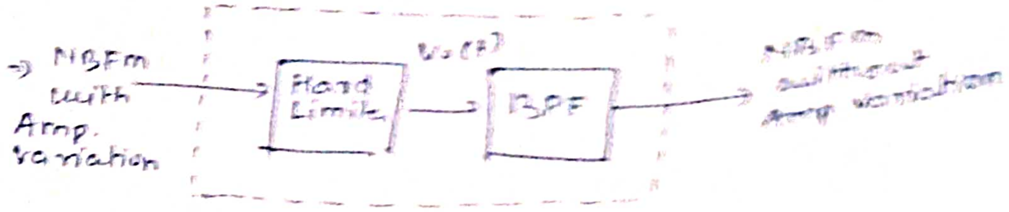
$\Rightarrow$ writing F.S. of $V_o(\theta)$

$$V_o(\theta) = \frac{4}{\pi}\left(\cos\theta - \frac{1}{3}\cos 3\theta + \frac{1}{5}\cos 5\theta - \cdots\right).$$

$$V_o(\theta(t)) = \frac{4}{\pi}\left(\cos(2\pi f_c t + k_f a(t)) - \frac{1}{3}\cos(6\pi f_c t + 3k_f a(t)) + \frac{1}{5}\cos(10\pi f_c t + 5k_f a(t)) \cdots\right)$$

We can pass this $v_o(\theta(t))$ into a BPF to get
For Example if we retain the 1st term and suppress others

$$\Rightarrow \varphi_{NBFM}(t) = \frac{4}{\pi}\cos(2\pi f_c t + k_f a(t))$$



Bandpass Limiter
(Amp. variation removed)

$$\boxed{\varphi_{NBFm}(t) = \frac{4}{\pi}\cos(2\pi f_c + k_f a(t))}$$ for $k_f a m_p \ll 1$

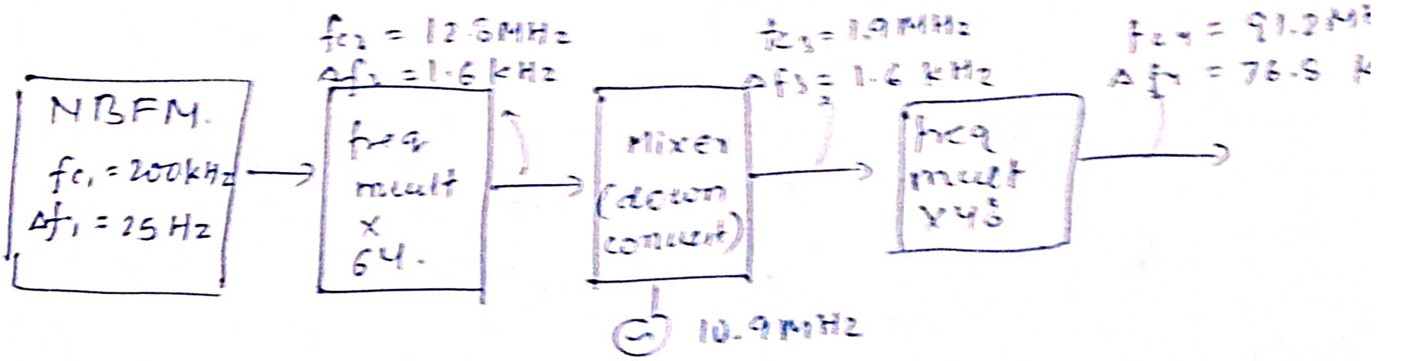
$$\boxed{\begin{array}{l} f_c = 91.2\ MHz \\ \Delta f = 75\ kHz \end{array}}$$

desired.

$m(t) \in (50\,Hz$ to $20\,kHz)$

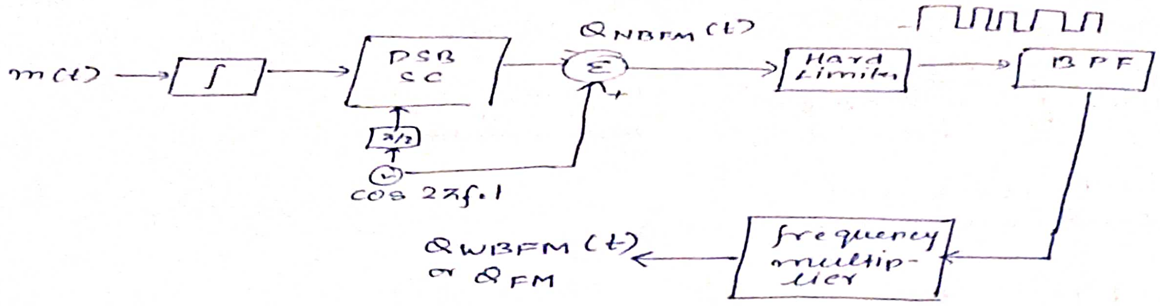
$$\beta = \frac{\Delta f_c}{f_m} = \frac{25}{50} = 0.5 \leftarrow \underline{\max \beta}$$

$\Rightarrow \beta$ will be $\ll 1$ every time



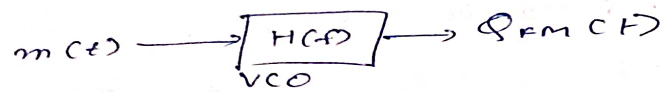
Now factor $= \dfrac{\Delta f}{\Delta f_1} = 3000 \approx 3072 = \underset{(2^6)}{64} \times \underset{(3^1 \cdot 2^4)}{48}$

Indirect Method



(+ mixer {down convert} if required).

Direct Method



oscillator freq $= \dfrac{1}{2\pi\sqrt{LC}}$ → [variable capacitance] $C = C_0 - k.m(t)$

$$f_i(t) = f_c + \frac{k_f}{2\pi} m(t)$$

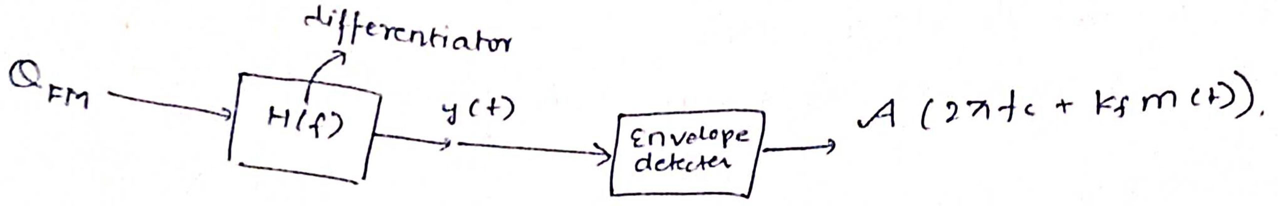
$$f_0 = \frac{1}{2\pi\sqrt{L}\sqrt{C_0 - k.m(t)}} = \frac{1}{2\pi\sqrt{L}}\left[C_0 - k\,m(t)\right]^{-1/2}$$

$$f_i = \underbrace{\frac{1}{2\pi\sqrt{LC_0}}}_{f_0}\left[1 - \frac{k}{C_0} m(t)\right]^{-1/2}$$

$$f_i(t) = f_0\left(1 + \frac{k}{2C_0} m(t)\right) \quad \text{if} \quad \frac{k_f\, m(t)}{C_0} \ll 1$$

$$\boxed{f_i(t) = f_0 + \frac{f_0 k}{2C_0} m(t)} \quad \text{with cond.} \quad \frac{k_f\, m(t)}{C_0} \ll 1$$

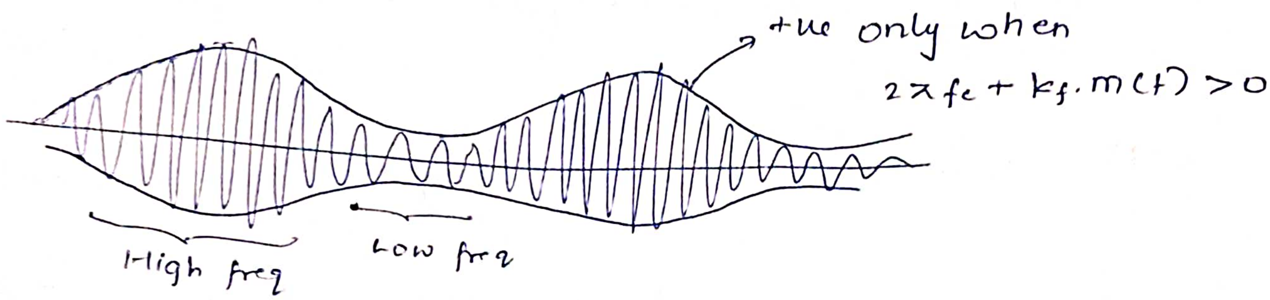
FM Demodulation (Slope detector)



$$Q_{FM} = A\cos\left(2\pi f_c t + k_f \int_{-\infty}^{t} m(\tau)\,d\tau\right)$$

$$Q'_{FM} = -A\sin\left(2\pi f_c t + k_f \int_{-\infty}^{t} m(\tau)\,d\tau\right).\left[2\pi f_c + k_f . m(t)\right]$$

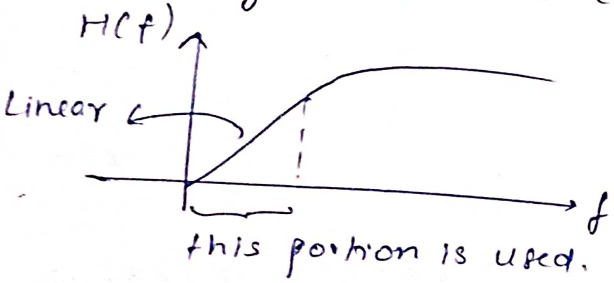
$$y(t) = \underbrace{A\left(2\pi f_c + k_f m(t)\right)}_{AM}.\ \underbrace{\sin\left(2\pi f_c t + k_f \int_{-\infty}^{t} m(\tau)\,d\tau - \pi\right)}_{FM.}$$

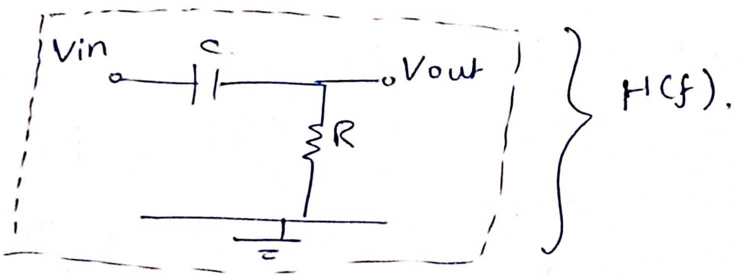


This signal has AM + FM modulation, but its envelope is still $m(t)$ (linear fun).

⇒ We can find $m(t)$ using envelope detecter.

* To Realize $H(f)$, we can use linear portion of a BPF





$$V_{out} = \frac{V_{in} . R}{R + \frac{1}{j\omega C}} = \frac{V_{in} R . j\omega C}{j\omega RC + 1}$$

when $\omega RC << 1$ or $\boxed{2\pi f << \frac{1}{RC}}$

This acts as differentiator. OR $\boxed{f << \frac{1}{2\pi RC}}$

▶ Features of F.M.

$y(t) = a_0 + a_1 x(t) + a_2 x^2(t) + \ldots$

* let $x(t) = A\cos(2\pi f_c t + k_f a(t))$ $\longrightarrow$ $\boxed{FM}$

$+ k_f a(t)$

$y(t) = c_0 + c_1 \cos(2\pi f_c t) + c_2(\cos 4\pi f_c t + 2k_f(a)(t)) + \ldots$

$\downarrow$

$\boxed{BPF}$ $\longrightarrow$ $\boxed{c_1 \cos(2\pi f_c t + k_f a(t))}$

$\cos 3x = 4\cos^3 x - 3\cos x$

$\frac{\cos 3x + 3\cos x}{4} = \cos^3 x$

* let $x(t) = m(t)\cos(2\pi f_c t)$ $\longrightarrow$ $\boxed{AM}$

$y(t) = a_1 x(t) + a_3 x^3(t)$

$y(t) = a_1 m(t)\cos(2\pi f_c t) + \frac{a_3 m^3(t)}{4}\left[\cos(6\pi f_c t + 3k_f m(t)) + 3\cos(2\pi f_c t + k_f m(t))\right]$

$y(t) = a_1 m(t)\cos(2\pi f_c t) + \frac{3a_3 m^3(t)}{4} \cdot \cos(2\pi f_c t + \cancel{k_f m(t)})$

$+ \frac{a_3 m^3(t)}{4}\cos(6\pi f_c t + \cancel{3k_f m(t)})$

Thus, FM signals handle non-linearity better than AM signals

Interference.

$y(t) = A\cos(2\pi f_c t) + I\cos(2\pi(f_c + f)t)$

$= [A + I\cos(2\pi f t)]\cos(2\pi f_c t) - \underbrace{I\sin(2\pi f t)}_{b} \cdot \sin(2\pi f_c t)$

$R(t) \cdot \cos(2\pi f_c t + \phi(t))$

$R(t) \to \sqrt{a^2 + b^2}$

$\phi(t) \to \tan^{-1}\left(\frac{b}{a}\right)$

where $\psi(t) = \tan^{-1} \dfrac{I \sin(2\pi f t)}{A + I\cos(2\pi f t)}$

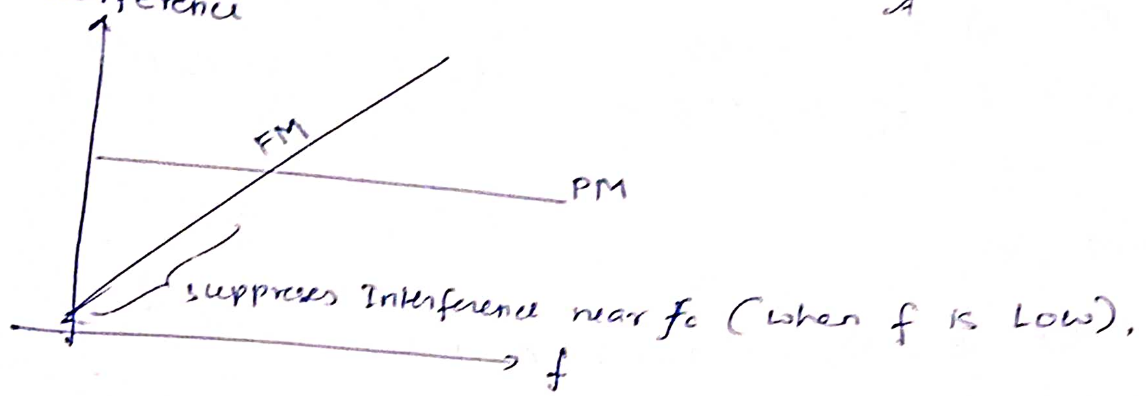
* case: $A \gg I$

then $\psi(t) = \tan^{-1} \dfrac{I \sin(2\pi f t)}{\cancel{A+}\ A} \approx \dfrac{I}{A} \sin(2\pi f t)$.

$\Rightarrow$ $\boxed{R(t) \cos(2\pi f_c t - \psi(t))} \longrightarrow \boxed{\text{PM Modulation}} \longrightarrow z(t) = \dfrac{I}{A} \sin(2\pi f t)$

$\downarrow$

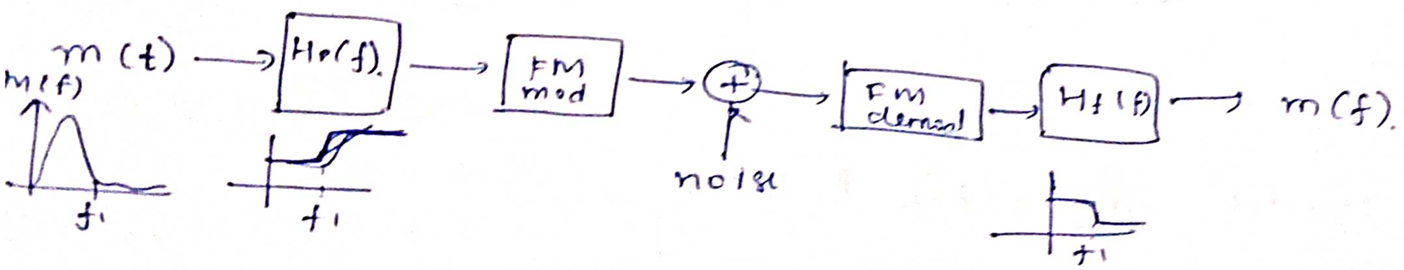
$\boxed{\text{FM Modulation}} \longrightarrow z(t) = \dfrac{I}{A} 2\pi f \cos(2\pi f t)$

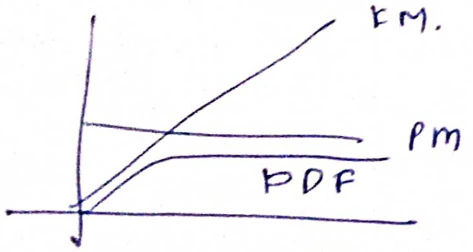


Thus if f is small, we must use F.M.
else we must use P.M.

* Thus F.M. signal is better in handling interference

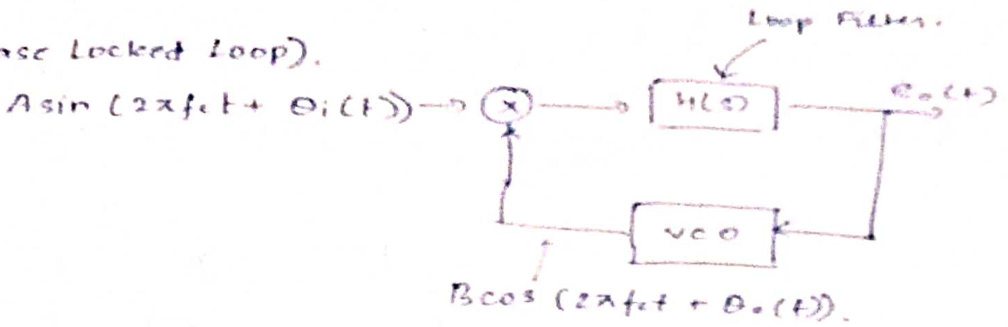
* for smaller freq. spectrum: FM is better
* for larger freq spectrum: P.M. is better





13/03/2023

PLL : (Phase Locked Loop).



VCO instantaneous freq :-

$$\left.\begin{aligned} f_i(t) &= f_c + c \cdot e_o(t) \\ \text{also } f_i(t) &= f_c + \frac{1}{2\pi} \dot{\theta}_o(t) \end{aligned}\right\} \Rightarrow \boxed{\dot{\theta}_o(t) = \underbrace{2\pi c}_{c'} e_o(t)} \text{ --- (1)}$$

output of multiplier

$$= AB \sin(2\pi f_c t + \theta_i(t)) \cos(2\pi f_c t + \theta_o(t))$$

$$= \frac{AB}{2}[\sin(\theta_i(t) - \theta_o(t)) + \sin(4\pi f_c t + \theta_i(t) + \theta_o(t))]$$

supressed

after passing through H(s). [finite freq. response] (say low pass)

$$\Rightarrow e_o(t) = \frac{AB}{2} \sin(\theta_i(t) - \theta_o(t)) * h(t) = \frac{AB}{2} \sin(\theta_e(t)) \text{ (say)}$$

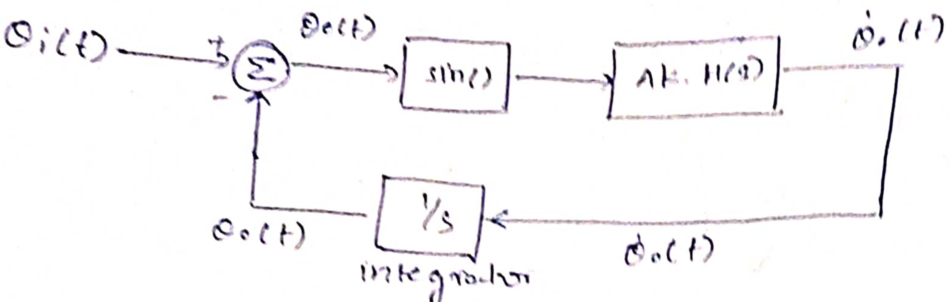
(where $\theta_e(t) = \theta_i(t) - \theta_o(t)$)

$$\Rightarrow. \; e_o(t) = \frac{AB}{2} \sin(\theta_e(t)) * h(t)$$

using eq (1).

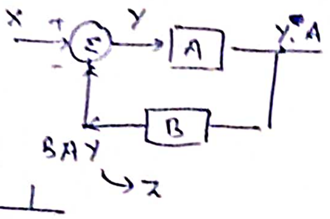
$$\boxed{\dot{\theta}_o(t) = \frac{ABc'}{2} \sin(\theta_e(t)) * h(t)}$$

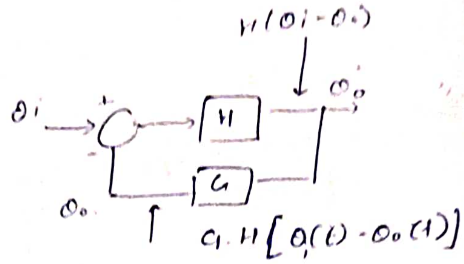
Redrawing the block diagram



assuming $\theta_e(t)$ is small, we can ignore $\sin()$ block

we know





$$\frac{Y}{X} = \frac{1}{1+AB}$$

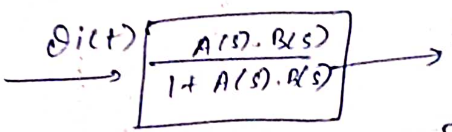
$$\Rightarrow \frac{(BAY)}{(X)} = \frac{(AB).1}{1+AB} \Rightarrow \boxed{\frac{Z(s)}{X(s)} = \frac{A(s).B(s)}{1+A(s).B(s)}}$$

$Y = A[X - BY]$

$Y = AX - ABY$

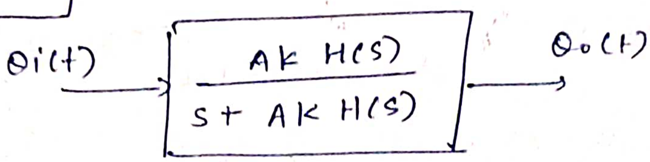
$Y(1+AB) = AX$

$\Rightarrow$ if we have



$\Rightarrow$ $A(s) = AK\,H(s)$

$B(s) = 1/s$ $\Rightarrow$



$$\theta_e(t) = \theta_i(t) - \theta_o(t) = \theta_i(t)\left[1 - \frac{\theta_o(t)}{\theta_i(t)}\right]$$

$$\theta_e(s) = \theta_i(s) - \theta_o(s) = \theta_i(s)\left[1 - \frac{\theta_o(s)}{\theta_i(s)}\right]$$ } Laplace transform (Linear operator)

$$\boxed{\theta_e(s) = \theta_i(s) . \frac{s}{s + AK.H(s)}}$$

Let input signal: $A \sin(2\pi f_o t + \varphi)$

$$A \sin(2\pi f_c t + \underbrace{2\pi(f_o - f_c)t + \varphi}_{\theta_i(t)})$$

$$\theta_i(s) = \frac{2\pi(f_o - f_c)}{s^2} + \frac{\varphi}{s}$$

$$\Rightarrow \quad \theta_e(s) = \frac{s}{s + AKH(s)}\left[\frac{2\pi(f_o - f_c)}{s^2} + \frac{\varphi}{s}\right]$$

* Let $\boxed{H(s) = 1}$

$$\theta_e(s) = \frac{s}{s + AK}\left[\frac{2\pi(f_o - f_c)}{s^2} + \frac{\varphi}{s}\right].$$

Now

$$\lim_{t \to \infty} \theta_e(t) = \lim_{s \to 0} s \cdot \theta_e(s).$$

$$\Rightarrow \lim_{t \to \infty} \theta_e(t) = \frac{s^2}{s + AK}\left(\frac{2\pi(f_o - f_c)}{s^2} + \frac{\varphi}{s}\right) = \frac{2\pi(f_o - f_c)}{AK}.$$

$$\boxed{\lim_{t \to \infty} \theta_e(t) = \frac{2\pi(f_o - f_c)}{AK}} \longrightarrow \text{Result 1}$$

* Let $H(s) = \dfrac{s + a}{s}$

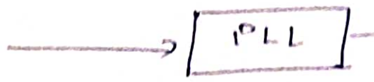
$$\theta_e(s) = \frac{s^2}{s^2 + AK(s + a)}\left[\frac{2\pi(f_o - f_c)}{s^2} + \frac{\varphi}{s}\right].$$

$$\Rightarrow \lim_{t \to \infty} \theta_e(t) = \lim_{s \to 0} s \cdot \theta_e(s)$$

$$\lim_{t \to \infty} \theta_e(t) = \lim_{s \to 0} \frac{s^3}{s^2 + AK(s + a)}\left[\frac{2\pi(f_o - f_c)}{s^2} + \frac{\varphi}{s}\right] = 0$$

$$\boxed{\lim_{t \to \infty} \theta_e(t) = 0} \longrightarrow \text{Result 2}$$

PLL as F.M. demodulator



$$e_o(t) = \frac{1}{c'}\,\dot{\theta}_o(t)$$

$$\sin(2\pi f_c t + \underbrace{k_f\,a(t) + \tfrac{\pi}{2}}_{\theta_i(t)})$$

$$\theta_e(t) = \theta_i(t) - \theta_o(t)$$

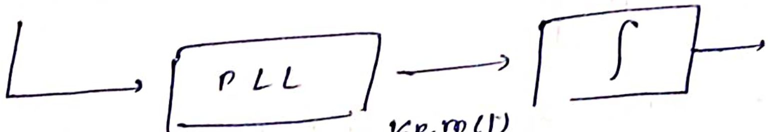
$$e_o(t) = \frac{1}{c'}\left[\frac{d}{dt}\right]\left[\theta_i(t) - \theta_e(t)\right]$$

$$e_o(t) = \frac{K_f}{c'}\cdot \underbrace{m(t)}_{\downarrow}$$ when PLL is locked (i.e. $\theta_e(t) \equiv 0$)

message signal demodulated.

PLL as P.M. demodulator

$$\sin(2\pi f_c t + k_p\, m(t) + \tfrac{\pi}{2})$$



$K_p . m(t)$ at large t

$m(t)$

---

* A2D conversion
* Quantization Noise