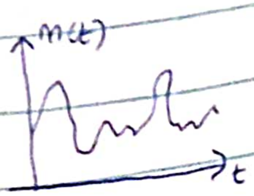
$$s(t) \rightarrow y(t)$$

Source of information $m(t)$ $\xrightarrow[(T_x)]{\text{Transmitter}}$ Channel $\xrightarrow[(R_x)]{\text{Reciever}}$ Destination

Channel:
- Wireless link
- Coaxial Cable
- Optical Fibre
- Satellite link



Building blocks of

a) Transmitter:

(i) Modulator ⇒ Transforms the signal such that it becomes suitable for given type of channel.

(ii) A/D converter (ADC)

(iii) Encoder

b) Reciever:

(i) Demodulator

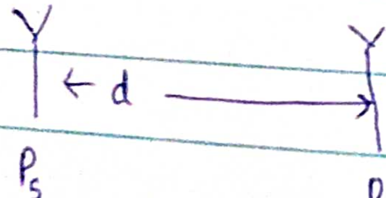
(ii) DAC

(iii) Decoder

Channel Imperiments: $(s(t) \neq y(t))$

(i) Attenuation:

Consider wireless channel, then:



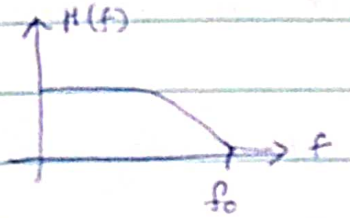
$P_r \propto \frac{P_s}{d^\alpha}$ where 'α' depends on environment

Similarly, in any coaxial cable, the resistance and dielectric constant also responsible for attenuation

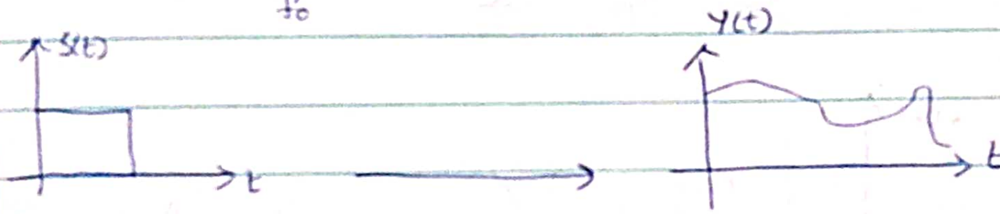
(ii) Distortion:

When signal s(t) is passed through channel then y(t) will be it's distorted version as follows:

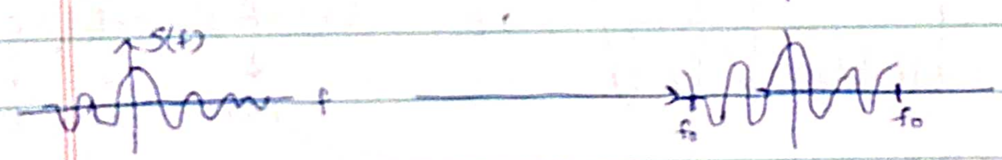
Ex: Consider the channel to be LPF, whose H(f) is given as follows:



(distortion depends on fo)

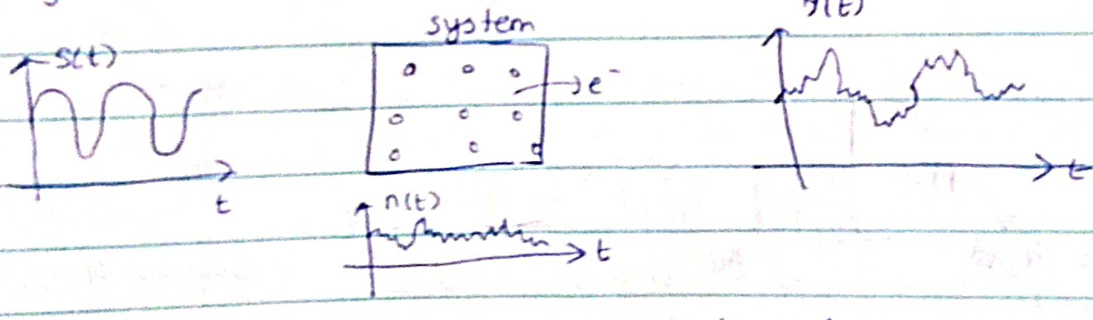


(distorted Version)



(iii) Noise:

Any unwanted signal contributes to noise. Ex:



n(t) can be due to internal agitation (thermal noise), external factors, etc.

Modulator:

Need:

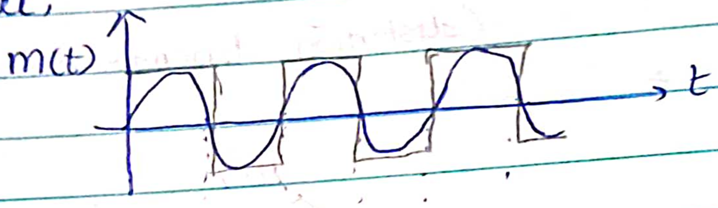
(i) To make the input signal suitable for given channel.

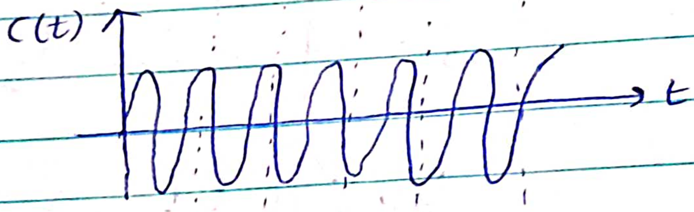
(ii) To change the frequency of signal so that it can be recieved at the other end, with antenna of optimal dimensions.

(iii) Multiplexing (FDM)

Consider Amplitude modulation (AM),

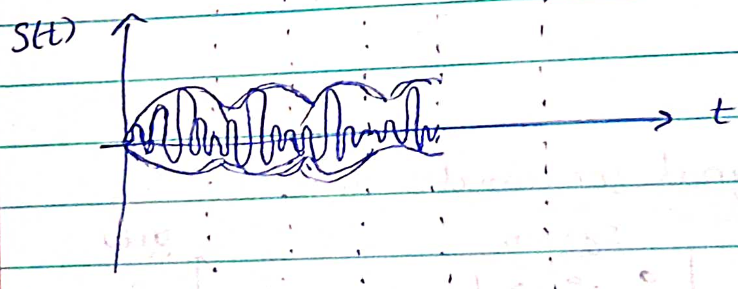
let,

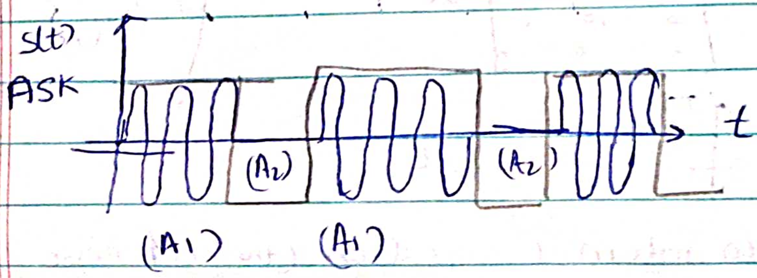




$c(t) = A\cos(2\pi f_c t + \phi)$

$\downarrow$ AM, $\downarrow$ FM, $\downarrow$ PM





for digital m(t)

How to convert analog signal to digital signal?

(i) Sampling

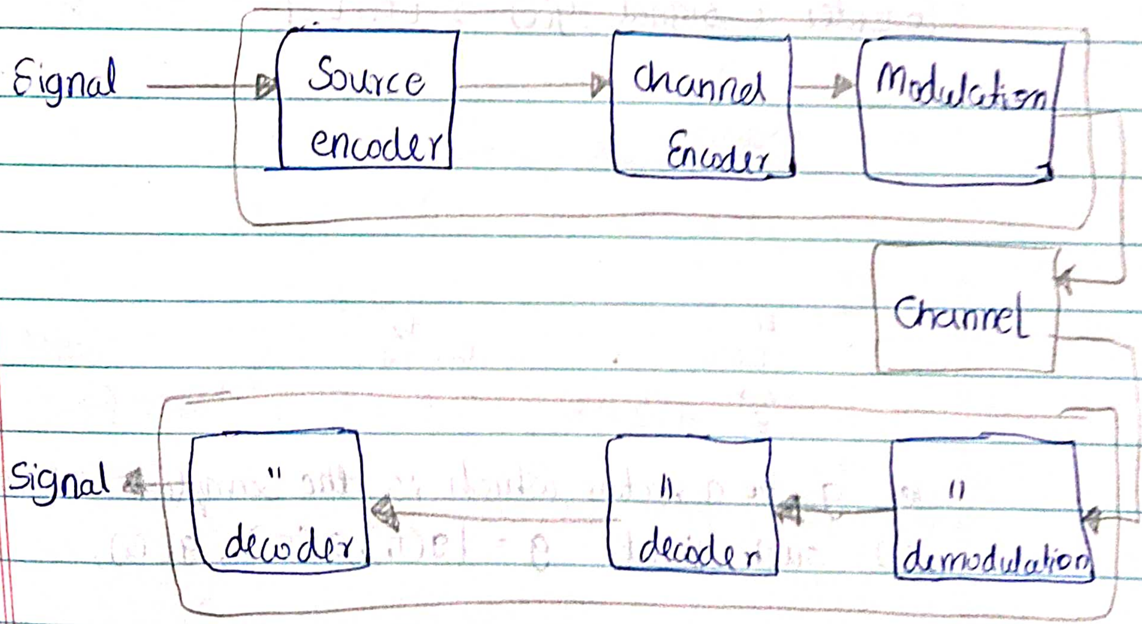
(ii) Quantization

While sampling, if $T_s \not> \frac{1}{2B} \Rightarrow$ aliasing doesn't occur.

let $R_s = \frac{1}{T_s}$

also while quantization, if there are 'L' levels of quantization then no. of bits generated are $\log_2 L$

let $R_b = \frac{\log_2 L}{T_s}$ [∵ No of bits transmitted per second]



* SQNR or SNR : $\frac{P_s}{N_o} = \frac{\text{power of signal}}{\text{power of noise}}$

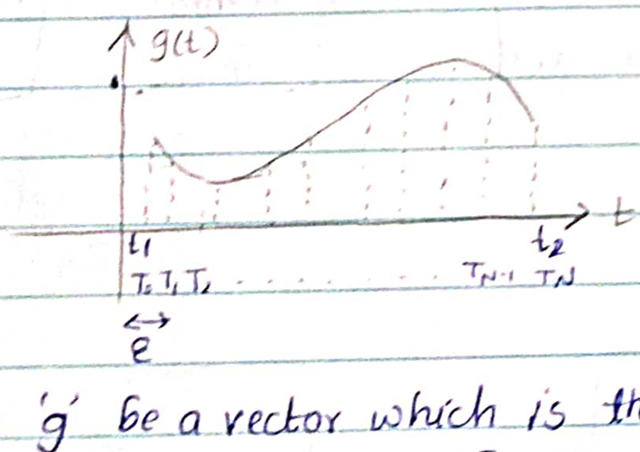
Consider a signal '$x(t)$' defined over $[t_1, t_2]$, then energy and power of the signal is given by:

$$E_x = \int_{t_1}^{t_2} |x(t)|^2 dt \quad < \infty$$

$$P_s = \lim_{T_0 \to \infty} \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} |x(t)|^2 dt$$

* if $x(t)$ is finite then $E_x < \infty$, $P_s = 0$, since $T_0 \to \infty$ $1/T_0 \to 0$. Such signal is called energy signal.
* if $x(t)$ is infinite then $P_s \neq 0$. And if $E_x = 0$ then that signal is called power signal.

Consider a signal $g(t) \to [t_1, t_2]$



let 'g' be a vector which is the sampled version of $g(t)$, such that $g = [g(T_1), g(T_2), g(T_3), \ldots, g(T_N)]$

if $\varepsilon \to 0 \Rightarrow N \to \infty \Rightarrow g = g(t)$

$\therefore \lim_{N \to \infty} g = g(t)$

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Vectors revisited:

→ Scalar product : $\langle g, g \rangle = \|g\|^2$

$\langle g, x \rangle = \|x\| \|g\| \cos\theta$

→ Component of 'g' along 'x'

$c\|x\| = \|g\| \cos\theta$

$c = \frac{\|g\| \cos\theta}{\|x\|}$

$= \frac{\|x\| \|g\| \cos\theta}{\|x\|^2} = \frac{\langle x, g \rangle}{\langle x, x \rangle}$

→ If g, x are orthogonal ; $\langle g, x \rangle = 0$ and $\theta = \pi/2$

W.K.T.

$\langle g, x \rangle = \sum_i g_i x_i = \sum_{i=0}^{\infty} g(t_i) x(\tau_i)$

$$\boxed{\langle g, x \rangle = \int_{t_1}^{t_2} g(t) x(t)\, dt}$$

Also,

$$\|x(t)\|^2 = \boxed{\langle x, x \rangle = \int_{t_1}^{t_2} |x(t)|^2 dt = E_x}$$

* Consider $g(t) = cx(t) + e(t)$ where '$e(t)$' is the error signal. Find optimal value of 'c' such that $g(t) \cong x(t)$.

→ Given, $g(t) = cx(t) + e(t)$

$e(t) = g(t) - cx(t)$

$E_e = \int_{t_1}^{t_2} (g(t) - cx(t))^2 \, dt$

For $g(t) \cong x(t)$, $\min_{c} \int_{t_1}^{t_2} (g(t) - cx(t))^2 \, dt$

Consider,

For 'E_e' to be minimum, $\frac{dE_e}{dc} = 0$

i.e., $\frac{d}{dc}\left[\int_{t_1}^{t_2} (g(t) - cx(t))^2 \, dt\right] = 0$

$\Rightarrow c = \frac{\int g(t)x(t)\,dt}{\int x^2(t)\,dt} = \frac{\langle g(t), x(t)\rangle}{\langle x(t), x(t)\rangle}$

Consider,

$$g(t) = \lim_{N\to\infty} g$$

where, $g = [g(\tau_1), g(\tau_2), \ldots\ldots, g(\tau_N)]$

Inner product $\langle g, x\rangle = \|g\| \, \|x\| \cos\theta$

$$\langle g(t), x(t)\rangle = \int_{t_1}^{t_2} g(\tau)x(\tau)d\tau$$

Orthogonality:

$\theta = 90°$

$\Rightarrow \langle g, x\rangle = 0$

If 'x' and 'g' are complex,

$$\langle g(t)\cdot x^*(t)\rangle = 0$$

$$\Rightarrow \int_{t_1}^{t_2} g(t)x^*(t)\,dt = 0$$

also,

$$c(t) = \frac{\int g(t)\cdot x^*(t)\,dt}{\int x(t)\cdot x^*(t)\,dt}$$

$$= \frac{\int g(t)\cdot x^*(t)dt}{\int \|x(t)\|^2 dt} = \frac{\int g(t)\cdot x^*(t)dt}{E_x}$$

Correlation :

$\langle g, x \rangle = \|x\| \, \|g\| \cos\theta$

if $\theta = 0$, then $g = cx$

* $\rho = \cos\theta = \dfrac{\langle g, x \rangle}{\|g\| \|x\|}$ $\quad$ [$\because \rho \rightarrow$ Imp quantity that captures correlation]

$$\boxed{\rho_{g,x} = \frac{1}{\sqrt{E_g \cdot E_x}} \cdot \int_{t_1}^{t_2} g(t) \cdot x^*(t) \, dt}$$

$$\rho_{g,x} = \begin{cases} 1, & \text{if } g = x \\ 0, & \text{if } g \perp x \end{cases}$$

$$\psi_{x,g}(\tau) = \int_{t_1}^{t_2} g(t) \cdot x^*(t+\tau) \, dt$$

$\uparrow$

correlation function

$$\psi_{x,x}(\tau) = \int_{t_1}^{t_2} x(t) \cdot x^*(t+\tau) \, dt$$

$\uparrow$

auto-correlation function

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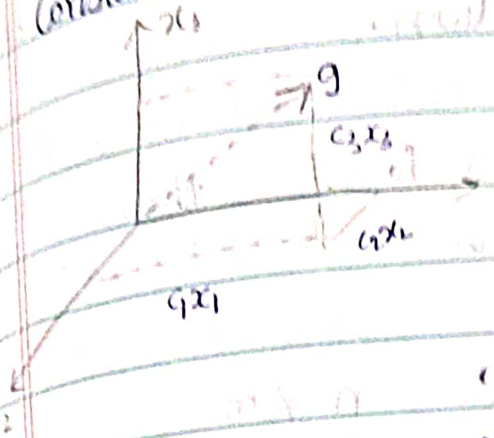
Date:

Consider,



$c_1x_1 \rightarrow$ projection of 'g' on 'x_1'

$c_2x_2 \rightarrow$ " " " " 'x_2'

$c_3x_3 \rightarrow$ " " " " 'x_3'

where x_1, x_2, x_3 perpendicular to each other.

here, $g = c_1x_1 + c_2x_2 + c_3x_3$

and $c_i = \dfrac{\langle g, x_i \rangle}{\langle x_i, x_i \rangle}$

$\Rightarrow \{x_1, x_2, x_3\} \Rightarrow$ complete basis set.

also, $g(t) = \sum_{i=1}^{N} c_i x_i(t) + e_N$ [$\because e_N \rightarrow$ error]

[for N - dimensions]

if $N \rightarrow \infty$,

$\|e_N\|^2 \rightarrow 0$, also $\|e_N\| > \|e_{N+1}\|$ (for $n/n+1 \rightarrow$ dim)

also, $g(t) = \sum_{i=-\infty}^{\infty} c_i x_i(t)$

* Any signal is implemented as summation of orthogonals signals.

* If $x_i(t)$ is exponential, then $g(t)$ looks like the fourier series representation.

If $x(t) = \exp(j2\pi n f_0 t)$, $f_0 = \frac{1}{T_0}$ then,
$n = -\infty$ to ∞

$$\int_0^{T_0} x_n(t) \cdot x_m^*(t)\, dt = \begin{cases} 0 & , n \neq m \\ T_0 & , n = m \end{cases}$$

similarly,

$$\int_0^{T_0} \exp(j2\pi(n-m)f_0 t)\, dt = \begin{cases} 0 & , m \neq n \\ T_0 & , m = n \end{cases}$$

W.K.T.

$$g(t) = \sum_{n=-\infty}^{\infty} x_n(t) \cdot C_n$$

$$= \sum_{n=-\infty}^{\infty} C_n \cdot \exp(j2\pi n f_0 t), \quad f_0 = \frac{1}{T_0}$$

also, W.K.T

$$C_n = \frac{\langle g(t), x^*(t) \rangle}{\langle x(t), x^*(t) \rangle}$$

$$= \frac{\int_0^{T_0} g(t) \cdot \exp(-j2\pi n f_0 t)\, dt}{\left(\int_0^{T_0} \exp(j2\pi n f_0 t) \exp(-j2\pi n f_0 t)\, dt\right) \rightarrow T_0}$$

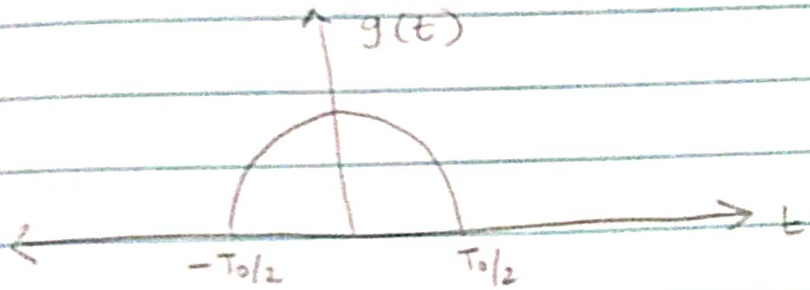
* Let $X = x_1 + x_2$;

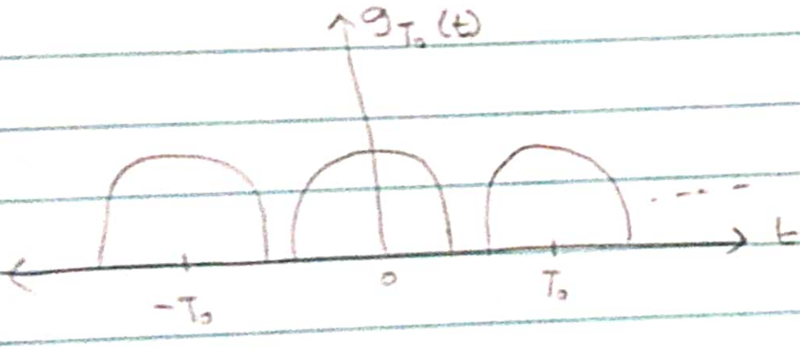
If x_1, x_2 are orthogonal,

$$\|x\|^2 = \|x_1\|^2 + \|x_2\|^2$$

$$E_x = E_{x_1} + E_{x_2}$$

Consider,





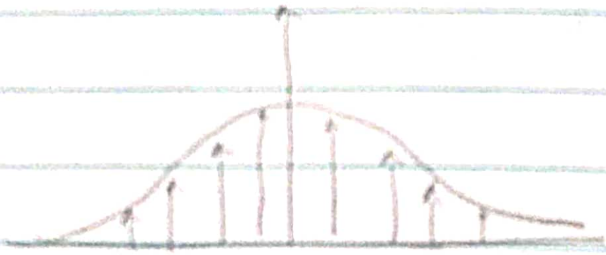
$$\lim_{T_0 \to \infty} g_{T_0}(t) = g(t)$$

(*) $$g_{T_0}(t) = \sum_{n=-\infty}^{\infty} D_n \exp(j2\pi n f_0 t); \quad f_0 = \frac{1}{T_0}$$

where, $$D_n = \frac{1}{T_0} \int_{-\infty}^{\infty} g(t) \exp(-j2\pi n f_0 t)\, dt \Big|_{f_n = nf_0}$$

$D_n = \frac{1}{T_0} G(nf_0)$, $D_n = \Delta f \, G(n\Delta f)$

$\Delta f = \frac{1}{T_0}$



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F.T. of $g(t)$: $G(f) = \int_{-\infty}^{\infty} g(t) e^{-j2\pi ft} dt$

IFT of $G(f)$: $g(t) = \int_{-\infty}^{\infty} G(f) e^{j2\pi ft} df$

$g(t) \xleftrightarrow{F.T.} G(f)$

* If $g(t)$ is real,

$$g(t) = \int_{-\infty}^{\infty} G(f) e^{j2\pi ft} df \Rightarrow g^*(t) = \int_{-\infty}^{\infty} G^*(f) e^{-j2\pi ft} df$$

$\Rightarrow \boxed{G(-f) = G^*(f)}$

* $|G(-f)| = |G(f)|$

$\theta_g(-f) = -\theta_g(f)$

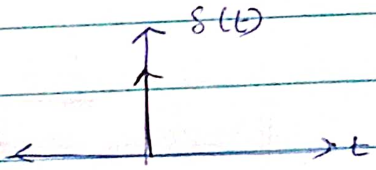
Linearity:

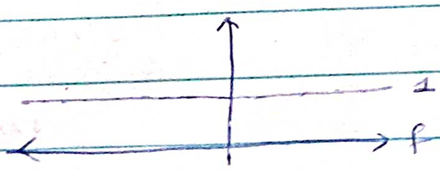
$$\alpha_1 g_1(t) + \alpha_2 g_2(t) \xrightarrow{F.T.} \alpha_1 G_1(f) + \alpha_2 G_2(f)$$

* $g(t) = \delta(t)$

$G(f) = \int_{-\infty}^{\infty} \delta(t) e^{-j2\pi ft} dt$

$= 1, \ \forall f$



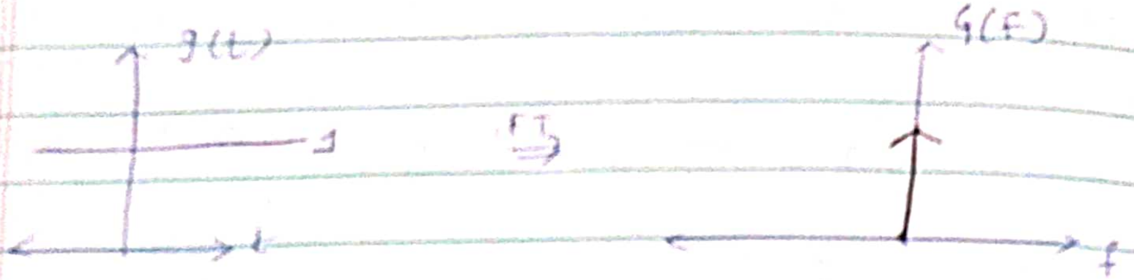


If $G(f)$ is $\delta(f)$ then,

$$g(t) = \int_{-\infty}^{\infty} G(f) e^{j2\pi f t} df$$

$$= \int_{-\infty}^{\infty} e^{j2\pi f t} df$$

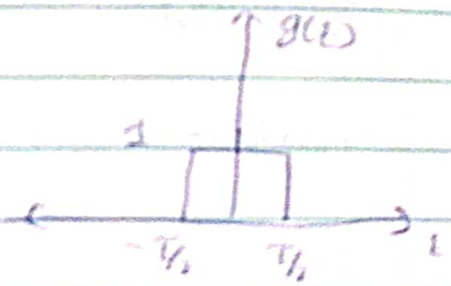
$$= 1$$



* If $G(f) \xrightarrow{I.F.T.} g(t)$

then $\boxed{G(f-f_0) \xrightarrow{I.F.T} g(t)\, e^{j2\pi f_0 t}}$

* If $g(t)$ is Square pulse



$$G(f) = \int_{-T/2}^{T/2} e^{-j2\pi f t} dt$$

$$= \frac{\left[e^{-j2\pi f t}\right]_{-T/2}^{T/2}}{-j2\pi f}$$

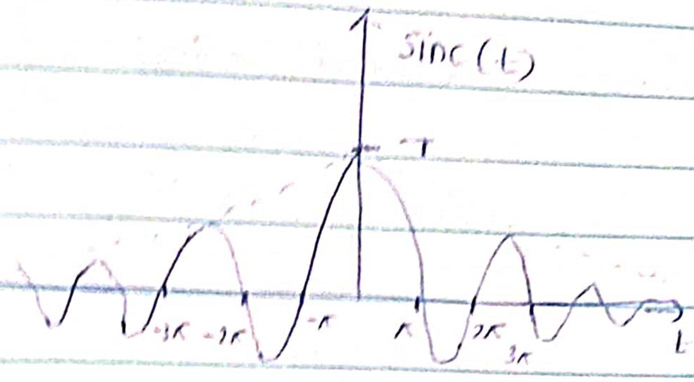
$$= \frac{e^{j2\pi f T/2} - e^{-j2\pi f T/2}}{j2\pi f}$$

$$= \frac{\cancel{2j}\sin(2\pi f T/2)}{\cancel{2j}\pi f} = \frac{\sin(\pi f T)}{\pi f T} \cdot T$$

$$\boxed{G(f) = T\,\mathrm{sinc}(\pi f T)}$$

Properties of sinc function:

1) Even function
2) Zero at $x = \pm k\pi,\ k > 0$
3) $Sinc(0) = 1$
4) Amplitude $\propto \frac{1}{x}$



* if $g(t) \longleftrightarrow G(f)$

then $\boxed{g(at) \Rightarrow \frac{1}{|a|} G\left(\frac{f}{|a|}\right)}$

* Time shifting:

$$\boxed{g(t-t_0) \xleftrightarrow{F.T.} G(f)e^{-j2\pi f t_0}}$$

Note:
Amplitude spectrum remains unchanged.
Phase spectrum changes by $-2\pi f_0$.

* Frequency shifting:

$$\boxed{G(f-f_0) \xrightarrow{I.F.T.} g(t)e^{j2\pi f_0 t}}$$

Q.) Consider $g'(t) = g(t)\cos(2\pi f_0 t)$ [∵ Modulation Property]

$$g'(t) = g(t)\left(\frac{e^{j2\pi f_0 t} + e^{-j2\pi f_0 t}}{2}\right)$$

$$G'(f) = \int_{-\infty}^{\infty} \frac{g(t)e^{-j2\pi(f-f_0)t} + g(t)e^{-j2\pi(f+f_0)t}}{2} = \frac{G(f-f_0)+G(f+f_0)}{2}$$

Convolution Property:

$$z(t) = x(t) * g(t) = \int_{-\infty}^{\infty} x(\tau)\, g(t-\tau)\, d\tau$$

then $\boxed{x(t) * g(t) \xrightarrow{F.T.} X(f)G(f)}$

Similarly,
if $Z(f) = X(f) * G(f)$, then

$$\boxed{X(f) * G(f) \xrightarrow{I.F.T.} x(t) \cdot g(t)}$$

Also, if $\underset{\underset{B_z}{\downarrow}}{Z(t)} = \underset{\underset{B_x}{\downarrow}}{x(t)} \cdot \underset{\underset{B_g}{\downarrow}}{g(t)}$

then $\boxed{B_z = B_x + B_g}$ [$B \rightarrow$ bandwidth of a signal]

LTI system:

$$x(t) \longrightarrow \boxed{\begin{matrix}\text{LTI system} \\ h(t)\end{matrix}} \longrightarrow y(t)$$

$$\boxed{\begin{aligned} y(t) &= x(t) * h(t) \\ Y(f) &= X(f) \cdot H(f) \end{aligned}}$$

where, $H(f) \longrightarrow$ transfer function

Distortionless system / Filter:

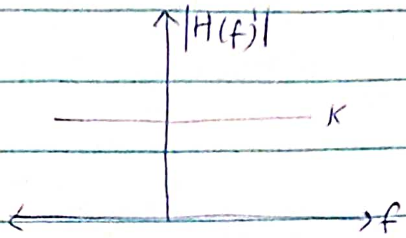
The system which doesn't cause any distortion in the shape of input frequency spectrum is called distortion system.

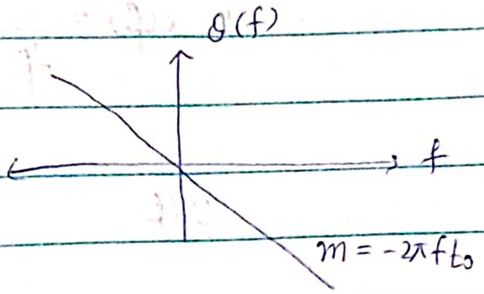
This is possible when,

$y(t) = K \cdot x(t - t_0)$

$Y(f) = K \cdot X(f) e^{-j2\pi t_0 f}$

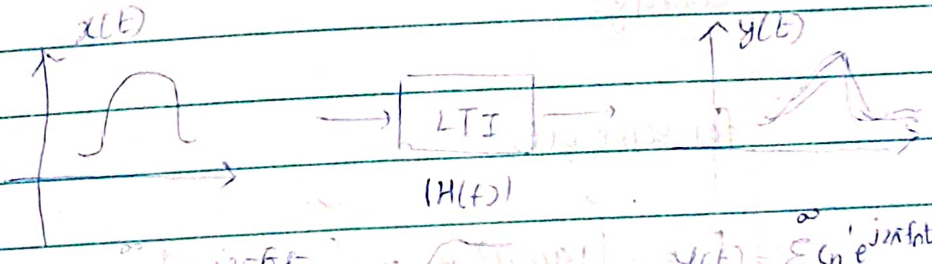
$\frac{Y(f)}{X(f)} = K e^{-j2\pi t_0 f}$





$$\boxed{\therefore H(f) = K e^{-j2\pi f t_0}}$$

If $|H(f)| = g(f)$ and $\theta_h(f) = -2\pi f t_d$, then:



$x(t) = \sum_{n=-\infty}^{\infty} C_n e^{j2\pi f_n t}$ $y(t) = \sum_{n=-\infty}^{\infty} C_n' e^{j2\pi f_n t}$

$x(t) \neq y(t)$ [∵ This type of distortion is linear distortion]

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let,

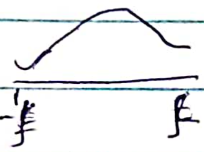
$x(t) \longrightarrow \boxed{sys} \longrightarrow y(t) = g(x(t))$

where, $y(t) = a_0 + a_1 x(t) + a_2 x(t)^2 + \ldots\ldots$

$\downarrow$ $\qquad$ $\downarrow$

$B.W = B \qquad B.W = 2B$

B.W increases

$\therefore$ if $x(f) =$ 

$y(f) =$ 

This kind of distortion is called non-linear distortion.

Energy:

for some $g(t)$,

$$E_g = \int_{-\infty}^{\infty} |g(t)|^2 dt = \int_{-\infty}^{\infty} g(t) \cdot g^*(t)\, dt$$

W.K.T $\quad g(t) = \int_{-\infty}^{\infty} G(f) e^{j2\pi f t} df$

$$g^*(t) = \int_{-\infty}^{\infty} G^*(f) e^{-j2\pi f t} df$$

$$= \int_{-\infty}^{\infty} G^*(-f) e^{j2\pi f t} dt$$

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then, $E_g = \int_{-\infty}^{\infty} g(t) \cdot \left(\int_{-\infty}^{\infty} G^*(f) e^{-j2\pi f t} df \right) dt$

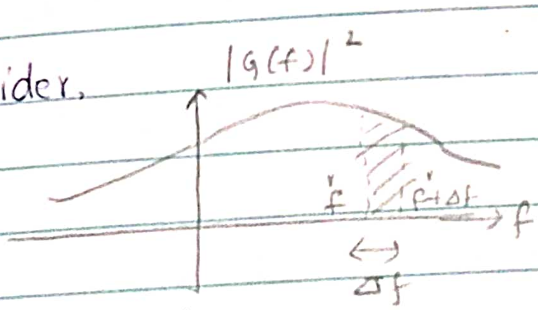
$= \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} G^*(f) g(t) e^{-j2\pi f t} dt \right] df$

$= \int_{-\infty}^{\infty} G^*(f) \left(\int_{-\infty}^{\infty} g(t) e^{-j2\pi f t} dt \right) df$

$= \int_{-\infty}^{\infty} G^*(f) \cdot G(f) df = \int_{-\infty}^{\infty} |G(f)|^2 df$

$\therefore E_g = \int_{-\infty}^{\infty} |g(t)|^2 dt = \int_{-\infty}^{\infty} |G(f)|^2 df$

Consider,



→ Energy spectral density

$$\underset{(\text{shaded})}{E_g} = \int_{f'}^{f'+\Delta f} |G(f)|^2 df$$

$$\lim_{\Delta f \to 0} \underset{(\text{shaded})}{E_g} = \lim_{\Delta f \to 0} \int_{f'}^{f'+\Delta f} |G(f)|^2 df \approx |G(f')|^2 \Delta f$$

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W.K.T $R_g(\tau) = \int_{-\infty}^{\infty} g(t) \cdot g^*(t-\tau)\,dt$

if $\tau = 0$, then $R_g(0) = E_g = \int_{-\infty}^{\infty} g(t) \cdot g^*(t)\,dt$

Consider,

$$R_g(\tau) = \int_{-\infty}^{\infty} g(t) \cdot g^*(t-\tau)\,dt$$

let $\Psi_g(f) = \int_{-\infty}^{\infty} R_g(\tau) e^{-j2\pi f\tau}\,d\tau$

$$= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(t) g^*(t-\tau)\,dt\; e^{-j2\pi f\tau}\,d\tau$$

$$= \int_{-\infty}^{\infty} g(t) \int_{-\infty}^{\infty} g^*(t-\tau) e^{-j2\pi f\tau}\,d\tau\;dt$$

$$= \int_{-\infty}^{\infty} g(t) \int_{-\infty}^{\infty} g^*(\tau) \cdot e^{-j2\pi f(t-\tau)}\,d\tau\;dt$$

$$= \int_{-\infty}^{\infty} g(t) e^{-j2\pi ft} \int_{-\infty}^{\infty} g^*(\tau) e^{+j2\pi f\tau}\,d\tau\;dt$$

$$= \int_{-\infty}^{\infty} g(t) e^{-j2\pi ft} \cdot \left(\int_{-\infty}^{\infty} g(\tau) e^{-j2\pi(-f)\tau}\,d\tau\right)^* dt$$

$$= \int_{-\infty}^{\infty} g(t) e^{-j2\pi ft}\, G^*(-f)\,dt$$

$$= \left[\int_{-\infty}^{\infty} g(t) e^{-j2\pi(-f)t}\,dt\right]^* \cdot G(-f)$$

$$= G(-f) \cdot G^*(-f)$$

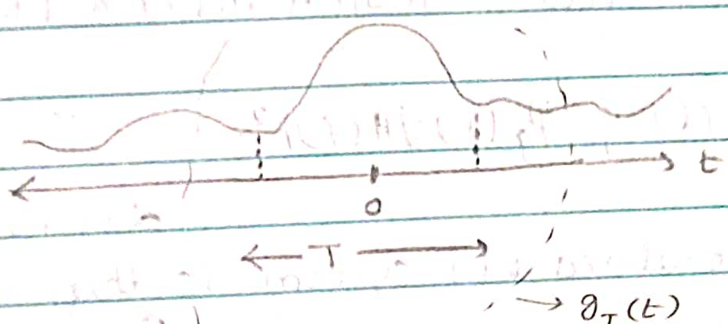
$\therefore R_g(t) \xrightarrow{F.T} \Psi_g(f)$

Then, $R_g(\tau) = \int_{-\infty}^{\infty} |G(f)|^2 df \, e^{j2\pi f \tau} df$

Verification By putting $\tau = 0$, $R_g(0) = \int_{-\infty}^{\infty} |G(f)|^2 df$

As energy can be found only for finite signals, we are interested to find power for infinite duration signals.

Consider:



$$P_g = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} |g_T(t)|^2 dt$$

$$= \lim_{T\to\infty} \frac{1}{T} \cdot E_{g_T} = \int_{-\infty}^{\infty} \lim_{T\to\infty} \frac{|G_T(f)|^2}{T} df$$

Let $\Psi_g(t) = \lim_{T\to\infty} \frac{|G_T(t)|^2}{T}$

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Ex: $g(t) \longrightarrow \boxed{\underset{H(f)}{LTI}} \longrightarrow y(t)$.

Given E.S.D of $g(t)$ as $|G(f)|^2$ and $H(f)$ given.
Find E.S.D of $Y(t)$.

W.K.T.

$$Y(f) = H(f)\,G(f)$$
$$|Y(f)| = |H(f)G(f)|$$
$$|Y(f)|^2 = |H(f)G(f)|^2 = |H(f)|^2 \cdot |G(f)|^2$$

$\therefore$ ESD of $Y(t)$ = [ESD of $H(f)$] $\times$ [ESD of $G(f)$]

$$\therefore \psi_y(f) = \psi_g(f) \cdot |H(f)|^2$$

Ex: If the above LTI system is differentiator then find $\psi_y(f)$.

$$y(t) = \frac{d}{dt}x(t)$$

W.K.T. $$x(t) = \int_{-\infty}^{\infty} X(f)\,e^{j2\pi ft}\,df$$

$$\frac{dx(t)}{dt} = j2\pi f \int_{-\infty}^{\infty} X(f)\,e^{j2\pi ft}\,df$$

$$y(t) = j2\pi f \cdot x(t)$$
$$Y(f) = j2\pi f \cdot X(f) \Rightarrow H(f) = j2\pi f$$

Energy signal

$$E_g = \int_{-\infty}^{\infty} g^2(t)\,dt$$

$$\psi_g(\tau) = \int_{-\infty}^{\infty} g(t)\,g(t+\tau)\,dt$$

$$\psi_g(0) = E_g$$

$$\psi_g(\tau) \xrightarrow{F.T.} \phi_g(f)$$

$$ESD: \phi_g(f) = |G(f)|^2$$

$$E_g = \int_{-\infty}^{\infty} \phi_g(f)\,df$$

Power signal

$$P_g = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} g_T^2(t)\,dt$$

$$P.S.D.: \phi_{g_T}(f) = \lim_{T\to\infty} \frac{|G_T(f)|^2}{T}$$

$$\Big\updownarrow \text{F.T.}$$

Autocorrelation: $\psi_g(t)$

$$P_g = \int_{-\infty}^{\infty} \phi_{g_T}(f)\,df = \psi_g(0)$$

Ex:

$$x(t) \longrightarrow \boxed{\begin{matrix} LTI \\ h(t), H(f) \end{matrix}} \longrightarrow y(t)$$

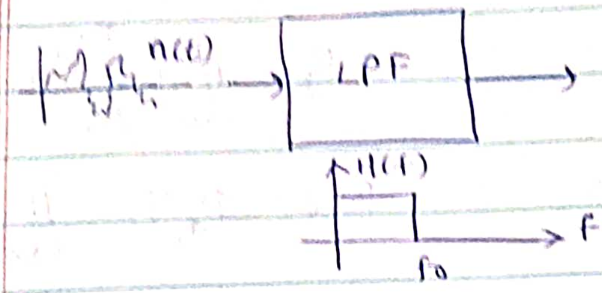
$$y(t) = h(t) * x(t)$$

$$Y(f) = H(f) \cdot X(f)$$

$$|Y(f)|^2 = |H(f)|^2 \cdot |X(f)|^2$$

$$\Phi_{y_T}(f) = |H(f)|^2 \cdot \phi_{x_T}(f)$$

Ex: Consider a noise signal to an LPF:



$$R_n(\tau) = \int_{-\infty}^{\infty} n(t) \cdot n(t+\tau)\,dt$$

as two different noise signals are uncorrelated, $R_n = 0$ for $\tau \neq 0$.

$$\therefore R_n(\tau) = \begin{cases} 0, & \tau \neq 0 \\ N_0, & \tau = 0 \end{cases}$$

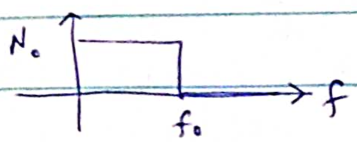
where, N_0 is the variance of the signal.

$$\therefore \boxed{R_n(\tau) = N_0\,\delta(\tau)}$$

$$\Rightarrow \phi_n(f) = F.T.(R_n(t))$$
$$= F.T.(N_0\,\delta(t))$$
$$= N_0, \quad \forall f$$

W.K.T. $\phi_y(f) = |H(f)|^2 \cdot \phi_n(f)$

$$= |H(f)|^2 \cdot N_0$$



$\Rightarrow R_y(\tau)$ is a 'sinc' function. Which has non-zero correlation for all 'τ'.

$\therefore$ When we give completely uncorrelated signal (r.p.) to an LTI system, the we get a non-zero correlated function for all 'τ'.

Modulation:-

Consider a message signal $m(t)$ and a carrier signal of high frequency $c(t)$, given by

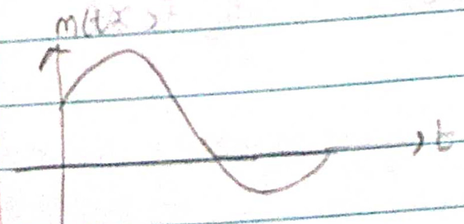
$$c(t) = A\cos(2\pi f_c t + \phi)$$

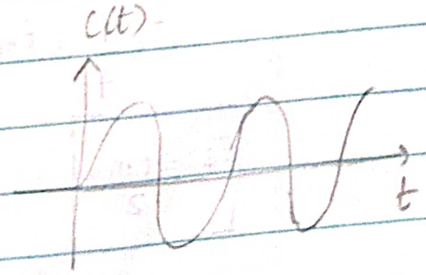
$$\left.\begin{array}{l} \text{if, } A(t) = g_1(m(t)) \Rightarrow \text{A.M.} \\ \left.\begin{array}{l} f_c(t) = g_2(m(t)) \Rightarrow \text{F.M.} \\ \phi(t) = g_3(m(t)) \Rightarrow \text{P.M.} \end{array}\right\} \text{Angle Modulation} \end{array}\right\} \text{Carrier modulation}$$

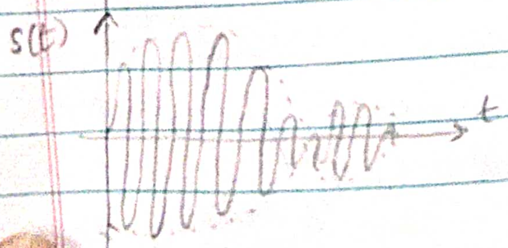
Amplitude Modulation:-

Here, $A(t) = m(t)$, Then $s(t) = \underbrace{m(t)}_{\text{modulating signal}} \underbrace{\cos(2\pi f_c t)}_{\text{carrier signal}}$

Ex: Consider $m(t)$:



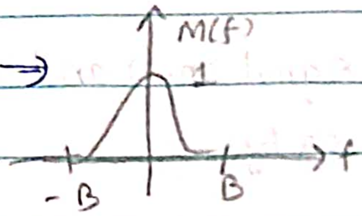




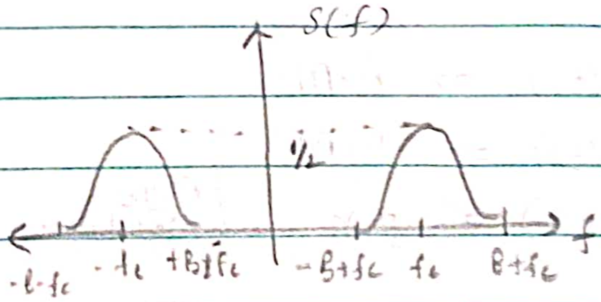
W.K.T., as $s(t) = m(t)\cos 2\pi f_c t$, from modulation property

$$S(f) = \frac{M(f+f_c) + M(f-f_c)}{2}$$

let, $M(f) \Rightarrow$



then, $S(f) \Rightarrow$



If $f_c \geq B$

W.K.T. $E_m = \int_{-\infty}^{\infty} |M(f)|^2 df$

also, $E_s = \int_{-\infty}^{\infty} |S(f)|^2 df$

$$= \int_{-\infty}^{\infty} \frac{1}{4} \left(M(f+f_c)^2 + M(f-f_c)^2 + 2 \cdot M(f-f_c) \cdot M(f+f_c) \right) df$$

$$= \frac{1}{4}(E_m + E_m + 0), \quad \text{if } f_c \geq B$$

$$\boxed{E_s = \frac{E_m}{2}}$$

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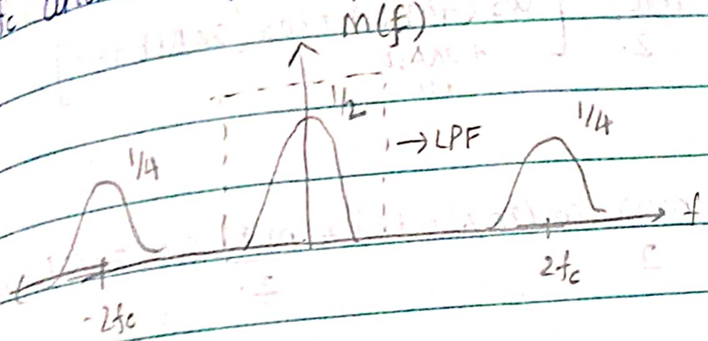
Demodulation:

multiply the modulated signal with the same carrier freq signal so that we can shift the spectrum of modulated signal by $\pm f_c$ and then apply a low pass filter.



Hence,

$$s(t) \rightarrow \otimes \rightarrow \boxed{LPF} \rightarrow \hat{m}(t)$$

$$\uparrow c(t)$$

$$\hat{m}(t) = s(t)\cos(2\pi f_c t)$$

$$= m(t)\cos^2(2\pi f_c t)$$

$$= m(t)\frac{(1+\cos 4\pi f_c t)}{2}$$

$$= \frac{m(t)}{2} + \frac{m(t)\cdot\cos 4\pi f_c t}{2}$$

$$\hat{M}(f) = \frac{M(f)}{2} + \frac{1}{4}\left(M(f-2f_c) + M(f+2f_c)\right)$$

$\downarrow$ L.P.F

$\frac{M(f)}{2}$

$\therefore$ This type of demodulation is called synchronised demodulation (or coherent demodemulation)

If we multiply the modulated signal with non-synchronous carrier signal,

let $\hat{m}(t) = s(t) \cdot \cos(2\pi f_c t + 2\pi \Delta f t + \theta)$

$= m(t) \cos(2\pi f_c t) \cdot \cos(2\pi f_c t + 2\pi \Delta f t + \theta)$

$= \frac{m(t)}{2} \left[\cos(4\pi f_c t + 2\pi \Delta f t + \theta) + \cos(2\pi \Delta f t + \theta) \right]$

$\hat{m}(t) = \frac{m(t)}{2} \cos(2\pi \Delta f t + \theta) + \frac{m(t)}{2} \cos(2\pi (2f_c + \Delta f) t + \theta)$

let filter this by LPF

$\therefore \hat{m}(t) = \frac{m(t)}{2} \cos(2\pi \Delta f t + \theta)$

Case-1: If $\Delta f = 0$,

$\hat{m}(t) = \frac{m(t)}{2} \cos\theta$ (attenuated)

Case-2: If $\theta = 0$

$\hat{m}(t) = \frac{m(t)}{2} \cos(2\pi \Delta f t)$

for different 't', $\hat{m}(t)$ will be different from $m(t)$. Hence we observe distortion.

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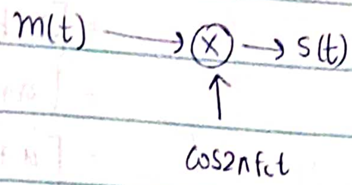
Recap: (DSB - SC)

Modulation :- $s(t) = m(t) \cos 2\pi f_c t$

$B_s = 2B_m$

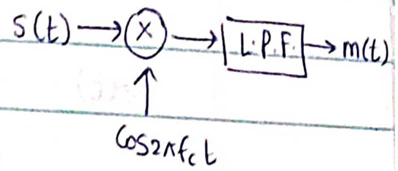
$P_s = \frac{1}{2} P_m$



Demodulation :- $\hat{m}(t) = s(t) \cos 2\pi f_c t$

$= m(t) \cos^2 2\pi f_c t$

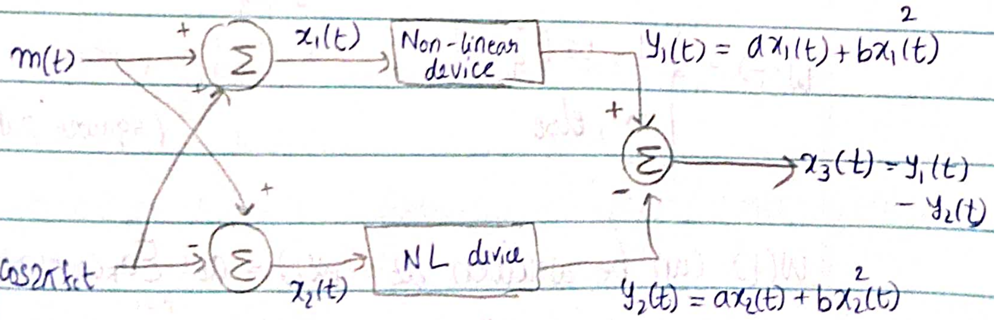
$= \frac{m(t)}{2} + \frac{m(t) \cos 4\pi f_c t}{2}$



But here $\cos(2\pi f_c t)$ should be synchronous. It depends on at what time you start giving $\cos(2\pi f_c t)$. Then there may be some phase shift.

Other methods of modulation :-

a) Non-linear model :



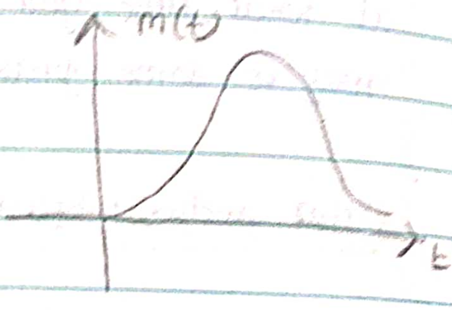
$$x_3(t) = (a x_1(t) + b x_1^2(t)) - (a x_2(t) + b x_2^2(t))$$
$$= a(x_1(t) - x_2(t)) + b[x_1^2(t) - x_2^2(t)]$$
$$= a(x_1(t) - x_2(t)) + b(x_1(t) - x_2(t))(x_1(t) + x_2(t))$$
$$= [a + b(x_1(t) + x_2(t))][x_1(t) - x_2(t)]$$
$$= \cancel{[a + b(2m(t))][}$$
$$= [a + b(2\cos w_c t)][2m(t)]$$
$$= 2am(t) + 4bm(t)\cos w_c(t)$$

$$x_3(t) \longrightarrow \boxed{\text{B.P.F. } f_c} \longrightarrow 4b\, m(t)\cos(2\pi f_c t) = s(t)$$

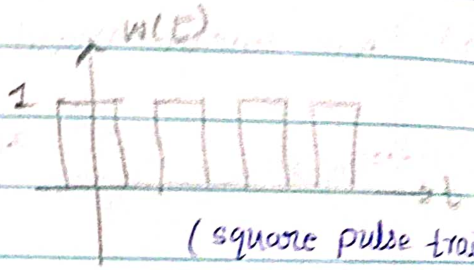
depends on non-linearity of the device.

b) Switching Modulators:

Consider $m(t)$ as:



and consider $w(t)$ as:



$$w(t) = \begin{cases} 1, & t \in [-\frac{T}{4}, \frac{T}{4}] \\ 0, & \text{else} \end{cases}$$

(square pulse train)

$w(t)$ can be written as: $w(t) = a_0 + \sum_{k=1}^{\infty} a_k \cos k w_0 t + \sum_{k=1}^{\infty} b_k \sin k w_0 t$
where a_k, b_k are F.S. coefficients.

$$a_0 = \frac{1}{T}\int_{-T/2}^{T/2} f(t)\,dt \; ; \; a_k = \frac{2}{T}\int_{-T/2}^{T/2} f(t)\cos k w_0 t\,dt$$

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$$b_K = \frac{2}{T}\int_{-T/2}^{T/2} f(t)\sin K\omega_0 t\, dt.$$

As above signal $w(t)$ is even signal, $b_K = 0$, $\forall K$, and a_K is given by:

$$a_K = \frac{2}{T}\int_{-T/4}^{T/4} \cos K\omega_0 t\, dt \qquad a_0 = \frac{1}{T}\int_{-T/4}^{T/4} 1\, dt = \frac{1}{2}$$

$$= \frac{2}{T}\cdot\frac{(2\sin K\omega_0 T/4)}{(K\omega_0)}$$

$$a_K = \text{sinc}(K\omega_0 T/4) \Rightarrow a_1 = 2/\pi,\ a_2 = 0,\ a_3 = -2/3\pi, \ldots$$

$$= \text{sinc}(K\pi/2)$$

$$\therefore a_K = \begin{cases} 0 & , K \in 2\mathbb{Z} \\ \frac{2}{\pi}\cdot\left(\frac{1}{K}\right)\cdot(-1)^{\left(\frac{K+3}{2}\right)} & , \text{else} \end{cases}$$

$$\therefore w(t) = \frac{1}{2} + \frac{2}{\pi}\left(\cos\omega_c t - \frac{1}{3}\cos 3\omega_c t + \frac{1}{5}\cos 5\omega_c t - \cdots\right)$$

let $g(t) = w(t) \times m(t)$

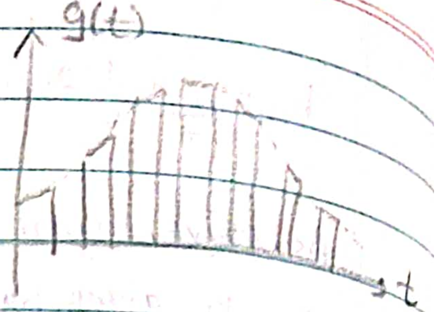
$$g(t) = \frac{1}{2}m(t) + \frac{2}{\pi}m(t)\cos\omega_c t - \frac{2}{3\pi}m(t)\cos 3\omega_c t \cdots$$

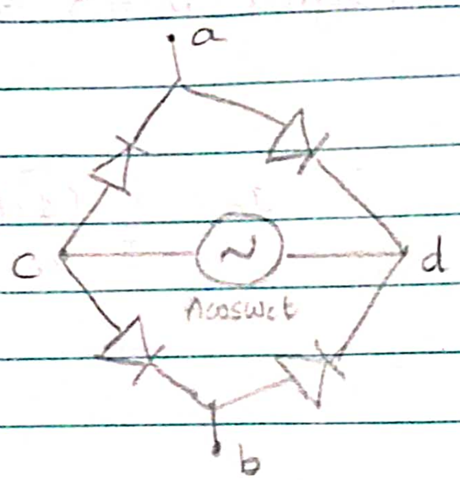
$$G(f) = \frac{M(f)}{2} + \frac{1}{\pi}\left(M(f+f_c) + M(f-f_c)\right) - \frac{1}{3\pi}\left(M(f+3f_c) + M(f-3f_c)\right)$$

$$+ \cdots\cdots$$

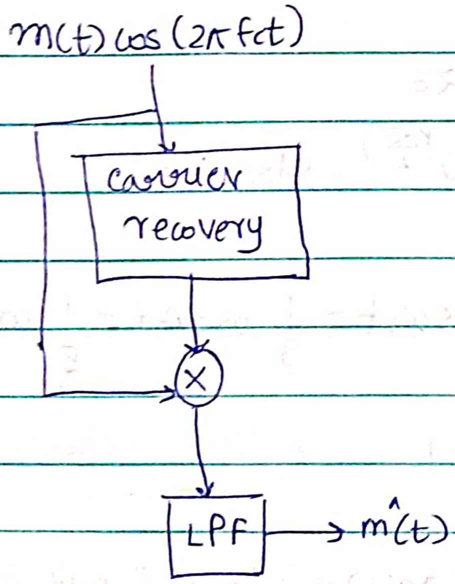
So, $g(t) \longrightarrow \boxed{\text{B.P.F}}_{\pm(2K+3)f_c} \longrightarrow \frac{2}{\pi}\left(\frac{1}{K}\right)(-1)^{\left(\frac{K+3}{2}\right)} m(t)\cos 2\pi K f_c t$

→ for switching operators of $w(t)$, we can use bridge circuits.

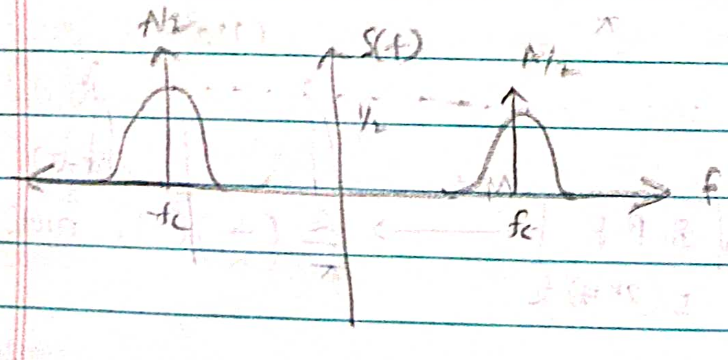




Carrier recovery:



It is difficult to ensure zero phase shift. Alternative method: add a term '$A \cos 2\pi f_c t$' to '$S(t)$'.



Extra

$s(t) = A\cos(2\pi f_c t) + m(t)\cos(2\pi f_c t)$

$s(t) = [A + m(t)]\cos(2\pi f_c t)$

then, $S(f)$ will be:

$$S(f) = \frac{A}{2}\left[\delta(f+f_c) + \delta(f-f_c)\right] + \frac{1}{2}\left[M(f+f_c) + M(f-f_c)\right]$$

W.K.T:

$$P_s = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} |S_T(f)|^2\, df$$

$$= \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} \frac{A^2}{4}\left(\delta^2(f+f_c) + \delta^2(f-f_c) + \cancel{2\delta(f+f_c)\cdot\delta(f-f_c)}\right) + \frac{1}{4}\left(M^2(f+f_c) + M^2(f-f_c) + \cancel{2M(f-f_c)M(f+f_c)}\right)$$

$$+ \frac{A}{2}\left[M(f+f_c)\left(\delta(f+f_c) + \delta(f-f_c)\right) + M(f-f_c)\left(\delta(f-f_c) + \delta(f+f_c)\right)\right] df$$

$$= \left[\lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} \frac{A^2}{4}\left(\delta^2(f+f_c) + \delta^2(f-f_c)\right) df\right] + \left[\lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} \frac{1}{4}\left(M^2(f+f_c) + M^2(f-f_c)\right) df\right]$$

$$+ \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} \frac{A}{2}\left[M(0) + 0 + M(0) + 0\right] df$$

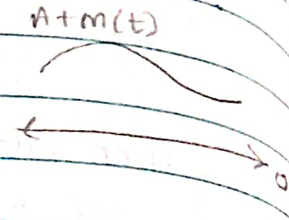
$$= \frac{A^2}{4}\cdot 2 + \frac{1}{4}\cdot 2\cdot P_m + \lim_{T\to\infty} \frac{1}{T}\cdot \frac{M(0)\cdot A\cdot 2\cdot T}{2}$$

$$= \frac{P_m}{2} + \frac{A^2}{2} + M(0)\cdot A$$

$$\boxed{\therefore P_s \approx \frac{P_m}{2} + \frac{A^2}{2}}$$

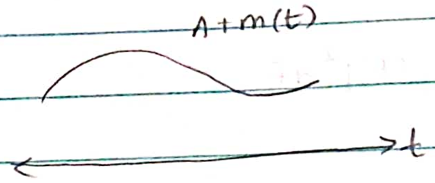
$[\because A^2 \gg A$ as A is very high$]$

Extra power due to additional term 'A' in A.M.

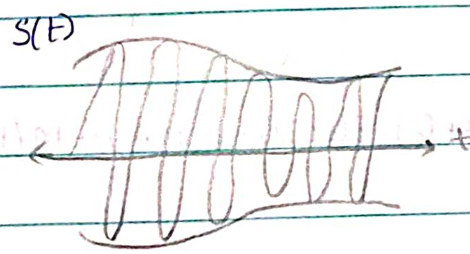


We got, $s(t) = (A+m(t))\cos(2\pi f_c t)$

If $A+m(t) > 0 \ \forall t$

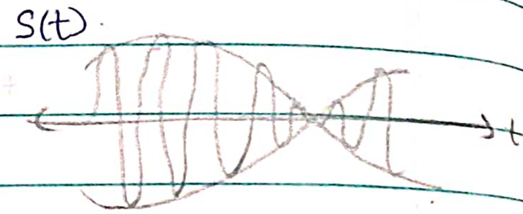


$\Downarrow$



If $A+m(t) \ngtr 0, \forall t$

$\Downarrow$



Cannot be recovered from envelope.

$\therefore$ We are interested only in $A+m(t) > 0$ case.

Let $m(t) = B\cos(2\pi f_c t)$.

Let 'm_p' be defined as peak amplitude of $m(t)$.

Then, we can define modulation index (μ) as follows

$$\boxed{\mu = \frac{B}{A}}$$

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as $\mu = \frac{B}{A} \Rightarrow B = \mu A$

W.K.T $m(t) = B\cos 2\pi f_m t$

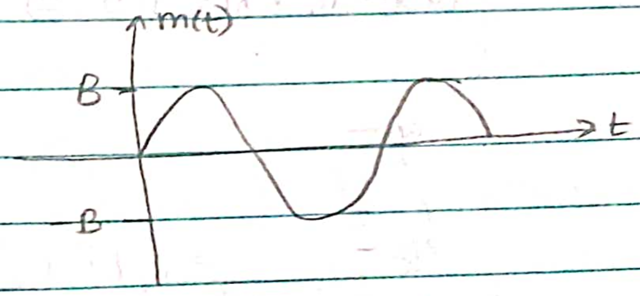
$= \mu A \cos 2\pi f_m t$

$S(t) = (A + B\cos 2\pi f_m t)\cos 2\pi f_c t$

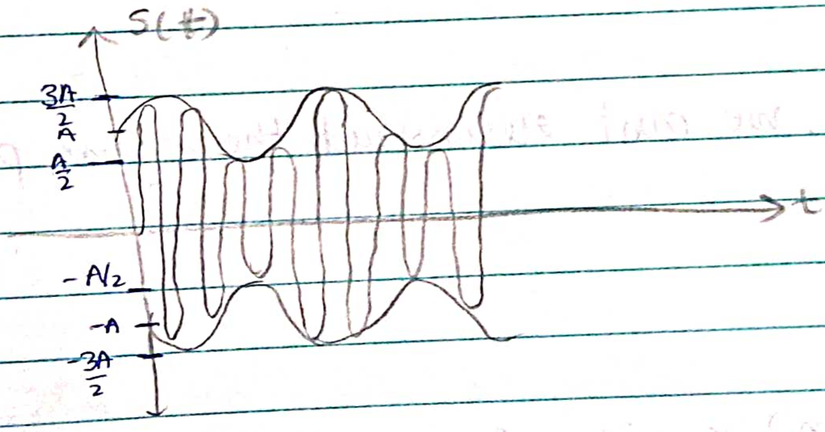
$= (A + \mu A \cos 2\pi f_m t)\cos 2\pi f_c t$

$s(t) = A(1 + \mu \cos 2\pi f_m t)\cdot \cos 2\pi f_c t \quad -(1)$

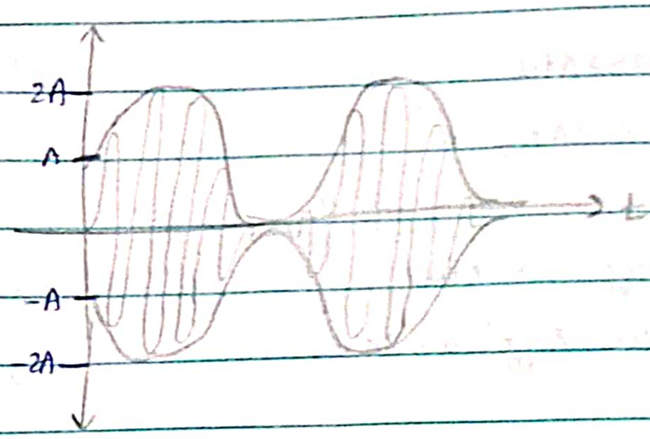
Consider $m(t)$ as :



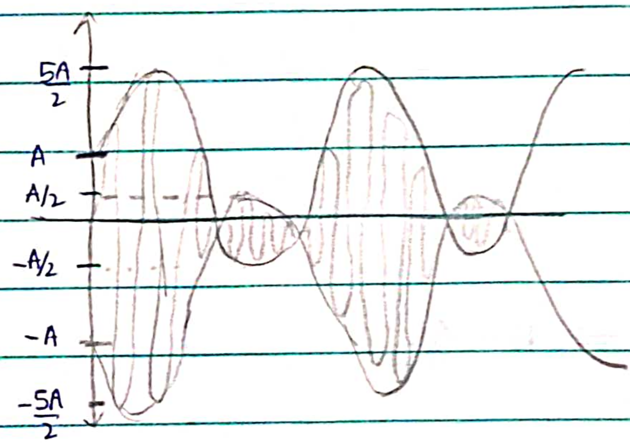
Case-I : $\mu = 0.5 \Rightarrow B = A/2$



Case-II : $\mu = 1 \Rightarrow A = B$



Case-III : $\mu = 1.5 \Rightarrow B = 3A/2$



$\therefore$ If $\mu > 1$, we cant reconstruct the signal perfectly

Efficiency:

Power Efficiency (η) is given by:

$$\eta = \frac{\text{desired power}}{\text{Total Power}} \text{ (or) } \frac{\text{useful power}}{\text{total power}}$$

$$\eta = \frac{P_m/2}{\frac{P_m}{2} + \frac{A^2}{2}}$$

$$\boxed{\eta = \frac{P_m}{P_m + A^2}}$$

Tone modulation (special case) :
If message signal is a sinusoid then, this type of modulation is called toned modulation.

i.e., $m(t) = B\cos 2\pi f_m t = \mu A \cos 2\pi f_m t$

$$P_m = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} |M_T(f)|^2 \, df \quad , \quad M_T(f) = \frac{\mu A}{2}\left(\delta(f+f_m) + \delta(f-f_m)\right)$$

$$P_m = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} \frac{\mu^2 A^2}{4} \left(\delta^2(f+f_m) + \delta^2(f-f_m) + \cancel{2\delta(f+f_m)\cdot\delta(f-f_m)}\right) df$$

$$= \frac{\mu^2 A^2}{4} \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} \delta^2(f+f_m) + \delta^2(f-f_m) \, df$$

$$= \frac{\mu^2 A^2}{4}(1+1) = \frac{\mu^2 A^2 \cdot \cancel{2}}{\cancel{4}_{\,2}}$$

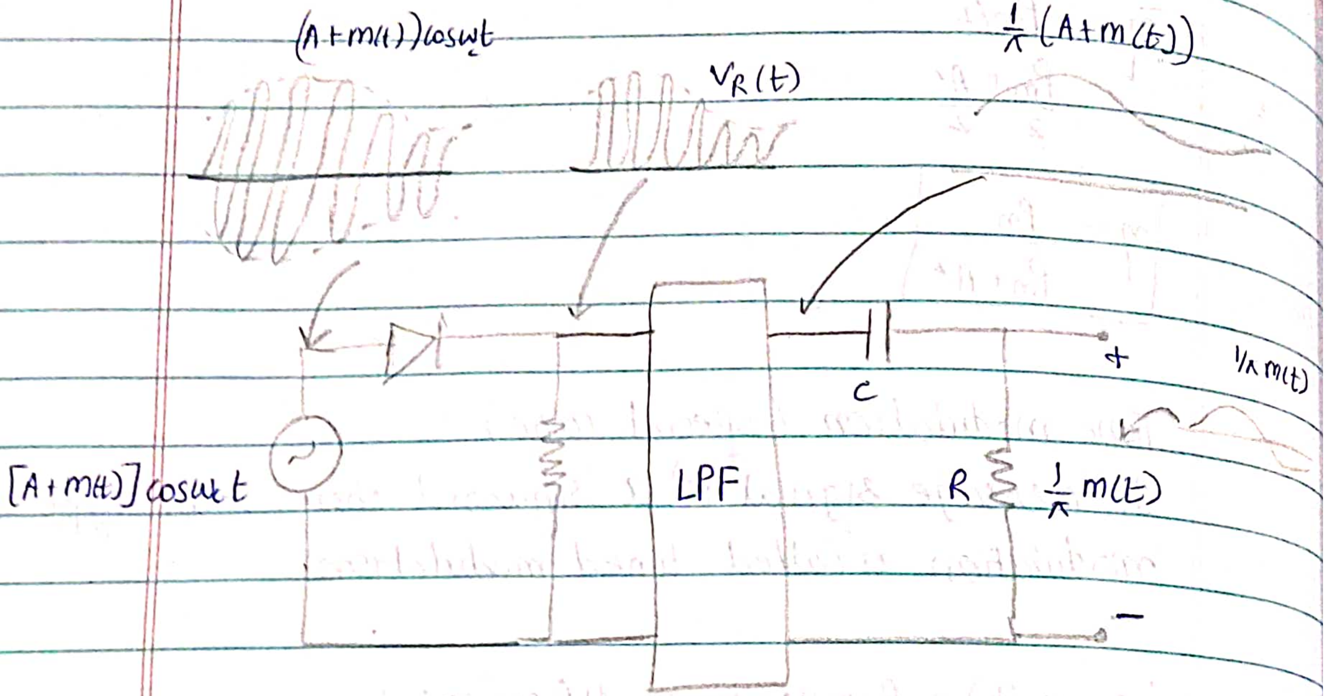
power efficiency for tone modulation

$$\therefore P_m = \frac{\mu^2 A^2}{2} \Rightarrow \eta = \frac{\mu^2 A^2/2}{\mu^2 A^2/2 + A^2} \Rightarrow \boxed{\eta = \frac{\mu^2}{\mu^2 + 2}}$$

for perfect reconstruction, $0 \le \mu \le 1$, i.e., $\mu_{max} = 1$

$$\boxed{\therefore \eta_{max} = \frac{1}{3}}$$

Rectifier Demodulator:



→ The diode-resistor combination works as half-wave rectifier. Mathematically, we can analyze as follows:

$$V_R(t) = s(t)\cdot w(t) \quad , \quad w(t) \Rightarrow$$

$$V_R(t) = \left[(A+m(t))\cos(2\pi f_c t)\right]\left[\frac{1}{2}+\frac{2}{\pi}\left(\cos 2\pi f_c t - \frac{1}{3}\cos 6\pi f_c t + \cdots\right)\right]$$

$$= \frac{(A+m(t))}{\pi} + \text{other terms of higher frequencies}$$

→ Pass $V_R(t)$ to LPF so that $\left(\frac{A+m(t)}{\pi}\right)$ is obtained. Later, the newly obtained signal is passed through capacitor-resistor network, so that the DC component is removed. Finally '$\frac{m(t)}{\pi}$' is obtained.

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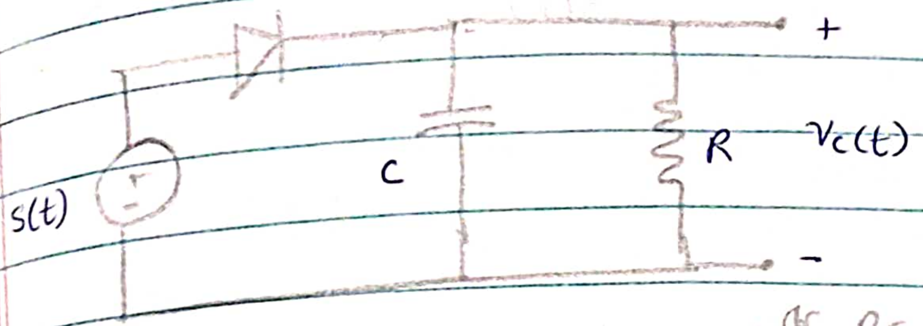
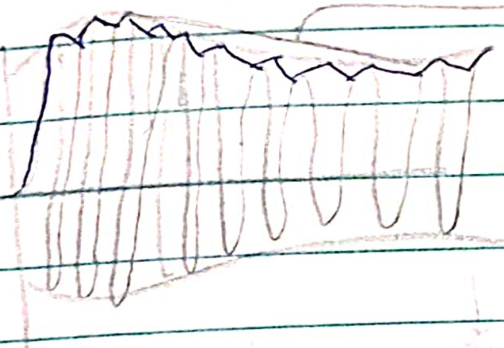
Envelop demodulation:





→ +ve half cycle of s(t) ⟹ diode is on, capacitor charges.

→ -ve half cycle of s(t) ⇒ s(t) will be less than $v_c(t)$ hence diode is turned off. Then $v_c(t)$ gets discharged in the R-C network with time constant 'RC'.

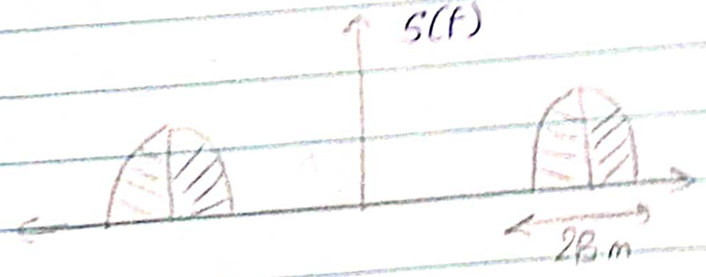
→ As soon as $v_c(t)$ is less than s(t), diode turns on and capacitor gets charged and the process continues.

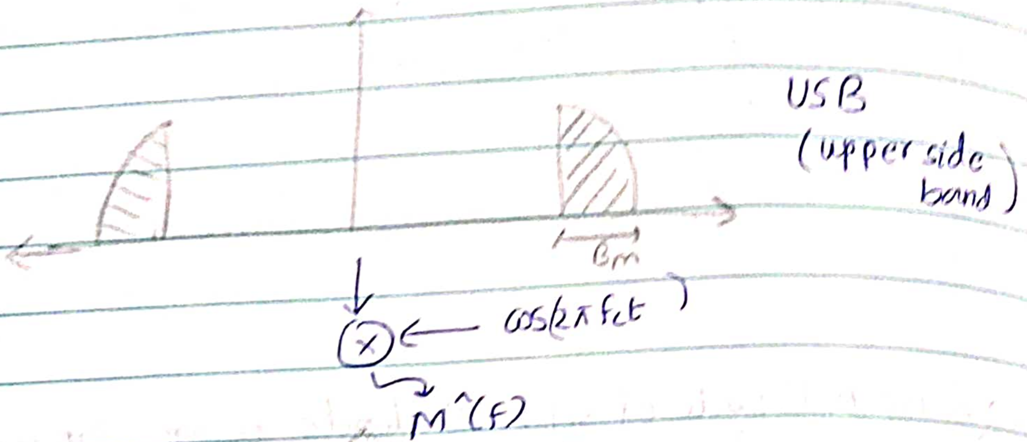
→ Capacitor discharge between positive peaks cause ripples which can be reduced by choosing 'RC' value high but less than $1/2\pi B$.

$$\boxed{\therefore \frac{1}{\omega_c} \ll R_c < \frac{1}{2\pi B}}$$

As $v_c(t) = A + m(t)$, 'A' can be removed by simple RC - HPF.

Single Sideband (SSB) modulation: (LSB/USB)





→ SSB requires only half the bandwidth of DSB signal to recover the message signal.

Hilbert Transform:

$$x_h(t) = H(x(t)) = x(t) * \frac{1}{\pi t}$$

$$= \frac{1}{\pi}\, p.v. \int_{-\infty}^{\infty} \frac{x(\tau)}{t-\tau}\, d\tau$$

[Principle value] → [As the integral is N.D. at $t = \tau$, we take other values of t] let $H(t) = \dfrac{1}{\pi t}$, then

$$x(t) \longrightarrow \boxed{\begin{matrix} H.T. \\ H(t) \end{matrix}} \longrightarrow x_h(t)$$

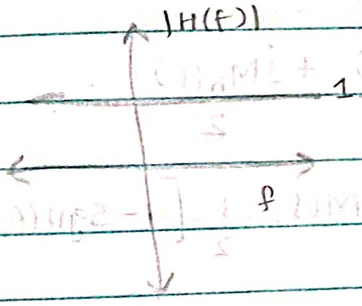
W.K.T,

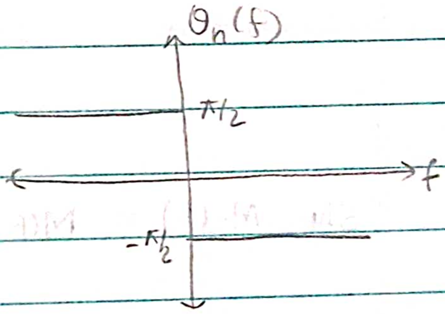
$$x_h(t) = x(t) * H(t)$$

$$X_h(f) = X(f) \cdot F.T\big(H(t)\big)$$

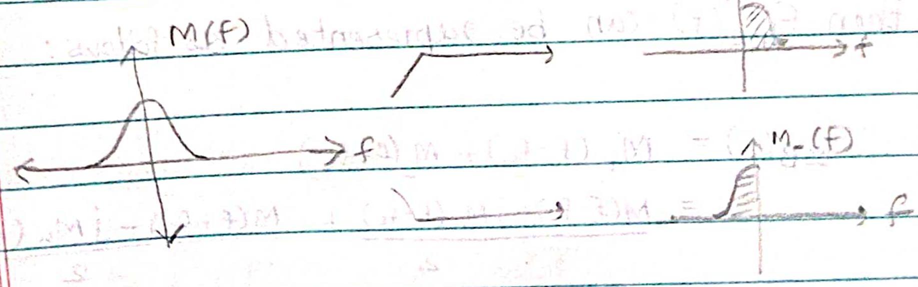
$$X_h(f) = -jX(f)\,sgn(f) \qquad [\because F.T(H(t)) = -j\,sgn(f)]$$

$$H(f) = \begin{cases} e^{-j\pi/2}, & f>0 \\ e^{j\pi/2}, & f<0 \\ 0, & f=0 \end{cases} = -j\,sgn(f) \Rightarrow jH(f) = sgn(f)$$

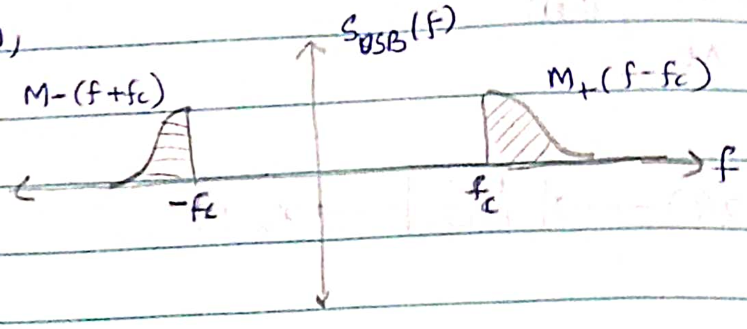


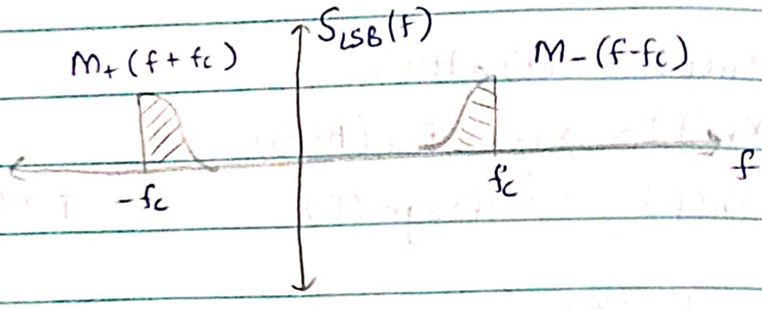


W.K.T.



then,





$M_+(f)$ and $M_-(f)$ is given by:

$$M_+(f) = M(f)\cdot U(f) = M(f)\cdot\frac{1}{2}\left[1+\operatorname{sgn}(f)\right]$$

$$= \frac{M(f)}{2} + \frac{jM_h(f)}{2}$$

sly, $$M_-(f) = M(f)\cdot U(-f) = M(f)\cdot\frac{1}{2}\left[1-\operatorname{sgn}(f)\right]$$

$$= \frac{M(f) - jM_h(f)}{2}$$

then $S_{USB}(f)$ can be represented as follows:

$$S_{USB}(f) = M_+(f-f_c) + M_-(f+f_c)$$

$$= \frac{M(f-f_c) + jM_h(f-f_c)}{2} + \frac{M(f+f_c) - jM_h(f+f_c)}{2}$$

$$= \frac{1}{2}\left(M(f+f_c) + M(f-f_c)\right) - \frac{1}{2j}\left(M_h(f-f_c) - M_h(f+f_c)\right)$$

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on taking inverse fourier Transform, we get:

$$s_{SSB}(t) = m(t)\cos(2\pi f_c t) \mp m_h(t)\sin(2\pi f_c t) \quad \left[\begin{matrix} - \Rightarrow USB \\ + \Rightarrow LSB \end{matrix}\right]$$

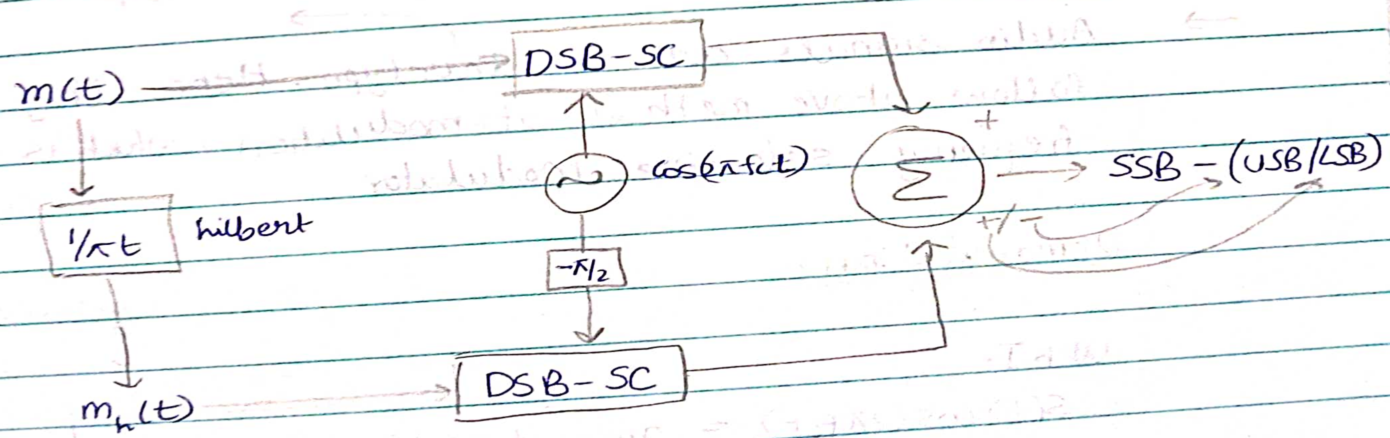
Demodulation:

$$s_{SSB}(t)\cdot\cos(\omega_c t) = [m(t)\cos(2\pi f_c t) \mp m_h(t)\sin(2\pi f_c t)]\cos(2\pi f_c t)$$

$$= \frac{m(t)}{2} + \underbrace{\frac{m(t)\cos(4\pi f_c t)}{2} \mp \frac{m_h(t)\sin(4\pi f_c t)}{2}}_{\text{SSB with carrier } 2f_c}$$

using LPF, we get $m(t)$.

We can generalise above process in a block diagram as follows:



(phase shift modulation)

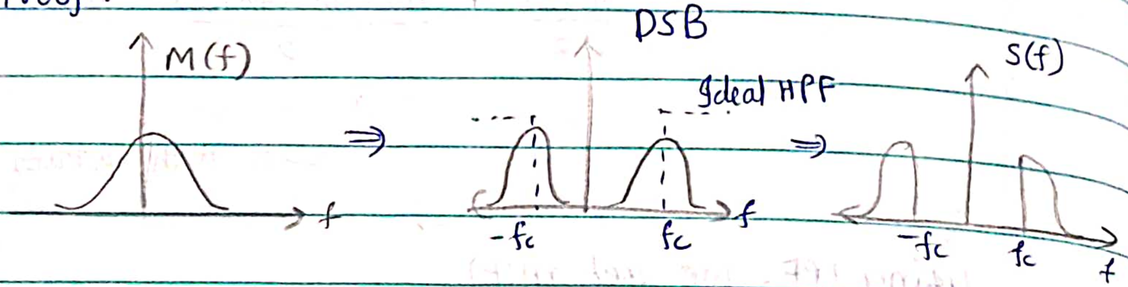
$B_s = B_m$, $P_s = \frac{P_m}{4}$, but high modulator design complexity

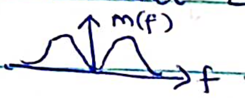
→ Above modulator design can be done in alternative way :

$$m(t) \longrightarrow \otimes \longrightarrow \boxed{\text{H.P.F}\ (f_c)} \longrightarrow s(t)$$

$$\uparrow \cos(2\pi f_c t)$$

Proof :



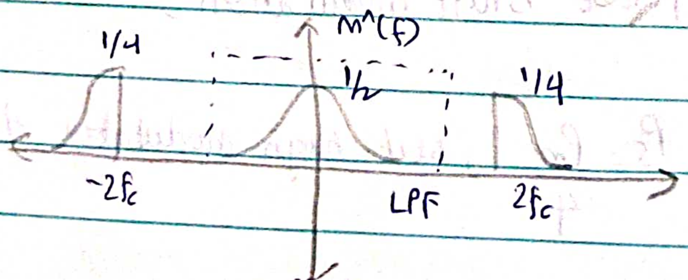
→ Only suitable if the amplitude around zero frequency is too small. Ex : Try for 

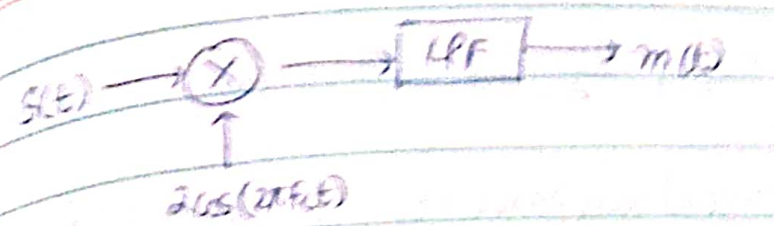
→ Audio signals are of this type. Hence they can follow above method of modulation, which is called frequency selective modulator.

Demodulation :

W.K.T.

$$s(t)\cos(2\pi f_c t) = \frac{m(t)}{2} + \overbrace{\frac{m(t)\cos(4\pi f_c t)}{2} \mp \frac{\hat{m}(t)\sin(4\pi f_c t)}{2}}^{0 \text{ by LPF}}$$





[Coherent demodulation]

Demodulation using envelop detection :

Consider,

$$s(t) = [m(t)\cos(2\pi f_c t) \mp m_h(t)\sin(2\pi f_c t)] + A\cos(2\pi f_c t)$$
$$= [(A + m(t))\cos(2\pi f_c t) \mp m_h(t)\sin(2\pi f_c t)]$$

[∵ SSB with a carrier]

* If a function is represented in the form :

$$y = a\cos x + b\sin x$$

then, $$y = \left[\left(\frac{a}{\sqrt{a^2+b^2}}\right)\cos x + \left(\frac{b}{\sqrt{a^2+b^2}}\right)\sin x\right]\sqrt{a^2+b^2}$$

$$= \sqrt{a^2+b^2}\cdot\cos(x-\alpha)$$
$$= R\cos(x-\alpha)$$

where, $R = \sqrt{a^2+b^2} \longrightarrow$ envelope of 'y'.

envelope of $s(t) \Rightarrow$

$$s(t) = \sqrt{(A+m(t))^2 + m_h(t)^2}\quad\cos(2\pi f_c t + \theta)$$

$$R = [(A+m(t))^2 + m_h(t)^2]^{1/2} = [A^2 + m^2(t) + 2Am(t) + m_h^2(t)]^{1/2}$$

$$= A\left[\left(\frac{m(t)}{A}\right)^2 + \left(\frac{m_h(t)}{A}\right)^2 + \frac{2m(t)}{A}\right]^{1/2} \approx A\left[1 + \frac{2m(t)}{A}\right]^{1/2}$$

$+1$

[∵ A is too large]

$$\therefore R \approx A\left[1 + \frac{1}{2}\left(\frac{2m(t)}{A}\right)\right] \approx A + m(t)$$

$$s(t) \approx [A + m(t)]\cos(2\pi f_c t + \theta) \quad \left[\because \text{ if } A \gg [m(t)]_{max}\right]$$

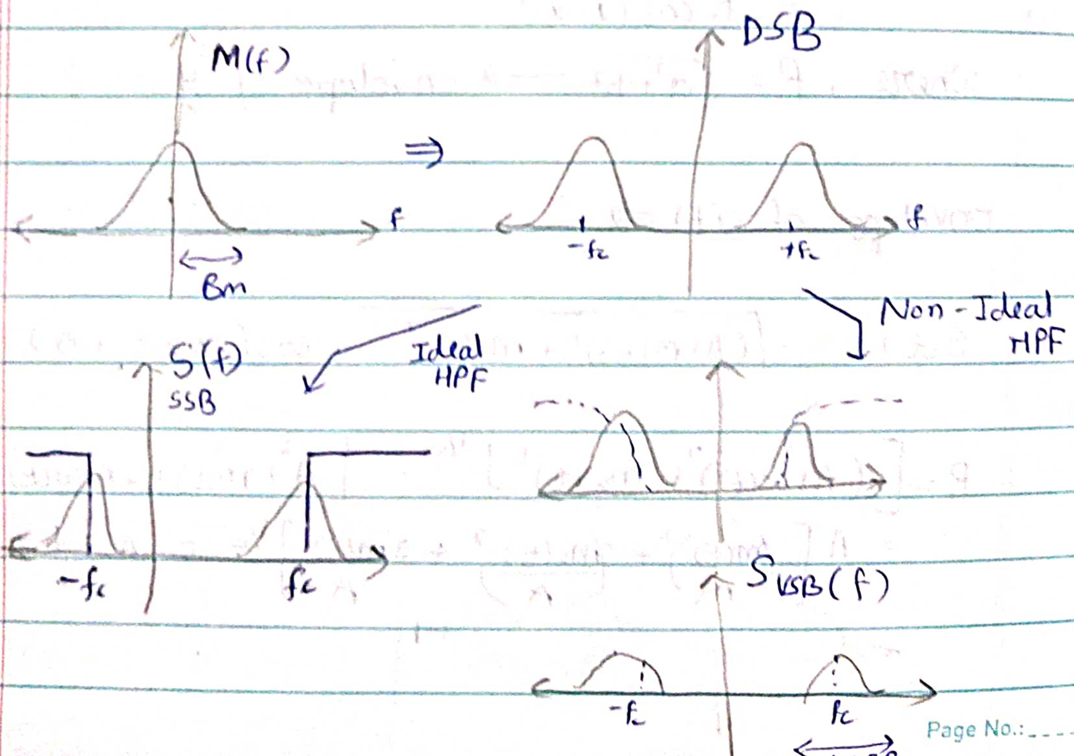
→ Drawbacks of above method: Highly power inefficient method as you are adding an additional carrier whose amplitude is high.

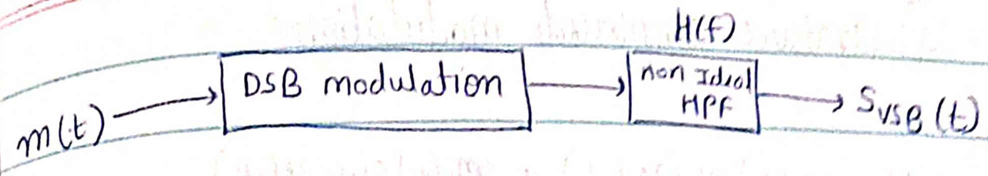
DSB (vs) SSB:

DSB → simpler structure, higher bandwidth
SSB → Higher complexity, Lower bandwidth

Vestigial Sideband Modulation:

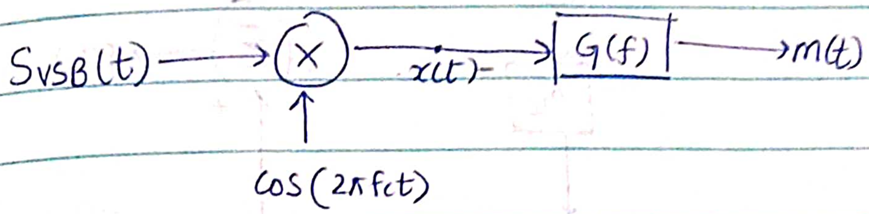
Consider:





$$S_{VSB}(f) = \left(\frac{M(f+f_c) + M(f-f_c)}{2}\right) H(f)$$

Consider demodulation (coherent):



$$X(f) = \frac{S_{VSB}(f+f_c) + S_{VSB}(f-f_c)}{2}$$

$$= \frac{1}{4}\left[\left(M(f+2f_c) + M(f)\right) H(f+f_c) + \left(M(f) + M(f-2f_c)\right) H(f-f_c)\right]$$

$$= \frac{M(f)}{4}\left[H(f+f_c) + H(f-f_c)\right] + \frac{M(f+2f_c) \cdot H(f+f_c)}{4}$$

$$+ \frac{M(f-2f_c) \cdot H(f-f_c)}{4}$$

We prefer $G(f) \approx \dfrac{1}{H(f+f_c) + H(f-f_c)}$ such that

$$X(f) \approx \frac{1}{4}\left[M(f)\left[H(f+f_c) + H(f-f_c)\right]\right]$$

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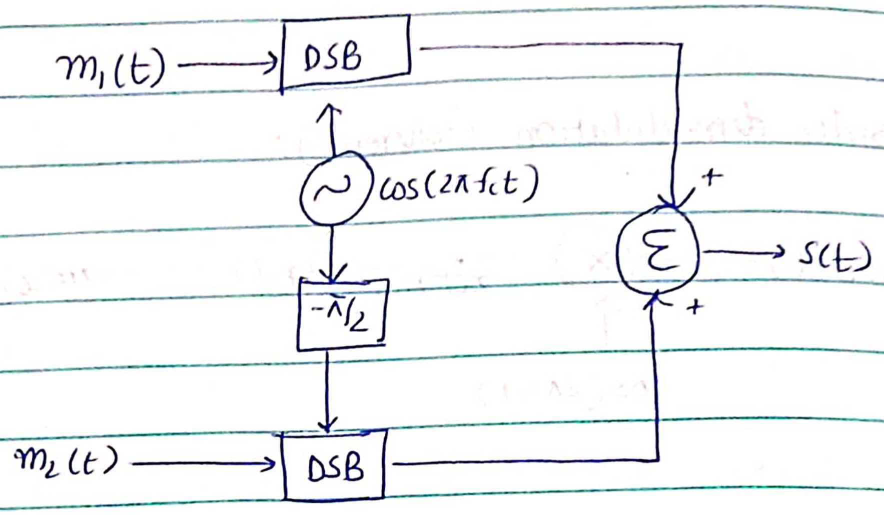
Quadrature amplitude modulation:

$$s(t) = \underset{(2B_m)}{m_1(t)\cos(2\pi f_c t)} + \underset{(2B_m)}{m_2(t)\sin(2\pi f_c t)}$$



Recap:

Type	P_s	B_s	Complexity
DSB-SC	$\frac{P_m}{2}$	$2B_m$	Carrier recovery required
AM (envelop det) (DSB-FC)	$\frac{P_m}{2} + \frac{A^2}{2}$	$2B_m$	Reduced complexity via envelop detection.
SSB-SC	$\frac{P_m}{4}$	B_m	Modulation requires Hilbert's transform which increases complexity.
VSB	$\frac{P_m}{2} > P_s > \frac{P_m}{4}$	$2B_m > B_s > B_m$	Similar to DSB-SC. A bit easier.

Frequency Modulation:

Frequency is function of $m(t)$

$\therefore f_i(t) = f_c + K_f\, m(t)$

W.K.T. $-m_p < m(t) < m_p, \forall t$

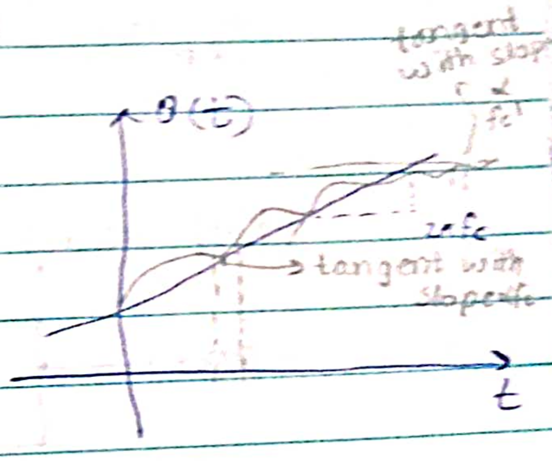
$f_c - K_f m_p < f_i(t) < f_c + K_f m_p$

then Band width $B = 2K_f m_p$

Consider, $A\cos(2\pi f_c t + \theta_0)$

let $\theta(t) = 2\pi f_c t + \theta_0$

$\therefore f_i(t) = \left(\dfrac{d\theta(t)}{dt}\right)\dfrac{1}{2\pi}$



also,

$$\theta(t) = 2\pi \int_{-\infty}^{t} f_i(\tau)\, d\tau$$

→ Instantaneous frequency is the derivative of instantaneous angle.

Phase modulation:

let, $\theta(t) = 2\pi f_c t + \theta_0 + 2\pi k_p m(t)$ $\quad [\because \theta_0 = 0]$

$S_{pm}(t) = A\cos(2\pi f_c t + 2\pi k_p m(t))$

$f_i(t) = \dfrac{d\theta(t)}{dt} \cdot \dfrac{1}{2\pi}$

$\therefore f_i(t) = f_c + k_p \left(\dfrac{dm(t)}{dt}\right)$

How to determine the type of modulation based on angle of the signal $f_i(t)$? :-

$\Rightarrow$ If ~~$\theta(t)$~~ $\propto$

If $f_i(t) \propto m(t)$ [FM signal]

If $f_i(t) \propto \frac{d}{dt}(m(t))$ [PM signal]

$$S_{FM}(t) = A\cos\left(2\pi f_c t + K_f \int_{-\infty}^{t} m(\tau)\, d\tau\right)$$

PM using FM :

$$m(t) \longrightarrow \boxed{\frac{d}{dt}} \xrightarrow{m'(t)} \boxed{FM} \longrightarrow S_{PM}(t)$$

Similarly,

FM using PM :

$$m(t) \longrightarrow \boxed{\int_{-\infty}^{t} dt} \xrightarrow{\int_{-\infty}^{t} m(t)} \boxed{PM} \longrightarrow S_{FM}(t)$$

Consider ;

$$m(t) \longrightarrow \boxed{\begin{matrix} L.T.I. \\ h(t) \end{matrix}} \longrightarrow \theta(t) = m(t) \circledast h(t)$$

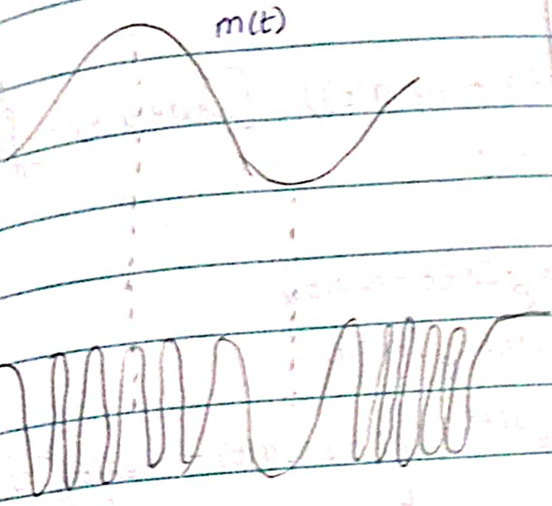
for FM : $h(t) = K_f u(t) \Rightarrow \theta(t) = K_f \int_{-\infty}^{t} m(\tau)d\tau$

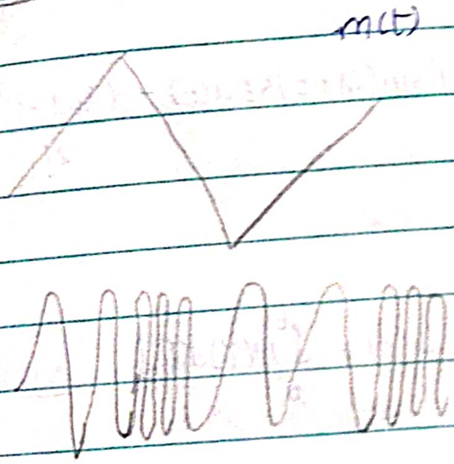
PM : $h(t) = K_p \delta(t) \Rightarrow \theta(t) = K_p \delta(t)$

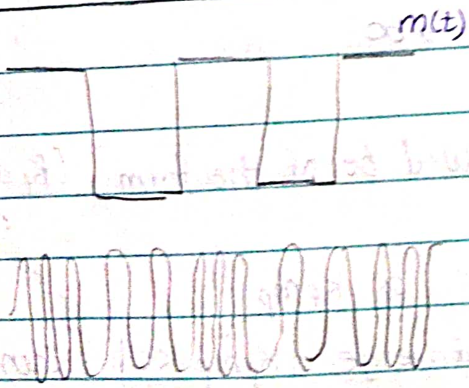
$\hookrightarrow$ special cases of angle modulation

FM signal:

$f_i(t) = f_c + \frac{K_f}{2\pi} m(t)$

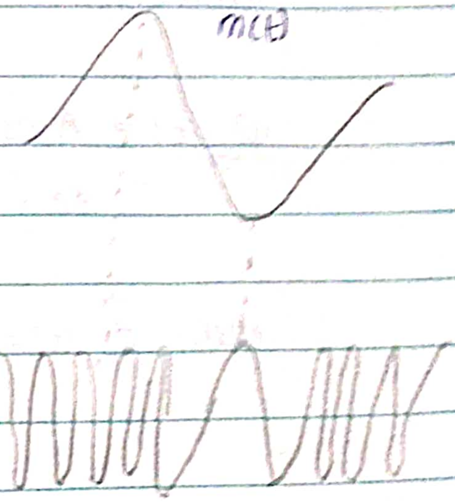


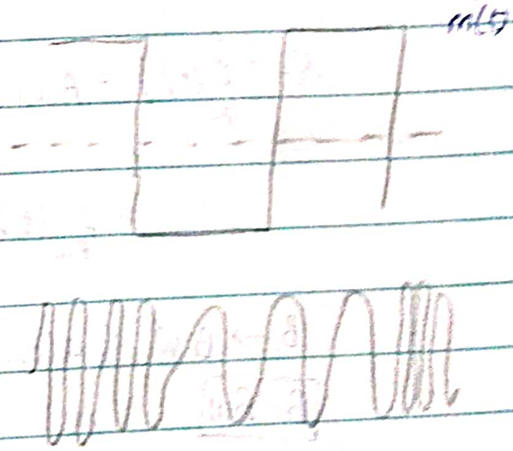




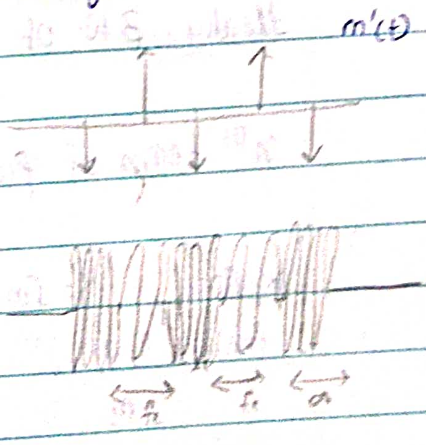
PM signal:

$f_i(t) = f_c + K_p m'(t)$





Only two frequency component



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$P_{PM} = P_{FM} = \frac{A^2}{2}$ irrespective of 'θ'.

Consider,

$\varphi_{FM}(t) = A\cos(2\pi f_c t + k_f\, a(t))$ [where $a(t) = \int_{-\infty}^{t} m(\tau)\,d\tau$]

$= Re\{\hat{\phi}_{FM}(t)\}$

then, $\hat{\phi}_{FM}(t) = A e^{j(2\pi f_c t + k_f a(t))}$

$= A e^{j2\pi f_c t} \cdot e^{jk_f a(t)}$

$= A e^{j2\pi f_c t}\left[1 + jk_f a(t) - \frac{(k_f a(t))^2}{2!} - j\frac{(k_f(a(t))^3}{3!}\right]$

$Re\{\hat{\phi}_{FM}(t)\} = A\cos(2\pi f_c t) - A\sin(2\pi f_c t)\underbrace{k_f a(t)}_{2B} - \underbrace{\frac{(k_f a(t))^2}{2!}}_{4B} A\cos 2\pi f_c t + \underbrace{\ldots\ldots}_{2nB}$

$= \varphi_{FM}(t)$

$B \rightarrow$ B.W. of $a(t)$. [$\because a(t) = \int_{-\infty}^{t} m(\tau)\,d\tau$]

$\boxed{B = B_m}$

Ideally, B.W. of $\varphi(t) \rightarrow \infty$

n^{th} term of $\varphi(t)$ will be of the form $\frac{(k_f a(t))^n}{n!}$

$k_f a(t) = k_f \int m(t)\,dt \;\; \alpha \;\; k_f m_p$

for $\frac{(k_f a(t))^n}{n!}$ to converge $|k_f m_p| << 1$, then

(Narrow band FM) $\Longleftrightarrow$ NBFM

$\varphi_{FM}(t) \cong A\cos 2\pi f_c t - Ak_f\, a(t) \sin 2\pi f_c t$ $\quad \boxed{\therefore B_F = 2B_m}$

A.M. signal with 90° phase diff btw the carrier

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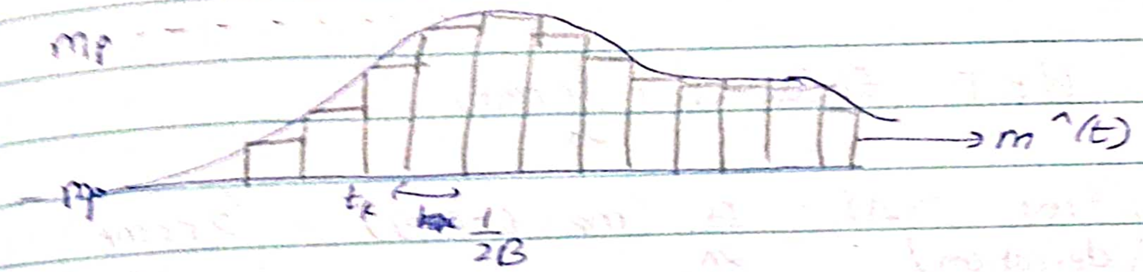
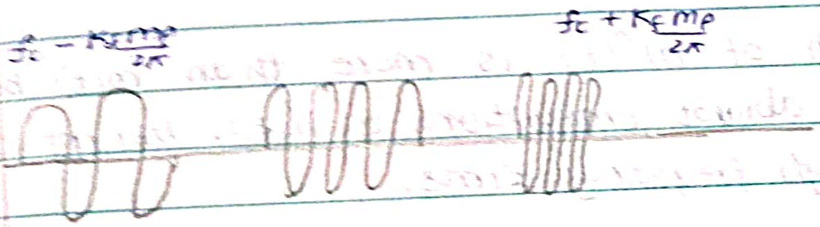
WBFM (wide band FM):

consider m(t):



$$f_i(t_k) = f_c + \frac{k_f}{2\pi} m^{\wedge}(t_k)$$

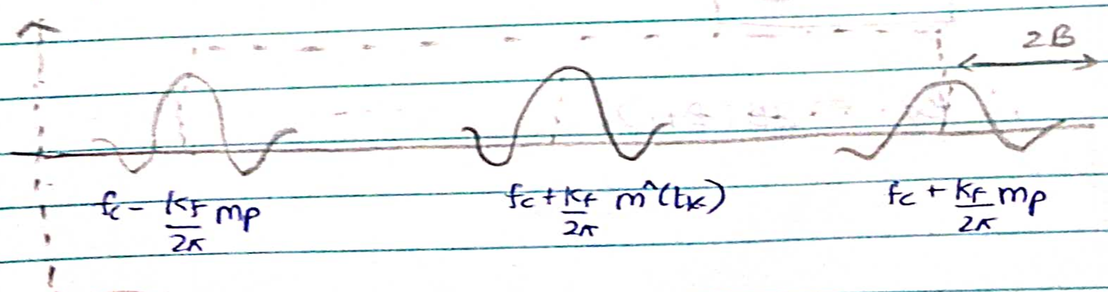
$$\Pi(t/\tau) = \begin{cases} 1 & ; |t| < \tau \\ 0 & ; \text{else} \end{cases}$$
$$\updownarrow \text{F.T.}$$
$$\tau \,\text{sinc}(f\tau/2)$$



each signal component can be written as:

$$\Pi(t \cdot 2B) \cos(2\pi f_c t + k_f m^{\wedge}(t_k)) \quad \forall\ t_k$$
$$\updownarrow \text{F.T.}$$
$$\frac{1}{2}\ \text{sinc}\left(\frac{2\pi f \pm (2\pi f_c + k_f m^{\wedge}(t_k))}{4B}\right)$$

The spectrum for $\hat{m}(t)$ will be:



$\therefore$ B.W of $\hat{m}(t)$ is given by :

$$\boxed{B_{\hat{\phi}} = \frac{2k_f m_p}{2\pi} + 4B}$$

WKT: $f_i(t) = f_c + \frac{k_f}{2\pi} m(t)$

$$\left(\begin{matrix}\text{freq} \\ \text{deviation}\end{matrix}\right) \Delta f = \frac{\frac{k_f}{2\pi}\left(m_p - (-m_p)\right)}{2} = \left(\frac{2k_f m_p}{2\pi}\right)\frac{1}{2}$$

$$\boxed{\therefore B_{\hat{\phi}} = 2\Delta f + 4B}$$

Note: Bandwidth of $\hat{m}(t)$ is more than $m(t)$ because there are abrupt transitions in $\hat{m}(t)$. Abrupt changes means high frequency terms.

Case I : ~~Δf~~ $\Delta f \approx 0 \Rightarrow B_{\hat{\phi}} = 4B$

$\Rightarrow |k_f m_p| \ll 1 \Rightarrow$ NBFM $\Rightarrow$ $B.W_{NBFM} = 2B$

Hence we approximate $\boxed{B_{\phi} \approx 2\Delta f + 2B}$

~~Case II~~ : also, $B_{\phi} \in [2\Delta f, 2\Delta f + 4B]$

Let $\boxed{\beta = \frac{\Delta f}{B}}$ $\Rightarrow \beta \to$ modulation.

$\therefore B_{\phi} = 2B(\beta + 1)$

$\Pi\left(\frac{t}{\tau}\right)$ $\tau \operatorname{sinc}(f\tau/2)$