Communication Theory

Spring-2023

Assignment 2 | Deadline: 27^{th} Mar, 11:55pm

Instructions:

- All questions are compulsory.
- Clearly state the assumptions (*if any*) made that are not specified in the questions.
- The simulation code and output must be clearly explained.
- Submission Format : Rollnumber.zip .The zip file should contain 2 folders.
  1st Folder : should contain all MATLAB files .
  Folder name : **Code**
  2nd folder : should contain Reports for Theoretical and MATLAB questions .
  Folder name : **Report**
- For simulation part, submit a report (pdf format) clearly depicting the generated plots (if any) with answers to questions asked as part of simulation exercise. State the parameter values used for simulation in the report clearly. Marks obtained will depend upon clarity in report writing.

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1. Show that any periodic signal with period T_p can be used as a carrier to generate a DSB-SC signal. A DSB-SC signal is generated by multiplying the message signal $m(t)$ with the periodic rectangular waveform shown Figure 1 and filtering the product with a bandpass filter tuned to the reciprocal of the period T_p, with bandwidth $2W$, where W is the bandwidth of the message signal. Demonstrate that the output $u(t)$ of the BPF is the desired DSB-SC AM signal.

$$u(t) = m(t)\sin(2\pi f_c t) \tag{1}$$

Hint: Assume that s(t) is a periodic signal with period T_p, i.e.$\sum_n x(t - nTp)$. Then $v(t) = m(t)s(t)$.

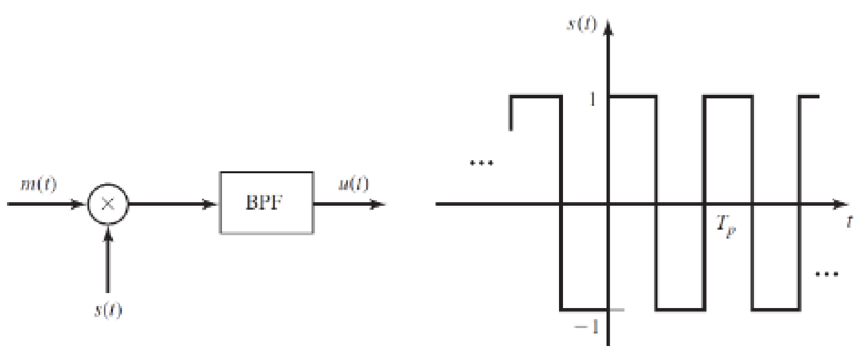

Figure 1: Rectangular pulse

2. Figure 2 shows the phase deviation of a bandpass FM signal modulated by a sinusoidal message.

1. Find the modulation index (assume that it is an integer multiple of π for your estimate).
2. Find the message bandwidth.
3. Estimate the bandwidth of the FM signal using Carson's formula.

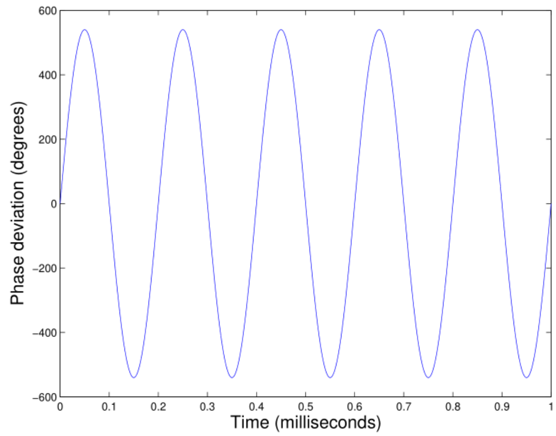

Figure 2: Phase deviation of FM signal

3. The input $m(t)$ to an FM modulator with $k_f = 1$ has Fourier transform

$$M(f) = \begin{cases} j2\pi f & |f| < 1 \\ 0 & \text{else} \end{cases}$$

The output of the FM modulator is given by

$$u(t) = A\cos\left(2\pi f_c t + \phi(t)\right)$$

where f_c is the carrier frequency.

(a) Find an explicit time domain expression for $\phi(t)$ and carefully sketch $\phi(t)$ as a function of time.

(b) Find the magnitude of the instantaneous frequency deviation from the carrier at time $t = \frac{1}{4}$.

(c) Using the result from (b) as an approximation for the maximum frequency deviation, estimate the bandwidth of $u(t)$

4. Let $p(t) = I_{\left[-\frac{1}{2},\frac{1}{2}\right]}(t)$ denote a rectangular pulse of unit duration. Construct the signal

$$m(t) = \sum_{n=-\infty}^{\infty} (-1)^n p(t-n)$$

The signal $m(t)$ is input to an FM modulator, whose output is given by

$$u(t) = 20\cos\left(2\pi f_c t + \phi(t)\right)$$

Consider Phase deviation of FM signal from Figure 2 where

$$\phi(t) = 20\pi \int_{-\infty}^{t} m(\tau)d\tau + a$$

and a is chosen such that $\phi(0) = 0$.

(a) Carefully sketch both $m(t)$ and $\phi(t)$ as a function of time.

(b) Approximating the bandwidth of $m(t)$ as $W \approx 2$, estimate the bandwidth of $u(t)$ using Carson's formula.

(c) Suppose that a very narrow ideal BPF (with bandwidth less than 0.1) is placed at $f_c + \alpha$. For which of the following choices of α (if any), you will get a nonzero component at the output of the BPF: (i) $\alpha = 0.5$, (ii) $\alpha = 0.75$, (iii) $\alpha = 1$.

5. Answer the following questions.

   (a) Consider a modulating signal $m(t)$ to be a periodic sawtooth signal as shown in Figure 4. Sketch $\psi_{FM}(t)$ and $\psi_{PM}(t)$ for signal $m(t)$ if $w_c = 2\pi 10^6$, $K_f = 2000\pi$ and $K_p = \pi/2$. Explain why it is necessary to use $k_p < \pi$ in this case.
   (b) Estimate the bandwidth of $\psi_{FM}(t)$ and $\psi_{PM}(t)$. Assume the bandwidth of modulating signal till the fifth harmonic frequency of $m(t)$.

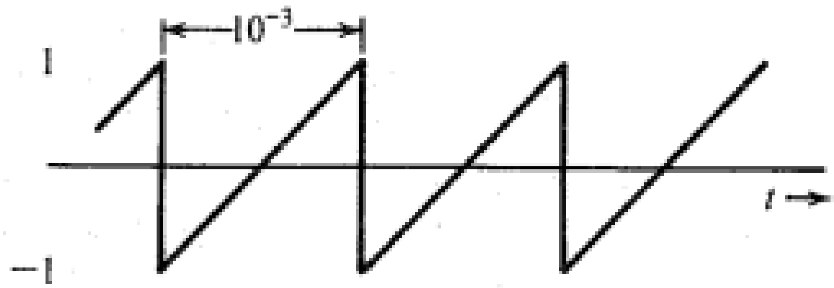

Figure 3: Sawtooth signal

MATLAB Simulation

6. Provide a MATLAB code for the following questions

   (a) Generate a sinusoidal message signal of frequency 1KHz having peak amplitude as 1 and a sinusoidal carrier signal of frequency 10KHz having peak amplitude of 10. Choose $K_f = 1$. Plot the frequency modulated waveform along with the message and carrier signals using subplots. Do not use the inbuilt function for perform the modulation.

   (b) Demodulate the FM wave using the crude discriminator based on differentiation and envelope detection. Plot the rectified FM and the recovered signal in a single figure.

   (c) Plot the spectrum of the message signal, modulated FM signal, rectified FM signal and the recovered message signal on a single figure.

   (d) Repeat (a), (b) and (c) if the message signal is as shown in Figure 3.

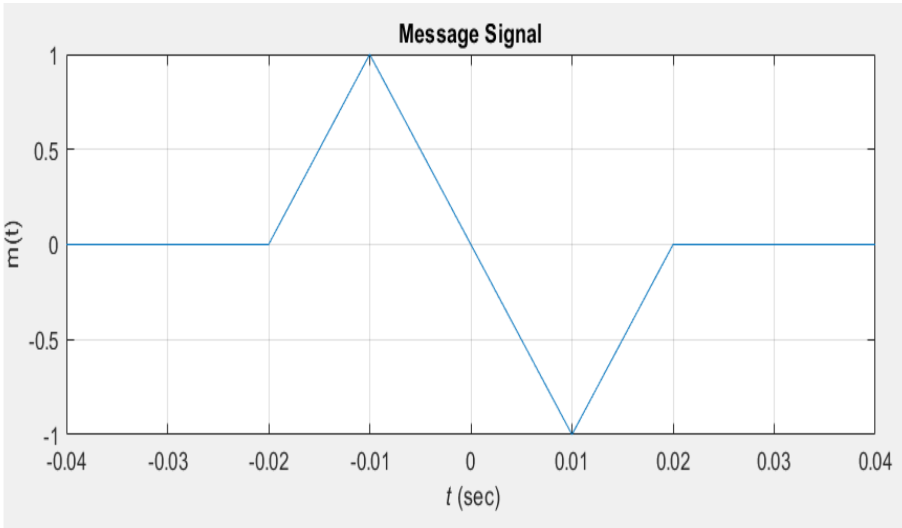

Figure 4: Message signal