Science 1 - Assignment 1

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(1) Let's consider the scenerio described in the question, where frame S_0' is moving with speed v along the positive X axis of frame S.

In frame S, let an even occur at point $P(x,y,z)$ and in frame S_0', let the same event occur at point $P_0(x',y',z',t')$. The coordinates are measured at the same time in both frames, $t = t_0 = 0$.

We take the transformation relating two events.

$x' = Ax + Bt$, A, B are some constants.

$y' = y$

$z' = z$

$t' = Cx + Dt$ C, D - Constants.

Without voialating Einsting 1st postulates, we c which describes in all inertial frame laws of physics have same form, we can take linear transformation.

Consider $S \equiv (x=0, t)$ and $S' \equiv (x' = -vt, t')$
From earlier equation, on comparing coefficients,

$$-vt' = Bt \quad , \quad t' = Dt$$

$$-vD\cancel{t} = B\cancel{t}$$

$$\boxed{B = -vD}$$

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Same we can do from s' frame,

then $S \equiv (x = vt, t)$ and $s' \equiv (x' = 0, t')$

From earlier equations, $(Av + B)t = 0$ — ①

$Cv + D = t'$ — ⑪.

gives $Av + B = 0$ and $D = A$.

Now acc to question let, say there is an isotropic light pulse is emmitted from origin $t = t' = 0$.

The pulse is observed along x axis.

let $S \equiv (x = ct, t)$ and $s' \equiv (x' = ct', t')$ wher c is speed of light.

Substituting the x cordinates in transformation equation

$$\boxed{ct' = Act + Bt}$$

$$\Rightarrow \frac{t'}{t} = \frac{Ac + B}{c} \quad \text{— } (I)$$

and $t' = (cC + D)t$.

$$\Rightarrow \boxed{\frac{t'}{t} = cC + D} \quad \text{— } (4)$$

c — speed of light
C — coefficient.

using the above ② equation. $ct' = Act + Bt$

$= Act - vDt$

$= Act - vAt$ — (IV)

and $ct' = c^2 tC + ctA$ — ②

Replacing v with c and equative above ②. $C = \left(\frac{-v}{c^2}\right)A$ — ②.

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Find

Now let the light pulse observed along the y axis.
$S(x=0, y = ct, t)$ and $S' = (x = -vt, y' = \sqrt{ct'^2 - vt'^2}, t')$

From the initial transformation, $y = y'$

$$\Rightarrow ct = \sqrt{ct'^2 - ct'^2}$$

$$\Rightarrow \boxed{c^2t^2 = (c^2 - v^2)t'^2}$$

We can also express $D = \dfrac{1}{\sqrt{1 - v^2/c^2}}$.

After substitution, $x' = \dfrac{1}{\sqrt{1-v^2/c^2}}(x - vt)$

$y = y'$
$z = z'$ } {No motion No change}

$$t' = \frac{1}{\sqrt{1-v^2/c^2}}\left(t - \frac{vx}{c^2}\right)$$

taking $\gamma = \dfrac{1}{\sqrt{1 - \dfrac{v^2}{c^2}}}$ {Lorentz constant}

On following the 1st postulate, the laws of physics are same in all inertial frame. Hence $x' = \dfrac{1}{\sqrt{1-v^2/c^2}}$

$y' = y$, $z = z'$

and $t' = \dfrac{1}{\sqrt{1-(v/c)^2}}\left(t - \dfrac{v}{c^2}x\right)$

Ans.

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(2)

Given $\begin{pmatrix} t' \\ x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \cosh\phi & \sinh\phi & 0 & 0 \\ \sinh\phi & \cosh\phi & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0. & 0 & 1 \end{pmatrix} \begin{pmatrix} t \\ x \\ y \\ z \end{pmatrix}$

From given data the equation we get are,

$$ct' = ct\cosh\phi + x\sinh\phi \quad -(1)$$
$$x' = ct\sinh\phi + x\cosh\phi \quad -(2)$$
$$y' = y \quad -(3)$$
$$z' = z \quad (4)$$

We can use space time interval invariance.

$$t^2 - x^2 - y^2 - z^2 = t'^2 - x'^2 - y'^2 - z'^2.$$

using above. $\boxed{ct^2 - x^2 - y^2 - z^2 = ct'^2 - x'^2 - y'^2 - z'^2}$

, using give eq. above. $\boxed{ct^2 - x^2 - (ct'^2 - x'^2 = 0}$ (5).

taking square (I) and (II) and adding both. , we will get.

$$ct'^2 = c^2t^2\cosh^2\phi + x^2\sinh^2\phi + 2xct\sinh\phi\cosh\phi$$
$$- x'^2 = c^2t^2\sinh^2\phi + x^2\cosh^2\phi + 2xct\sinh\phi\cosh\phi$$

$\Rightarrow \boxed{ct' - x'^2 = ct^2 - x^2}$ $\left\{ \begin{array}{l} \text{Given} \\ \because \cosh^2\phi - \sinh^2\phi = 1 \end{array} \right]$ Adding, - eq. (V).

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From eq. 5, $\boxed{ct'^2 - x'^2 = ct^2 - x^2}$, which one is correct.

And hence the matrix given in the question represents a Lorentz Transformation.

③.d.

Given $x = 10\,km$, $y = 5\,km$, $z = 2\,km$ $t = 2 \cdot 10^{-3}$ s.

ⓐ s' is moving with speed $0.9c$, in $-x$ direction. And hence $v = -0.9c$.

$$\gamma \text{ (Lorentz constant)} = \frac{1}{\sqrt{1-(v/c)^2}}$$

$$= \frac{1}{\sqrt{1-\left(\frac{0.9c}{c}\right)^2}} = \frac{1}{\sqrt{1-(0.9)^2}} = 2.294$$

$$\boxed{\begin{aligned} y' &= y \\ z' &= z \end{aligned}} \Big\} \text{ No Changing, No motion}$$

$$x' = \gamma(x - vt)$$

$$= 2.294(10000 - (-0.9c) \times (2 \cdot 10^{-3}))$$

$$\boxed{x' = 1261.17\,km.}$$

and

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and $t' = \gamma\left(t - \frac{vx}{c^2}\right)$

$= 2.294\left(2.10^{-3} - \left(-\frac{0.9c}{c^2} \cdot 10^4\right)\right)$

$= 2.294\left(2\times10^{-3} - \left(-\frac{0.9}{c}\times10^4\right)\right)$

$= 4.65$ ms.

(3.b) For the frame s'', the even take place at (t'', x'', y'', z'') (let say) and given $t'' = 10$ ms.

~~Let~~ With the help of LT equations, $t'' = \gamma\left(t - \frac{vx}{c^2}\right)$

$\Rightarrow 10\times10^{-3} = \gamma\left(2\times10^{-3} - \frac{v.10^4}{c^2}\right)$

let the for $v/c = k$ for calculation, then we can write.

$10\times10^{-3} = \frac{1}{\sqrt{1-k^2}}\left(2 - \frac{v.10^4 k}{c}\right)$

$\Rightarrow \boxed{\sqrt{1-k^2} = 0.2 - \frac{k}{c}.10^6}$

$\Rightarrow 1-k^2 = \left(0.2 - \frac{k}{c}10^6\right)^2.$

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$\Rightarrow$ solving the previous eq. with $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

we get, $K = 0.9805, -0.9791$

Hence $\frac{v''}{s} = 0.9805c$ and $\frac{v_s''}{s} = -0.9791c$ -ve axis.

Now, using Lorentz velocity addition and transforming frame

$$\frac{v_s''}{s'} = \frac{\frac{v_s''}{s} - \frac{v_s'}{s}}{1 - \frac{v_s''/s \cdot \frac{v_s'}{s}}{c^2}} = \frac{0.9805c - (-0.9c)}{1 - \frac{(0.9805c)(-0.9c)}{c^2}}$$

$$\boxed{\frac{v_s''}{s'} = 0.9989c} \quad - (1)$$

Ans

using second ~~result~~ root,

$$\frac{v_s''}{s'} = \frac{-0.9791c - (-0.9c)}{1 - \frac{(-0.9791c)(-0.9c)}{c^2}}$$

$$\boxed{\frac{v_s''}{s'} = -0.6657c}$$ 2 Ans.

Final result

$$\frac{v_s''}{s} = 0.9805c \text{ and } 0.9791c \quad \text{(opposite direction)}$$

$$\text{and } \frac{v_s''}{s'} = 0.9989c \text{ and } 0.6657c$$