(۹) نشان دهید اگر تابع چگالی احتمال نرمال با ماتریس کوواریانس Σ و میانگین μ معلوم باشد، تخمین های ML برای [illegible] از رابطه زیر بدست می‌آید:

$$\hat{\mu} = \frac{1}{N} \sum_{k=1}^{N} x_k$$

$$\hat{\Sigma} = \frac{1}{N} \sum_{k=1}^{N} (x_k - \hat{\mu})(x_k - \hat{\mu})^T$$

می‌دانیم: تابع چگالی احتمال برای توزیع نرمال چند متغیره $\longrightarrow p(x|\mu,\Sigma) = \frac{1}{(2\pi)^{d/2}|\Sigma|^{1/2}} e^{-\frac{1}{2}(x-\mu)^T \Sigma^{-1}(x-\mu)}$

به ازای N نمونه: $\log(L)$ $\quad L = \ln\left(\prod_{k=1}^{N} p(x_k|\mu,\Sigma)\right) = \sum_{k=1}^{N} \ln\left(p(x_k|\mu,\Sigma)\right)$

از طرفی: $\ln(p(x_k|\mu,\Sigma)) = -\frac{1}{2}(x-\mu)^T \Sigma^{-1}(x-\mu) - \frac{1}{2}\ln\left((2\pi)^d|\Sigma|\right)$

پس $\Rightarrow L = -\frac{1}{2}\sum_{k=1}^{N}(x_k-\mu)^T\Sigma^{-1}(x_k-\mu) - \frac{N}{2}\ln\left((2\pi)^d|\Sigma|\right)$

حال $\rightarrow$ برای تخمین [illegible] max $\longrightarrow$ مشتق $= 0$
نسبت به μ و Σ

یادآوری: $\frac{\partial}{\partial x} x^T M x = (M + M^T)x$

ماتریس متقارن

$$\Sigma\left(\frac{\partial L}{\partial \mu} = \sum_{k=1}^{N} \Sigma^{-1}(x_k - \mu) = 0\right)$$

$\rightarrow \sum_{k=1}^{N} x_k - \mu \cdot N = 0$ یا $\mu = \frac{1}{N}\sum_{k=1}^{N} x_k \longrightarrow$ یعنی $\mu =$ میانگین x ها

حال باید [illegible] برای $\Sigma \longrightarrow$ ابتدا [illegible] Σ^{-1} را [illegible]:

$$L(M) = -\frac{1}{2}\sum_{k=1}^{N}(x_k-\mu)^T M (x-\mu) - \frac{N}{2}\ln\left((2\pi)^d\right) + \frac{N}{2}\ln(|M|)$$

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یادآوری : $\frac{\partial \ln(|M|)}{\partial M} = (M^{-1})^T = (M^T)^{-1} = M^{-1}$ و M ، M^{-1} هر دو متقارن هستند

$\frac{\partial}{\partial M}(a^T M b) = ab^T$

پس $\frac{\partial L}{\partial M} = -\frac{1}{2}\sum_{k=1}^{N}(x_k-\mu)(x_k-\mu)^T + \frac{N}{2}M^{-1} = 0$

$M^{-1} = \frac{1}{N}\sum_{k=1}^{N}(x_k-\mu)(x_k-\mu)^T$ صفر!

Σ

بی‌طرفانه ~~Bias~~

۳) ثابت کنید که تخمین‌گر زیر، unbiased است $\hat{\mu} = \frac{1}{N}\sum x_k$

۲۳ـ متغیر تصادفی x دارای تابع توزیع Erlang می‌باشد.

$$p(x;\theta)=\theta^2 x\, e^{(-\theta x)}u(x)$$

unit step function : $u(x)=\begin{cases}1 & x\geq 0\\ 0 & x<0\end{cases}$

نشان دهید که تخمین ماکزیمم درستنمایی θ به ازای $x_1 \ldots x_N$:

$$\hat{\theta}_{ML}=\frac{2N}{\sum_{k=1}^{N} x_k}$$

حل: فرض کنیم $x_1 \ldots x_N$ (نمونه‌های مستقل) نمونه‌هایی از تابع بالا باشند:

$$P(X;\theta)=\prod_{i=1}^{N}\theta^2 x_i\, e^{-\theta x_i}\,u(x_i)$$

$u(x_i)=1 \rightarrow x_i \geq 0 \quad i=1,\ldots,N$ فرض

$$L(\theta)=\ln(P(X;\theta))=\sum_{i=1}^{N}(2\ln(\theta)+\ln(x_i)-\theta x_i)$$

$$=2\ln(\theta)N+\sum_{i=1}^{N}\ln(x_i)-\theta\sum_{i=1}^{N}x_i$$

مشتق $=0 \rightarrow$ max

$$\frac{dL}{d\theta}=0 \rightarrow \frac{2N}{\theta}-\sum_{i=1}^{N}x_i=0 \rightarrow \theta=\frac{2N}{\sum_{i=1}^{N}x_i}$$

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۲۳) در تخمین ML، مشتق log تابع چگالی احتمال را بر حسب صفر قرار دادیم. با استفاده از

مشتق دوم نشان دهید که جواب ما max است نه min

$$L(\mu) = -\frac{N}{2}\ln\left((2\pi)^{l}|\Sigma|\right) - \frac{1}{2}\sum_{k=1}^{N}(x_k-\mu)^T\Sigma^{-1}(x_k-\mu)$$

$$\frac{\partial L(\mu)}{\partial \mu} = \Sigma^{-1}\sum_{k=1}^{N}(x_k-\mu) = \Sigma^{-1}\sum_{k=1}^{N}x_k - N\Sigma^{-1}\mu = 0 \longrightarrow max$$

$$\frac{\partial^2 L(\mu)}{\partial \mu^2} = -N\Sigma^{-1}$$

+ + −

$x \sim N(\mu_x, \sigma_x^2)$

۲۴) ثابت کنید ... ۲ متغیر تصادفی مستقل ($z = x+y$

$$\mu_N = \frac{N\sigma_0^2 \bar{x}_N + \sigma^2 \mu_0}{N\sigma_0^2 + \sigma^2} \qquad \bar{x}_N = \frac{1}{N}\sum_{k=1}^{N} x_k$$

$$\sigma_N^2 = \frac{\sigma^2 \sigma_0^2}{N\sigma_0^2 + \sigma^2}$$

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(۲۵) روابط زیر را نشان دهید ... $p(x|X)$ ... $N(\mu_N, \sigma^2+\sigma_N^2)$

[illegible]

[illegible] $N(\mu_0, \sigma_0^2)$

$$p(\mu|X) = \frac{p(X|\mu)\,p(\mu)}{p(X)} \qquad p(X|\mu) = \prod_{k=1}^{N} p(x_k|\mu)$$

dataset [illegible] ، $p(X)$ ثابت

$$p(\mu|X) = \alpha\, p(\mu) \prod_{k=1}^{N} p(x_k|\mu)$$

$$= \alpha \left(\frac{1}{\sqrt{2\pi}\sigma_0} \exp\left\{-\frac{1}{2}\frac{(\mu-\mu_0)^2}{\sigma_0^2}\right\}\right) \prod_{k=1}^{N}\left(\frac{1}{\sqrt{2\pi}\sigma}\exp\left\{-\frac{1}{2}\frac{(x_k-\mu)^2}{\sigma^2}\right\}\right)$$

$$= \alpha' \exp\left[-\frac{1}{2}\left(\sum_{k=1}^{N}\left(\frac{\mu^2}{\sigma^2} - \frac{2x_k\mu}{\sigma^2} + \frac{x_k^2}{\sigma^2}\right) + \frac{\mu^2 - 2\mu\mu_0 + \mu_0^2}{\sigma_0^2}\right)\right]$$

$$= \alpha' \exp\left[-\frac{1}{2}\left(\frac{\mu^2 N}{\sigma^2} - \frac{2\mu}{\sigma^2}\sum_{k=1}^{N} x_k + \frac{1}{\sigma^2}\sum_{k=1}^{N} x_k^2 + \frac{\mu^2}{\sigma_0^2} - 2\frac{\mu\mu_0}{\sigma_0^2} - \frac{\mu_0^2}{\sigma_0^2}\right)\right]$$

$$= \alpha'' \exp\left[-\frac{1}{2}\left(\frac{N}{\sigma^2} + \frac{1}{\sigma_0^2}\right)\mu^2 - 2\left(\frac{1}{\sigma^2}\sum_{k=1}^{N} x_k + \frac{\mu_0}{\sigma_0^2}\right)\mu\right]$$

$$\frac{(\mu-\mu_0)^2}{\sigma_0^2}$$

ضرایب را با ... میانه و واریانس ... [illegible]:

$$\frac{1}{\sigma_N^2} = \frac{N}{\sigma^2} + \frac{1}{\sigma_0^2} \rightarrow \sigma_N^2 = \frac{1}{\frac{N}{\sigma^2} + \frac{1}{\sigma_0^2}} = \frac{\sigma^2\sigma_0^2}{N\sigma_0^2 + \sigma^2}$$

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$$\bar{x} = \frac{1}{N}\sum_{k=1}^{N} x_k$$

$$\frac{\mu_N}{\sigma_N^2} = \frac{N\bar{x}}{\sigma^2} + \frac{\mu_0}{\sigma_0^2}$$

$$\mu_N = \left(\frac{\sigma_0^2\sigma^2}{N\sigma_0^2+\sigma^2}\right)\left(\frac{N\bar{x}}{\sigma^2}+\frac{\mu_0}{\sigma_0^2}\right) = \frac{N\bar{x}\sigma_0^2+\mu_0\sigma^2}{N\sigma_0^2+\sigma^2}$$

$$= \left(\frac{N\sigma_0^2}{N\sigma_0^2+\sigma^2}\right)\bar{x} + \frac{\sigma^2}{N\sigma_0^2+\sigma^2}\mu_0$$

$$\Rightarrow p(\mu|X) = \alpha' \exp\left\{-\frac{1}{2}\left(\frac{\mu^2}{\sigma_N^2} - \frac{2\mu_N\mu}{\sigma_N^2}\right)\right\}$$

میانگین

$$= \alpha'' \exp\left\{-\frac{1}{2}\left(\frac{\mu^2-2\mu_N\mu+\mu_N^2}{\sigma_N^2}\right)\right\} = \alpha''\exp\left\{-\frac{1}{2}\frac{(\mu-\mu_N)^2}{\sigma_N^2}\right\}$$

واریانس

$$p(x|X) = \int p(x,\mu|X)\,d\mu = \int p(x|\mu,X)\,p(\mu|X)\,d\mu$$

$$= \int p(x|\mu)\,p(\mu|X)\,d\mu = \int_{-\infty}^{\infty}\left(\frac{1}{\sqrt{2\pi}\,\sigma}e^{-\frac{1}{2}\frac{(x-\mu)^2}{\sigma^2}}\right)\left(\frac{1}{\sqrt{2\pi}\,\sigma_N}e^{-\frac{1}{2}\frac{(\mu-\mu_N)^2}{\sigma_N^2}}\right)d\mu$$

$$= \frac{1}{2\pi\sigma\sigma_N}\int_{-\infty}^{\infty}\exp\left\{-\frac{1}{2}\left[\frac{x^2-2x\mu+\mu^2}{\sigma^2}+\frac{\mu^2-2\mu\mu_N+\mu_N^2}{\sigma_N^2}\right]\right\}d\mu$$

$$-\frac{1}{2}\left[\left(\frac{1}{\sigma_N^2}+\frac{1}{\sigma^2}\right)\mu^2 - 2\left(\frac{x}{\sigma^2}+\frac{\mu_N}{\sigma_N^2}\right)\mu + \left(\frac{x^2}{\sigma^2}+\frac{\mu_N^2}{\sigma_N^2}\right)\right]$$

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$$P(x|X) = \frac{1}{2\pi\sigma\sigma_N}\left(\sqrt{2\pi}\,\frac{1}{\sqrt{\frac{1}{\sigma^2}+\frac{1}{\sigma_N^2}}}\,e^{-\frac{1}{2}\frac{(x-\mu_N)^2}{\sigma^2+\sigma_N^2}}\right)$$

میانگین

$$= \frac{1}{\sqrt{2\pi}\sqrt{\sigma^2+\sigma_N^2}}\,e^{-\frac{1}{2}\frac{(x-\mu_N)^2}{\sigma^2+\sigma_N^2}}$$ حکم ۱

واریانس

[illegible]

$$P(\mu) = \frac{\mu \exp(-\mu^2 / 2\sigma_m^2)}{\sigma_m^2}$$

توزیع Rayleigh $\leftarrow$ prior

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(۲۸) متغیر تصادفی x که توزیع نرمال دارد $N(\mu, \sigma^2)$

نشان دهید که ماکسیمم احتمال پسین μ:

$$\hat{\mu}_{map} = \frac{z}{2R}\left(1+\sqrt{1+\frac{4R}{z^2}}\right)$$

$$z = \frac{1}{\sigma^2}\sum_{k=1}^{N} x_k \qquad R = \frac{N}{\sigma^2} + \frac{1}{\sigma_m^2}$$

حل

$$l(\mu) = \left(\prod_{i=1}^{N} p(x_i \mid \mu, \sigma^2)\right) p(\mu)$$

لگاریتم می‌گیریم

$$L(\mu) = \ln(l(\mu)) = \ln(P(\mu)) + \sum_{i=1}^{N} \ln(P(x_i \mid \mu, \sigma^2))$$

$$= \ln(\mu) - \frac{\mu^2}{2\sigma_m^2} - \ln(\sigma_m^2) + \sum_{i=1}^{N}\left[\ln\left(\frac{1}{\sqrt{2\pi}\sigma}\right) - \frac{(x_i-\mu)^2}{2\sigma^2}\right]$$

$$= \ln(\mu) - \frac{\mu^2}{2\sigma_m^2} - \ln(\sigma_m^2) + N\ln\left(\frac{1}{\sqrt{2\pi}\sigma}\right) - \frac{1}{2\sigma^2}\sum_{i=1}^{N}(x_i-\mu)^2$$

مشتق:

$$\frac{\partial L}{\partial \mu} = \frac{1}{\mu} - \frac{\mu}{\sigma_m^2} + \frac{1}{\sigma^2}\sum_{i=1}^{N}(x_i-\mu) = 0$$

$$\left(\frac{1}{\sigma_m^2} + \frac{N}{\sigma^2}\right)\mu^2 - \left(\frac{1}{\sigma^2}\right)\left(\frac{1}{N}\sum_{i=1}^{N} x_i\right)\mu - 1 = 0 \xrightarrow{\text{معادله ۲}} R\mu^2 - z\mu - 1 = 0$$

R $\qquad$ z

$$\mu = \frac{z \pm \sqrt{z^2+4R}}{2R} = \frac{z}{2R}\left(1 \pm \sqrt{1+\frac{4R}{z^2}}\right)$$

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Lognormal
لگ نرمال

Subject:

۲۹) نشان دهید تخمین ML برای توزیع لگ نرمال زیر $\hat{\theta}_{ML} = \frac{1}{N}\sum_{k=1}^{N} \ln x_k$ است.

$$\Rightarrow p(x) = \frac{1}{\sigma x\sqrt{2\pi}} \exp\left(-\frac{(\ln x-\theta)^2}{2\sigma^2}\right) \quad ; \quad x>0$$

با استقلال $p(x)$ ، x_i ها

$$\Rightarrow L(\theta) = \sum_{i=1}^{N} \ln(p(x_i))$$

$$= -\sum_{i=1}^{N} \left(\ln(\sigma x_i\sqrt{2\pi}) + \frac{(\ln(x_i)-\theta)^2}{2\sigma^2}\right)$$

$$\frac{dL}{d\theta} = -\sum_{i=1}^{N} \frac{(\ln(x_i)-\theta)}{\sigma^2} = 0$$

$$\Rightarrow \theta = \frac{1}{N}\sum_{i=1}^{N} \ln(x_i)$$

$\mu = \int_{-\infty}^{+\infty} x\,p(x)\,dx$ $\quad$ $\sigma^2 = \int_{-\infty}^{+\infty} (x-\mu)^2 p(x)\,dx$

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۳) نشان دهید [illegible] PDF [illegible]
توزیع نرمال است $N(\mu, \sigma^2)$

[illegible] $-\int p(x) \ln(p(x))$ [illegible]

$$\begin{cases} \int p(x)\,dx = 1 \\ \int x\,p(x)\,dx = \mu \\ \int (x-\mu)^2 p(x)\,dx = \sigma^2 \end{cases}$$

[illegible] $\min\left(-\int\right)$ [illegible]

$$H_L = \int p(x) \ln(p(x))\,dx - \lambda_1 \left(\int p(x)\,dx - 1 \right)$$

$$-\lambda_2 \left(\int x\,p(x)\,dx - \mu \right) - \lambda_3 \left(\int (x-\mu)^2 p(x)\,dx - \sigma^2 \right)$$

$$\frac{\partial H_L}{\partial p} = \int \ln(p(x))\,dx + \int dx - \lambda_1 \int dx - \lambda_2 \int x\,dx - \lambda_3 \int (x-\mu)^2 dx = 0$$

$$= -(1 - \lambda_1 - \lambda_2 x - \lambda_3 (x-\mu)^2) = 0 \Rightarrow p(x) = e^{-(1-\lambda_1-\lambda_2 x - \lambda_3 (x-\mu)^2)}$$

A

$$\begin{cases} \int e^{-(1-\lambda_1-\lambda_2 x - \lambda_3 (x-\mu)^2)}\,dx = 1 \\ \int x\,e^{-(1-\lambda_1-\lambda_2 x - \lambda_3 (x-\mu)^2)}\,dx = \mu \\ \int (x-\mu)^2 e^{-(1-\lambda_1-\lambda_2 x - \lambda_3 (x-\mu)^2)}\,dx = \sigma^2 \end{cases}$$

[illegible]

$N(\mu, \sigma^2)$

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$A \longrightarrow \int \frac{e^{-1} \times e^{\lambda_1}}{d} \times e^{\lambda_2 x} \times e^{\lambda_3 x^2 + \mu^2 \lambda_3 - 2x\mu\lambda_3}\, dx$

$e^{\lambda_1 - 1} x$ $\quad \frac{1}{\lambda_2} e^{\lambda_2 x}$ $\quad \frac{2x}{\lambda_3} e^{\lambda_3 x^2 + \mu^2 \lambda_3 x - \mu \lambda_3 x}$

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$$p(x) = e^{-\left(\ln\sqrt{2\pi}\sigma + \frac{1}{2}(x-\mu)^2\right)} = e^{\ldots}$$

$$= \frac{1}{\sqrt{2\pi}\,\sigma}\, e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

[illegible]

$$\mu_j(t+1) = \frac{\sum_{k=1}^{N} P(j \mid x_k;\theta(t))\, x_k}{\sum_{k=1}^{N} P(j \mid x_k;\theta(t))}$$

$$\sigma_j^2(t+1) = \frac{\sum_{k=1}^{N} P(j \mid x_k;\theta(t))\, \|x_k-\mu_j(t+1)\|^2}{l\sum_{k=1}^{N} P(j \mid x_k;\theta(t))}$$

$$P_j(t+1) = \frac{1}{N}\sum_{k=1}^{N} P(j \mid x_k;\theta(t))$$

[illegible]

$$p(x_k \mid j;\theta) = N(x_k \mid \mu_j, C_j)$$

$P(j_k \mid x_k;\theta(t))$ [illegible] E-step [illegible]

$$Q(\theta;\theta(t)) = \sum_{k=1}^{N}\sum_{j_k=1}^{J} P(j_k \mid x_k;\theta(t)) \ln\left(p(x_k \mid j_k;\theta)\, P_{j_k}\right)$$

$$= \sum_{k=1}^{N}\sum_{j=1}^{J} P(j \mid x_k;\theta(t)) \ln\left(p(x_k \mid j;\theta)\, P_j\right)$$

$$\ln\left(p(x_k \mid j;\theta)\right) = -\frac{1}{2}\ln\left((2\pi)^l |C_j|\right) - \frac{1}{2}(x_k-\mu_j)^T C_j^{-1}(x_k-\mu_j)$$

$$Q(\theta;\theta(t)) = \sum_{k=1}^{N}\sum_{j=1}^{J} P(j \mid x_k;\theta(t))\left(-\frac{1}{2}\ln(|C_j|) - \frac{1}{2}(x_k-\mu_j)^T C_j^{-1}(x_k-\mu_j) + \ln P_j\right)$$

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ادامه ۳۱: [illegible] P_j, C_j [illegible]

[illegible] $\sum_j P_j = 1$ [illegible]

[illegible] $Q(\theta, \theta(t))$ [illegible]

[illegible] (λ) [illegible] $\sum P_j = 1$ [illegible]

$$Q'(\theta;\theta(t)) = \sum_{k=1}^{N} \sum_{j=1} [P(j \mid m_k;\theta(t)) \ln(N(m_k \mid \mu_j, C_j)) + P(j \mid m_k;\theta(t)) \ln(P_j)] - \lambda\left(\sum_j P_j - 1\right)$$

[illegible] $\max$ [illegible]

$$\frac{\partial}{\partial \mu_j} N(m_k \mid \mu_j, C_j) = N(m_k \mid \mu_j, C_j) \frac{\partial}{\partial \mu_j}\left(-\frac{1}{2}(m_k - \mu_j)^T C_j^{-1} (m_k - \mu_j)\right)$$

$$= \frac{1}{2} N(m_k \mid \mu_j, C_j)(C_j^{-1} + C_j^{-T})(m_k - \mu_j) = N(m_k \mid \mu_j, C_j) C_j^{-1} (m_k - \mu_j)$$

[illegible]: $$\frac{\partial Q'(\theta;\theta(t))}{\partial \mu_j} = \sum_{k=1}^{N} P(j \mid m_k;\theta(t)) \left(\frac{1}{N(m_k \mid \mu_j, C_j)}\right)\left(\frac{\partial}{\partial \mu_j} N(m_k \mid \mu_j, C_j)\right)$$

پس: $$\frac{\partial Q'(\theta;\theta(t))}{\partial \mu_j} = \sum_{k=1}^{N} P(j \mid m_k;\theta(t)) C_j^{-1}(m_k - \mu_j) = 0$$

$$\Rightarrow \mu_j = \frac{\sum_{k=1}^{N} P(j \mid m_k;\theta(t)) m_k}{\sum_{k=1}^{N} P(j \mid m_k;\theta(t))}$$

تذکر : $\frac{\partial}{\partial C_j^{-1}} N(x_k|\mu_j, C_j) = \frac{1}{2}\frac{1}{(2\pi)^{l/2}}\frac{1}{|C_j|^{1/2}} \exp\left\{-\frac{1}{2}(x_k-\mu_j)^T C_j^{-1}(x_k-\mu_j)\right\} C_j$

$-\frac{1}{2} N(x_k|\mu_j, C_j)(x_k-\mu_j)^T(x_k-\mu_j) =$

$\frac{1}{2} N(x_k|\mu_j, C_j)\left(C_j - (x_k-\mu_j)(x_k-\mu_j)^T\right)$

بنابراین $\longrightarrow \frac{\partial}{\partial C_j^{-1}} \ln\left(N(x_k|\mu_j, C_j)\right) = \frac{1}{2}\left(C_j - (x_k-\mu_j)(x_k-\mu_j)^T\right)$

$\longrightarrow \frac{\partial}{\partial C_j^{-1}} Q(\Theta;\Theta(t)) = \sum_{k=1}^{N} P(j|x_k;\Theta(t))C_j - \sum_{k=1}^{N} P(j|x_k;\Theta(t))(x_k-\mu_j)(x_k-\mu_j)^T = 0$

$\longrightarrow C_j = \frac{\sum_{k=1}^{N} P(j|x_k;\Theta(t))(x_k-\mu_j)(x_k-\mu_j)^T}{\sum_{k=1}^{N} P(j|x_k;\Theta(t))}$

حال می‌خواهیم احتمال پیشین کلاس‌ها را بدست آوریم ، P_j :

$\frac{\partial Q(\Theta;\Theta(t))}{\partial P_j} = 0 \longrightarrow \sum_{k=1}^{N} P(j|x_k;\Theta(t)) - \lambda = 0$

ضرب در P_j

احتمال تعلق x_k به کلاس j است

$\longrightarrow \lambda P_j = \sum_{k=1}^{N} P(j|x_k;\Theta(t))$ ، $j=1,\dots,J$ ، $\sum_{j=1}^{J} P_j = 1$

$\lambda = \sum_{j=1}^{J}\sum_{k=1}^{N} P(j|x_k;\Theta(t)) = \sum_{k=1}^{N}\sum_{j=1}^{J} P(j|x_k;\Theta(t)) = \sum_{k=1}^{N} 1 = N$

$\longrightarrow P_j = \frac{1}{N}\sum_{k=1}^{N} P(j|x_k;\Theta(t))$ حل!

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۳۳ـ ۳ PDF تو سه متغیره … $N(\mu_1, \sigma_1)$, $N(\mu_2, \sigma_2)$, $N(\mu_3, \sigma_3)$ … [illegible]

از N_4 و … از N_1 و … از N_2 … [illegible]

$\frac{1}{4}\ \frac{2}{4}\ \frac{1}{4} = P$

$N_4 N_4,\ N_1 N_4,\ N_4 N_4 N_1 N_4,\ N_4 N_4 N_4 N_1 N_2$ … [illegible]

… (PDF) … $\sum_{i=1}^{3} N(\mu_i, \sigma_i^2) P_i$

در EM … μ_i و σ_i^2 و P_i … [illegible]

… [illegible]

EM, GMM … [illegible] … P … [illegible]

… [illegible] … P … [illegible]

۳۴ـ P … [illegible] … h … N … [illegible]

احتمال … K … h … [illegible]

$$prob(K) = \frac{N!}{K!(N-K)!} P^K (1-P)^{N-K}$$

$\binom{N}{K}$

… N … [illegible]

… $E[K/N] = P$ … [illegible]

$$\sigma^2 = E[(K/N - P)^2] = P(1-P)/N \quad \text{و} \quad \hat{P} = K/N$$

… unbiased … [illegible]

$$E\left[\frac{K}{N} \mid P\right] = \frac{1}{N} E[K \mid P]$$

… P … K … N … [illegible]

… h … [illegible] … E

… K … [illegible]

$E(K) = PN$ … (P, N)

… $\frac{1}{N} E[K \mid P] = P$

… P … [illegible]

[illegible] ($e = \hat{P} - P$) [illegible]

$$\sigma_e^2(P) = E[(e - E[e])^2 \mid P]$$

$$= E\left[\left(\frac{1}{N}K - P\right)^2 \mid P\right] = \frac{1}{N^2} E\left[(K - NP)^2 \mid P\right]$$

$$= \frac{1}{N^2}\left(NP(1-P)\right) = \frac{P(1-P)}{N}$$

[illegible] (N, P) [illegible] $NP(1-P)$ [illegible] K/N [illegible]

[illegible]

۳۴ [illegible]

[illegible]

[illegible] ۳۲ [illegible] N [illegible]

[illegible] NN [illegible] KNN [illegible] K [illegible]

[illegible] K [illegible]

[illegible] K [illegible]

۳۵ [illegible] (۱۲) [illegible]

[illegible] NN [illegible] NNe [illegible]

$NN \rightarrow$ [illegible]

[illegible]

[illegible]

[illegible]

$$P_B \le P_{NN} \le P_B\left(2 - \frac{M}{M-1}P_B\right) \le 2P_B$$

$$P_{NN} \approx 2P_B$$

$$P_{3NN} \approx P_B + 3P_B^2$$

$$P_{3NN} \approx 12 \times 2$$

$$N \uparrow \; P_B \downarrow \Rightarrow P_{3NN} \approx P_B$$

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$P(m) = \begin{cases} \frac{1}{4} & \text{for } |m| < 2 \\ 0 & \end{cases}$

(۳۶) [illegible] $(\cdots)$ [illegible]

[illegible] $N=256$ [illegible] $N=1024$ [illegible] $N=\ldots$ [illegible]

[illegible] $h=2$ [illegible]

[illegible] $P(m)$ [illegible]

[illegible] h [illegible] $P(m)$ [illegible]

[illegible] h [illegible] Smith [illegible]

(۳۷) [illegible] $N=5000$ [illegible] $K=100, 1024, 256$ [illegible]

[illegible] $K\uparrow$ [illegible] $P(m)$ [illegible]

$\hat{P}(m) = \frac{K}{N\,V(m)}$ [illegible]

[illegible] K [illegible]

(۳۸) [illegible] $\sigma_N^2(m)$ [illegible] pdf

$$\hat{P}(m) = \frac{1}{h}\left(\frac{1}{N}\left(\sum_{i=1}^{N}\phi\left(\frac{m_i - m}{h}\right)\right)\right)$$ ۳.۱۱۰

supremum

$$\sigma_N^2(m) \leq \frac{\sup(\phi)\, E[\hat{P}(m)]}{Nh^2}$$

[illegible] $h\uparrow \rightarrow \sigma^2\downarrow$

$\begin{cases} N \rightarrow \infty \\ Nh^2 \rightarrow \infty \end{cases}$ [illegible] $h\downarrow \rightarrow \sigma^2\downarrow$

۴۱) در مثال شکل ۲.۲۸ ... H_1 ...

$P(C = True \mid H_1 = True)$

S	True	False
True	٫۲	٫۸
False	٫۱	٫۹

$P(C|S)$

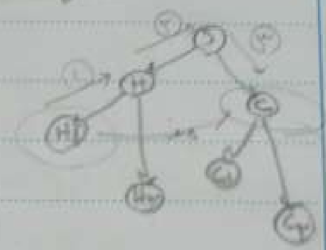
$P(H_{۲}|H_{(1)}) = ٫۹۵$

$P(H_{(1)}|S_{(1)}) = ٫۶$

$P(S_{(1)}) = ٫۲$

$P(C_{(1)}|S_{(1)}) = ٫۲$

$P(C_{(1)}|H_{(1)}) = ?$



$P(C_{(1)})$

$A_C = \%S_1$

$P(C|S)$

$P(C_1|S) = P(C_{(1)}|S_{(1)}) + P(C_{(1)}|S_{(2)}) = ٫۱۱ + ٫۲ = ٫۳۱$

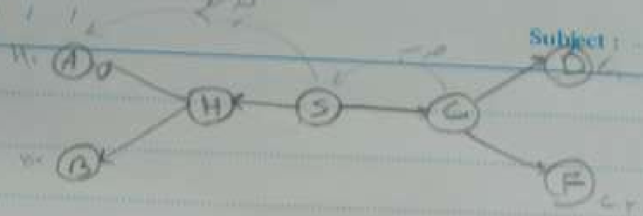
$P(S|H_{(1)}) = \dfrac{P(H_{(1)}|S_{(1)})P(S_{(1)})}{P(H_{(1)})} + \dfrac{P(H_{(1)}|S_{(2)})P(S_{(2)})}{P(H_{(1)})}$

$H_1^+ = \cancel{٫۹۵}$

٪۲۸۵

$H_1^+ = ٫۰۱$

Date: / /   Subject:



$P(S=1) = ۰٫۴$, $P(H_1|S_1) = ۰٫۴$

$P(H_0|S_1) = ۰٫۶$

$P(A_1|H_1) = ۰٫۹۵$ $P(A_1|H_0) = ۰٫۰۱$

$P(A_0|H_1) = ۰٫۰۵$ $P(A_0|H_0) = ۰٫۹۹$

$P(B_1|H_1) = ۰٫۹۸$ $P(B_1|H_0) = ۰٫۰۵$

$P(B_0|H_1) = ۰٫۰۲$ $P(B_0|H_0) = ۰٫۹۵$

$P(C_1|S_1) = ۰٫۲$

$P(C_0|S_1) = ۰٫۸$

$P(D_1|C_1) = ۰٫۹۹$ $P(D_1|C_1) = ۰٫۱$

$P(D_0|C_1) = ۰٫۰۱$ $P(D_0|C_0) = ۰٫۹$

$P(F_1|C_1) = ۰٫۹۸$ $P(F_1|C_0) = ۰٫۰۵$

$P(F_0|C_1) = ۰٫۰۲$ $P(F_0|C_0) = ۰٫۹۵$

$P(S=0) = ۰٫۶$, $P(H_1|S_0) = ۰٫۱۵$ $P(C_1|S_0) = ۰٫۱۱$

$P(H_0|S_0) = ۰٫۸۵$ $P(C_0|S_0) = ۰٫۸۹$

$$P(C_1|A_1) = \overbrace{P(C_1|S_1)}^{۰٫۲} P(S_1|A_1) + \overbrace{P(C_1|S_0)}^{۰٫۱۱} P(S_0|A_1)$$

$$P(S_1|A_1) = \frac{P(A_1|S_1)\,P(S_1)^{۰٫۴}}{P(A_1)} = \frac{۰٫۳۸۶ \times ۰٫۴}{۰٫۹۶}$$

$$P(A_1) = P(A_1|H_1) + P(A_1|H_0) = ۰٫۹۶$$

$$P(A_1|S_1) = \underbrace{P(A_1|H_0)}_{۰٫۰۱}\underbrace{P(H_0|S_1)}_{۰٫۶} + \underbrace{P(A_1|H_1)}_{۰٫۹۵}\underbrace{P(H_1|S_1)}_{۰٫۴} = ۰٫۳۸۶$$

$۰٫۰۰۶$ $۰٫۳۸۰$