Lab 2 Solution

NOTE: The values provided are approximate guidelines. Marks should not be deducted if student uses a correct procedure but the values are off.

Use your vertical acceleration data (in the z direction) to estimate the frequency of the motor ω and the corresponding **displacement amplitude** $\mathbb{X}$ of the platform for each motor speed.

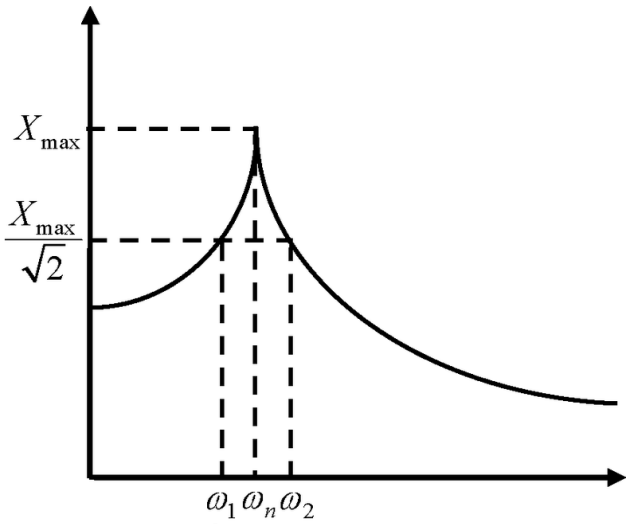
1. **(1 pt)** Estimate the natural frequency of the system by determining the motor frequency that results in the largest measured displacement amplitude.

$$\mathbb{X} = \frac{\ddot{\mathbb{X}}}{\omega^2}$$

   p is best estimated by the ω corresponding to the largest $\mathbb{X}$

2. **(2 pts)** Plot $\mathbb{X}$ vs. ω/p to obtain the frequency response curve of the system.

While the logarithmic decrement can be used to estimate the damping ratio for damped free vibrations, the half-power bandwidth method can be used to estimate the damping of a forced system, as shown below:



The damping ratio can be estimated using:

$$\zeta = \frac{\omega_2 - \omega_1}{2\omega_n}$$

Where ω_n is the natural frequency (ie. $\omega_n = p$), and ω_1 and ω_2 correspond to the frequencies that result in an amplitude that is $1/\sqrt{2}$ times the amplitude at resonance.

3. **(3 pts)** Using your plot of $\mathbb{X}$ vs. ω/p, estimate the damping ratio using the half-power bandwidth method. ω_1 and ω_2 can be determined using linear interpolation.

   Interpolation between the points defining the interval containing $\frac{X_{max}}{\sqrt{2}}$, for both $\omega < p$ and $\omega > p$:

$$y = \frac{(y_2 - y_1)}{(x_2 - x_1)}(x - x_1)$$

4. **(2 pts)** Using your calculated damping ratio, estimate the mass of a single imbalance (remember that there are two imbalances, not one), if they each have an eccentricity of 25 mm. Assume the total mass of the system is 13 kg.

$$\frac{M\mathbb{X}}{\tilde{m}e} = \frac{\left(\frac{\omega}{p}\right)^2}{\sqrt{\left[1 - \left(\frac{\omega}{p}\right)^2\right]^2 + \left(\frac{2\zeta\omega}{p}\right)^2}}$$

$$\tilde{m} \cong 50 \text{ g}$$

5. **(2 pts)** Estimate the natural frequency of the system using the vertical acceleration data recorded during beating of the platform and the motor frequency measured with the stroboscope.

$$\tau_{beat} = \frac{2\pi}{\omega - p}$$

$$p \cong 25 \text{ rad/s}$$