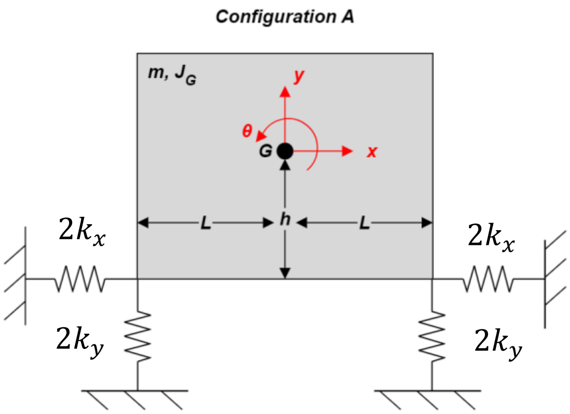
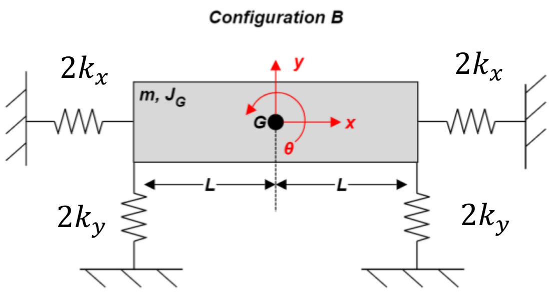
Lab 3 Solution

NOTE: The values provided are approximate guidelines. Marks should not be deducted if student uses a correct procedure but the values are off.

Each configuration of the platform can be modelled as shown:





1. **(3 pts)** Determine the system of equations of motion using the coordinates $\begin{Bmatrix} x \\ y \\ \theta \end{Bmatrix}$ for Configuration A.

$$\begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & J_G \end{bmatrix} \begin{Bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{\theta} \end{Bmatrix} + \begin{bmatrix} 4k_x & 0 & 4k_x h \\ 0 & 4k_y & 0 \\ 4k_x h & 0 & 4k_y L^2 + 4k_x h^2 \end{bmatrix} \begin{Bmatrix} x \\ y \\ \theta \end{Bmatrix} = \{0\}$$

2. **(1 pt)** Determine the system of equations of motion using the coordinates $\begin{Bmatrix} x \\ y \\ \theta \end{Bmatrix}$ for Configuration B.

$$\begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & J_G \end{bmatrix} \begin{Bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{\theta} \end{Bmatrix} + \begin{bmatrix} 4k_x & 0 & 0 \\ 0 & 4k_y & 0 \\ 0 & 0 & 4k_y L^2 \end{bmatrix} \begin{Bmatrix} x \\ y \\ \theta \end{Bmatrix} = \{0\}$$

Assume that $m = 13$ kg, $L = 270$ mm, and $k_y = 2.8$ kN/m.

Using your collected data for the platform in Configuration B:

3. **(1 pt)** Estimate the lateral spring stiffness k_x.

Estimate p_x by estimating the oscillation period as the distance between adjacent peaks for the translational acceleration vs. time data collected for configuration B:

$$k_x = \frac{mp_x^2}{4}$$

$$k_x \cong 2.4 \text{ kN/m}$$

4. **(1 pt)** Estimate the moment of inertia J_G of the platform.

Estimate p_θ by estimating the oscillation period as the distance between adjacent peaks for the angular velocity vs. time data collected for configuration B:

$$J_G = \frac{4k_y L^2}{p_\theta^2}$$

$$J_G \cong 0.25 \text{ kgm}^2$$

For your collected data for the platform in Configuration A:

5. **(2 pts)** Use your data to estimate the natural frequencies p_1 and p_2. Estimate the frequencies using both the translational acceleration and angular velocity data sets and report your natural frequencies as an average of the two values.
   For each mode, you should obtain two estimates for the natural frequency (one from the translational acceleration vs. time, and one from the angular velocity vs. time). Use of the average of the two values:

$$p_1 \cong 4 \text{ Hz}$$

$$p_2 \cong 9 \text{ Hz}$$

6. **(2 pts)** Calculate the amplitudes of $x(t)$ and $\theta(t)$ in each mode using their corresponding data sets. Use your calculated amplitudes to estimate the magnitude of the ratio $\frac{\mathbb{X}}{\Theta}$ in the first and second modes.

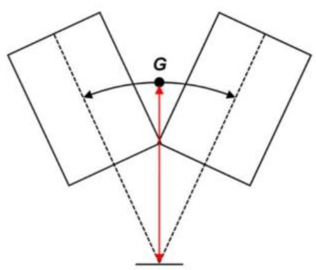
In each mode, the acceleration and angular velocity magnitudes can be converted to magnitudes for linear displacement and rotation:

$$\mathbb{X}_1 = \frac{\ddot{\mathbb{X}}_1}{p_1^2}, \qquad \mathbb{X}_2 = \frac{\ddot{\mathbb{X}}_2}{p_2^2}, \qquad \Theta_1 = \frac{\dot{\Theta}_1}{p_1}, \qquad \Theta_2 = \frac{\dot{\Theta}_2}{p_2}$$

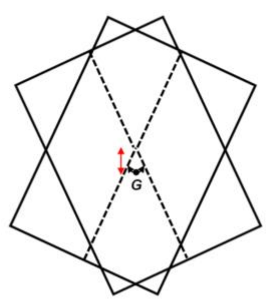
$$\left(\frac{\mathbb{X}}{\Theta}\right)^{①} = \frac{\mathbb{X}_1}{\Theta_1} \cong 0.7, \qquad \left(\frac{\mathbb{X}}{\Theta}\right)^{②} = \frac{\mathbb{X}_2}{\Theta_2} \cong 0.10$$

7. **(1 pt)** Based on your visual observations of each mode shape and nodal locations for Configuration A, determine the sign of $\frac{\mathbb{X}}{\Theta}$ (positive or negative) for each mode **based on the provided coordinate system.**

In mode 1, the platform appears to be rotating about a point located below G. Therefore, $\left(\frac{\mathbb{X}}{\Theta}\right)$ should be negative (i.e a positive (CCW) value of θ will cause a displacement of G to the left (corresponding to a negative value of x)).



In mode 2, the platform appears to be rotating about a point located above G. Therefore, $\left(\frac{\mathbb{X}}{\Theta}\right)$ should be positive (i.e a positive (CCW) value of θ will cause a displacement of G to the right (corresponding to a positive value of x)).



For the following questions, use your equations of motion from Question 1 and values from Question 3 to Question 7.

8. **(2 pts)** Estimate the value of h using your values for the first mode.
9. **(2 pts)** Estimate the value of h using your values for the second mode.

$$\begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & J_G \end{bmatrix}\begin{Bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{\theta} \end{Bmatrix} + \begin{bmatrix} 4k_x & 0 & 4k_x h \\ 0 & 4k_y & 0 \\ 4k_x h & 0 & 4k_y L^2 + 4k_x h^2 \end{bmatrix}\begin{Bmatrix} x \\ y \\ \theta \end{Bmatrix} = \{0\}$$

Focusing on x, θ:

$$\ddot{x}(t) = -p^2 x(t), \qquad \ddot{\theta} = -p^2 \theta(t)$$

$$\ddot{\mathbb{X}} = -p^2 \mathbb{X}, \qquad \ddot{\Theta} = -p^2 \Theta$$

$$\begin{bmatrix} 4k_x - mp^2 & 4k_x h \\ 4k_x h & 4k_y L^2 + 4k_x h^2 - J_G p^2 \end{bmatrix}\begin{Bmatrix} \mathbb{X} \\ \Theta \end{Bmatrix} = \{0\}$$

Provides two equations (either one can be used to solve for h, using known values and mode shapes $\left(\frac{\mathbb{X}}{\Theta}\right)$ determined in question 6 and 7):

$$(4k_x - mp^2)\mathbb{X} + (4k_x h)\Theta = 0, \qquad \left(\frac{\mathbb{X}}{\Theta}\right) = \left(\frac{4k_x h}{mp^2 - 4k_x}\right)$$

$$(4k_x h)\mathbb{X} + \left(4k_y L^2 + 4k_x h^2 - J_G p^2\right)\Theta = 0, \qquad \left(\frac{\mathbb{X}}{\Theta}\right) = \frac{\left(J_G p^2 - 4k_y L^2 - 4k_x h^2\right)}{4k_x h}$$

Actual value of h is approximately 130 mm.