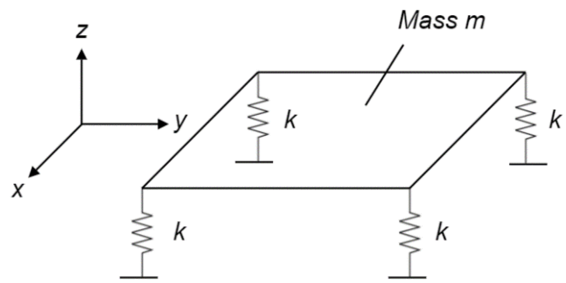
Lab 1 Solution

Please type your submission and submit your answers as a PDF with the Excel data file <u>separately</u>.



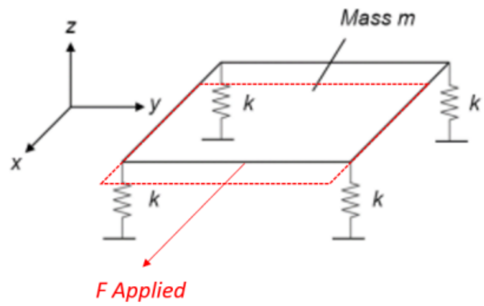
1. In this lab, the apparatus is a simple platform suspended by springs as shown above. When modeling a stiffness/elastic element as a spring, it is typically assumed that the spring provides a stiffness in only the axial direction (ie. in the z – direction).
   a) **(1 pt)** How much would the natural frequency of vibrations in the vertical direction increase if the stiffness of each spring is doubled?

   $$p = \sqrt{\frac{k_{eff}}{m_{eff}}}, \qquad p_{new} = \sqrt{\frac{2k_{eff}}{m_{eff}}} = \sqrt{2}p$$

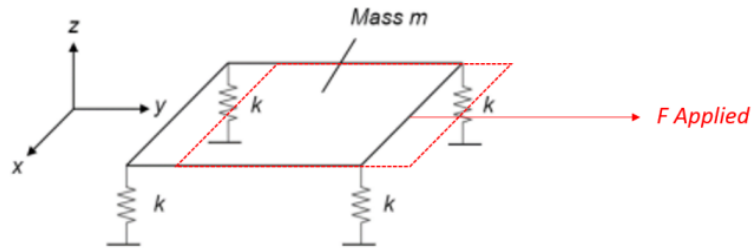
   b) **(3 pts)** The springs are manufactured such that they are also able to resist lateral forces (ie. in the x and y directions). If the springs have both an axial and lateral stiffness, determine how many degrees of freedom the system has and state each degree of freedom.

   The system is free to move in the x, y, z directions. It is also free to rotate about all axes even with linear springs, as the motion of the attachment point is approximately linear for small oscillations.

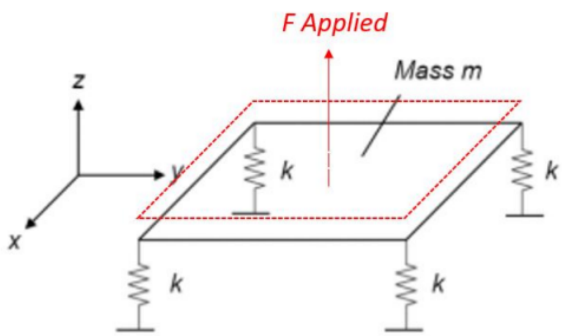
Translation in x Direction



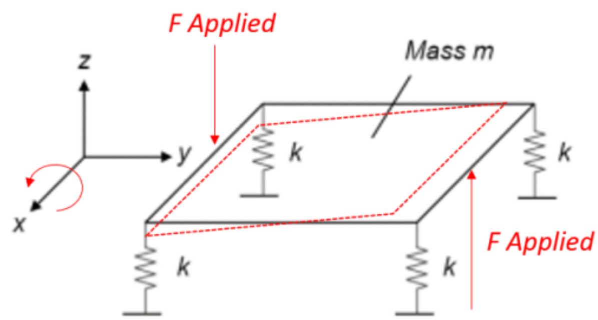
Translation in y Direction



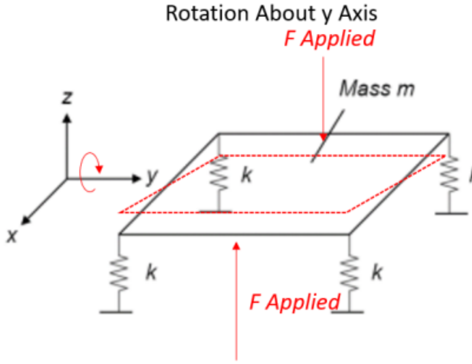
Translation in z Direction



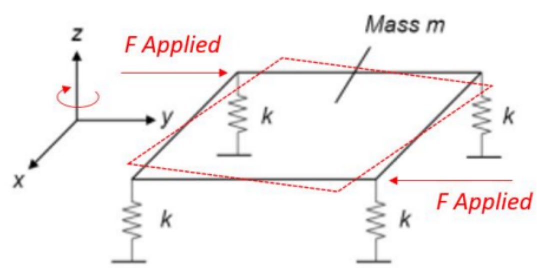
Rotation About x Axis



Rotation About y Axis



Rotation About z Axis



Plot the vertical acceleration versus time. Using your plot:

2. **(4 pts)** Calculate the damping ratio using the logarithmic decrement. Use a set of peaks away from the beginning of the measured response due to the initial lateral motion of the platform when it is released.

$$\delta = \frac{1}{n}\ln\left(\frac{\ddot{X}_0}{\ddot{X}_n}\right), \qquad n \text{ is the number of cycles between } \ddot{X}_0 \text{ and } \ddot{X}_n$$

$$\zeta = \frac{\delta}{\sqrt{(2\pi)^2 + \delta^2}}$$

$$\zeta \cong 0.01$$

3. **(3 pts)** If each spring has a stiffness of 2.8 kN/m, calculate the mass of the platform.

The mass of the platform corresponds to the mass of the system being suspended by springs, so $m = m_{eff}$.

$$p = \sqrt{\frac{k_{eff}}{m}}$$

$$m_{eff} = \frac{k_{eff}}{p^2}$$

$$m_{eff} \cong 6 \text{ kg}$$

4. **(2 pts)** Determine the effective damping of the system.

$$\zeta = \frac{c_{eff}}{c_C} = \frac{c_{eff}}{2m_{eff}p}$$

$$c_{eff} = 2\zeta m_{eff} p$$

5. **(2 pts)** If the metal springs were replaced with rubber springs of the same stiffness, would you expect the oscillation period to increase or decrease? Why?

$$\tau = \frac{1}{p\sqrt{1-\zeta^2}}, \qquad p \text{ is in Hz}$$

$$k_{rubber} = k_{metal}, \qquad c_{rubber} \gg c_{metal}, \qquad \tau \text{ increases}$$