chapter 13

13.1) setting the intracellular & extracellular concentrations as:

$C_{Int}(Na^+) = 10$ mM $\quad C_{Int}(K^+) = 147.5$ mM $\quad C_{Int}(Cl^-) = 10$ mM

$C_{Ext}(Na^+) = 145$ mM $\quad C_{Ext}(K^+) = 4.5$ mM $\quad C_{Ext}(Cl^-) = 115$ mM

The transmembrane potential can be found from:

$$\Delta V = V_2 - V_1 = \frac{K_B T}{ze} \ln \frac{c_1}{c_2} = \left(\frac{25}{z}\right) mV \times \ln \frac{c_1}{c_2}$$

$Na^+ \xrightarrow{z=+1} \Delta V = +66.85$ mV

$K^+ \xrightarrow{z=+1} \Delta V = -87.24$ mV

$Cl^- \xrightarrow{z=-1} \Delta V = -61.06$ mV

b) $C_{Sea}(Na^+) = 470$ mM $\quad C_{Sea}(K^+) = 10$ mM $\quad C_{Sea}(Cl^-) = 550$ mM

$Na^+ \rightarrow \Delta V = 96.25$ mV

$K^+ \rightarrow \Delta V = -67.28$ mV

$Cl^- \rightarrow \Delta V = -100.18$ mV

c) for part a) $\rightarrow V_{qss} = \dfrac{\gamma_{Na} V_{Na} + \gamma_K V_K + \gamma_{Cl} V_{Cl}}{\gamma_{Na} + \gamma_K + \gamma_{Cl}} = -73.22$ mV

for part b) $\rightarrow V_{qss} = -79.13$ mV

13.2) $D = \dfrac{\varepsilon_m}{\varepsilon_0}$ (dielectric const.) $\rightarrow \dfrac{C}{A} = \mathcal{C} = \dfrac{1}{d} \times D \times \varepsilon_0 = 8.85 \times 10^{-12} \times \dfrac{D}{d}$

lower range: $D = 3$, $d = 5nm \rightarrow \mathcal{C} = 8.85 \times 10^{-12} \frac{F}{m} \times \dfrac{3}{5 \times 10^{-9}} = 5.31 \times 10^{-3}$ F

upper range: $D = 6$, $d = 4nm \rightarrow \mathcal{C} = 8.85 \times 10^{-12} \times \dfrac{6}{4 \times 10^{-9}} = 13.28 \times 10^{-3}$ F

b) $Q = CV \rightarrow \sigma_{low} = CV = 3.71 \times 10^{-4} \frac{C}{m^2}$

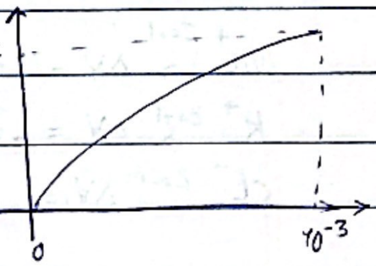
$\sigma_{high} = CV = 9.29 \times 10^{-4} \frac{C}{m^2}$

$\xi = \frac{\text{elementary charge } e}{\text{lipid head group}} \rightarrow \xi_{low} = 3.71 \times 10^{-4} \frac{C}{m^2} \times \frac{1}{1.6 \times 10^{-19}} \frac{e}{C} \times \frac{0.5 \times 10^{-18}}{1} \frac{m^2}{lipid} = 1.15 \times 10^{-3} \frac{e}{unit}$

$\xi_{high} = 9.29 \times 10^{-4} \times \frac{0.5 \times 10^{-18}}{1.6 \times 10^{-19}} = 2.9 \times 10^{-3} \frac{e}{lipid}$

13.3) $\underset{\text{length scale}}{\lambda} \cong \left(\frac{rK_{ax}}{2\gamma}\right)^{1/2}$ $\underset{\text{time scale}}{\tau} \cong \frac{\ell}{\gamma}$

$v = \frac{\lambda}{\tau} = \frac{\gamma\sqrt{rK_{ax}}}{\ell\sqrt{2\gamma}} = \frac{\sqrt{r\gamma K_{ax}}}{\sqrt{2}\,\ell} = 2.23 \times 10^{2} \sqrt{r} \ \frac{m}{s}$



13.4) 13.50) $\rightarrow \frac{dC_A}{dt} = -\lambda C_A + \frac{\gamma}{(1+KC_B^n)}$ $\frac{dC_B}{dt} = -\lambda C_B + \frac{\gamma}{(1+KC_A^n)}$

$\frac{du}{d\tau} = -u + \frac{\alpha}{1+v^n}$ $\frac{dv}{dt} = -v + \frac{\alpha}{1+u^n}$

$C_B \propto v \rightarrow C_B = K^{-1/n} v$

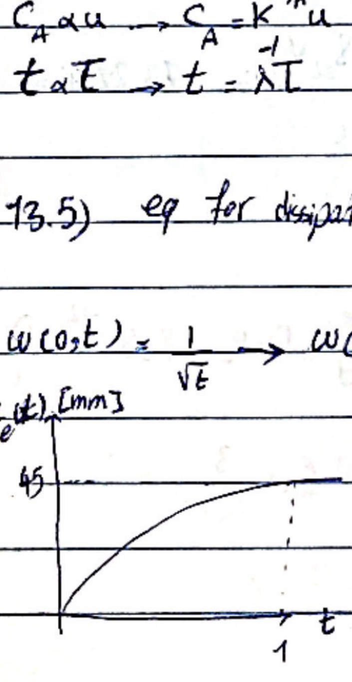
$C_A \propto u \rightarrow C_A = K^{-1/n} u$ $\frac{dC_A}{dt} \propto -\lambda C_A \rightarrow \left(K^{-1/n} \frac{du}{dt} \propto -\lambda K^{-1/n} u\right) \times \frac{d\tau}{du}$

$t \propto \tau \rightarrow t = \lambda^{-1} \tau$

$\frac{d\tau}{dt} = -\lambda$

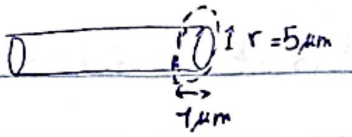
13.5) eq for dissipation of sub threshold signal is $w(x,t) = \frac{1}{\sqrt{t}} e^{(-x^2\tau/4\lambda^2 t)}$

$w(0,t) = \frac{1}{\sqrt{t}} \rightarrow w(x_e,t) = \frac{1}{e\sqrt{t}} = \frac{1}{\sqrt{t}} e^{\left(-\frac{x_e^2\tau}{4\lambda^2 t}\right)} \rightarrow \frac{x_e^2 \tau}{4\lambda^2 t} = 1 \rightarrow x_e = \frac{2\lambda\sqrt{t}}{\sqrt{\tau}}$



$x_e(t) = \frac{2 \times 10^{-3}}{0.045} \sqrt{t} = 0.045 \sqrt{t}$ m

$= 44.72 \sqrt{t}$ mm

13.6) 

$C = CA = 10^{-2} \times (\pi(5\times10^{-6})^2) = \overset{0.785}{\quad}$ pF

a) The charge passing through the axon during a 100mV potential change is

$$\Delta Q = C\Delta V = \frac{3.14\times10^{-12}}{4} \times 10^{-1} = \frac{0.314\times10^{-12}}{4}\,C = \frac{1.96\times10^{6}}{4}\,e = 0.49\times10^{6}\,e$$

b) The current moves outward at membrane $V_2^{mem} > V_1^{mem}$

c)
$$\frac{\text{ion flow}}{Na^{+}\text{ ions}} = \frac{\overset{0.49}{\times10^{6}}}{\underset{25\pi\times10^{-20}\times6.02\times10^{23}}{10^{-2}\times6.02\times10^{23}\times(\pi(5\times10^{-6})^2\times10^{-6})}} = \frac{0.49\times10^{6}}{4.72\times10^{5}} = 1.03$$

13.7) to substitute $\omega(x,t) = v(x,t)\cdot\exp(+t/\tau)$ into 13.26 we

$$\left(\lambda^2 \frac{d^2v}{dx^2} - \tau\frac{dv}{dt} = v\right)\times e^{at} = \lambda^2 e^{at}\frac{dv^2}{dx^2} - \left(\tau\frac{dv}{dt}e^{at} + v e^{at}\right) = 0$$

$$\lambda^2 e^{at}\frac{dv^2}{dx^2} - \tau\left(\frac{dv}{dt}e^{at} + \frac{v}{\tau}e^{at}\right) = \lambda^2 e^{\frac{t}{\tau}}\frac{dv^2}{dx^2} - \tau\frac{d}{dt}\left(v(x,t)\cdot e^{\frac{t}{\tau}}\right) = 0 \rightarrow a = \frac{1}{\tau}$$

$$= \lambda^2\frac{d^2}{dx^2}\left(v e^{\frac{t}{\tau}}\right) - \tau\frac{d}{dt}\left(v e^{t/\tau}\right) = 0 \quad \text{where } \boxed{v(x,t)\,e^{t/\tau} = \omega(x,t)}$$

b) substituting $\omega(x,t) = \frac{1}{\sqrt{t}}\exp(-x^2\tau/4\lambda^2 t)$ in the eq. we have:

$$\lambda^2\frac{d^2\omega}{dx^2} = \frac{\lambda^2}{\sqrt{t}}\frac{d}{dx}\left(\frac{-2x\tau}{4\lambda^2 t}e^{-\frac{x^2\tau}{4\lambda^2 t}}\right) = \frac{\lambda^2}{\sqrt{t}}\times\frac{-2\tau}{4\lambda^2 t}e^{-\frac{x^2\tau}{4\lambda^2 t}}\left(1 - \frac{2x\tau}{4\lambda^2 t}\right)$$

$$\tau\frac{d\omega}{dt} = e^{-\frac{x^2\tau}{4\lambda^2 t}}\left(-\frac{1}{2}t^{-\frac{3}{2}} + \frac{1}{\sqrt{t}}\frac{x^2\tau}{4\lambda^2 t^2}\right)$$

$$= \lambda^2\frac{d\omega^2}{dx^2} - \tau\frac{d\omega}{dt} = 0 \ ✓$$

13.8) the four steady state solutions of eq 13.31 are:

case 1) u_{ss} is small: $u_{ss} = \alpha^{1-n}$ $v_{ss} = \alpha$

case 2) v_{ss} is small: $u_{ss} = \alpha$ $v_{ss} = \alpha^{1-n}$

case 3) u_{ss} and v_{ss} are large: ($\alpha \gg 1$) $u_{ss} = v_{ss} = \alpha^{\frac{1}{1+n}}$

case 4) u_{ss} & v_{ss} are small ($\alpha \ll 1$) $u_{ss} = v_{ss} = \alpha$

for case 1) $\rightarrow g(x) = \dfrac{\alpha}{1+x^n} \xrightarrow{\frac{d}{dx}} g'(x) = \dfrac{-n\alpha x^{n-1}}{(1+x^n)^2}$

$$\xi(x_i, x_j) = g'(x_i)\, g'(x_j) = (n\alpha)^2 \times \frac{x_j^{n-1}\, x_i^{n-1}}{(1+x_j^n)^2\,(1+x_i^n)^2}$$

$u_{ss} = \alpha^{1-n}$, $v_{ss} = \alpha \rightarrow (n\alpha)^2 \times \dfrac{(\alpha^{1-n})^{n-1}}{(1+\alpha)^2} \times \dfrac{\alpha^{n-1}}{(1+\alpha^n)^2} = (n\alpha)^2 \dfrac{\alpha^{n-1}}{(1+\alpha)^2(1+\alpha^n)^2} = \dfrac{n^2\alpha^{n+1}}{(1+\alpha)^2(1+\alpha^n)^2}$

for $\alpha \gg 1 \rightarrow \xi(u_{ss}, v_{ss}) \approx n^2 \dfrac{\alpha^{n+1}}{\alpha^{2n+2}} \rightarrow 0$ stable ✓

for case 2) $\xi(u_{ss}, v_{ss}) = n^2 \dfrac{\alpha^{n+1}}{(1+\alpha)^2(1+\alpha^n)^2}$ Same as case 1, stable ✓

for case 3) $\xi(u_{ss}, v_{ss}) = \dfrac{(n\alpha)^2\, \alpha^{-(1+n)(n-1)}}{(1+\alpha^{-(1+n)n})^4} = \dfrac{n^2\alpha^2\, \alpha^{-(n^2-1)}}{(1+\alpha^{-n^2-1})} = n^2$ stable for $n^2 < 1$

$\alpha \ll 1$ case 4) $\xi(u_{ss}, v_{ss}) = \dfrac{(n\alpha)^2\, \alpha^{2n-2}}{(1+\alpha^n)^4}$ unstable

13.9) for steady state $\rightarrow \dfrac{du}{dt} = \dfrac{dv}{dt} = 0$

$u_{ss} = \dfrac{\alpha}{(1+v_{ss}^n)}$ $v_{ss} = \dfrac{\alpha}{(1+u_{ss}^n)} \rightarrow u_{ss}\left(1 + \dfrac{\alpha}{1+u_{ss}}\right) = \alpha$

$u_{ss}(1+u_{ss}^n+\alpha) = \alpha + \alpha u_{ss}^n \rightarrow u_{ss} + u_{ss}^{n+1} + \alpha u_{ss} - \alpha - \alpha u_{ss}^n$

(?) $\alpha(u_{ss} - u_{ss}^n + 1) + u_{ss}(1+u_{ss}^n$