

chapter 6

6.1) The filament density can be found using:

$$\begin{aligned} & \text{molecular mass of each actin monomer} \rightarrow 42 \frac{\text{kDa}}{\text{monomer}} \times 370 \frac{\text{monomer}}{\mu\text{m}} \times 1.66 \times 10^{-27} \frac{\text{kg}}{\text{Da}} \times 5 \frac{\mu\text{m}}{\text{filament}} = 12.9 \times 10^{-20} \frac{\text{kg}}{\text{filament}} \\ & 3 \frac{\text{kg}}{\text{m}^3} \times \frac{1}{12.9 \times 10^{-20} \frac{\text{kg}}{\text{filament}}} = 0.23 \times 10^{20} \frac{\text{filament}}{\text{m}^3} \end{aligned}$$

nominal value of $\rho k_B T$ is then found to be: $0.23 \times 10^{20} \times 300 \times 1.38 \times 10^{-23} = 0.095 \frac{\text{J}}{\text{m}^3}$

b) equation D.35 states $\frac{1}{\beta k_V} = \frac{\langle (\Delta V)^2 \rangle}{V_0}$

$$\frac{\langle \Delta V^2 \rangle^{1/2}}{V} = \left(\frac{1}{\beta k_V} \right)^{1/2} \times \frac{1}{V^{1/2}} = \rho^{-1/2} \times V^{-1/2} = \left(\frac{4}{3} \pi (5 \times 10^{-6})^3 \right)^{-1/2} \times \rho^{-1/2} \approx 0.01$$

6.2) The persistence length of the peptide can be found from

$$\langle r_{ee}^2 \rangle = 2 \xi_p L_c \rightarrow \xi_p = \frac{\langle r_{ee}^2 \rangle}{2 L_c} = 0.21 \text{ nm}$$

and spring constant k_{sp} from:

$$k_{sp} = \frac{3 k_B T}{2 \xi_p L_c} = \frac{3 \times 1.38 \times 10^{-23} \times 300}{2 \times 0.21 \times 10^{-9} \times 4 \times 10^{-9}} = 7.4 \times 10^{-3} \frac{\text{J}}{\text{m}^2}$$

b) $k_V = \frac{k_{sp}}{\epsilon b} = \frac{7.4 \times 10^{-3}}{8 \times 10^{-9}} = 9.25 \times 10^5 \frac{\text{J}}{\text{m}^3}$

c) The bond density is $\rho = \frac{3}{2 a^2 b}$ so the value of $\rho k_B T = 36.7 \times 10^5 \frac{\text{J}}{\text{m}^3}$

compare \rightarrow

3D

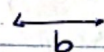
6.4) simple cubic $\rightarrow \rho^* = \frac{2d}{z} = 1 \rightarrow z = 6$

body-centered cubic $\rho^* = \frac{3}{4} \rightarrow z = 8$

face-centered cubic $\rho^* = \frac{1}{2} \rightarrow z = 12$

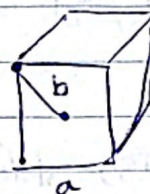


for simple cubic $\rightarrow V = b^3$



for body centric

$\frac{\sqrt{3}}{2} a = b \rightarrow V = \frac{2^3}{3\sqrt{3}} b^3$



$\frac{\sqrt{2}}{2} a = b \rightarrow V = \frac{b^3}{2\sqrt{2}}$

6.5) The concentration of filaments in the system is: $\rho = 1600 \pm 600 \frac{\text{filament}}{\mu\text{m}^3}$

$D_f \text{ actin} = 8 \text{ nm}$ $L_c = 1 \mu\text{m}$

$= 1.6 \pm 6 \times 10^{20} \frac{\text{filament}}{\text{m}^3}$

\rightarrow Then transition number densities for rigid rods is

$\rho^* L_c^{-3} = 10^{18}$

$\rho^{**} \sim (D_f L_c^2)^{-1} \approx 10^{20} \times 1.25$


so the system may be isotropic or nematic since the concentration of filaments is in fact far above ρ^{**} , so they could be in a nematic phase

b) assuming the thermal energy of each particle to be $\frac{3}{2} k_B T$, for this concentration the pressure is found to be:

$P = 2.94 \times 10^{20} \frac{\text{J}}{\text{m}^3}$

Hilroy

6.6) $D_f = 0,9 \mu\text{m}$ $L_c = 2,7 \mu\text{m}$



The densities are evaluated to be $\rho^* = L_c^{-3} = 5,1 \times 10^{16} \frac{\text{bacteria}}{\text{m}^3}$

$$\rho^{**} = (D_f L_c^2)^{-1} = 75,2 \times 10^{16} \frac{\text{bacteria}}{\text{m}^3}$$

b) Onsager's condition for the existence of a nematic phase is that $\frac{L_c}{D_f} > \sim 3$ and this condition is met here.

6.7) Onsager's condition: $\frac{L_c}{D_f} > \sim 3$

$$\frac{L_c}{D_f} = \frac{260 \text{ nm}}{18 \text{ nm}} \gg 3 \quad \checkmark \text{ satisfies}$$

b) $\rho^* = L_c^{-3} = 5,7 \times 10^{19} \frac{\text{virus}}{\text{m}^3}$

$$\rho^{**} = (L_c^2 D_f)^{-1} = 7,07 \times 10^{21} \frac{\text{virus}}{\text{m}^3}$$

$$\rho_N = \frac{4,25}{D_f L_c^2} = 4,55 \times 10^{21} \frac{\text{virus}}{\text{m}^3}$$

to find the concentrations we must find the mass of each virus:

$$\rightarrow \lambda_L \left[\frac{\text{mass}}{\text{length}} \right] = 140,000$$


mass of each virus is then found to be: $\lambda_L \left[\frac{\text{mass}}{\text{length}} \right] \times \text{length of virus} = \text{mass of virus}$

$$\rightarrow \lambda_L \times L = 140000 \times 260 \times 10^{-9} = 364 \times 10^{-4} \frac{\text{mass}}{\text{virus}}$$

$$\rightarrow C^* = 2,01 \times 10^{18} \frac{\text{kg}}{\text{m}^3} \text{ or } \frac{\text{mg}}{\text{ml}}$$

$$\rightarrow C^{**} = 3,9 \times 10^{19} \frac{\text{kg}}{\text{m}^3} \text{ or } \frac{\text{mg}}{\text{ml}}$$

$$C_N = 1,66 \times 10^{20} \frac{\text{kg}}{\text{m}^3} \text{ or } \frac{\text{mg}}{\text{ml}}$$

6.8)  $D \approx 8.1 \mu\text{m}$

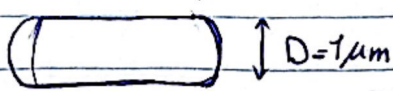
$$140 \mu\text{m}^3 = \omega \times \pi \left(\frac{D}{2}\right)^2 \Rightarrow \omega = \frac{140}{\pi (4.05)^2} = 2.72 \mu\text{m}$$

$$\frac{T}{D} = \frac{2.72}{8.1} = 0.336 > 0.2 \quad \checkmark \rightarrow \text{The disks can form ordered phases}$$

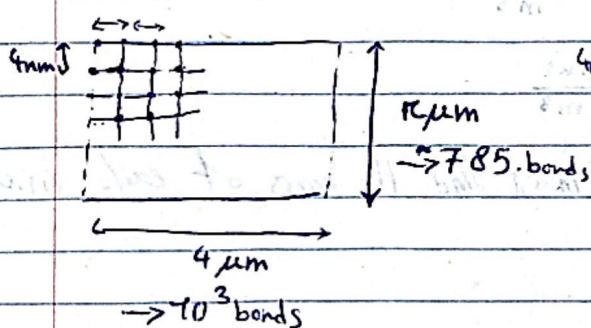
b) approx concentration: $5.4 \times 10^6 \frac{\text{cell}}{\text{mL}} \times 10^9 = 5.4 \times 10^{15} \frac{\text{cell}}{\text{m}^3}$

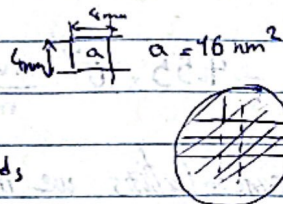
$$\frac{4}{D^3} = \frac{4}{(8.1)^3 \times 10^{-18}} = 7.5 \times 10^{15} \frac{\text{cm}^3}{\text{m}^3}$$

ρ is not greater than $\frac{4}{D^3}$, so the condition for ordered phase is not met.

6.9)  $\text{length} = 4 \mu\text{m}$
 $D = 1 \mu\text{m}$
 four-fold network & $b = 4 \text{ nm}$

to find the total number of bonds on the surface of this bacterium

 $4 \mu\text{m}$
 $4 \mu\text{m}$
 $\rightarrow 10^3 \text{ bonds}$

 $a = 16 \text{ nm}$
 $\rightarrow 785 \text{ bonds}$

$$(2A = 2 \times \frac{4\pi R^2}{2} = 4\pi R^2) / a = \frac{2\pi \times 10^{-12}}{16 \times 10^{-18}} = 4.2 \times 10^5$$

the approximate number of bonds is around $7.86 \times 10^5 + 4.2 \times 10^5$
 $= 8.25 \times 10^5$

so we need $8.25 \times 10^5 \text{ bond} \times \frac{1}{6.022 \times 10^{23}} = 1.4 \times 10^{-18} \text{ mole of this drug}$

6.10) $\Delta U = -\pi a^2 \sigma^2 / Y$ for a crack of length $2a$

critical
stress
for fracture

$$\sigma_c \sim (2Y\gamma_s / \pi a)^{1/2} \rightarrow \Delta U = \frac{-\pi a^2 \sigma_c^2 (2Y\gamma_s)}{Y\pi a} = -2a\gamma_s$$

$$\sigma_c = \left(\frac{2 \times 10^9 \times 5 \times 10^{-2}}{\pi \times 10^{-8}} \right)^{1/2} = (0,31 \times 10^{16})^{1/2} = 0,56 \times 10^8 \text{ J/m}$$