Chapter 10

2D tension
"stress resultant" [energy/A]
= mean stress x shell thickness
$\langle\sigma\rangle \times d_p$
=

tension [N]
stress [N/A]

pressure Δp:

10.1) $\Delta p = 1\,atm = 10^5\, J/m^3$

thickness of plate

$K_V = 3\times10^9\, J/m^3$ & for solid thin plates : $K_A \sim K_V d_p$

pressure difference $p \rightarrow$ lateral tension

for a sphere with thickness $d_p \rightarrow \tau = \frac{rp}{2}$

i) for a spherical cell with $r = 10^{-5}\,m \rightarrow \tau = \frac{10^{-5}\times10^{5}}{2} = 0{,}5\, J/m^2$

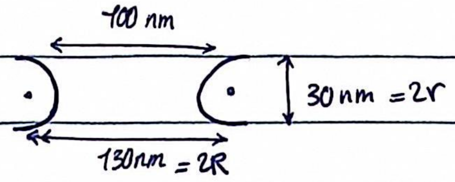
maximum allowed tension $= 1.05\, K_A = 1.05 \times K_V d_p = 3.15\times10^9 d_p = 0{,}5$

$\rightarrow d_p = 1{,}6\times10^{-10}\,m$ minimum thickness for cell

ii) for a spherical weather ballon with $r = 10\,m \rightarrow \tau = \frac{10\times10^5}{2} = 0{,}5\times10^6\, J/m^2$

max tension $\rightarrow = 3.15\times10^9 d_p = 0{,}5\times10^6$

$\rightarrow d_p = 1.6\times10^{-4}\,m$ min thickness for ballon

10.2) 

$\rho = \frac{13}{3}$

bending energy per pore $= \pi K_b(-8+\pi\rho) - 4\pi K_G = 20\pi K_BT\,(\frac{13}{3}-8) = -230.4\, k_BT$

Total bending energy $= 8.06\times10^5\, K_BT$

energy per ATP hydrolysis $= 8\times10^{-20}\,J \approx 20\, K_BT$

$\rightarrow$ we need around 4×10^4 ATP molecules

10.3) The bending energy of the structure is the sum of bending energies of the "pancake" & "neck" structures

• B.E of 3 pancakes with $\rightarrow D = 60r$: $E_b^{P_1} = 20\pi k_BT(8 + 30\pi) = 5419\ k_BT$

• " " 4 pancakes " $\rightarrow D = 20r$: $E_b^{P_2} = 20\pi k_BT(10\pi - 8) = 468\ k_BT$

• " " 4 necks " with $D = 10r$: $E_b^{N} = 20\pi k_BT(-8 + 5\pi) = 154\ k_BT$

$$E_{Total}^{bending} = 3E_b^{P_1} + 4E_b^{P_2} + 4E_b^{N} = 18745\ k_BT$$

10.4) bending energy of the pancake can be found by noting

$C_m^{edge-pancake} = \frac{1}{r} = 5\ \frac{1}{\mu m} \rightarrow r = 0.2\ \mu m$ $\quad E_{bend}^{pancake} = \pi K_b(8 + 50\pi) + 4\pi K_G$

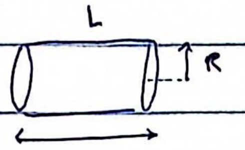
$C_m^{bud} = 1\ \mu m^{-1} \rightarrow r = 1\ \mu m$ $\quad E_{bend}^{bud} = \pi^2 k_b \frac{\varphi^2}{\sqrt{(\varphi^2 - 1)}} = \pi^2 k_b \cdot \frac{9}{2\sqrt{2}}$

$$E_{Total}^{bend} = \pi k_b\left(8 + 50\pi + \frac{9\pi}{2\sqrt{2}}\right) + 4\pi k_G$$

ii) $E_{Sphere} = 4\pi(2k_b + k_G)$

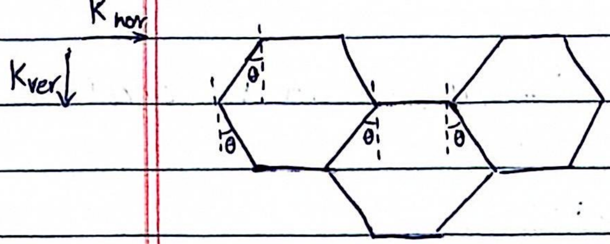
$$\frac{\Delta E}{k_bT} = \pi^2 \frac{k_b}{k_BT}\left(50 + \frac{9}{2\sqrt{2}}\right) = 10497$$

(10.5) The effective spring constant of a cylindrical rod is $K_{sp} = \frac{\pi R^2 Y}{L}$



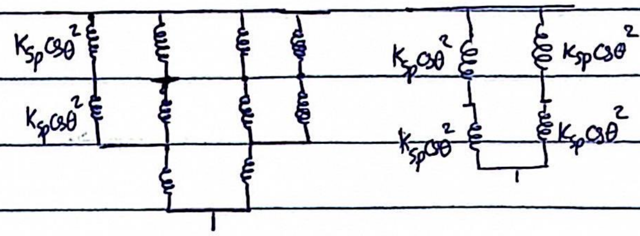
$$K_{sp} = \frac{\pi \times (3.5\times10^{-9})^2 \times 10^{9}}{17\times10^{-9}} = 2.26\ J/m^2$$

To find the effective spring constant we break K_{sp} into vertical & horizontal components.



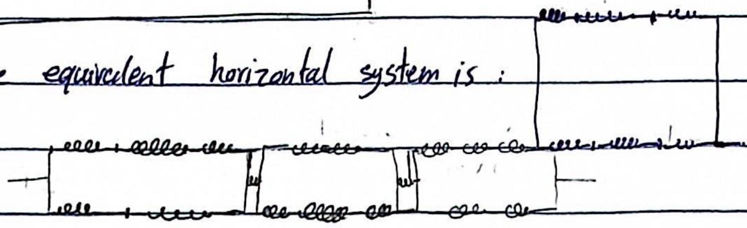
$K_{ver} =$

The equivalent vertical system would be:



$$\boxed{K_{ver} = K_{sp}\cos\theta^2 = \frac{3}{4}K_{sp}}$$

The equivalent horizontal system is:



for three springs in series: $K_{sp}\sin\theta^2 \quad K_{sp} \quad K_{sp}\sin\theta^2$

$$\frac{1}{K_T} = \frac{1}{K_{sp}} + \frac{2}{K_{sp}\sin\theta^2} \rightarrow K^{(3)}_{Total} = K_{sp}\left(1 + \frac{2}{\sin\theta^2}\right)^{-1} = \frac{\sin\theta^2}{2+\sin\theta^2}K_{sp}$$

for two springs in parallel:

$$K^{(2)}_{Tot} = \frac{\sin\theta^2}{1+\sin\theta^2}K_{sp}$$

The system can now be simplified as:

I don't think I'm doing this right

but after finding $K_{eff} \rightarrow K_A = \left(\frac{K_{sp}}{2\sqrt{3}}\right)\left(1 - \frac{\sqrt{3}\tau}{K_{sp}}\right)$

$$\xrightarrow{\tau=0} K_A$$

$$[E] = [K_A]$$

Ksp
$S_0 =$
r sphere
$p = ?$
τ_{exp}

10.6) The spring length under tension is found from the relation:

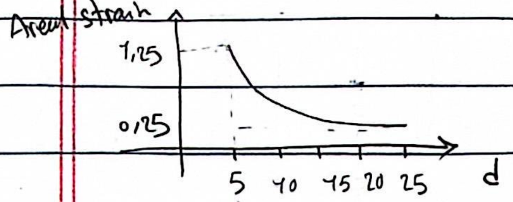
$$S_\tau = \frac{S_0}{1 - \frac{\sqrt{3}\tau}{K_{sp}}} \qquad \tau_{exp} = \frac{K_{sp}}{\sqrt{3}} \approx 5{,}8 \times 10^{-3}\ \mathrm{J/m^2} \rightarrow \tau \text{ for expansion without limit}$$

$$\tau = S d_{sh}$$

p internal pressure
r
area

10.7) The change in relative area of the cell wall can be calculated as follows:

$$\frac{a - a_0}{a} = u_{xx} + u_{yy} = \frac{\tau}{K_A} = \frac{rp}{2K_A} = \frac{rp}{2d_{sh}K_V} = \frac{0{,}5 \times 10^{-6} \times 5 \times 10^{5}}{2 \times 2 \times 10^{7} \times d_{sh}} = \frac{6{,}25 \times 10^{-5}}{d_{sh}}$$



10.8) the reduced volume of a torus is:

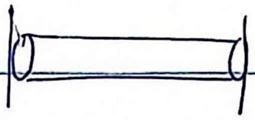
$$\nu_{red} = \frac{6\sqrt{\pi}V}{A^{3/2}} = 6\sqrt{\pi} \times \frac{2\pi^2 r^2 R}{(4\pi^2 r R)^{3/2}} = \frac{12\pi^2\sqrt{\pi}\, r^2 R}{8\pi^3 r^{3/2} R^{3/2}} = \frac{3}{2} \times \frac{\sqrt{r}}{\sqrt{\pi R}}$$

$$\rightarrow \nu_{red} = \frac{3}{2\sqrt{\pi}} \times \sqrt{\frac{r}{R}}$$

and the bending energy is

$$E_{bending} = 2\pi^2 K_b \times \frac{\left(\frac{R}{r}\right)^2}{\left(\left(\frac{R}{r}\right)^2 - 1\right)^{1/2}} = 2\pi^2 K_b \frac{\left(\frac{3}{2\sqrt{\pi}\,\nu_{red}}\right)^4}{\left(\left(\frac{3}{2\sqrt{\pi}\,\nu_{red}}\right)^4 - 1\right)^{1/2}}$$

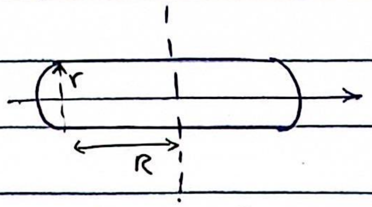
10.9)



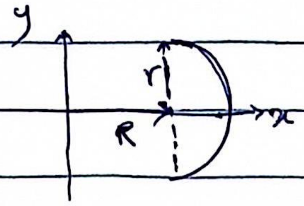
$$F_{buckle} = \frac{\pi^2 Y I}{L_c^2} = \frac{1}{2}\mu R^2 \times \left(\frac{\pi}{L_c}\right)^2 \times Y = 4.9\times10^{7}\,M \quad \text{for } L_c = 20\,\mu m$$

$$= 1.97\times10^{6}\,\mu \quad \text{for } L_c = 100\,\mu m$$

10.10)



The area of the pancake is the sum of the areas of the two plates, plus the area of the circular edge. $A = 2\pi R^2 + A_{edge}$



$x^2 + y^2 - 2Rx + R^2 = r^2$

equation of curve: $(R-x)^2 + y^2 = r^2$

$\rightarrow f(x) = \sqrt{r^2 - (x^2 - R^2)}$

$g(y) = \sqrt{r^2 - y^2} + R$

the surface area can be found from relation: $S = 2\pi \int x\sqrt{1 + (f'(x))^2}\,dx$

$$S = 2\pi \int x \times \sqrt{1 + \left(-\frac{x}{y}\right)^2}\,dx = 2\pi \int \frac{x}{y}\sqrt{r^2 - R^2 + 2Rx}\,dx$$

$$S = 2\pi \int g(y) \times \sqrt{1 + \left(\frac{y}{g(y)}\right)^2}\,dy = 2\pi \int \sqrt{g(y)^2 + y^2}\,dy = 2\pi \int (R^2 + (r^2 - y^2) + y^2 + R\sqrt{r^2$$

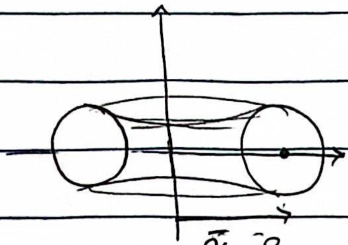
$$= 2\pi \int (R^2 + r^2 - 2R\sqrt{r^2 - y^2})^{\frac{1}{2}}\,dy$$

$$V = \int dV = \int dy \times \pi \times (R + \sqrt{r^2 - y^2})^2 = \pi R^2 (2r) + 2R\pi \int \sqrt{r^2 - y^2}\,dy + \pi \times \frac{4}{3} r^3$$

$$= 2\pi R^2 r + 2\pi R \times \frac{\pi r^2}{2} + \frac{4}{3}\pi r^3$$

$$\rightarrow V = 2\pi r^3 \left[\left(\frac{R}{r}\right)^2 + \left(\frac{\pi}{2}\right)\frac{R}{r} + \frac{2}{3}\right]$$

10.11) using pappus's Theorem we can find the volume & Area of the torus by knowing the centroid of the circle whos revolution around the y-axis results in the torus.



$$V = 2\pi\bar{x}A = 2\pi R \times \pi r^2 = 2\pi^2 r^2 R$$

$$A = 2\pi\bar{x}S = 2\pi R \times 2\pi r = 4\pi^2 r R$$

$$V_{red} = \frac{6\sqrt{\pi}\,V}{A^{3/2}} = 6\sqrt{\pi} \times \frac{2\pi^2 r^2 R}{(4\pi^2 r R)^{3/2}} = \frac{12\pi^2\sqrt{\pi}\, r^2 R}{8\pi^3 r^{3/2} R^{3/2}} = \frac{3}{2\sqrt{\pi}}\frac{\sqrt{r}}{\sqrt{R}} = \frac{3}{2\sqrt{\pi}}\sqrt{\frac{r}{R}}$$