Section 9

according to section 9.2.2

9.1) The counter ions reside on one side of the plate, so we have:

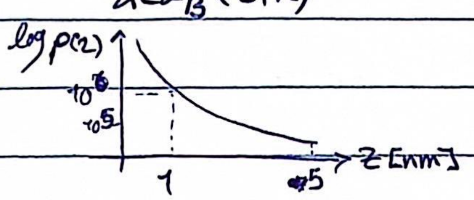
$\boxed{\sigma_s = 0{,}3\, C/m^2}$ $\quad \varepsilon = 80\varepsilon_0 \quad T = 300\,K \quad q = e$

a) $$l_B = \frac{q^2}{4\pi\varepsilon K_B T} = \frac{(1.6\times10^{-19})^2}{4\pi\times80\varepsilon_0\times K_B T} = 6.95\times10^{-10}\,m$$

$$\chi = \frac{q}{-2\pi l_B \sigma_s} = -1.22\times10^{-10}\,m$$

$$\rho = \frac{1}{2\pi l_B (z+\chi)^2} \xrightarrow{z=0} \frac{1}{2\pi l_B \chi^2} = 1.54\times10^{28}\,\frac{unit}{m^3} \approx 2.5\times10^{4}\,\frac{Mole}{m^3} \approx 2.5\times10^{7}\,M$$

b)



c) $\rho[z] = 0{,}2\,M$

$\rightarrow \boxed{z = 1.38\,\mu m}$

at values $z = 1.38\,\mu m$ and above, ρ falls bellow 0,2 molar

d) $$E = \frac{\sigma_s z}{2\varepsilon} = \boxed{0{,}21\,\frac{V}{m}}$$ magnitude of the electric field is 0,21

9.2)

a) $$l_B = \frac{q^2}{4\pi\varepsilon K_B T} \xrightarrow[\varepsilon \text{ as } 9.1]{\text{using the same}} = 6.95\times10^{-10}\,m$$

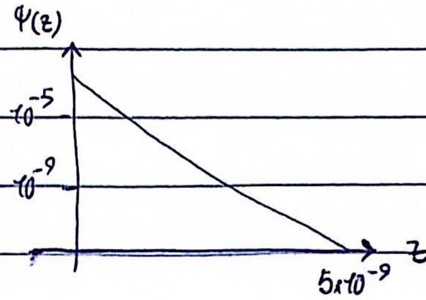
$$l_D = (8\pi l_B \rho_s)^{-1/2} = (8\pi\times6.95\times10^{-10}\times0{,}2\times6.022\times10^{23}\times\underbrace{10^{3}}_{l^{-3}\to m^{-3}})^{-1/2} = 2.18\times10^{-10}\,m$$

page 336

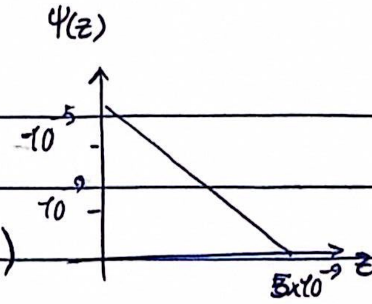
b) $$\sigma_s = \frac{\varepsilon\psi_0}{l_D} = 4{,}06\times10^{-2}\,\frac{C}{m^2}$$

c) from debye approximation:

$$\psi(z) = -0{,}0125\times e^{\left(\frac{-z}{l_D}\right)}$$



$$-\frac{d}{dD}\left(\frac{V(D_s)}{A}\right)\left(\frac{-\pi\rho^2 C_{vdw}}{12 D_s^2}\right) = +2\pi\rho^2 C_{vdw} \times \frac{1}{12 D_s^3} =$$

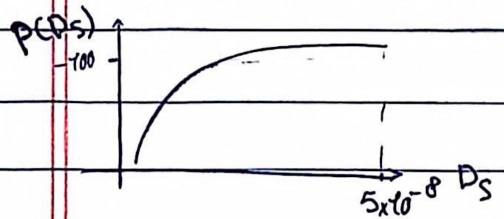


using the result of 9.12)

$$\Psi(z) = 2\ln\left(\frac{1+\alpha}{1-\alpha}\right) \quad \text{where } \alpha = \tanh(\Psi_0/4)\cdot\exp(-z/\ell_D)$$

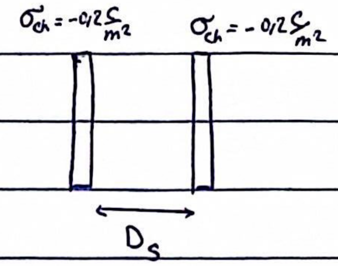
9.3) Using the potential energy density 9.20(b)? $\xrightarrow{\text{probably}}$ 9.15(b)

$$\frac{V(D_s)}{A} = \frac{-\pi C_{vdw}\rho^2 d_{sh}^2}{2D_s^4} \quad P = \xrightarrow[\text{van der walls pressure}]{} -\frac{d}{dD_s}\left(\frac{V(D_s)}{A}\right) = -\frac{4\pi C_{vdw}\rho^2 d_{sh}^2}{2D_s^5}$$



compare to eq 8.38?

that part does not make sence



9.4)

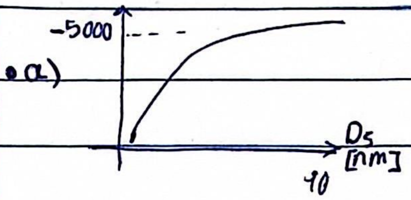
a) The pressure between charged plates in the absent of salts has the form

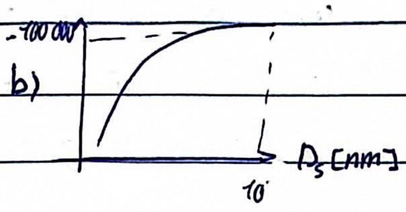
$P = -2\sigma_s k_B T/(q D_s)$ in the ideal gas limit

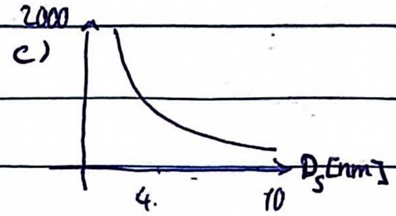
$P = \pi k_B T/2\ell_B D_s^2$ for large D_s

9.5)

- for small D_s : $p = -\frac{d}{dD_s}\left(\frac{-\pi \rho^2 C_{vdw}}{12 D_s^2}\right) = -\pi \rho^2 C_{vdw} / 6D_s^3$
  semi-infinate slabs
- for large D_s : $p = -\frac{d}{dD_s}\left(\frac{-\pi C_{vdw} \rho^2 d_{sh}^2}{2D_s^4}\right) = -\frac{2\pi C_{vdw} \rho^2 d_{sh}^2}{D_s^5}$
  two sheets
- repulsion from undulations $P_{entropic} = \frac{2C_{fl}(k_BT)^2}{k_b D_s^3}$







9.6)

Top view   side view

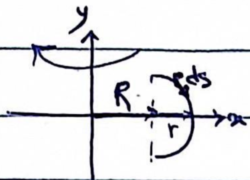
9.8) 

The total area of the vesicle is the sum of the top and bottom plates plus the area on the sides.

$$A_{Total} = 2 \times (\pi R^2) + A_{Side}$$

The side area can be calculated using formula $S = 2\pi \int |x| \, ds = A_{side}$

rotation about the y-axis



where ds is the arc element on curve $(x-R)^2 + y^2 = r^2$ for $R < x < R+r$ & $-r < y < r$

and integrating over the bounds gives us the desired surface area.

we have:

$$x = \sqrt{(r^2 - y^2)} + R \qquad ds = \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \, dx$$

$$A_{side} = 2\pi \int |x| \sqrt{1 + \left(\frac{x-R}{y}\right)^2} \, dx = 2\pi \int_0^r |x| \times \frac{r}{(r^2 - (x-R)^2)^{1/2}} \, dx \times 2$$

$$\frac{dy}{dx} = \frac{d}{dx}\left(\left(r^2 - (x-R)^2\right)^{1/2}\right) = \frac{1}{2} \, \frac{\times -2 \times (x-R)}{(r^2 - (x-R)^2)^{1/2}} = -\left(\frac{x-R}{y}\right)$$

setting $x - R = u$ we have:

$dx = du$

$$A_{side} = 4\pi r \int_R^{r+R} \frac{u+R}{R(r^2-u^2)^{1/2}} \, du = 4\pi r \left[-\sqrt{r^2-u^2} \Big|_R^{r+R} + R \tan^{-1}\left(\frac{u}{\sqrt{r^2-u^2}}\right) \Big|_R^{r+R} \right]$$

$= 4\pi r\,($