Section 8

8.1) deformation energy $E = \pi \kappa_b \left(\frac{h_0}{w}\right)^2$

$E = \pi \times 10 K_B T \times \left(\frac{h_0}{1\mu m}\right)^2 = K_B T \rightarrow h_0 \simeq 0{,}18 \mu m$

b) we can find the bending contribution from bending energy density,

$F = \left(\frac{K_b}{2}\right)(C_1 + C_2)^2 + K_G C_1 C_2$

where $C_1 + C_2 = h_{xx} + h_{yy}$ & $C_1 C_2 = h_{xx} h_{yy} - h_{xy}^2$

and calculate E_b as $\int (C_1+C_2)^2 dA \times \left(\frac{K_b}{2}\right) + K_G \int C_1 C_2 dA$

we have: $\underbrace{\frac{K_b}{2} \int (C_1+C_2)^2 \sqrt{1+h_x^2+h_y^2}\, dx dy}_{\mathrm{I}} + \underbrace{K_G \int C_1 C_2 \sqrt{1+h_x^2+h_y^2}\, dx dy}_{\mathrm{II}}$

$h(x,y) = h_0 \exp(-[x^2+y^2]/2w^2)$

$h_{xx} = + h_0 \times \frac{\partial}{\partial x} \left[\exp\left(\frac{-(x^2+y^2)}{2w^2}\right) \times \left(\frac{-2x}{2w^2}\right)\right] =$

$= \frac{-2h_0}{2w^2} \exp\left(\frac{-(x^2+y^2)}{2w^2}\right) + \frac{4h_0 x^2}{4w^4} \exp\left(\frac{-(x^2+y^2)}{2w^2}\right) = h_0 \left(\frac{x^2}{w_0^4} - \frac{1}{w_0^2}\right) \exp\left(\frac{-(x^2+y^2)}{2w_0^2}\right)$

$h_{yy} = h_0 \left(\frac{y^2}{w_0^4} - \frac{1}{w_0^2}\right) \exp\left(\frac{-(x^2+y^2)}{2w_0^2}\right)$

$h_{xy} = \frac{h_0 \times xy}{w_0^4} \exp\left(\frac{-(x^2+y^2)}{2w_0^2}\right)$

$h_x = \frac{\partial h}{\partial x} = -h_0 \frac{x}{w^2} \exp\left(\frac{-(x^2+y^2)}{2w^2}\right) \quad h_y = \frac{\partial h}{\partial y} = -h_0 \frac{y}{w^2} \exp\left(\frac{-(x^2+y^2)}{2w^2}\right)$

$\sqrt{g} = \sqrt{1+h_x^2+h_y^2} = \sqrt{1 + \frac{h_0^2}{w^4}(x^2+y^2) \exp\left(\frac{-(x^2+y^2)}{2w^2}\right)}$

(I) $$\frac{K_b}{2}\int\sqrt{g}\,(C_1+C_2)^2\,dx\,dy = \frac{h_0^2}{w^8}\iint e^{\frac{-(x^2+y^2)}{w^2}}(-2w^2+x^2+y^2)^2\sqrt{1+\frac{e^{\frac{-(x^2+y^2)}{2w^2}}h_0^2(x^2+y^2)}{w^4}}\,dx\,dy$$

(II) $$KG\int\sqrt{g}\,C_1C_2\,dx\,dy \xrightarrow[\text{into mathematica}]{\text{Plugging it}} \frac{4h_0^2}{16w^4}xy\exp\left(-\frac{(x^2+y^2)}{w^2}\right)$$

$$+\frac{\sqrt{2\pi}y}{2w}e^{\frac{-y^2}{2w^2}}\mathrm{Erf}\left[\frac{x}{\sqrt{2}w}\right]-\frac{2\sqrt{\pi}x}{8w^3}e^{\frac{-x^2}{2w^2}}\mathrm{Erf}\left[\frac{y}{w}\right]$$

$$+\frac{\mathrm{Erf}\left[\frac{x}{w}\right]}{16w^4}\left(-2e^{\frac{-y^2}{w^2}}h_0\sqrt{\pi}wy+h_0\pi w^2\,\mathrm{Erf}\left[\frac{y}{w}\right]\right)+8e^{\frac{-x^2}{2w^2}}\sqrt{2\pi}w^3\times\mathrm{Erf}\left[\frac{y}{\sqrt{2}w}\right]$$

in theory we should be able to find the term for bending energy using the integrations above and comparing it to the E but the integration above is non-trivial and I'm not sure how I should proceed.