Assignment-2 REPORT

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QUESTION-6

Contents

PCA for Another Image Dataset

1 Finding Mean, Covariance and Eigen vectors,values

Similiar to the previous question analysis, We have the data set of 16 fruits each of which is a **19200x1** column vector.So our total data set is represented by a single matrix **D** = **19200x16**, each column represents a fruit.

All the steps of finding mean,Covariance etc are Exactly same as **Q4**, Mean μ = average of all the 16 columns = (sum of all 16 columns)/16.So mean can be found easily and same as previous questions.Also the covariance matrix can be found from **D** and μ. If **S = D** - μ, then **C = S*S**^{T}**/N**.Finding them is same.

Now using **C** I found the top-4 Eigen **vectors** of **C** and the top-10 **Eigen values** of C.

Displaying the **mean** and the 4-**eigen vectors** as Images:

I reshaped those vectors to 80x80x3 and scaled to [0,1] by dividing with 255.Then used the image() function to visualise those images. Also I used imwrite() to save those images in the current directory.I used the subplot() to plot the 4 eigen-vector images in a single plot.

The obtained images are:

Figure 1: Mean of all the Fruits

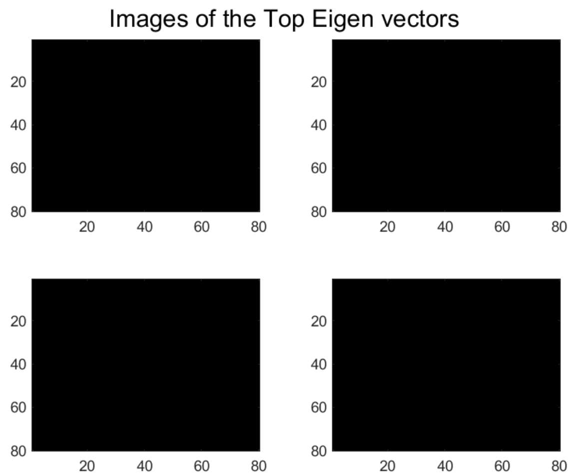


Figure 2: Top-4 eigen vectors as images

Plotting the Top-10 eigen values:

I found the top10 eigen values of **C** using the eigs(C) function.And plotted them. The obtained plot is:

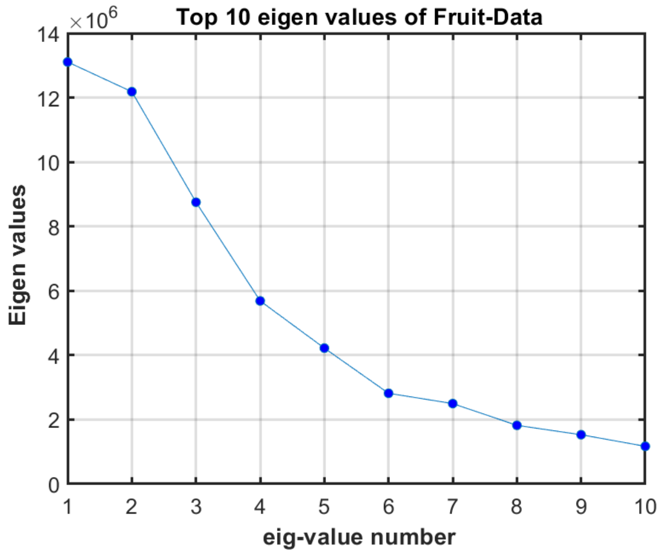


Figure 3: Top-10 eigen values

2 Closest Representations of the fruit images.

Let $\mathbf{u_1}, \mathbf{u_2}, \mathbf{u_3}, \mathbf{u_4}$ be the **unit** eigen vectors (The top-4 eigen unit vectors) and let μ be the mean.So now we need to find coefficients a_1, a_2, a_3, a_4 so that the vector **c** $= \mu + a_1 u_1 + a_2 u_2 + \ldots + a_4 u_4$ is as close to the original Fruit data vector **x** as possible. Here we perform computations assuming all are Column vectors, i.e, we represent the images as 19200x1 column vectors.

For closeness, we need the norm ||**x**-**c**|| to be as small as possible.norm is nothing but square root of the sum of squares of the vector.So we need the sum of squares of components of **x**-**c** to be as small as possible.

$$\sum_{i=1}^{19200} (x_i - \mu_i - a_1 u_{1i} + \ldots + a_4 u_{4i})^2 \qquad \textbf{minimum}$$

Now if we expand the sum, then we get the following

$$||\mathbf{x}||^2 + ||\mu||^2 + \sum_{k=1}^{4} [(a_k)^2 ||\mathbf{u_k}||^2 - 2a_k(\mathbf{u_k} \cdot \mathbf{x})] + \sum_{i=1}^{4}\sum_{j=1}^{4} a_i a_j (\mathbf{u_i} \cdot \mathbf{u_j})$$

First 2 terms are constant and also the 4rd term is zero since $\mathbf{u_i}$, $\mathbf{u_j}$ are orthogonal eigen vectors.So we need the 3rd term to be minimum for all k = 1,4. Also $||u_k|| = 1$as we have choosen **unit** eigen vectors.

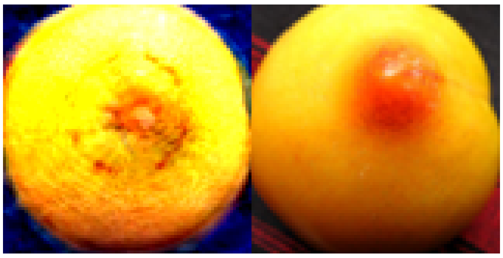
So it will be minimum if $a_k^2 - 2a_k(u_k.x)$ is minimum.Therefore $\mathbf{a_k} = \mathbf{u_k} \cdot \mathbf{x}$

Hence the closest vector to **x** is **c** $= \mu + \sum_{k=1}^{4} (\mathbf{u_k}.\mathbf{x})\mathbf{u_k}$

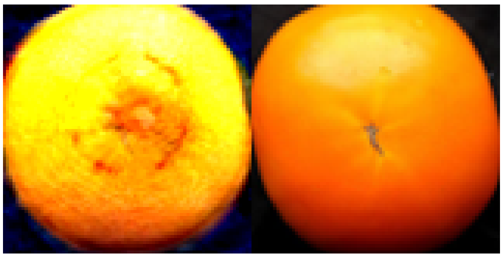
So I implemented the same for all images in a loop and Plotted the image of both **x** and **c** side-by-side for comparison using the image() function after rescaling and reshaping them to 80x80x3. I also saved those images in the directory using the imwrite() function.The images that I obtained are:

Fruit: 1

Fruit: 2



Fruit: 3

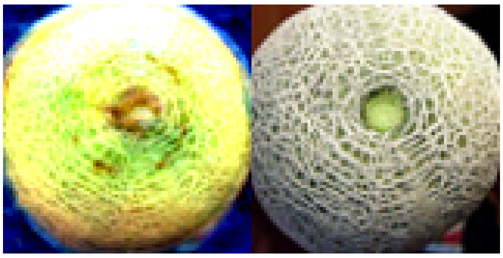


Fruit: 4

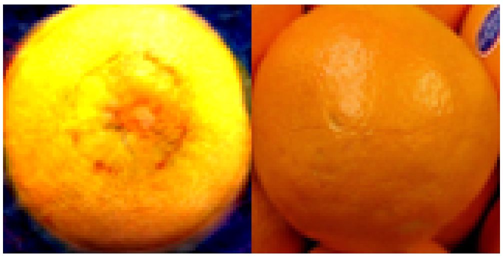
Fruit: 5

Fruit: 6

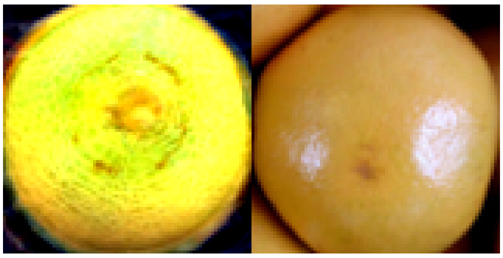
Fruit: 7



Fruit: 8



Fruit: 9



Fruit: 10

Fruit: 11

Fruit: 12

Fruit: 13

Fruit: 14

Fruit: 15

Fruit: 16

3 Sampling new fruit images.

So now we need to sample random images from the same distribution given by the μ and **C**. We have to use the 4 eigen vectors $\mathbf{u_1}, \mathbf{u_2}, \mathbf{u_3}, \mathbf{u_4}$ and the mean to sample new 'fruits'. So we have to generate vectors **x** $= \mu + \mathbf{a_1 u_1} + \mathbf{a_2 u_2} + ... + \mathbf{a_4 u_4}$. where a_1, a_2, a_3, a_4 are the scalar real coefficients of the 4 **unit** eigen vectors.

So our job is essentially to generate the 4 coefficients randomly *based on the given fruit distribution data*.

Now when we project all our data(in the 19200 dimensions) on a line in the direction of $\mathbf{u_1}$, what we are capturing is the coefficient of $\mathbf{u_1} = a_1$ in all our data points.So when we project data vectors **x** along a **Mode of variation** i.e, along the eigen vectors, We just require the coefficient of $\mathbf{u_1}$ in **x** - μ which determines the *distance* of the projected point on the line from the mean μ.

Also when we caluculate the variance of the **projected** data along the Mode of variation determined by say $\mathbf{u_1}$, what we are finding is essentially the **variance of the component** $\mathbf{a_1}$ among our whole data points. And we know that variance along a direction is equal to the **eigen value** of the eigen-vector along that direction.

Hence for a **large data sample**, the coefficients vary as following:

Variance($\mathbf{a}_k$) $= \lambda_k(eigen\ value\ of\ \mathbf{u_k})$ and the Mean($\mathbf{a}_k$) $= 0$ for all k = 1,2,3,4 because a_k is the *displacement from the mean* hence it's average/mean = 0.

So now as we know the variance and means of the 4 coefficients, we try to **model** or **sample** them from their distributions. We only know the **Variance** and the **mean** of the coefficients we don't know any exact distribution, so let's **assume** that it is a **Gaussian** with that Variance and mean(= 0).

So we **sample** draws for $\mathbf{a_k}$ from a **Gaussian** with Variance = $\boldsymbol{\lambda_k}$ and mean = 0 for k = 1,2,3,4

IMAGES OF THE TOP EIGEN VECTORS XD