Name :- Yoseen Islam Mohamed

Id: 9032 التاريخ G1-sec2 الموضوع

$$y = \Theta_0 + \Theta_1 x \qquad \Theta_1 = \frac{\Sigma y \Sigma x^2 - \Sigma x \Sigma xy}{N \Sigma x^2 - (\Sigma x)^2}$$

x	y	x^2	xy
-1	0	1	0
0	2	0	0
1	4	1	4
2	5	4	10

$$\Theta_0 = \frac{\Sigma y - \Theta_1 \Sigma x}{N}$$

$\Sigma x = -1+1+0+2 = 2$

$\Sigma y = 2+4+5 = 11$

$\Sigma x^2 = 1+0+1+4 = 6$

$\Sigma xy = 0+0+4+10 = 14$

$N = 5$ ← No. of Training samples

$$\Theta_1 = \frac{\Sigma y \Sigma x^2 - \Sigma x \Sigma xy}{N \Sigma x^2 - (\Sigma x)^2} = \frac{11 \times 6 - 2 \times 14}{5 \times 6 - 4}$$

$$= \frac{38}{26} = 1.4615 = 19/13$$

$$\theta_0 = \frac{\Sigma y - \theta_1 \Sigma x}{N} = \frac{N\Sigma xy - \Sigma x \Sigma y}{N(\Sigma x^2) - (\Sigma x)^2}$$

$$= \frac{5 \times 14.3 - 2 \times 11}{5 \times 6 - 4} = 1.8462$$

$$\theta = (X^T X)^{-1} X^T t$$

$$X = \begin{bmatrix} 1 & -1 \\ 1 & 0 \\ 1 & 1 \\ 1 & 2 \end{bmatrix}_{4\times 2} \qquad X^T = \begin{bmatrix} 1 & 1 & 1 & 1 \\ -1 & 0 & 1 & 2 \end{bmatrix}_{2\times 4}$$

$$X^T X = \begin{bmatrix} 4 & 2 \\ 2 & 6 \end{bmatrix} \qquad 1+1+4$$

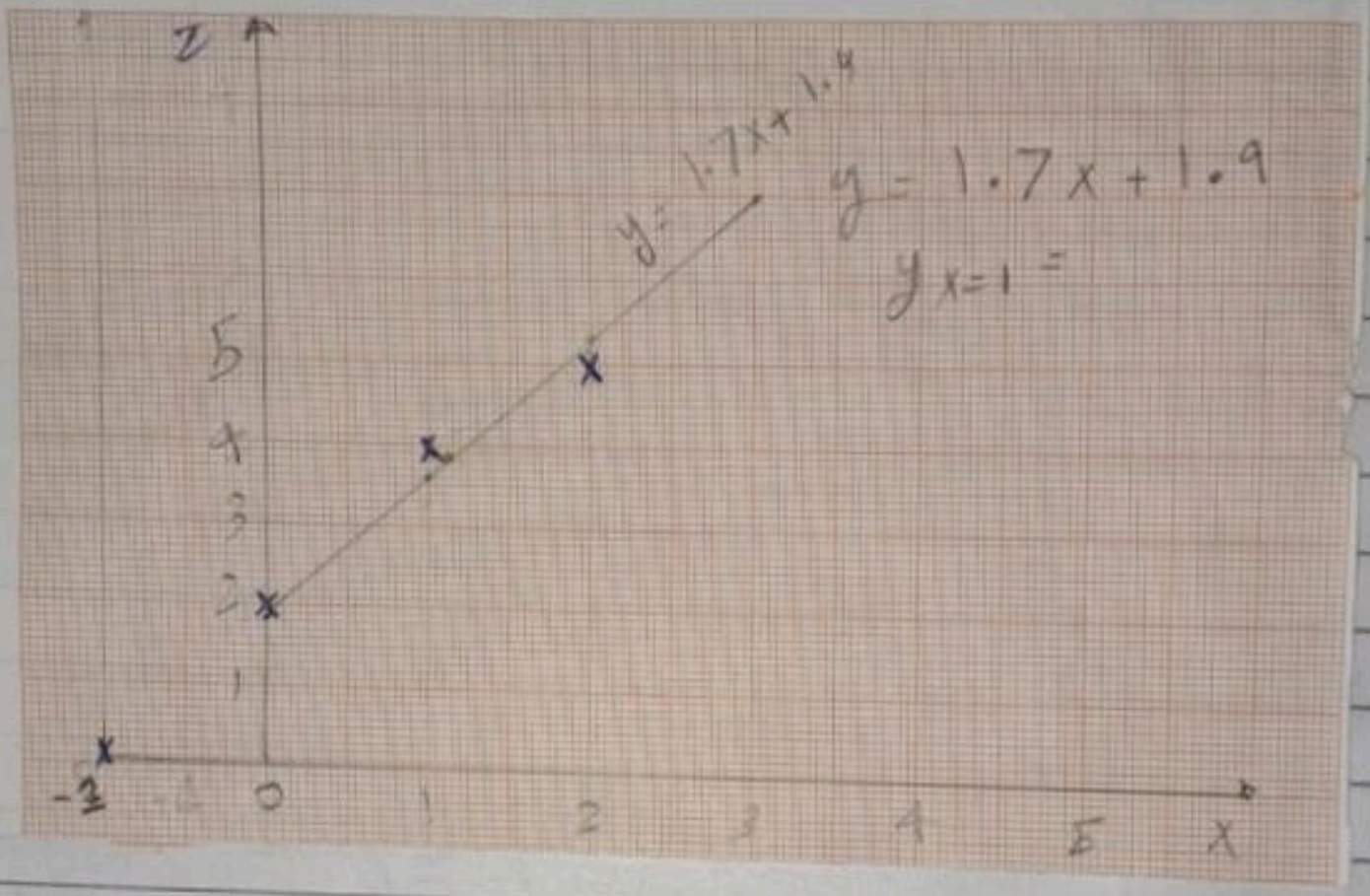
$$[X^T X]^{-1} = \frac{1}{20} \begin{bmatrix} 6 & -2 \\ -2 & 4 \end{bmatrix} = \begin{bmatrix} 6/20 & -2/20 \\ -2/20 & 4/20 \end{bmatrix}$$

$$X X^T = \begin{bmatrix} 0.4 & 0.3 & 0.2 & 0.1 \\ -0.3 & -0.1 & 0.1 & 0.3 \end{bmatrix}_{2\times 4} \qquad 2\times 2$$

$$X t = \begin{bmatrix} 1.9 \\ 1.7 \end{bmatrix}_{2\times 1}$$

$\theta_0 = 1.9 \quad \theta_1 = 1.7$

$y = 1.7x + 1.9$



x	y	x^2	xy
0	2	0	0
1	3	1	3
2	5	4	10
3	4	9	12
4	6	16	24
10	20	30	49

$y = ax + b, \quad N = 5$

$$a = \frac{\Sigma y \Sigma x^2 - \Sigma x \Sigma xy}{N\Sigma x^2 - (\Sigma x)^2}$$

$$a = \frac{20 \times 30 - 10 \times 49}{5 \times 30 - (10)^2}$$

$a = 2.2$

$$b = \frac{N\Sigma xy - \Sigma x \Sigma y}{N\Sigma x^2 - (\Sigma x)^2}$$

$$b = \frac{5 \times 49 - 10 \times 20}{5 \times 30 - (10)^2} = 0.9$$

$y = 2.2x + 0.9$

$y(x=10) = 2.2 \times 10 + 0.9 = 22.9$

③ $X = \begin{bmatrix} 1 & 2 \\ 1 & 0 \\ 1 & -1 \\ 1 & -2 \\ 1 & 2 \end{bmatrix}_{6\times 2}$ $X^T = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 2 & 0 & -1 & 1 & -2 & 2 \end{bmatrix}_{2\times 6}$

$Z = \begin{bmatrix} 2 \\ 0 \\ \vdots \\ \vdots \end{bmatrix}$ ② $= (X^T X)^{-1} X^T Z$

$X^T X = \begin{bmatrix} 6 & 2 \\ 2 & 14 \end{bmatrix}_{2\times 2} = \begin{bmatrix} n & \Sigma x \\ \Sigma x & \Sigma x^2 \end{bmatrix}$

$(X^T X)^{-1} = \frac{1}{n\Sigma x^2 - (\Sigma x)^2} = \frac{1}{80} \begin{bmatrix} 14 & -2 \\ -2 & 6 \end{bmatrix}$

$= \begin{bmatrix} 14/80 & -2/80 \\ -2/80 & 6/80 \end{bmatrix} = \frac{1}{n\Sigma x^2 - (\Sigma x)^2} \begin{bmatrix} \Sigma x^2 & -\Sigma x \\ -\Sigma x & n \end{bmatrix}$

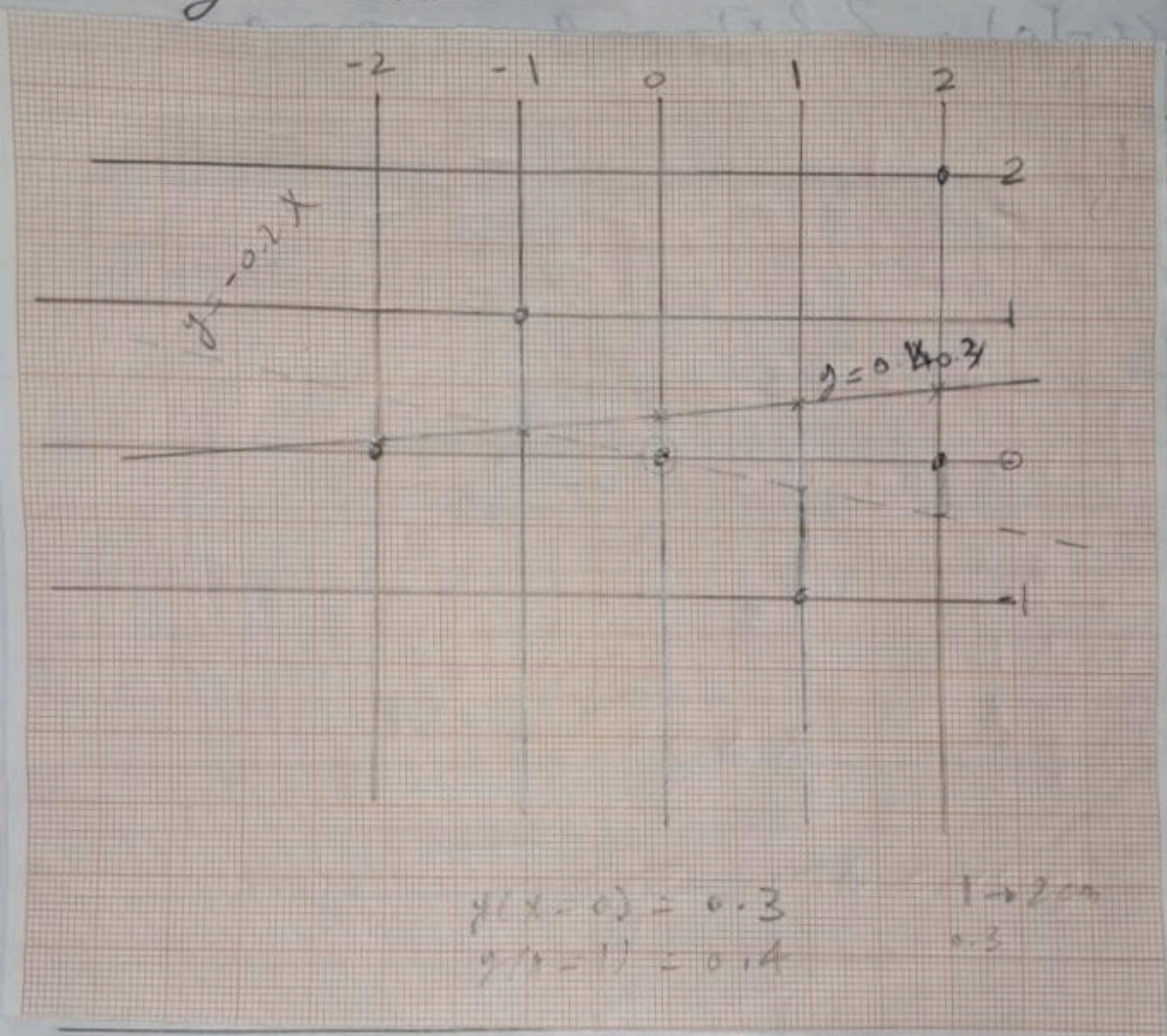
$_{2\times 2}$

$(X^T X)^{-1} X = \begin{bmatrix} 0.125 & 0.175 & 0.2 & 0.15 & 0.225 & 0.125 \\ 0.125 & -0.025 & -0.1 & 0.05 & -0.175 & 0.125 \end{bmatrix}$

$X Z = \begin{bmatrix} 0.3 \\ 0.1 \end{bmatrix}$

$y = 0.1x + 0.3$

$y = 0.1x + 0.3$



$y(x=0) = 0.3$

$y(x=1) = 0.4$

$1 \rightarrow 2 = 0.3$

x	z	$y = 0.1x + 0.3$	$\|y - z\|$	$(y - \bar{y})^2$
2	2	0.5	1.5	2.25
0	0	0.3	0.3	0.09
-1	1	0.2	0.8	0.64
1	-1	0.4	1.4	1.96
-2	0	0.1	0.1	0.01
2	0	0.5	0.5	0.25

(c) $SSE = \sum (y - \bar{y})^2 = 2.25 + 0.09 + 0.64 + 1.96 + 0.01 + 0.25$

$= 5.2$

(e) $y(x = 1.5) = .1 \times 1.5 + 0.3 = 0.45$

$y(x = 0.5) = .1 \times 0.5 + 0.3 = 0.35$

$y(x = -0.5) = .1 \times -0.5 + 0.3 = 0.25$

$$X = \begin{bmatrix} 1 & 0 \\ 1 & -1 \\ 1 & 1 \\ 1 & -2 \\ 1 & 2 \end{bmatrix} \qquad X^T = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & -1 & 1 & -2 & 2 \end{bmatrix}$$

$$X^T X = \begin{bmatrix} 5 & 0 \\ 0 & 10 \end{bmatrix} \qquad (X^T X)^{-1} = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.1 \end{bmatrix}$$

$$(X^T X)^{-1} X^T = \begin{bmatrix} 0.2 & 0.2 & 0.2 & 0.2 & 0.2 \\ 0 & -0.1 & 0.1 & -0.2 & 0.2 \end{bmatrix}$$

$$\downarrow \quad X Z = \begin{bmatrix} 0 \\ -1/5 \end{bmatrix} \quad \Rightarrow$$

$$y = -0.2x$$

x	y	$(y-\hat{y})^2$
0	0	0
-1	1	0.2
1	-1	0.2
-2	0	0.4
2	0	0.4

changing the slope and causing the intercept to drop to zero

$$\sum (y-\hat{y})^2 = 2.4$$

x	y	$\hat{y}$	$y-\hat{y}$	$(y-\hat{y})^2$
0	0	0	0	0
-1	0.2	1	-0.8	0.64
1	-0.2	-1	0.8	1.44
-2	0.4	0	0.4	0.16
2	-0.4	0	-0.4	0.16

The cost is reduced by 46% so the model is very sensitive to outliers