
Sheet 7: Special Continuous Probability Distributions

Question (1):

The number of flaws in a fiber optic cable follows a Poisson distribution with an average of 0.6 per 100 feet.

- (a) Find the probability of at most 2 flaws in a 200-foot cable.
- (b) Find the length of a cable (x) such that the probability of observing no flaws is equal to 0.9.
- (c) Find the standard deviation of the number of flaws in a cable which is 100-foot long.
- (d) Find the probability that the distance between two flaws is between 0.5 foot and 1 foot.

Question (2):

The number of telephone calls that arrive at a phone exchange is often modeled as a Poisson random variable. Assume that on average there are 10 calls per hour.

- (a) What is the probability there are exactly 15 calls in two hours?
- (b) What is the probability that the time until the next call arrives exceeds 20 minutes?
- (c) Calculate the length of an interval of time such that the probability there are no calls arriving at the phone exchange during this interval is 90%.
- (d) If no calls arrived during the past 10 minutes, what is the probability that the first call arrives within the next 20 minutes?
- (e*) What is the probability that the time until two calls have arrived does not exceed 20 minutes?

Question (3):

The time between arrivals of taxis at a busy intersection is exponentially distributed with a mean of 10 minutes.

- (a) What is the probability that you wait more than one hour for a taxi?
- (b) What is the probability that three taxis arrive in one hour?
- (c) If no taxis appeared during the last 30 minutes, what is the probability that the first taxi appears within the next 20 minutes?
- (d*) What is the probability that the time until two taxis have arrived at the intersection does not exceed 20 minutes?
- (e) What is the mean time for observing the arrival of 10 taxis at the intersection?

Question (4):

The life of automobile voltage regulators has an exponential distribution with a mean of six years.

- (a) What is the probability that the voltage regulator fails within the next six years?
- (b) Calculate the length of an interval of time such that the probability the voltage regulator is still working during this interval is 0.9.
- (c) If 10 voltage regulators are purchased, what is the probability that exactly 7 of them are still functioning by the end of six years?

Question (5):

The diameter of the dot produced by a printer is uniformly distributed between 0.0015 and 0.0025 inch. Suppose that the specifications require the dot diameter to be between 0.0017 and 0.0023 inch.

- (a) What is the probability that the dot diameter meets specifications?
- (b) Determine the diameter that is exceeded by 90% of the dots produced.

Question (6):

Suppose the length of computer cables is uniformly distributed from 1200 to 1210 mm.

- (a) If buyers' specify that cables should have a length between 1205 and 1208 mm, what proportion of cables conform to specifications?
- (b) If 10 cables are purchased, what is the probability that exactly 5 of them meet the specifications?

Question (7):

The life of a semiconductor laser operating at a constant power is normally distributed with a mean of 5000 hours and a standard deviation of 200 hours. Assume that lasers fail independently.

- (a) What is the probability that a laser fails before 4500 hours?
- (b) What is the probability that the life of a laser exceeds 5000 hours?
- (c) If five lasers are used in a product, what is the probability that three of them are still operating after 5200 hours? (Hint: $\Phi(1.0) = 0.8413$, $\Phi(1.5) = 0.9332$, $\Phi(2.5) = 0.9938$)

Question (8):

The time until recharge for a battery in a laptop computer under common conditions is normally distributed with a mean of 260 minutes and a standard deviation of 50 minutes.

- (a) What is the probability that a battery lasts more than 4 hours?
 - (b) What is the probability that the battery is recharged in less than 260 minutes?
 - (c) If the battery has been recharged 110 minutes ago, what is the probability it will need to be recharged within the next 150 minutes?
- (Hint: $\Phi(1.25) = 0.89435$, $\Phi(1.5) = 0.933193$, $\Phi(0.4) = 0.655422$, $\Phi(1.2) = 0.88493$)

Question (9):

The length of a plastic case that holds a magnetic tape is normally distributed with a mean length of 90.2 mm and a standard deviation of 0.1 mm.

- (a) What is the probability that the length of a part is less than 90 mm?
 - (b) If seven parts are examined, what is the probability that exactly 3 of them will have a length exceeding 90.3 mm?
- (Hint: $\Phi(1.0) = 0.841345$, $\Phi(1.25) = 0.89435$, $\Phi(1.5) = 0.933193$, $\Phi(2) = 0.9772$)