Probability and Statistics

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Outline

1.3 Probability computation

- Basics
- Counting techniques

1.4 Conditional probability

- Notion and definition
- Multiplication rule
- Law of total probability
- Bayes' theorem
- Independent events
- Some reliability problems

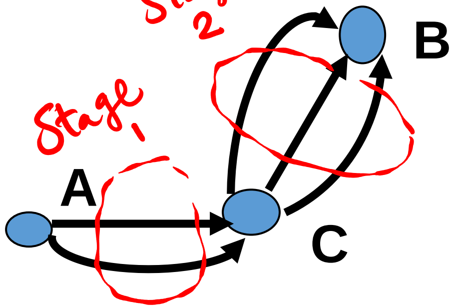
This Lecture

Next Lecture

Techniques to determine the number of elements in a set without listing them

ناتج ال stage لا يتوقف على ال Next stages

Multiplication Principle



Stages

"independent stages"

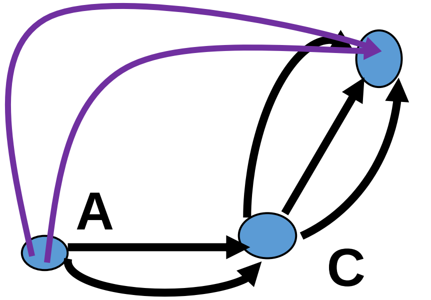
$2 * 3$

stage 1   stage 2

≡ And "∩"

In how many ways can one goes from A to B?

Addition Principle



Alternatives

"Mutually exclusive"

special case

$2*3$ or $\cancel{+}$ 2

≡ Union "∪"

$A + B - (A \cap B)$

"General Case"

How many possible password of 4 or 5 numbers?

4 number   Mutually exclusive $\boxed{\text{or}}$ 5 numbers

10	10	10	10

10	10	10	10	10

Stages "Independent"

$N_s = 10^4 + 10^5$

Permutation and combination

A, B, C → *In how many ways, can we select two* ***Different*** *items?*

{AB, BA, AC, CA, BC, CB}

Permutation

- Permutation is the arrangement of items in which **order matters**
- Number of ways of **selection and arrangement of items** in which Order Matters

$$^{n}P_{r} = \frac{n!}{(n-r)!}$$

{AB, AC, BC}

Combination

- Combination is the selection of items in which **order does not matters** .
- Number of ways of **selection of items** in which Order does not Matters

*Main Concept

To remove repetitions

$$^{n}C_{r} = \frac{1}{r!}\frac{n!}{(n-r)!}$$

$^{n}P_{r}$

More problems

Example

An instructor gives her class a set of 10 problems with the information that the final exam will consist of a random selection of 5 of them. If a student has figured out how to do 7 of the problems, what is the probability that he or she will answer correctly

a. All 5 problems?

b. At least 4 of the problems?

10 problems $\xrightarrow{\text{select}}$ 5

$N_s = {}^{10}C_5$

No specifications

→ Grouping

Important

(a)

7 G	→ 5
3 B	→ 0

stages

$$P_a = \frac{{}^{7}C_5 * {}^{3}C_0}{{}^{10}C_5}$$

(b)

4 prob.

7 G	→ 4
3 B	→ 1

or

+

5 probs.

7 G	→ 5
3 B	→ 0

$$P_b = \frac{{}^{7}C_4 * {}^{3}C_1 + {}^{7}C_5 * {}^{3}C_0}{{}^{10}C_5}$$

Ex: A conference on bioinformatics includes six talks for six different speakers (3 biologists and 3 engineers). However, the three biologists must speak first, in any order, followed by the three engineers, in any order.
How many different schedules are possible for the six talks in this conference?

6	5	4	3	2	1

$\rightarrow N_s = 6! = {}^6P_6$

Bio. Eng.

3	2	1	3	2	1

$\rightarrow N_a = 3! * 3! = 36$

stages

How many schedules are possible if biologists can come first or second

Bio. Eng. $\rightarrow 36$

2! or

Eng. Bio. $\rightarrow 36$

$N_a = 36 + 36$

$= 36 * 2!$

Ex: Consider the experiment of flipping two coins then a fair dice is tossed. Determine the number of possible outcomes in this experiment.

Specific order of tossing

Coin	Coin	dice
2	2	6

$N_s = 2*2*6 = 24$

What if tossing is allowed in any order?

3! possib.

Coin1 Coin2 dice ---> 24

Coin1 dice Coin2 ---> 24

Coin2 Coin1 dice --->

Coin2 dice Coin1 →

dice Coin1 Coin2 →

dice Coin2 Coin1 → 24

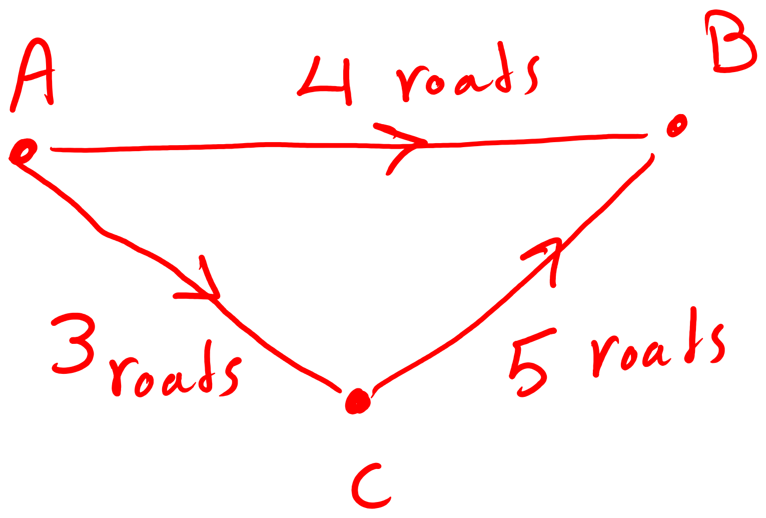
$$\frac{24 * 3!}{2!}$$

To remove repetition of the similar 2 coins

if 2 coins & 2 dices

$$\frac{2*2*6*6 * 4!}{2!\ 2!}$$

Ex: A driver from city A can directly reach city B using four different roads. Moreover, he can also reach city B by passing by city C first. In this latter case, he can drive along any of three roads connecting cities A and C, then choose any of the five roads from C to B.
How many different choices are available to the driver?



$$N_s = 4 + 3 * 5 = 19$$

Ex: If each coded item in a catalogue begins with three distinct letters (a-z) followed by four distincit non-zero digits (1-9).

Find the probability of randomly selecting one of these coded items with the first letter a vowel (a, e, I, o, u) or the last digit is even. B

A

Alternative

A&B are not mutually exclusive

$\underbrace{26 \mid 25 \mid 24}_{\text{letters}} \mid \underbrace{9 \mid 8 \mid 7 \mid 6}_{\text{digits}}$

$N_s = {}^{26}P_3 * {}^{9}P_4$

A: $24 \mid 25 \mid 5 \mid 9 \mid 8 \mid 7 \mid 6$

Vowel letter

$P_A = \dfrac{24*25*5*{}^{9}P_4}{{}^{26}P_3 * {}^{9}P_4}$

B: $26 \mid 25 \mid 24 \mid 6 \mid 7 \mid 8 \mid 4$

even {2,4,6,8}

$P_B = \dfrac{{}^{26}P_3 *6*7*8*4}{{}^{26}P_3 * {}^{9}P_4}$

$A \cap B$:

$24 \mid 25 \mid 5 \mid 6 \mid 7 \mid 8 \mid 4$

$P_{A\cap B} = \dfrac{24*25*5*6*7*8*4}{{}^{26}P_3 * {}^{9}P_4}$

$P_{A\cup B} = P_A + P_B - P_{A\cap B}$

Seven balls are randomly withdrawn from an urn that contains 12 red, 16 blue, and 18 green balls. Find the probability that

a. 3 red, 2 blue, and 2 green balls are withdrawn.

b. At least 2 red balls are withdrawn.

c. All withdrawn balls are the same color.

d. Either exactly 3 red balls or exactly 3 blue balls are withdrawn.

46 = 12 R, 16 B, 18 G ⇒ 7

$N_s = {}^{46}C_7$

No specifications ≡ Grouping

(a) 12 R → 3, 16 B → 2, 18 G → 2

$$P_a = \frac{{}^{12}C_3 * {}^{16}C_2 * {}^{18}C_2}{{}^{46}C_7}$$

(b) 2 Red: 12 R → 2; 16 B, 18 G (34) → 5

or 3 Red: 12 R → 3; 16 B, 18 G → 4

or ------ 7 red: 12 R → 7; 16 B, 18 G → 0

$$P_b = \frac{{}^{12}C_2 * {}^{34}C_5 + {}^{12}C_3 * {}^{34}C_4 + \dots + {}^{12}C_7 * {}^{34}C_0}{{}^{46}C_7}$$

or using Complement

0 Red: 12 R → 0, 34 → 7

or

1 Red: 12 R → 1, 34 → 6

$$P_b = 1 - \frac{{}^{12}C_0 * {}^{34}C_7 + {}^{12}C_1 * {}^{34}C_6}{{}^{46}C_7}$$

Seven balls are randomly withdrawn from an urn that contains 12 red, 16 blue, and 18 green balls. Find the probability that

a. 3 red, 2 blue, and 2 green balls are withdrawn.
b. At least 2 red balls are withdrawn.
c. All withdrawn balls are the same color.
d. Either exactly 3 red balls or exactly 3 blue balls are withdrawn.

12 R
16 B
18 G

7 red $\boxed{\text{or}}$ 7 blue $\boxed{\text{or}}$ 7 green

12 Red	→ 7
34	→ 0

remaining

16 B	→ 7
12+18 =30	→ 0

18 G	→ 7
12+16=28	→ 0

$$P = \frac{{}^{12}C_7 * {}^{34}C_0 + {}^{16}C_7 * {}^{30}C_0 + {}^{18}C_7 * {}^{28}C_0}{{}^{46}C_7}$$

Seven balls are randomly withdrawn from an urn that contains 12 red, 16 blue, and 18 green balls. Find the probability that

a. 3 red, 2 blue, and 2 green balls are withdrawn.

b. At least 2 red balls are withdrawn.

c. All withdrawn balls are the same color.

d. Either exactly 3 red balls or exactly 3 blue balls are withdrawn.

Alternatives

$\underbrace{\text{exactly 3 red balls}}_{A}$ or $\underbrace{\text{exactly 3 blue}}_{B}$

$A \cap B \neq \phi$

A: | 12 R | → 3 |
| 34 | → 4 |

$P_A = \dfrac{{}^{12}C_3 * {}^{34}C_4}{{}^{46}C_7}$

B: | 16 B | → 3 |
| 30 | → 4 |

$P_B = \dfrac{{}^{16}C_3 * {}^{30}C_4}{{}^{46}C_7}$

$A \cap B$: | 12 R | → 3 |
| 16 B | → 3 |
| 18 G | → 1 "The remaining" |

$P_{A\cap B} = \dfrac{{}^{12}C_3 * {}^{16}C_3 * {}^{18}C_1}{{}^{46}C_7}$

$P_{A\cup B} =$

$P_A + P_B - P_{A\cap B}$

Exercise problems

Example

A lot of 15 monitors contains **2** defective ones. **Three** monitors are chosen at random. What is the probability that **at least one is defective**?

$\boxed{15} \longrightarrow 3$ $\qquad N_s = {}^{15}C_3$

one def. $\quad \boxed{\text{or}} \quad$ 2 def.

$\begin{array}{|c|}\hline 2D \\ \hline 13G \\ \hline\end{array} \begin{array}{l} \rightarrow 1 \\ \rightarrow 2 \end{array} \qquad \begin{array}{|c|}\hline 2D \\ \hline 13G \\ \hline\end{array} \begin{array}{l} \rightarrow 2 \\ \rightarrow 1 \end{array}$

$$P = \frac{{}^{2}C_1 * {}^{13}C_2 + {}^{2}C_2 * {}^{13}C_1}{{}^{15}C_3}$$

Example

A password consists of **at most 3 letters** “Assume small letters”.

How many passwords are there?

one letter $\boxed{\text{or}}$ 2 letters $\boxed{\text{or}}$ 3 letters

$\boxed{26}$ $\boxed{26 \mid 26}$ $\boxed{26 \mid 26 \mid 26}$

$$N_s = 26 + 26^2 + 26^3$$

Example

A three-digit number is constructed at random from {**0,**2, 3, 5, 8, 9}. What is the probability that it is

(a) even? (b) divisible by five?

"Assume repetition is not allowed"

5	5	4

zero is not allowed

$N_S = 5*5*4$

(a)

0 is not allowed {0,2,8}

Think of the complement "odd no."

Stages are not indep.

step 2	step 3	step 1
4	4	2

0 is not allowed {2,8}

or

5	4	1

{0}

$$P = \frac{4*4*2+5*4*1}{5*5*4}$$

(b)

0 is not allowed {0,5}

or think of the complement

4	4	1

0 is not allowed {5}

5	4	1

0 is not allowed {0}

$$P = \frac{4*4*1+5*4*1}{5*5*4}$$