Electric Potential Due to a Dipole

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1 Introduction

Recall that an electric dipole is merely two charges of the same magnitude but different signs separated by a finite distance. We define the dipole moment vector as the vector pointing from the negative to the positive charge with magnitude as

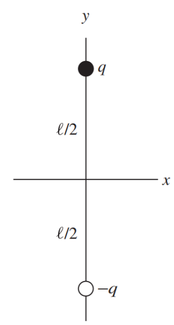

Figure 1: An electric dipole

the product of the charge and distance of separation.

$$\vec{p} = q\vec{\ell} \tag{1}$$

In this case, we can also write $\vec{p} = q\ell\hat{y}$.

2 Calculating the Potential

Potentials are easy to calculate due to their scalar nature. They also let us calculate the electric field by simply computing the gradient. In radially symmetric systems, we have that

$$\vec{E} = -\frac{\partial V}{\partial r}\hat{r} \tag{2}$$

Let us finally calculate the potential due to a dipole at a point P in the figure. Using the principle of superposition, we can conclude that the net potential at P is the sum of the potentials due to the negative and positive charges.

$$V = V_{+} + V_{-} \tag{3}$$

$$V = \frac{kq}{r_1} + \frac{k(-q)}{r_2} \tag{4}$$

Now the issue lies only in calculating r_1 and r_2. There are two ways to do it. If we approximate $\ell << r$, things become easier. However, the exact method shall also be discussed. We shall also use radial coordinates (r, θ) as things are going to be messy using ordinary rectangular coordinates (x, y).

Since the distance of separation is very less compared to r, we can assume that r, r_1 and r_2 are parallel. From the figure, we can immediately write down

$$r_1 = r - (\ell/2)\cos\theta \tag{5}$$

$$r_2 = r + (\ell/2)\cos\theta \tag{6}$$

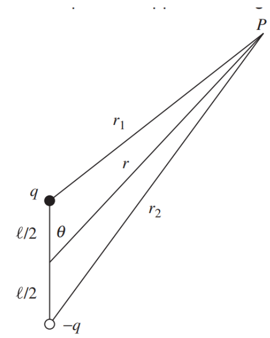

Figure 2: Caption

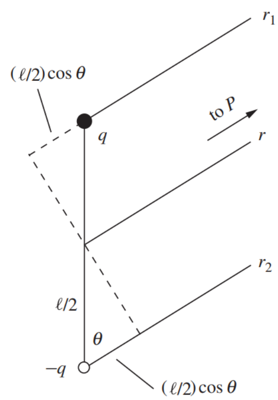

Figure 3: Caption

Plugging this in the potential equation gives

$$V = kq\left(\frac{1}{r - \frac{\ell\cos\theta}{2}} - \frac{1}{r + \frac{\ell\cos\theta}{2}}\right) = \frac{kq}{r}\left(\frac{1}{1 - \frac{\ell\cos\theta}{2r}} - \frac{1}{1 + \frac{\ell\cos\theta}{2r}}\right) \tag{7}$$

Now we make use of the fact that $(1+x)^n \approx 1 + nx$ if $x << 1$. Since $\ell << r$, $\ell/r << 1$ and thus we can write

$$\frac{kq}{r}\left(\frac{1}{1 - \frac{\ell\cos\theta}{2r}} - \frac{1}{1 + \frac{\ell\cos\theta}{2r}}\right) = \frac{kq}{r}\left[\left(1 - \frac{\ell\cos\theta}{2r}\right)^{-1} - \left(1 + \frac{\ell\cos\theta}{2r}\right)^{-1}\right] = \frac{kq}{r}\left[1 + \frac{\ell\cos\theta}{2r} - 1 + \frac{\ell\cos\theta}{2r}\right] = \frac{kq\ell\cos\theta}{r^2} \tag{8}$$

Thus, when $\ell << r$, the potential due to the dipole is

$$V = \frac{p\cos\theta}{4\pi\varepsilon_0 r^2} \tag{9}$$

Notice that $\vec{p}\cdot\hat{r} = p|\hat{r}|\cos\theta = p\cos\theta$. Thus we can write

$$\boxed{V = \frac{p\cdot\hat{r}}{4\pi\varepsilon_0 r^2}} \tag{10}$$

Where $\vec{r}$ is the position vector of the point in question w.r.t the centre of the dipole. Observe that $V \propto 1/r^2$ rather than the usual $V \propto 1/r$. This means that the field due to the dipole is $E \propto 1/r^3$. What is the field due to this arrangement? As mentioned, we simply need to take the negative gradient of this expression in polar coordinates, however, that is beyond the scope of this introductory analysis.