Differentiation Basics

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1 MA161 Questions

These are basic calculus questions taken from the Mathematical Studies Module most(all?) first years in the school of science take.

1.1 2024-2025, Paper 1 First Sitting Q4

a) Consider the following piecewise defined function

$$f(x) = \begin{cases} cx^2 + 2x, & x < 2 \\ x^3 - cx, & x \geq 2 \end{cases}$$

i) Find the value of c for which f is continuous at $x = 2$

ii) For this vale of c. does the derivative of f exist at $x = 2$? Justify your answer, motivate this with a graph.

b) Find the derivative of each of the following functions

i) $f(x) = \dfrac{x^2 + 1}{x^2 - 1}$

ii) $f(x) = sin^2(x) - cos^2(x)$

c) Consider the function $f(x) = 2 + 3x - x^3$

i) Find the derivative $\frac{d}{dx}f(x)$ and the second $\frac{d^2}{dx^2}f(x)$.

ii) Find all critical points of f.

iii) Find the intervals where f is increasing and where f is decreasing.

iv) For each critical point, determine if it corresponds to a local minimum or local maximum.

v) Use the information obtained in (i)-(iv) to sketch the graph of f

1.2 2023-2024. Paper 1 First Sitting Q4

a) Consider the following piecewise defined function

$$\begin{cases} ax + 1, & x \leq 2 \\ ax^2 - 1, & x < 2 \end{cases}$$

i) Find the value of a for which f is continuous at $x = 2$.

For this value of a:

ii) Sketch the graph of f

iii) Does the derivate of f exist at $x = 2$? Justify your answer.

b) Find the derivate of each of the following functions

i) $f(x) = \ln(x^2 + x + 1)$

ii) $f(x) = \dfrac{1}{e^x + e^{-x}}$

c) Consider the function $f(x) = x^3 - 3x^2 - 9x + 11$

i) Find the derivative $\frac{d}{dx}f(x)$ and the second derivative $\frac{d^2}{dx^2}f(x)$.

ii) Find all critical points of f.

iii) Find all the intervals where f is increasing and where f is decreasing.

iv) For each critical point, determine if it corresponds to a local minima or local maxima.

v) Use the information obtained in (i)-(iv) to sketch the graph of f