Conceptual Report for the traditional Loan Pricing Tool

* Introduction

A loan is the most traditional banking product and, despite its importance, studies and practical examples on loan price is scarce. One of the greatest difficulties is that, as opposed to market instruments, there is no market information on a borrower available and the development of an elegant theory like arbitrage free pricing in the derivatives world is not feasible. The most common product in the Portuguese banking industry is the mortgage loan, which is almost always linked to a real collateral. In some countries, like Portugal, borrowers have prepayment rights on their loans, an option which is increasingly being exercised due to the economic recovery, increasing competition and lower spreads (the spreads are decreasing when compared with the peak of the sovereign crisis). By law every debtor as the right to pay the notional amount, so the prepayment right can be of considerable value.

The most prominent risk of the loans is the credit risk, so it is a very important element that needs to be included in the model. In our solution, the credit risk reflects:

* The default risk (i.e. debtor fails to pay the installments), which is introduced through a transition matrix that describes the upgrading/downgrading and default probabilities by the Markov Chain processes. In the traditional loans there is no market information for the majority of debtors, so we must base our transition matrix on objective probabilities (estimation based on observed rating migrations and defaults). In this version (1.0) the transition matrix will be an input and the probabilities extracted from this matrix will depend on the initial risk bucket. In the future, some improvements can be made in terms of incorporating non-homogeneity adjustments in the multi-period estimations, which can be very useful for pricing purposes,
* The loss risk, which is estimated by the loss given default parameter (LGD). In this tool the LGD will be an input.

As already stated, the prepayment option is also a relevant part of the fair value of the loan. In the real world, the prepayment option could be explained by 2 factors:

* The evolution of the interest rate in the future. The debtor will exercise this option only when it is beneficial, i.e, at the time of exercise, the interest rate is lower than the contractual interest rate. However, debtors in the loan market do not act perfectly rational because there are barriers that makes the early redemption more difficult (e.g. administrative costs of substituting the older loan with a new one, loyalty to the current bank, time burden, etc),
* Estimated early redemption rates (ERR). This parameter takes into account the historical behavior of debtors in terms of prepayment options exercise.

In the version v1.0, the prepayment option will be captured using only the estimated ERR. Ideally, the embedded option should conjugate the ERR with a stochastic model for interest rates (the trinomial model of Damiano Brigo is one of the most used approach in the derivatives pricing), however we haven’t still been able to implement the stochastic model in a feasible way. This could be an important development for version 2.0.

By giving this information all risk and cost components of a loan will be calculated to determine the interest rate margin a bank should charge to a debtor in order to cover all the costs:

* the expected loss of a loan is one of these components,
* Another issue that the bank concerning is to protect a loan portfolio against the risk of unexpected losses by computing economic capital.
* Funding costs in order to reflect the funding conditions of a Bank. In version 1.0 this will be incorporated as an unique constant spread over the risk free curve (it is an input). In the future some improvements can be made:
  + - The spread which serves as input can vary with the tenor (v2.0),
    - EY could develop a methodology that measures the “yield curve” of the bank (v3.0).
* Operating costs, like staff salaries or office cost, will be summarized by a spread that reflects an additional cost margin (it is an input).

In terms of risk management our innovation work combine credit risk modeling to build a generic framework for the following purposes:

* pricing (commercial divisions);
* risk management (risk divisions) because it allows the measurement of the return of the portfolio, e.g. risk adjusted return on capital (RAROC).

Economically RAROC is equal to

where *zs* is the hedging cost, *sf* is the funding cost, *sEL* is the expected loss, *sc* is the operating cost and *sf* is the funding cost. The economic capital which is allocated to the loan (*E*) generates a return *wr*, e.g. could be invested in high quality assets like government bonds.

The RAROC process should be integrated into the overall risk management process, especially in limits management.. Another advantage of this pricing tool is that it is based almost entirely on risk parameters that are already available in banks risk management system.

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* Pricing Loans without Prepayment Rights

We start with a simple loan that does not contain any options by considering bullet loan, a loan without amortization payment during its lifetime, and as well as for loans with amortization payments like annuity loans or installment loans. Furthermore, our pricing tool can be applied to all three types of bullet, xed-rate, and oating-rate loans. The notional of for example the bullet loan is denoted with N. The interest rate periods are de ned by the times Ti where [Ti; Ti+1] is an interest rate period, i = Ti Ti+1 is the year fraction of the period. In the case of a oating-rate loan which is our main focus here, the rate payment is (fi + s) iN, where we have assumed that the oating rate fi to namely be a Libor rate, is xed at the beginning of each interest rate period i and s is a spread that is constant throughout the loan s lifetime. where fi is the forward rate. To compute the forward rates we use swap-rates of 50 years from the interest rate market, then by using Bootstrapping technique we derive the corresponding zero-rates, in the next step we make bene t of interpolation to derive zero-curve in order to carefully calculate the relavant forward-rates.

We calculate the price of the loan based on expected present value of all future cash ows:

The value of a loan is de ned as the expected discounted value

|  |  |  |  |
| --- | --- | --- | --- |
| V (t) = t<Ti(Nizi iAi) (Ti) (Ti) + | t<Ti | Ti |  |
| NiRi Zmax(t;Ti 1) (u)d(1(u)); |  |
| X | X | |  |

where Ti are the payment times, Ni the outstanding notional in each period, Ri the recovery rate in case of default in period i, is the discount factor and the survival probability of the borrower.

Recovery rate re ect the degree of collateralization of a loan. One pragmatic way to include collateral in a loan pricing framework is to provide the collateral value as input. this collateral should include the loss given default (LGD) of the collateral, i.e. the expected loss in value in the case of a borrower default. However, recovery rate estimation process, as used in our pricing algorithm (R = 1- LGD), is quite complex and beyond the scope of our work at the moment.

Equivalent to the calculation of a survival probability (t) up to time t is the calculation of a default probability p(t) = 1 (t). We assume that a bank s rating system has n rating grades where the n-th grade is the default grade. Just in brief, there are typically two ways how banks extract statistical information about defaults from their rating system to estimate multi-year default probabilities. In the rst approach, banks directly estimate a term structure of default probabilities, i.e for each rating grade k a function pk(t) is estimated where pk(t) is the probability that a debtor in rating grade k will default within the next t years. This could be done using techniques from survival analysis. In the second approach which is our case applied to our tool, a one-year transition matrix is estimated. The entries of the matrix are denoted with pkl where pkl is the probability that a debtor in rating grade k moves to grade l within one year.

We note that our innovation in terms of risk combining credit risk modeling to build a generic framework for the pricing and the risk management of loan pricing portfolios through the measurment of the risk adjusted return on capital (RAROC). The price of a loan is given as the discounted expected value of all cash ows under the probability measure computed from the real-world probabilities of rating changes.

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Here we illustrate our pricing framework in detail:

The pricing algorithm apply to each of bullet, xed, and oating loan.

We compute the price V (kj; ti) of the loan depending on the rating grade kj of the debtor and time ti.

In this pricing algorithm we have used forward rates obtained from zero-rates by the Bootstrapping swap-rates given by the interest rate market to project the discount factor.

Given funding costs applied to the discount factor.

The expected losses implied by this algorithm (R:N:pkn). Furthermore, this ex-pected losses are forecasts of the losses that are expected in reality which is an important quantity in credit risk management.

We calculate the price of the loan based on expected present value of all future cash ows, like Amortization payment (Ai), Interest rate payment (zi i), Liquidation proceeds of collateral in the case of debtor default.

Rate of interest rate return (IRR) of the loan is approximated by using the nal price of the loan and the cash ow value.

The amount of both the expected losses (sEL) and the unexpected losses (sUL) are calculated by the pricing framework.

We obtained the spread of both the expected losses and unexpected losses in the same way of obtaining the IRR.

Di erent banks can have di erent option on default probabilities and the recovery rate.

We are interested in pointing out some remarks which it seems to be important to the practitioner.

Remark 1 The pricing framework that we have used for pricing it uses neither an ad-vanced interest rate model nor a state-of-the-art credit risk model.

Remark 2 We assume that there is no prepayment right applied to our algorithm. Since a loan is much more complex product than an interest rate derivative, we would have to make additional assumptions to price the prepayment rights to be applied to our pricing framework.

RAROC Scheme:

The main purpose of the RAROC scheme is to compute the fair interest rate of a loan which covers all costs and adequately compensates for the risks associated with a loan. The interest rate components are as follows:

Hedging costs (zs)

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Unexpected losses (SUL) Funding costs (sf )

Operating costs (sc) Basis Swap Costs (sb) Expected losses (sEL)

Some of cost components calculations are a part of our pricing tool:

Hedging Costs: For bank internal purposes it is very important to split the interest rate into its components, i.e. which part of the interest rate re ects hedging costs, which parts re ects expected losses, or which part re ects hedging costs. One of the calculated component of the RAROC scheme in our tool is the hedging cost which is relevant for a xed-rate loan because for a oating-rate loan this component is zero. The hedging cost is the xed-rate that has to be charged by a bank that leads to an identical present value as the stream of Ibor payment. This rate is needed as a refrence point to make oating-rate and xed-rate loans comparable.

Unexpected Losses: If a bank would be risk-neutral the calculation is done automat-ically. In reality, however, even if a banks shareholder are risk-neutral but the bank is forced to to be risk-averse by minimum capital requirement of regulators. These min-imum capital requirements (Basel Committee on Banking Supervision 2011) de ne the minimum capital bu ers that have to be held as a protection against unexpected losses in a loan portfolio. The unexpected loss margin is calculated as

E

SUL = (wt wr)N

where in our tool we calculate the value of the capital E which is allocated to the loan and can not be used for other investments. A bank de nes a target return wt on its equity capital. The level of wt is a political decision by a bank depending on the market environment it is operating in. The capital E is not lying in a safe but invested in assets like government bonds where it generate a return wr. The di erence to has to be generated by interest income of the loan. This leads to an additional interest rate margin sUL, the unexpected loss margin.

Expected Losses: coming soon!!

Putting all cost components together gives the hurdle rate zh of a loan, i.e. the interest rate that covers all costs and pro tability targets of a bank. It is obtained by

zh = zs + sEL + sUL + sf + sb + sc

Assuming that the interest rate z is given, the return on equity capital, equivalently, a loan s RAROC can be calculated as

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RAROC = z zs sf sEL sc + wr:

E=N

This equation allows a bank to measure the e ect of on the return of economic capital if it grants a loan at a di erent rate than the hurdle rate zh.

* Conclusions

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