Question 1) A company wants to sell two models of home computers at costs of $250 and $400, respectively. The $250 model yields a profit of $45 and the $400 model yields a profit of $50. The company estimates that the total monthly demand will not exceed 250 units. Find the number of units of each model that should be stocked in order to maximize profit. Assume that the company does not want to invest more than $70,000 in computer inventory.

Answer)

Objective function:

$P = 45x + 50y$

Constraints:

$250x + 400y <= 70000$

$x + y <= 250$

Augmented form:

$250x + 400y + s1 = 70000$

$x + y + s2 = 250$

Maximize:

$-45x -50y + P = 0$

Augmented form:

250x + 400y + s1 = 70000

x + y + s2 = 250

Maximize:

-45x -50y + P = 0

Setting up Simplex Tableau.

	x	y	S1	S2	P	RHS	rough work
R1	250	400	1	0	0	70000	
R2	1	1	0	1	0	250	
R3	-45	-50	0	0	1	0	

	x	y	S1	S2	P	RHS	rough work
R1	250	400	1	0	0	70000	175
R2	1	1	0	1	0	250	250
R3	-45	-50	0	0	1	0	

(1/400) R1→R1

	x	y	S1	S2	P	RHS	rough work
R1	0.625	1	1/400	0	0	175	
R2	1	1	0	1	0	250	
R3	-45	-50	0	0	1	0	

Iteration 1:

-1(R1) + R2 →R2

50 (R1) + R3 →R3

	x	y	S1	S2	P	RHS	rough work
R1	0.625	1	1/400	0	0	175	
R2	0.375	0	-0.0025	1	0	75	
R3	-13.75	0	0.125	0	1	8750	

	x	y	S1	S2	P	RHS	rough work
R1	0.625	1	1/400	0	0	175	280
R2	0.375	0	-(1/400)	1	0	75	200
R3	-13.75	0	0.125	0	1	8750	

(1/0.375) R2 → R2

	x	y	S1	S2	P	RHS	rough work
R1	0.625	1	1/400	0	0	175	280
R2	1	0	-0.0067	2.67	0	200	200
R3	-13.75	0	0.125	0	1	8750	

Iteration 2:

-0.625 (R2) + R1 → R1

13.75 (R2) + R3 → R3

	x	y	S1	S2	P	RHS	rough work
R1	0	1	0.0067	-1.67	0	50	
R2	1	0	-0.0067	2.67	0	200	200
R3	0	0	0.032	36.71	1	11500	

We stop as no negative elements in the Maximization row.

	x	y	S1	S2	P	RHS	rough work
R1	0	1	0.0067	-1.67	0	50	
R2	1	0	-0.0067	2.67	0	200	200
R3	0	0	0.032	36.71	1	11500	

S1 and S2 are non-basic variables, so they are equal to zero.

S1 = 0

S2 = 0

And,

x = 200

y = 50

P = 11500

Therefore, there needs to be 200 of Model x (costing $250 each) and 50 of Model y (costing $400 each), to be produced in order to achieve the maximum profit.

The maximum profit is $11,500

Question 2)

Objective function:

$$z = 2x_1 + x_2 + 3x_3$$

Constraints:

$$\begin{aligned} x_1 + x_2 + x_3 &\leq 59 \\ 2x_1 + 3x_3 &\leq 75 \\ x_2 + 6x_3 &\leq 54 \\ x_1, x_2, x_3 &\geq 0 \end{aligned}$$

Answer)

Augmented form:

x1 + x2 + x3 + S1 = 59

2x1 + 3x3 + S2 = 75

x2 + 6x3 + S3 = 54

To Maximize:

-2x1 – x2 – 3x3 + z = 0

Setting up Simplex Tableau.

	x1	x2	x3	S1	S2	S3	z	RHS	rough work
R1	1	1	1	1	0	0	0	59	59
R2	2	0	3	0	1	0	0	75	25
R3	0	1	6	0	0	1	0	54	9
R4	-2	-1	-3	0	0	0	1	0	

(1/6) (R3) → R3

	x1	x2	x3	S1	S2	S3	z	RHS	rough work
R1	1	1	1	1	0	0	0	59	
R2	2	0	3	0	1	0	0	75	
R3	0	0.167	1	0	0	0.167	0	9	
R4	-2	-1	-3	0	0	0	1	0	

Iteration 1

(-1) R3 + R1 → R1

-3R3 + R2 → R2

3R3 + R4 → R4

	x1	x2	x3	S1	S2	S3	z	RHS	rough work
R1	1	0.83	0	1	0	-0.167	0	50	
R2	2	-0.5	0	0	1	-0.5	0	48	
R3	0	0.167	1	0	0	0.167	0	9	
R4	-2	-0.499	0	0	0	0.501	1	27	

	x1	x2	x3	S1	S2	S3	z	RHS	rough work
R1	1	0.83	0	1	0	-0.167	0	50	50
R2	2	-0.5	0	0	1	-0.5	0	48	24
R3	0	0.167	1	0	0	0.167	0	9	
R4	-2	-0.499	0	0	0	0.501	1	27	

0.5(R2) → R2

	x1	x2	x3	S1	S2	S3	z	RHS	rough work
R1	1	0.83	0	1	0	-0.167	0	50	
R2	1	-0.25	0	0	0.5	-0.25	0	24	
R3	0	0.167	1	0	0	0.167	0	9	
R4	-2	-0.499	0	0	0	0.501	1	27	

P.T.O

Iteration 2:

-1 (R2) + R1 → R1

2 (R2) + R4 → R4

	x1	x2	x3	S1	S2	S3	z	RHS	rough work
R1	0	1.08	0	1	-0.5	0.083	0	26	
R2	1	-0.25	0	0	0.5	-0.25	0	24	
R3	0	0.167	1	0	0	0.167	0	9	
R4	0	-0.999	0	0	1	0.001	1	75	

	x1	x2	x3	S1	S2	S3	z	RHS	rough work
R1	0	1.08	0	1	-0.5	0.083	0	26	24.07
R2	1	-0.25	0	0	0.5	-0.25	0	24	
R3	0	0.167	1	0	0	0.167	0	9	53.892
R4	0	-0.999	0	0	1	0.001	1	75	

P.T.O

(1/1.08) R1 → R1

	x1	x2	x3	S1	S2	S3	z	RHS	rough work
R1	0	1	0	1/1.08	-0.4629	0.083/1.08	0	24.074	24.07
R2	1	-0.25	0	0	0.5	-0.25	0	24	
R3	0	0.167	1	0	0	0.167	0	9	53.892
R4	0	-0.999	0	0	1	0.001	1	75	

Iteration 3:

0.25 (R1) + R2 → R2

-0.167 (R1) + R3 → R3

0.999 (R1) + R4 → R4

	x1	x2	x3	S1	S2	S3	z	RHS	rough work
R1	0	1	0	1/1.08	-0.4629	0.083/1.08	0	24.074	
R2	1	0	0	0.2314	0.3842	-0.2307	0	30.018	
R3	0	0	1	-0.154	0.0773	0.0489	0	4.992	
R4	0	0	0	0.925	0.5375	0.077	1	99.04	

We stop now as there is no negative element in Maximize row.

S1, S2, S3 are non-basic variables so they are equal to 0

S1 = 0

S2 = 0

S3 = 0

And

x1 = 30

x2 = 24

x3 = 5

z = 99

Maximum profit is 99