Contents

1. Discrete random variable X has the pmf as

$P(X=0) = 1/10$, $P(X=1) = 6/10$, $P(X=2) = 3/10$

Compute EX and VarX. (Answer: 6/5; 9/25)

Answer)

Answer 1)

$P(X=0) = 1/10$
$P(X=1) = 6/10$
$P(X=2) = 3/10$

x	0	1	2
$P(X=x)$	$\frac{1}{10}$	$\frac{6}{10}$	$\frac{3}{10}$

Checking $\sum P(x) = 1$

i.e $P(X=0) + P(X=1) + P(X=2) = 1$

$\therefore \frac{1}{10} + \frac{6}{10} + \frac{3}{10} = 1$ ✓

$$EX = \sum x P(x)$$

$$= 0\left(\frac{1}{10}\right) + 1\left(\frac{6}{10}\right) + 2\left(\frac{3}{10}\right)$$

$$= \frac{6}{10} + \frac{6}{10}$$

$$= \frac{12\cancel{0}}{10\cancel{0}} = \frac{6}{5}$$

$$\therefore EX = \frac{6}{5}$$

-Yash

Now, to find $\underline{VarX}$

$$VarX = E(x^2) - E(x)^2$$

Finding $E(\cancel{x}^2)$

$$E(x^2) = \Sigma x^2 P(x)$$

$$= 0^2\left(\frac{1}{10}\right) + 1^2\left(\frac{6}{10}\right) + 2^2\left(\frac{3}{10}\right)$$

$$= \frac{6}{10} + \frac{12}{10} = \frac{180}{100} = \frac{9}{5}$$

We already know EX

$$\therefore E(x)^2 = \left(\frac{6}{5}\right)^2 = \frac{36}{25}$$

$$\therefore VarX = E(x^2) - E(x)^2$$

$$= \frac{9}{5} - \frac{36}{25}$$

$$= \frac{225 - 180}{125}$$

$$= \frac{\cancel{45}\,9}{\cancel{125}\,25}$$

$$= \frac{9}{25}$$

$$\therefore VarX = \frac{9}{25}$$

-Yash

Therefore, final answer is EX = 6/5 and VarX = 9/25

2. The pdf of X is

$f(x) = Cx^3, 0 < x < 1.$

•What is the constant C?

•Find a such that $P(X > a) = P(X < a)$

•Find b such that $P(X > b) = 0.01$

(Answer: 4; $0.5^{1/4}$; $0.99^{1/4}$)

Answers)

•What is the constant C?

$f(x) = C x^3 \ , 0 < x < 1$

find C

$C \int_0^1 f(x)\, dx = 1$

$\therefore \ C \int_0^1 (x^3)\, dx = 1$

$\therefore \ C \left[\frac{x^4}{4} \right]_0^1 = 1$

$\therefore \ C \left[\frac{(1)^4}{4} - \frac{(0)^4}{4} \right] = 1$

$\therefore \ C \left[\frac{1}{4} - \frac{0}{4} \right] = 1$

$\therefore \ C [0.25] = 1$

$\therefore \ C = \frac{1}{0.25}$

$\therefore \ C = 4$

Constant C is 4

-Yash

•Find a such that P(X > a) = P(X < a)

Answer)

Find a such that $P(X>a) = P(X<a)$

$$\therefore \int_a^{\infty} f(x)\,dx = \int_{-\infty}^{a} f(x)\,dx$$

$$\int_a^1 Cx^3\,dx = \int_0^a Cx^3\,dx$$

$$\int_a^1 4x^3 \cdot dx = \int_0^a 4x^3 \cdot dx \quad \text{[As we know } c=4\text{]}$$

$$\left[\cancel{4}\,\frac{x^4}{\cancel{4}}\right]_a^1 = \left[\cancel{4}\cdot\frac{x^4}{\cancel{4}}\right]_0^a$$

$$\left[1^4 - a^4\right] = \left[a^4 - 0^4\right]$$

$$1 - a^4 = a^4$$

$$1 - a^4 + a^4 = a^4 + a^4 \quad \text{[adding } a^4 \text{ on both sides]}$$

$$1 = 2a^4$$

$$\frac{1}{2} = a^4$$

$$\left(\frac{1}{2}\right)^{\frac{1}{4}} = a$$

$$\boxed{\therefore\ a = \left(\frac{1}{2}\right)^{\frac{1}{4}} = (0.5)^{\frac{1}{4}}}$$

-Yash

•Find b such that P(X > b) = 0.01

Answer)

Find b such that $P(X>b) = 0.01$

$$\int_b^\infty f(x)\,dx$$

$$\int_b^1 C x^3\,dx = 0.01$$

$$\int_b^1 4x^3\,dx = 0.01 \qquad [\text{As we know, } C = 4]$$

$$\left[\cancel{4}\frac{x^4}{\cancel{4}}\right]_b^1 = 0.01$$

$$(1)^4 - (b)^4 = 0.01$$

$$1 - b^4 = 0.01$$

$$-b^4 = 0.01 - 1$$

$$-b^4 = -0.99$$

$$b^4 = 0.99$$

$$b = 0.99^{1/4}$$

$$\therefore b = (0.99)^{1/4}$$

—Yash

3) If X ~ N(0, σ^2), calculate the expectation of $Y_1 = 3X + 5$ and $Y_2 = |X|$.

Answer)

$X \sim N(0, \sigma^2)$

$\therefore$ mean $\mu = 0$

and variance σ^2

Expectation of Y_1

$y_1 = 3x + 5$

$\therefore E(y_1) = E(3x+5)$

$= \cancel{E}$

$= 3E(x) + 5E(1)$

$= 3(0) + 5(1)$ [Since mean $= E(x) = 0$]

$= 5$

$\boxed{\therefore \text{ Expectation of } Y_1 \text{ is } 5}$

Expectation of Y_2

$y_2 = |x|$

$\therefore E(y_2) = E(|x|)$

Calculating both possibilities;

when x is positive

$E(y_2) = E(x)$

$= 0$

when x is negative

$E(y_2) = E(-x)$

$= -E(x)$

$= -0$

$= 0$

$\boxed{\therefore \text{ Expectation of } Y_2 \text{ is } 0}$

-Yash

4) Let X1, X2, X3 be a random sample from standard normal population N (0,1), and $Y = X_1^2 + X_2^2 + X_3^2$. Compute EY.

Answer)

From class notes:

X_1, X_2, X_3 are random sample from normal popl^n $N(0,1)$

$$Y = X_1^2 + X_2^2 + X_3^2$$

Y has 3 logically independent values ;

X_1, X_2 and X_3 $\therefore p = 3$

$\therefore$ Y is the squared of independent values.

$$Y = \Sigma (X_i)^2$$

$$Y \sim \chi^2_3$$

$\therefore$ Y has a chi squared distribution of 3 degrees of freedom.

$$Var(Y) = 2(3) = \underline{\underline{6}}$$

$$E(Y) = \underline{\underline{3}}$$

-Yash

5. The random variable X has a normal distribution with mean $\mu = 8.4$ and standard deviation $\sigma = 4$.

• Find $P(X \leq 8)$.

Answer)

• Find $P(X \leq 8)$

$$z = \frac{x - \mu}{\sigma}, \quad \mu = 8.4, \quad \sigma = 4$$

$$P(x \leq 8) = P\left(\frac{x - \mu}{\sigma} \leq \frac{8 - 8.4}{4}\right)$$

$$= P\left(z \leq \frac{-0.4}{4}\right)$$

$$= P(z \leq -0.1)$$

$$\boxed{= \Phi(-0.1)}$$

P.T.O

• Suppose $\overline{X}$ is the average of 25 independent measurements of X, what is P($\overline{X}$ $\leq$ 8)?

Answer)

• with ~~error~~ standard error $\sigma / \sqrt{n}$

Suppose $\bar{x}$ is the average of 25 independent measurements.

$$P(\bar{x} \leq 8) = P\left(\frac{\bar{x} - \mu}{\sigma/\sqrt{n}} \leq \frac{8 - 8.4}{4/\sqrt{25}}\right)$$

[Since, for average of sample 25,

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$]

$$= P\left(z \leq \frac{-0.4}{4/5}\right)$$

$$= P\left(z \leq \frac{-0.4}{0.8}\right)$$

$$= P(z \leq -0.5)$$

$$\boxed{= \Phi(-0.5)}$$

-Yash

References

[1] https://www.investopedia.com/terms/d/degrees-of-freedom.asp