Contents

1. Failing to reject the null hypothesis when it is false is:
   (a) α
   (b) Type I error
   (c) β
   (d) Type II error

Answer: (d)

2. A significance test based on a small sample may not produce a statistically significant result even if the true value differs substantially from the null value. This type of result is known as
   (a) the significance level of the test.
   (b) a low power of the study.
   (c) a Type 1 error.
   (d) a Type 2 error.

Answer: (d)

3. Out of 1,000 randomized controlled trials that tested two biologically equivalent therapies against each other, we would expect 50 of the trials to yield p-values less than 0.05.
   (a) True
   (b) False

Answer: (b)

4. An oceanographer wants to test, on the basis of the mean of a random sample of size $n = 35$ and at the 0.05 level of significance, whether the average depth of ocean in a certain area is 72.4 fathoms. What will she decide if she gets $\bar{x} = 73.2$ fathoms, and she can assume from information gathered in similar studies that $\sigma = 2.1$ fathoms?

4. answer)

$\bar{x} = 73.2$

$\sigma = 2.1$

$\mu = 72.4$

$H_0 : \mu = 72.4$

$H_1 : \mu \neq 72.4$

$Z|_{0.05} = 1.96$

$$Z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$$

$$= \frac{73.2 - 72.4}{2.1 / \sqrt{35}}$$

$$= \frac{0.8}{0.354} = 2.259$$

$Z_{cal} = 2.259$

$Z_{tab} = 1.96$ (From Z distribution table)

$Z_{cal} > Z_{tab}$

∴ Null Hypothesis is rejected.

∴ Cannot confirm that average depth of ocean in certain area is 72.4 fathoms.

-Yash

5. The yield of alfalfa from a random sample of six test plots is 1.4, 1.6, 0.9, 1.9, 2.2, and 1.2 tons per acre. If we can assume the sample is taken from a normal population, test at the 0.05 level of significance whether this supports the contention that the average yield for this kind of alfalfa is 1.5 tons per acre.

5. answer)

$n = 6$

mean $= 1.53$

SD $= 0.471$

Test at 0.05 level of significance.

$H_0: \mu = 1.5 \quad ; \quad H_1: \mu \neq 1.5$

Now,

$$t = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$$

$$= \frac{1.53 - 1.5}{0.471/\sqrt{6}}$$

$$= \frac{0.033}{0.192} = 0.171$$

$\therefore t_{cal} = 0.171$

$t_{tab} = 2.571$ (from t-distribution table)

$\because t_{cal} < t_{tab}$

$\therefore$ Null Hypothesis is not rejected.

-Yash

6. In a study to test whether or not there is a difference between the average heights of adult females in two different countries, random samples of size $n_1 = 120$ and $n_2 = 150$ yielded $\bar{x}_1 = 62.7$ inches and $\bar{x}_2 = 61.8$ inches. Extensive studies of a similar kind have shown that it is reasonable to let $\sigma_1 = 2.50$ inches and $\sigma_2 = 2.62$ inches. Test at the 0.05 level of significance whether the difference between the two population means is significant.

6. answer)

$n_1 = 120 \qquad \bar{x}_1 = 62.7 \qquad \sigma_1 = 2.50$

$n_2 = 150 \qquad \bar{x}_2 = 61.8 \qquad \sigma_2 = 2.62$

$H_0 : \bar{x}_1 = \bar{x}_2$

$H_1 : \bar{x}_1 \neq \bar{x}_2$

$$z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} \quad \frac{\sigma_2^2}{n_2}}}$$

$$= \frac{62.7 - 61.8}{\sqrt{\frac{6.25}{120} + \frac{6.86}{150}}}$$

$$= \frac{0.9}{0.3127}$$

$$= 2.878$$

$Z_{cal} > Z_{tab}$

$2.878 > 1.96$

∴ Null Hypothesis is rejected.

-Yash

7. The following random samples are measurements of the heat-producing capacity (in millions of calories per ton) of coal from two mines. Use the 0.05 level of significance to test whether the difference between the means of the two mines is significant.

Mine 1:	8,380	8,180	8,500	7,840	7,990
Mine 2:	7,660	7,510	7,910	8,070	7,790

7. answer)

here maximum times x is greater than y.

Therefore,

$H_0 : \mu_x = \mu_y$

$H_1 : \mu_x \neq \mu_y$

$d = x - y = 720, 670, 590, -230, 200.$

$\Sigma d = 1950$

$d^2 = 518400, 448900, 348100, 52900, 40000$

$\Sigma d^2 = 1408300$

$\bar{d} = \frac{\Sigma d}{n} = \frac{1950}{5} = 390$

$$S^2 = \frac{1}{n-1}\left[\Sigma d^2 - \frac{(\Sigma d)^2}{n}\right]$$

$$= \frac{1}{5-1}\left[1408300 - \frac{(1950)^2}{5}\right]$$

$$= \frac{1}{4}\left[1408300 - 760500\right]$$

$$= 161950$$

P.T.O

-Yash

$$t = \frac{\bar{d}}{\sqrt{s^2/n}}$$

$$= \frac{390}{\sqrt{161950/5}}$$

$$= 2.167$$

$$t_{cal} = 2.167$$

$$t_{tab} = 2.132$$ (from t distribution table)

$\because\ t_{cal} > t_{tab}$

$\therefore$ Null Hypothesis is rejected.

8. In the previous problem, use the 0.05 level of significance to test whether there is any evidence that the standard deviations of the two populations are not equal.

8. answer)

$\bar{x} = \frac{\Sigma x}{n} = 8178$ $\bar{y} = \frac{\Sigma y}{n} = 7788$

$d = x - \bar{x}$ <u>For Mine 1</u>

$= 202, 2, 322, -338, -188$

$\Sigma d = 0$

$d^2 = 40804, 4, 103684, 114244, 35344$

$\Sigma d^2 = 294080$

<u>For Mine 2</u>

$d = y - \bar{y}$

$= -128, -278, 122, 282, 2$

$\Sigma d = 0$

$d^2 = 16384, 77284, 14884, 79524, 4$

$\Sigma d^2 = 188080$

$$S_1^2 = \frac{1}{n_1 - 1}\left[\Sigma d^2 - \frac{\Sigma d^2}{n}\right]$$

$$= \frac{1}{4}(294080) = 73520$$

$$S_2^2 = \frac{1}{n_2 - 1}\left[\Sigma d^2 - \frac{\Sigma d^2}{n}\right]$$

$$= \frac{1}{4}(188080) = 47020$$

-Yash

$H_0 : S_1^2 = S_2^2$

$H_1 : S_1^2 \neq S_2^2$

$$F = \frac{S_1^2}{S_2^2} = \frac{73520}{47020} = 1.563$$

$F(n_1 - 1, n_2 - 1)$

$F(4, 4) = 6.39$ (From F distribution table)

$F_{cal} < F_{tab}$

$1.563 < 6.39$

$\therefore$ Null hypothesis is not rejected.

9. It has been claimed that more than 70% of the students attending a large state university are opposed to a plan to increase student fees in order to build new parking facilities. If 15 of 18 students selected at random at that university are opposed to the plan, test the claim at the 0.05 level of significance.

9. answer)

$P_0 = 0.7$

$q_0 = 0.3$

$n = 18$

$\bar{P} = 0.833$ $(\because 15 \div 18)$

```
> pbar = 15/18
> pbar
[1] 0.8333333
> p0 = 0.7
> n = 18
> z = (pbar-p0)/sqrt(p0*(1-p0)/n)
> z
[1] 1.234427
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$P(Z > 1.234) = 1 - P(Z < 1.234)$

$= 1 - 0.890$

$= 0.11$

Pvalue $= 0.11$

0.05 level of significance

$\alpha = 0.05$

Pvalue $> \alpha$

$0.11 > 0.05$

$\therefore$ Null Hypothesis is not rejected.

$\therefore$ Cannot confirm the claim.

10. A study showed that 56 of 80 persons who saw a spaghetti sauce advertised on television during a situation comedy and 38 of 80 other persons who saw it advertised during a football game remembered the brand name two hours later. At the 0.01 level of significance, what can we conclude about the claim that it is more cost effective to advertise this product during a situation comedy rather than during a football game? Assume that the cost of running the ad on the two programs is the same.

10. answer)

$n_1 = 80 \quad ; n_2 = 80$

$P_1 = \frac{56}{80} = 0.7 \quad ; P_2 = \frac{38}{80} = 0.475$

$$\hat{P} = \frac{n_1 P_1 + n_2 P_2}{n_1 + n_2} = \frac{(80 \times 0.7) + (80 \times 0.475)}{160}$$

$$= \frac{56 + 38}{160} = 0.5875$$

$\hat{q} = 1 - 0.5875 = 0.4125$

$$Z = \frac{0.7 - 0.475}{\sqrt{(0.5875)(0.4125)\left(\frac{1}{80} + \frac{1}{80}\right)}}$$

$$= \frac{0.225}{\sqrt{(0.242)(0.025)}}$$

$$= \frac{0.225}{0.0777} = 2.895$$

$\because Z_{cal} > Z_{tab}$

$2.895 > 2.58$ (From z distribution table)

$\therefore$ Null Hypothesis is rejected.

$\therefore$ It is more cost effective to run the advertisement during situation comedy than a football game

·Yah