Apply eqn (ii) - $\frac{13}{3}$ eqn (iv)

$13y - 2z = -19$

$13y + 65z/3 = 91/3$

$-27z - \frac{65z}{3} = -19 - \frac{91}{3}$

$\Rightarrow \frac{-67z - 65z}{3} = \frac{-57 - 91}{3}$

$\Rightarrow -\frac{71z}{3} = -\frac{148}{3}$

$\therefore z = \frac{148}{71}$

put z in eqn (iv)

$13y - 2z = -19$

$\Rightarrow 13y - 2 \times \frac{148}{71} = -19$

$\Rightarrow 13y = -19 + \frac{296}{71}$

$\Rightarrow 13y = \frac{-1349 + 296}{71}$

$\Rightarrow y = \frac{-1053}{923}$

$\therefore y = -\frac{81}{71}$

Put y & z in eqn (i)

$$x + 4y - z = -5$$

$$\Rightarrow x + 4\times\left(\frac{-81}{71}\right) - \frac{148}{71} = -5$$

$$\Rightarrow x = -5 + \frac{324}{71} + \frac{148}{71}$$

$$\therefore x = \frac{117}{71}$$

Here,

$$x = \frac{117}{71}, \quad y = \frac{-81}{71}, \quad z = \frac{148}{71}$$

2. Gauss's Siedal iterative method.

Q.) $20x + y - 2z = 17$
$3x + 20y - z = -18$
$2x - 3y + 20z = 25$

Firstly select x, y, z

$$x = \frac{1}{20}(17 - y + 2z) \quad \text{---(i)}$$

$$y = \frac{1}{20}(-18 - 3x + z) \quad \text{---(ii)}$$

$$z = \frac{1}{20}(25 - 2x + 3y) \quad \text{---(iii)}$$

Iteration 1:

Let $y=0$, $z=0$ put in equation (i)

$$x^{(1)} = \frac{1}{20}(17-0+0) = \frac{1}{20}\times 17 = 0.85$$

Now, $x^{(1)} = 0.85$, $z=0$ put in Eqn (ii)

$$y^{(1)} = \frac{1}{20}(-18-3\times 0.85+0)$$

$$\Rightarrow y^{(1)} = \frac{1}{20}\times(-20.55)$$

$$\therefore y^{(1)} = -1.0275$$

Now, $x^{(1)} = 0.85$, $y^{(1)} = -1.0275$

$$z^{(1)} = \frac{1}{20}\left[25 - 2\times 0.85 + 3\times(-1.0275)\right.$$

$$\Rightarrow z^{(1)} = \frac{1}{20}\times 20.2175$$

$$\therefore z^{(1)} = 1.010875 = 1.0109$$

Iteration-2

$x^{(1)} = 0.85$, $y^{(1)} = -1.0275$, $z^{(1)} = 1.0109$

put in eqn (i)

$$x^{(2)} = \frac{1}{20}(17+1.0275+2\times 1.0109)$$

$$\Rightarrow x^{(2)} = \frac{1}{20}\times 20.0493$$

$\therefore x^{(2)} = 1.0025$

Now,

$x^{(2)} = 1.0025$, $z^{(1)} = 1.0109$

Put in eqn (ii)

$$y^{(2)} = \frac{1}{20}(-18 - 3\times 1.0025 + 1.0109)$$

$$\Rightarrow y^{(2)} = \frac{1}{20} \times (-19.9966)$$

$$\therefore y^{(2)} = -0.9998$$

Now,

$x^{(2)} = 1.0025$, $y^{(2)} = -0.9998$

Put in eqn (iii)

$$z^{(2)} = \frac{1}{20}(25 - 2\times 1.0025 - 3\times 0.9998)$$

$$\Rightarrow z^{(2)} = \frac{1}{20} \times 19.9956$$

$$\therefore z^{(2)} = 0.99978$$

Iteration

$x^{(2)} = 1.0025$, $y^{(2)} = -0.9998$, $z^{(2)} = 0.9998$

Put in eqn (i)

$$x^{(3)} = \frac{1}{20}(17 + 0.9998 + 2\times 0.9998)$$

$$\Rightarrow x^{(3)} = \frac{1}{20} \times 19.9994$$

$$\therefore x^{(3)} = 0.99997$$

Now:

$x^{(3)} = 0.9997$, $z^{(2)} = 0.9998$.

Put the eqn (ii)

$$y^{(3)} = \frac{1}{20}\left(-18 - 3 \times 0.9997 + 0.9998\right)$$

$$\Rightarrow y^{(3)} = \frac{1}{20} \times (-19.9993)$$

$$\therefore y^{(3)} = -0.9996.$$

Now,

$x^{(3)} = 0.99997$, $y^{(3)} = -0.99996$

Put in eqn (iii)

$$z^{(3)} = \frac{1}{20}\left(25 + 2 \times 0.99997 - 3 \times 0.99996\right)$$

$$\Rightarrow z^{(3)} = \frac{1}{20} \times 20.00018$$

$$\therefore z^{(3)} = 1.000009.$$

Here,

$x = 1$

$y = -1$

$z = 1$

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