

1. Picard's Method.

$$y^{(n)} = y_0 + \int_{x_0}^x f(x, y^{(n-1)}) dx$$

Q.) Find the solution of $\frac{dy}{dx} = 1 + xy$ which passes through $(0, 1)$ at $x = 0.2$.

$$f(x, y) = 1 + xy, \quad x_0 = 0, \quad y_0 = 1, \quad x = 0.2$$

Iteration

$$y^{(1)} = y_0 + \int_{x_0}^x f(x, y^{(0)}) \cdot dx$$

$$\Rightarrow y^{(1)} = 1 + \int_0^x f(x, 1) dx$$

$$\Rightarrow y^{(1)} = 1 + \int_0^x (1+x) dx$$

$$\Rightarrow y^{(1)} = 1 + \left[x + \frac{x^2}{2} \right]_0^x$$

$$\Rightarrow y^{(1)} = 1 + x + \frac{x^2}{2}$$

$$\Rightarrow y^{(1)}(0.2) = 1 + 0.2 + \frac{0.04}{2}$$

$$\therefore y^{(1)}(0.2) = 1.22$$

Iteration-2

$$y^{(2)} = y_0 + \int_0^x f(x, y^{(1)}) dx$$

$$\Rightarrow y^{(2)} = y_0 + \int_0^x f\left(x, 1 + x + \frac{x^2}{2}\right) dx$$

$$\Rightarrow y^{(2)} = 1 + \int_0^x \left[1 + x\left(1 + x + \frac{x^2}{2}\right)\right] dx$$

$$\Rightarrow y^{(2)} = 1 + \int_0^x \left(1 + x + x^2 + \frac{x^3}{2}\right) dx$$

$$\Rightarrow y^{(2)} = 1 + \left[x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{8}\right]_0^x dx$$

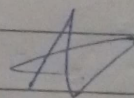
$$\Rightarrow y^{(2)} = 1 + x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{8}$$

$$\Rightarrow y^{(2)}(0.2) = 1 + 0.2 + 0.02 + 0.003 + 0.0002$$

$$\therefore y^{(2)}(0.2) = 1.2232$$

Here,

$$y = 1.22$$



2.) Euler's Method :-

$$y_n = y_{n-1} + h f(x_{n-1}, y_{n-1})$$

Q.) Find the solution of differential equation
 $\frac{dy}{dx} = x + y$ with initial condition $x_0 = 0, y_0 = 1$
by using Euler method at $x = 0.5$ taking
 $h = 0.1$.

$$f(x, y) = x + y, \quad x_0 = 0, \quad y_0 = 1, \quad h = 0.1$$

by Euler's Method.

$$y_n = y_{n-1} + hf(x_{n-1}, y_{n-1})$$

Now,

$$y_1 = y_0 + hf(x_0, y_0)$$

$$\Rightarrow y_1 = 1 + 0.1 \cdot f(0, 1)$$

$$\Rightarrow y_1 = 1 + 0.1 \times 1$$

$$\therefore y_1 = 1.1$$

$$x_1 = x_0 + h$$

$$\Rightarrow x_1 = 0 + 0.1$$

$$\therefore x_1 = 0.1$$

$$y_2 = y_1 + hf(x_1, y_1)$$

$$\Rightarrow y_2 = 1.1 + 0.1 \cdot f(0.1, 1.1)$$

$$\Rightarrow y_2 = 1.1 + 0.1 \times 1.2$$

$$\therefore y_2 = 1.22$$

$$x_2 = x_1 + h$$

$$\Rightarrow x_2 = 0.1 + 0.1$$

$$\therefore x_2 = 0.2$$

$$y_3 = y_2 + hf(x_2, y_2)$$

$$\Rightarrow y_3 = 1.22 + 0.1 \cdot f(0.2, 1.22)$$

$$\Rightarrow y_3 = 1.22 + 0.1 \times 1.42$$

$$\therefore y_3 = 1.362$$

$$x_3 = x_2 + h$$

$$\Rightarrow x_3 = 0.2 + 0.1$$

$$\therefore x_3 = 0.3$$

$$y_4 = y_3 + hf(x_3, y_3)$$

$$\Rightarrow y_4 = 1.362 + 0.1 f(0.3, 1.362)$$

$$\Rightarrow y_4 = 1.362 + 0.1 \times 1.662$$

$$\therefore y_4 = 1.5282$$

$$x_4 = x_3 + h$$

$$\Rightarrow x_4 = 0.3 + 0.1$$

$$\therefore x_4 = 0.4$$

$$y_5 = y_4 + hf(x_4, y_4)$$

$$\Rightarrow y_5 = 1.5282 + 0.1 f(0.4, 1.5282)$$

$$\Rightarrow y_5 = 1.5282 + 0.1 \times 1.9282$$

$$\therefore y_5 = 1.72102$$

$$x_5 = x_4 + h$$

$$\Rightarrow x_5 = 0.4 + 0.1$$

$$\therefore x_5 = 0.5$$

Now,

x	y
0	1
0.1	1.1
0.2	1.22
0.3	1.362
0.4	1.5282
0.5	1.72102