Relation

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Relation ⇒ Let A & B be non-empty sets, then any subset R of the A×B is called a relation from A to B.
It is denoted by R

$R = \{(x,y) : x \in A, y \in B \text{ and } x R y\}$

Total no. of Relations from A to B
If set A has m elements and set B has n elements then A×B has mn elements. So, total no. of Relation from A to B is 2^{mn}.

ex. Let $A = \{a,b,c\}$, $B = \{2,4,6,10\}$ &
$R = \{(a,2), (b,4), (c,10)\}$. Is R a relation from A to B?

Domain of R ⇒ Let $R = \{(x,y) : x \in A, y \in B\}$ be a relation from A to B then set of first co-ordinates of every elements of R is called Domain of R. It is denoted by dom(R)

ex. $R = \{(a,1), (a,3), (b,4), (c,3)\}$
$Dom(R) = \{a,b,c\}$

Range of R ⇒ Let $R = \{(x,y) : x \in A, y \in B\}$ be a relation from A to B then set of second co-ordinates of every element of R is called Range of R. It is denoted by Ran(R)

ex. $R = \{(a,1), (a,3), (b,1), (b,2), (b,3)\}$
$Ran(R) = \{1,2,3\}$

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Operations on Relation →

1) Complement of a Relation →

Consider a relation R from A to B then complement of relation R denoted by $\bar{R}$ or R' or R^c is a relation from A to B st

$$R' = \{(a,b) : (a,b) \notin R\}$$

ex- Let R be a relation from X to Y where
$X = \{1,2,3\}$, $Y = \{8,9\}$
$R = \{(1,8), (2,8), (1,9), (3,9)\}$. find R'.

2) Inverse Relation →

Consider a relation R from A to B then ~~compe~~ inverse of relation R is denoted by R^{-1} is a relation from B to A st

$$R^{-1} = \{(b,a) : (a,b) \in R\}$$

ex- Let R be a relation from A to B where
$A = \{1,2,4,8\}$, $B = \{2,4\}$
R = $x+y$ is divisible by 2
find inverse relation.

Sol^n

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Intersection and union of Relation:-

If R and S are two relations then

$R \cup S = \{(x,y) : x R y \text{ or } x S y\}$

$R \cap S = \{(x,y) : x R y \text{ and } x S y\}$

ex- Let $R_1 = \{(1,1), (2,2), (3,3), (4,4), (3,4), (4,3)\}$
$R_2 = \{(1,1), (2,2), (3,3), (4,4), (1,2), (2,1)\}$
then find $R_1 \cup R_2$ & $R_1 \cap R_2$

soln

Properties of Relation:-

A relation R on a set A satisfies certain properties are defined as -

1) Reflexive Relation:-

A relation R on a set A is reflexive if
$(a,a) \in R \quad \forall\, a \in A$ or $\boxed{a R a \ \forall\, a \in A}$

ex- Let $A = \{a,b\}$
& $R = \{(a,a), (a,b), (b,b)\}$

soln $(a,a) \in R \Rightarrow a R a$
$(b,b) \in R \Rightarrow \cancel{b} R a$
So, R is reflexive

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Irreflexive Relation :-

A relation R on set A is irreflexive if,
$(a,a) \notin R \quad \forall\, a \in A$
i.e. $a \not R a \quad \forall\, a \in A$

ex. Let $A = \{1,2\}$ & $R = \{(1,2),(2,1)\}$
$(1,1) \notin R$, and $(2,2) \notin R$
$\Rightarrow$ R is irreflexive.

Non-Reflexive Relation :-

A relation R on a set A is non-reflexive if R is neither reflexive nor irreflexive

ex. Let $A = \{1,2,3\}$
$R = \{(1,1),(1,2),(2,3)\}$
$\Rightarrow$ R is neither reflexive nor irreflexive

Symmetric Relation :- A relation R on set A is said to be symmetric relation if
$(a,b) \in R \Rightarrow (b,a) \in R \quad \forall\, (a,b) \in A$

Asymmetric Relation :- A relation R on set A is said to be asymmetric relation if
$(a,b) \in R$ but $(b,a) \notin R$

Anti-symmetric Relation :- A relation R on a set A is said to be antisymmetric if
$(a,b) \in R$ and $(b,a) \in R \Rightarrow \boxed{a=b}$

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Transitive Relation :- A relation on set A is said to transitive if $(a,b) \in R$ and $(b,c) \in R \Rightarrow (a,c) \in R$ $\forall\ a,b,c \in A$

Ex.- Give an example of a relation which is -

i) reflexive and transitive but not symmetric.

ii) symmetric and transitive but not reflexive

iii) reflexive and symmetric but not transitive.

iv) reflexive and transitive but neither symmetric nor antisymmetric.

Equivalence Relation :-

A relation on a set A is called an equivalence relation if it is reflexive, symmetric and transitive i.e R is an equivalence relation on A if it has following three properties.

1) $(a,a) \in R \quad \forall\ a \in A$

2) $(a,b) \in R \Rightarrow (b,a) \in R$

3) $(a,b) \in R$ & $(b,c) \in R \Rightarrow (a,c) \in R$

Classification of functions

1) **One-One function** ⇒ Let $f : A \to B$ be a function and it is called one-one if
$x \neq y \Rightarrow f(x) \neq f(y)$ for $x, y \in A$
or if $f(x) = f(y) \Rightarrow x = y$

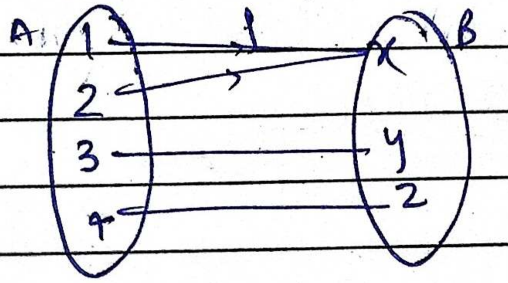
ex- Let $f : R \to R$ s.t $f(x) = ax + b$, $a \neq 0$

$f(x) = f(y)$
$ax + b = ay + b$
$ax = ay$
$\boxed{x = y}$

So, $f : R \to R$ is a ~~f~~ one-one function.

2) **Onto Function** ⇒ A function $f : A \to B$ is called onto if Range $f = B$

ex- Let $A = \{1, 2, 3, 4\}$ & $B = \{x, y, z\}$

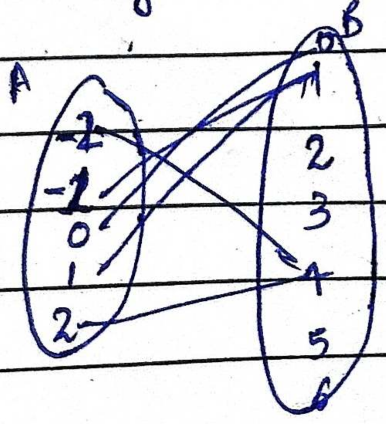


Here Range $f = \{x, y, z\} = B$ which is onto

3) **Many One Function** ⇒ If two or more than two elements of domain X have f-image in Y i.e. $x_1 \neq x_2 \Rightarrow f(x_1) = f(x_2)$

$f(x) = x^2$

ex-



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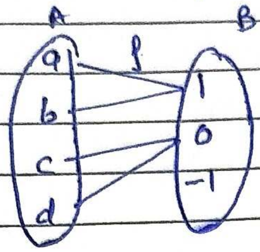
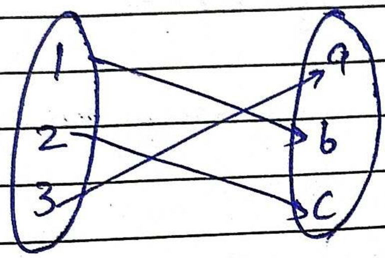
7) <u>Into Mapping</u> :- $f: A \to B$ is said to into mapping if $\{f(x): x \in A\} \subset B$



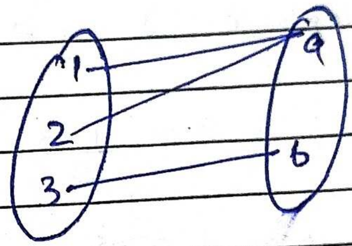
This is into mapping.

~~Bijective Function~~ :-

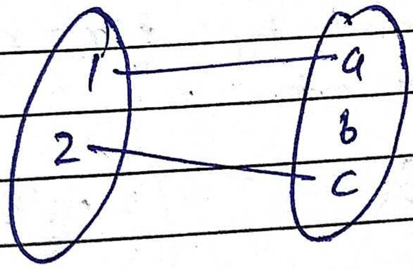
<u>Ex</u>:-



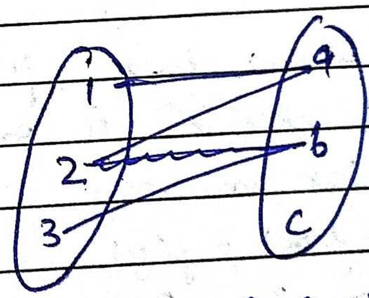
one-one, onto



many-one, onto



one-one, into



many-one, into

<u>Bijective Function</u> :- Let $f: A \to B$ be a function and it is called bijective if f is one-one and onto.

Ex- $f: R \to R$, $f(x) = 2x-3$. Is f is bijective?

Sol^n For one-one

$f(x) = f(y)$

$\Rightarrow 2x-3 = 2y-3$

$\Rightarrow 2x = 2y$

$\Rightarrow x = y$

$\Rightarrow f$ is one-one

For onto

suppose $\exists$ $y \in$ Ran f s.t. $y = 2x-3$

$\Rightarrow x = \frac{y+3}{2} \in R$ R→ real no.

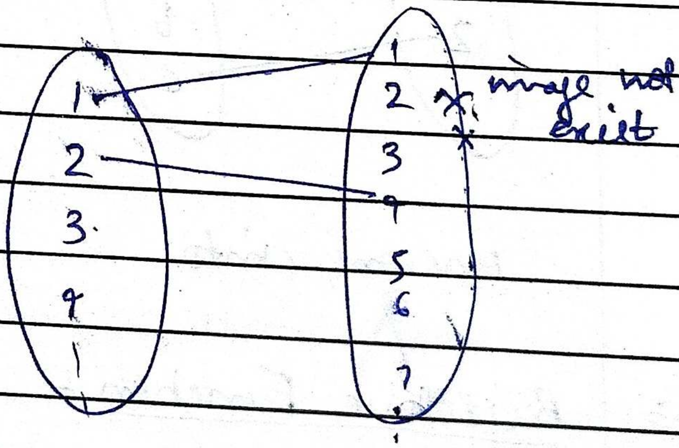
$\Rightarrow$ Ran $f = R$

$\Rightarrow f$ is onto function

So, f is bijective function.

Q Show that $f: Z^+ \to Z^+$ defined by $f(x) = x^2$ $\forall$ $x \in Z^+$ is one-one but not onto.

Sol^n For one-one



$f(x) = f(y)$

$\Rightarrow x^2 = y^2$

$\Rightarrow x = y$

f is one-one

for onto

$z = x^2$

$\Rightarrow x = \sqrt{z} \in Z^+$

for $z=2$

$\nexists$ x s.t. $f(x) = 2$

$\Rightarrow f(x)$ is not onto

$\Rightarrow f$ is one-one but not onto.

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Q Find the domain of the function

i) $f(x) = \dfrac{x}{x^2+1}$ ii) $f(x) = \sqrt{x-4}$