UNIT I :- Matrix

Matrix :-

A matrix A is a rectangular array in which elements are arranged in square bracket and column.

Ex :-

$$A = \begin{array}{cc} \begin{bmatrix} 2 & 3 & 4 & 2 \\ 5 & 6 & 7 & 1 \\ -1 & -2 & 3 & 4 \end{bmatrix} & \begin{array}{l} \rightarrow R_1 \\ \rightarrow R_2 \\ \rightarrow R_3 \end{array} \\ \begin{array}{cccc} \downarrow & \downarrow & \downarrow & \downarrow \\ C_1 & C_2 & C_3 & C_4 \end{array} & 3 \times 4 \end{array}$$

* Elements of matrix are represented by a_{11}, a_{12}, a_{13} .... and a_{21}, a_{22}, a_{23} .... in short elements ar represented by a_{ij} where $i = 1, 2, 3$ ...... and $j = 1, 2, 3$ ......

* Matrix is denoted by $A = [a_{ij}]_{m \times n}$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \cdots & a_{3n} \\ \vdots & \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & a_{m3} & & a_{mn} \end{bmatrix}_{m \times n}$$

$$A = [a_{ij}]_{3 \times 4}$$

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$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix}_{3 \times 4}$$

* In matrix A Horizontal lines represent = Row
vertical lines represent = column

Types of Matrices :-

(1) **Row Matrix :-** A matrix which contain only one row and any column is called row matrix

Ex :-

(i) $\begin{bmatrix} 1 & 2 & 3 & 4 \end{bmatrix}_{1 \times 4}$

(ii) $\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}_{1 \times 6}$ (iii) $\begin{bmatrix} 7 \end{bmatrix}_{1 \times 1}$

(ii) **Column Matrix :-** A matrix which contain only one column and any column is called column matrix

Ex :-

(i) $\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}_{4 \times 1}$ (ii) $\begin{bmatrix} 8 \end{bmatrix}_{1 \times 1}$ (iii) $\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}_{5 \times 1}$

(iii) Null Matrix or Zero matrix :-
—*—*—*—*—*—*—

A matrix in which all elements are zero is called null matrix and is denoted by O

Ex:-

$$A = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}_{3\times1} \quad B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}_{3\times3} \quad C = \begin{bmatrix} 0 \end{bmatrix}_{1\times1}$$

(iv) Square matrix :-
—*—*—*— A matrix is called square matrix if no of rows = no of column.

Ex:-

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 4 & 5 \end{bmatrix}_{3\times3} \quad B = \begin{bmatrix} 2 & 3 \\ 0 & 4 \end{bmatrix}_{2\times2} \quad C = \begin{bmatrix} 5 \end{bmatrix}_{1\times1}$$

(v) Diagonal Matrix :-
—*—*—*— A Square Matrix is said to be diagonal Matrix if its all non-diagonal entries are zero

Ex:-

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}_{3\times3} \quad \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}_{3\times3}$$

Principal diagonal

$a_{11}, a_{22}, a_{33} \ldots\ldots a_{nn}$ = diagonal element

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(vi) Scalar Matrix : A square matrix M is said to be scalar matrix if

(i) All non diagonal elements are zero

(ii) All diagonal elements are same

Ex :-

$$M = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}_{3\times 3} \qquad M = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}_{2\times 2}$$

$$M = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4\times 4}$$

(vii) Identity Matrix or Unit Matrix :-

A square matrix M is said to be unit matrix if

(i) All non diagonal elements are zero

(ii) All diagonal elements are equal to 1.

Ex :-

$$M = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}_{2\times 2} \qquad M = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4\times 4}$$

* unit matrix is denoted by I or In

(viii) Transpose of Matrix :
— * — * — * — * — * — Let M be any given matrix A matrix obtained by inter-changing corresponding row to corresponding column is known as transpose of matrix M and is denoted by M^T or M'

Ex :-

$$M = \begin{bmatrix} 2 & 4 & 6 \\ 0 & 7 & 2 \\ -1 & -2 & 3 \end{bmatrix}_{3\times3} \qquad M^T = \begin{bmatrix} 2 & 0 & -1 \\ 4 & 7 & -2 \\ 6 & 2 & 3 \end{bmatrix}_{3\times3}$$

$$A = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}_{3\times1} \qquad A^T = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}_{1\times3}$$

$$B = \begin{bmatrix} 2 & 3 & 4 \\ -1 & -2 & -3 \end{bmatrix}_{2\times3} \qquad B^T = \begin{bmatrix} 2 & -1 \\ 3 & -2 \\ 4 & -3 \end{bmatrix}_{3\times2}$$

(ix) Symmetric Matrix :
— * — * — * — * — A square Matrix is said to be symmetric Matrix if $\boxed{M = M^T}$

* In this Matrix element $a_{ij} = a_{ji}$

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Ex :-

$$M = \begin{bmatrix} 1 & -2 \\ -2 & 3 \end{bmatrix} \qquad M^T = \begin{bmatrix} 1 & -2 \\ -2 & 3 \end{bmatrix}$$

$$M = M^T$$

⇒ M is a symmetric matrix

$$A = \begin{bmatrix} a & b & c \\ b & d & f \\ c & f & g \end{bmatrix}_{3\times3} \qquad A^T = \begin{bmatrix} a & b & c \\ b & d & f \\ c & f & g \end{bmatrix}_{3\times3}$$

$$A = A^T$$

⇒ A is a symmetric Matrix

(x) Skew Symmetric Matrix :

—*—*—*—*—* A square Matrix is said to be skew symmetrix matrix if

$\boxed{M = -M^T}$

* In this matrix element $a_{ij} = -a_{ji}$

* Diagonal element must be zero

$$M = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix} \qquad M^T = -\begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$$

$$M = -M^T$$

⇒ M is skew symmetric matrix.

(xi) Upper triangular Matrix (U.T.M)

— * — * — * — * — * — * -

A square matrix whose all element below the principal diagonal are zero is called U.T.M

Ex:-

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 3 & 5 \\ 0 & 0 & 7 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix} \quad C = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

(xii) Lower triangular Matrix (L.T.M) :

— * — * — * — * — * — * A square Matrix whose all elements above the principal diagonal are zero is called L.T.M

Ex:-

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 3 & 0 \\ 2 & 5 & 7 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix} \quad C = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

(xiii) Orthogonal of a matrix :

— * — * — * — * — * — A square matrix M is said to be orthogonal if $MM^T = I$ (Identity Matrix)

Ex:-

$$A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

$$A^T = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

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$$AA^T = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2\theta + \sin^2\theta & -\cos\theta.\sin\theta + \sin\theta.\cos\theta \\ -\sin\theta.\cos\theta + \cos\theta.\sin\theta & \sin^2\theta + \cos^2\theta \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I \qquad AA^T = I$$

A is orthogonal matrix

(xiv) conjugate of a Matrix :

—*—*—*—*—*—

Ex:-

$$A = \begin{bmatrix} 4i-2 & 3 & 2i \\ 2i & -3 & -2+i \\ 4 & 2 & -3 \end{bmatrix}$$

$$\bar{A} = \bar{A} = \begin{bmatrix} -4i-1 & 3 & -2i \\ -2i & -3 & -2-i \\ 4 & 2 & -3 \end{bmatrix}$$

(xv) Matrix $A^\theta := (\bar{A})^T = \overline{(A^T)}$

—*—*— Transpose of a conjugate matrix

Ex:-

$$A = \begin{bmatrix} 2+i & 3 & 4 \\ 3i & 4 & 5i \\ 0 & 3 & -4i \end{bmatrix}$$

$$\bar{A} = \begin{bmatrix} 2-i & 3 & 4 \\ -3i & 4 & -5i \\ 0 & 3 & 4i \end{bmatrix}$$

$$(\bar{A})^T = (A^\theta) = \begin{bmatrix} 2-i & -3i & 0 \\ 3 & 4 & 3 \\ 4 & -5i & 4i \end{bmatrix}$$

(xvi) Hermitian Matrix :
— * — * — * — * $\boxed{A = A^\theta}$

Ex :-

$$A = \begin{bmatrix} 1 & 2+3i & 3+i \\ 2-3i & 2 & 1-2i \\ 3-i & 1+2i & 5 \end{bmatrix}$$

$$\bar{A} = \begin{bmatrix} 1 & 2-3i & 3-i \\ 2+3i & 2 & 1+2i \\ 3+i & 1-2i & 5 \end{bmatrix}$$

$$(\bar{A})^T = A^\theta = \begin{bmatrix} 1 & 2+3i & 3+i \\ 2-3i & 2 & 1-2i \\ 3-i & 1+2i & 5 \end{bmatrix}$$

since $A = A^\theta \Rightarrow A$ is ~~sym~~ Hermitian matrix

(xvii) Skew Hermitian Matrix :
— * — * — * — * — * — $\boxed{A = -A^\theta \text{ or } A^\theta = -A}$

Ex :-

$$A = \begin{bmatrix} 0 & 2-3i & 4+5i \\ -2-3i & 0 & 2i \\ -4+5i & 2i & 0 \end{bmatrix}$$

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$$\bar{A} = \begin{bmatrix} 0 & 2+3i & 4-5i \\ -2+3i & 0 & -2i \\ -4-5i & -2i & 0 \end{bmatrix}$$

$$(\bar{A})^T = A^\theta = \begin{bmatrix} 0 & -2+3i & -4-5i \\ 2+3i & 0 & -2i \\ 4-5i & -2i & 0 \end{bmatrix}$$

$$(\bar{A})^\theta = -\begin{bmatrix} 0 & 2-3i & 4+5i \\ -2-3i & 0 & 2i \\ -4+5i & 2i & 0 \end{bmatrix}$$

$\boxed{A^\theta = -A}$ $\Rightarrow$ A is Skew Hermitian matrix

(xviii) <u>Matrix multiplication :</u>

—*—*—*—*—

let $[A]_{m\times n}$ and $[B]_{n\times p}$

* Multiplications AB is Possible only when no of columns of matrix A = No of row of matrix B

Ex :-

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 2 & 3 & 4 \end{bmatrix}_{3\times 3} \qquad B = \begin{bmatrix} 1 & -2 \\ -1 & 0 \\ 2 & -1 \end{bmatrix}_{3\times 2}$$

$$AB = \begin{bmatrix} 0+(-1)+4 & 0+0+(-2) \\ 1+(-2)+6 & -2+0+(-3) \\ 2+(-3)+8 & -4+0+(-4) \end{bmatrix}$$

$$= \begin{bmatrix} 3 & -2 \\ 5 & -5 \\ 7 & -8 \end{bmatrix}_{3\times 2}$$

(xix) Idempotent matrix :- $\boxed{A^2 = A}$

—*—*—*—*

Ex :-

$$A = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$$

$$A^2 = A.A = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix} \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 4+2+(-4) & -4+(-6)+8 & -8+(-8)+12 \\ -2+3+4 & 2+9-8 & 4+12-12 \\ 2+2-3 & -2-6+6 & -4-8+9 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix} \qquad A^2 = A$$

$\Rightarrow$ A is an idempotent matrix

(xx) Invaluntary matrix :- $\boxed{A^2 = I}$ (identity matrix)

—*—*—*—*—

E/x :-

$$A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

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$$A^2 = \begin{bmatrix} 1+0 & 0+0 \\ 0+0 & 0+1 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

(xx) # Identity matrix is always an involuntary matrix.

— x — x — x — x — x — x —

Ex:-

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1+0+0 & 0+0+0 & 0+0+0 \\ 0+0+0 & 0+1+0 & 0+0+0 \\ 0+0+0 & 0+0+0 & 0+0+1 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

→ A is an involuntary matrix.

(xxi) Nilpotent Matrix :

— * — * — * — A matrix M is said to be nilpotent if $\boxed{M^k = 0}$, where K is a positive integer. The least positive integer K for which $\boxed{M^k = 0}$ is called index.

Ex:-

$$A = \begin{bmatrix} ab & b^2 \\ -a^2 & -ab \end{bmatrix}$$

$$A^2 = A \cdot A = \begin{bmatrix} ab & b^2 \\ -a^2 & -ab \end{bmatrix} \begin{bmatrix} ab & b^2 \\ -a^2 & -ab \end{bmatrix}$$

$$= \begin{bmatrix} \cancel{a^2b^2} - \cancel{a^2b^2} & \cancel{ab^3} - \cancel{ab^3} \\ -\cancel{a^3b} + \cancel{a^3b} & -\cancel{a^2b^2} + \cancel{a^2b^2} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \qquad \boxed{\text{Index} = 2}$$

(xxii) Unitary Matrix :-

— * — * — * — * — $\boxed{AA^\theta = I}$ or $\boxed{A^\theta A = I}$

Ex:-

$$A = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix}, \quad \bar{A} = 1/\sqrt{3} \begin{bmatrix} 1 & 1-i \\ 1+i & -1 \end{bmatrix}$$

$$(\bar{A})^T = A^\theta = 1/\sqrt{3} \begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix}$$

$$AA^\theta = 1/\sqrt{3} \begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix} \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix}$$

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$$= \frac{1}{3}\begin{bmatrix} 1+(1+i)(1-i) & 1+i+(-1)(1+i) \\ 1-i+(-1)(1-i) & (1-i)(1+i)+1 \end{bmatrix}$$

$$= \frac{1}{3}\begin{bmatrix} 1+1-i^2 & 1+i-1-i \\ 1-i-1+i & 1-i^2+1 \end{bmatrix}$$

$$= \frac{1}{3}\begin{bmatrix} 2-(-1) & 0 \\ 0 & 2-(-1) \end{bmatrix}$$

$$= \frac{1}{3}\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

$$AA^{\theta} = I$$

→ A is a unitary matrix.

(xxiii) Equal matrix or Equivalent matrix:

—*—*—*—*—*—*—*—

Two matrix A and B are said to be equal if

(i) Both matrix have same size.
(ii) corresponding elements of matrix A and B are equal.

Ex :-

$$A = \begin{bmatrix} 1 \\ 3 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

(xxiv) Elementary Transformations.

—*—*—*—*—*—*—*—

(1) We can interchange any two rows and column $(R_i \leftrightarrow R_j)$ or $(C_i \leftrightarrow C_j)$

Ex:-

$$A = \begin{bmatrix} 1 & 2 & 5 \\ 3 & 4 & 6 \end{bmatrix}$$

$C_1 \leftrightarrow C_3$

$$\sim \begin{bmatrix} 5 & 2 & 1 \\ 6 & 4 & 3 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 0 & 1 & 1 \end{bmatrix}$$

$R_1 \leftrightarrow R_2$

$$\sim \begin{bmatrix} 4 & 5 & 6 \\ 1 & 2 & 3 \\ 0 & 1 & 1 \end{bmatrix}$$

(ii) We can multiply any row or column by a non-zero number k (scalar)

Ex:-

$$A = \begin{bmatrix} 1 & 3 \\ 2 & 4 \\ 2 & 6 \end{bmatrix}$$

$R_1 \to 2R_1$

$$\sim \begin{bmatrix} 2 & 6 \\ 2 & 4 \\ 2 & 6 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 0 & 1 & 1 \end{bmatrix}$$

$R_3 \to 2R_3$

$$\sim \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 0 & 2 & 2 \end{bmatrix}$$

(iii) We can add any two rows or column ($R_i \to R_i + R_j$) or ($C_j \to C_j + C_k$)

Ex:-

$$A = \begin{bmatrix} 1 & 3 \\ 2 & 4 \\ 2 & 0 \end{bmatrix}$$

$R_1 \to R_1 + R_2$

$$\sim \begin{bmatrix} 3 & 7 \\ 2 & 4 \\ 2 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 0 & 1 & 1 \end{bmatrix}$$

$C_2 \to C_2 - 2C_1$

$$\sim \begin{bmatrix} 1 & 0 & 3 \\ 4 & -3 & 6 \\ 0 & 1 & 1 \end{bmatrix}$$

{[illegible]}

Rank by Echelon Form :

Rank (A) = $\rho(A)$ = No. of non-zero row in echelon form.

Echelon Form :

A matrix A is said to be in echelon form if A satisfy following conditions.

(i) The non-zero element in a row is unity (1)

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 5 \\ 0 & 0 & 1 \end{bmatrix}$$

(ii) All zero rows if any are at the bottom of the matrix

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 5 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

(iii) The no. of zero preceding the first non-zero element in a row is less than the no. of such zero in the succeeding row

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 5 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

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(iv) Matrix should be in U.T.M.

* Find Rank

Q. $$A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \\ 2 & 3 & 1 \end{bmatrix}$$

Soln..

$R_2 \to R_2 - 3R_1$, $R_3 \to R_3 - 2R_1$

$$\sim \begin{bmatrix} 1 & 2 & 3 \\ 0 & -5 & -7 \\ 0 & -1 & -5 \end{bmatrix}$$

$R_2 \to R_2 / (-5)$

$$\sim \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 7/5 \\ 0 & -1 & -5 \end{bmatrix}$$

$R_3 \to R_3 + R_2$

$$\sim \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 7/5 \\ 0 & 0 & -18/5 \end{bmatrix}$$

$R_3 \to R_3 / -18/5$

$$\sim \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 7/5 \\ 0 & 0 & 1 \end{bmatrix}$$

$\rho(A) =$ No. of Non-zero row $= 3$

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Q. $A = \begin{bmatrix} 4 & 1 & 3 & 8 \\ 6 & 2 & 6 & -1 \\ 10 & 3 & 9 & 7 \\ 16 & 4 & 12 & 15 \end{bmatrix}$

Solⁿ:-

$R_3 \to R_3 - R_2$,

$\sim \begin{bmatrix} 4 & 1 & 3 & 8 \\ 6 & 2 & 6 & -1 \\ 4 & 1 & 3 & 8 \\ 16 & 4 & 12 & 15 \end{bmatrix}$

$R_3 \to R_3 - R_1$

$R_3 \longleftrightarrow R_4$

$\sim \begin{bmatrix} 4 & 1 & 3 & 8 \\ 6 & 2 & 6 & -1 \\ 16 & 4 & 12 & 15 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$R_3 \to R_3 - 4R_1$

$\sim \begin{bmatrix} 4 & 1 & 3 & 8 \\ 6 & 2 & 6 & -1 \\ 0 & 0 & 0 & -17 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$R_1 \to \frac{1}{4} R_1$, $R_3 \to \frac{-1}{17} R_3$

$\sim \begin{bmatrix} 1 & 1/4 & 3/4 & 2 \\ 6 & 2 & 6 & -1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

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$R_2 \rightarrow R_2 - 6R_1$

$$\sim \begin{bmatrix} 1 & 1/4 & 3/4 & 2 \\ 0 & 1/2 & 3/2 & -13 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_2 \rightarrow 2R_2$

$$\sim \begin{bmatrix} 1 & 1/4 & 3/4 & 2 \\ 0 & 1 & 3 & -26 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\rho(A) = 3.$

Q. $A = \begin{bmatrix} 2 & 3 & 4 \\ 3 & 1 & 2 \\ -1 & 2 & 2 \end{bmatrix}$

Solⁿ:

$R_3 \leftrightarrow R_1$

$$\sim \begin{bmatrix} -1 & 2 & 2 \\ 3 & 1 & 2 \\ 2 & 3 & 4 \end{bmatrix}$$

$R_2 \rightarrow R_2 + 3R_1$

$R_3 \rightarrow R_3 + 2R_1$

$$\sim \begin{bmatrix} -1 & 2 & 2 \\ 0 & 7 & 8 \\ 0 & 7 & 8 \end{bmatrix}$$

$R_3 \rightarrow R_3 - R_2$

$$\sim \begin{bmatrix} -1 & 2 & 2 \\ 0 & 7 & 8 \\ 0 & 0 & 0 \end{bmatrix}$$

$R_2 \rightarrow \frac{1}{7} R_2$

$R_1 \rightarrow -1 R_1$

$$\sim \begin{bmatrix} 1 & -2 & -2 \\ 0 & 1 & 8/7 \\ 0 & 0 & 0 \end{bmatrix}$$

$\rho(A) = 2$

Normal Form of a matrix

—*—*—*—*—*—*—*—*

A matrix A can be reduced by elementary row and column transformation into one of the following equivalent matrix.

(i) $\begin{bmatrix} I_n & 0 \\ 0 & 0 \end{bmatrix}$ (ii) $\begin{bmatrix} I_n \\ 0 \end{bmatrix}$ (iii) $\begin{bmatrix} I_n & 0 \end{bmatrix}$ (iv) $\begin{bmatrix} I_n \end{bmatrix}$

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Q. Reduce the following matrix to the normal form and hence find its rank:

$$A = \begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$$

Solⁿ:-

$R_1 \longleftrightarrow R_2$

$$\sim \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & -3 & -1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$$

$R_3 \rightarrow R_3 - 3R_1$

$R_4 \rightarrow R_4 - R_2$

$$\sim \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & -3 & -1 \\ 0 & 1 & -3 & -1 \\ 0 & 1 & -3 & -1 \end{bmatrix}$$

$R_3 = R_3 - R_2$

$R_4 = R_4 - R_2$

$$\sim \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & -3 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$C_4 \rightarrow C_4 - C_1 + C_2$

$C_3 \rightarrow C_3 - C_1 + 3C_2$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} I_2 & 0 \\ 0 & 0 \end{bmatrix}$$