UNIT-V. SOLUTION OF DIFFERENTIAL EQUATION

1. Picard's Method.

$$y^{(n)} = y_0 + \int_{x_0}^{x} f(x, y^{n-1})\, dx$$

Q.) Find the solution of $\frac{dy}{dx} = 1 + xy$ which passes through $(0,1)$ at $x = 0.2$.

$f(x,y) = 1 + xy$, $x_0 = 0$, $y_0 = 1$, $x = 0.2$

Iteration 1

$$y^{(1)} = y_0 + \int_{x_0}^{x} f(x, y^{(0)})\, dx$$

$$\Rightarrow y^{(1)} = 1 + \int_{0}^{x} f(x, 1)\, dx$$

$$\Rightarrow y^{(1)} = 1 + \int_{0}^{x} (1 + x)\, dx$$

$$\Rightarrow y^{(1)} = 1 + \left[x + \frac{x^2}{2} \right]_0^x$$

$$\Rightarrow y^{(1)} = 1 + x + \frac{x^2}{2}$$

$$\Rightarrow y^{(1)}(0.2) = 1 + 0.2 + \frac{0.04}{2}$$

$$\therefore y^{(1)}(0.2) = 1.22$$

Iteration-2

$$y^{(2)} = y_0 + \int_0^x f(x, y^{(1)})\,dx$$

$$\Rightarrow y^{(2)} = y_0 + \int_0^x f\left(x, 1 + x + \frac{x^2}{2}\right) dx$$

$$\Rightarrow y^{(2)} = 1 + \int_0^x \left[1 + x\left(1 + x + \frac{x^2}{2}\right)\right] dx$$

$$\Rightarrow y^{(2)} = 1 + \int_0^x \left(1 + x + x^2 + \frac{x^3}{2}\right) dx$$

$$\Rightarrow y^{(2)} = 1 + \left[x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{8}\right]_0^x dx$$

$$\Rightarrow y^{(2)} = 1 + x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{8}$$

$$\Rightarrow y^{(2)}(0.2) = 1 + 0.2 + 0.02 + 0.003 + 0.0002$$

$$\therefore y^{(2)}(0.2) = 1.2232$$

Here,

$$y = 1.22$$

Ans

2.) Euler's Method :-

$$\left[\; y_n = y_{n-1} + h f(x_{n-1}, y_{n-1}) \;\right]$$

Q.) Find the solution of differential equation $\frac{dy}{dx} = x + y$ with initial condition $x_0 = 0, y_0 = 1$ by using Euler method at $x = 0.5$ taking $h = 0.1$.

$f(x,y) = x+y$, $x_0 = 0$, $y_0 = 1$, $h = 0.1$

by Euler's Method.

$y_n = y_{n-1} + hf(x_{n-1}, y_{n-1})$

Now,

$y_1 = y_0 + hf(x_0, y_0)$

$\Rightarrow y_1 = 1 + 0.1 \cdot f(0, 1)$

$\Rightarrow y_1 = 1 + 0.1 \times 1$

$\therefore y_1 = 1.1$

$x_1 = x_0 + h$

$\Rightarrow x_1 = 0 + 0.1$

$\therefore x_1 = 0.1$

$y_2 = y_1 + hf(x_1, y_1)$

$\Rightarrow y_2 = 1.1 + 0.1 f(0.1, 1.1)$

$\Rightarrow y_2 = 1.1 + 0.1 \times 1.2$

$\therefore y_2 = 1.22.$

$x_2 = x_1 + h$

$\Rightarrow x_2 = 0.1 + 0.1$

$\therefore x_2 = 0.2$

$y_3 = y_2 + hf(x_2, y_2)$

$\Rightarrow y_3 = 1.22 + 0.1 f(0.2, 1.22)$

$\Rightarrow y_3 = 1.22 + 0.1 \times 1.42$

$\therefore y_3 = 1.362$

$x_3 = x_2 + h$

$\Rightarrow x_3 = 0.2 + 0.1$

$\therefore x_3 = 0.3$

$y_4 = y_3 + hf(x_3, y_3)$

$\Rightarrow y_4 = 1.362 + 0.1 f(0.3, 1.362)$

$\Rightarrow y_4 = 1.362 + 0.1 \times 1.662$

$\therefore y_4 = 1.5282$

$x_4 = x_3 + h$

$\Rightarrow x_4 = 0.3 + 0.1$

$\therefore x_4 = 0.4$

$y_5 = y_4 + hf(x_4, y_4)$

$\Rightarrow y_5 = 1.5282 + 0.1 (0.4, 1.5282)$

$\Rightarrow y_5 = 1.5282 + 0.1 \times 1.9282$

$\therefore y_5 = 1.72102$

$x_5 = x_4 + h$

$\Rightarrow x_5 = 0.4 + 0.1$

$\therefore x_5 = 0.5$

Now,

x	y
0	1
0.1	1.1
0.2	1.22
0.3	1.362
0.4	1.5282
0.5	1.72102