UNIT-I ROOTS OF EQUATIONS

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1. Bisections Method :-

$$\left[x_2 = \frac{x_0 + x_1}{2} \right]$$

Q.) Find the real root of the given correct to 1 decimal place

$$f(x) = x^3 - x - 1$$

$f(x) = x^3 - x - 1$

$f(0) = -1$

$f(1) = 1-1-1 = -1$

$f(2) = 8-2-1 = 5$

$f(1) \cdot f(2) < 0$

hence root lies between 1 & 2

Iteration-1

$x_0 = 1$, $x_1 = 2$ by bisection method

$$x_2 = \frac{x_0 + x_1}{2}$$

$$\Rightarrow x_2 = \frac{1+2}{2}$$

$$\therefore x_2 = 1.5$$

$f(1.5) = (1.5)^3 - 1.5 - 1$
$= 0.875$

-ve +ve ve

1 1.5 2

$f(1) \cdot f(1.5) < 0$

hence root lies between 1 & 1.5

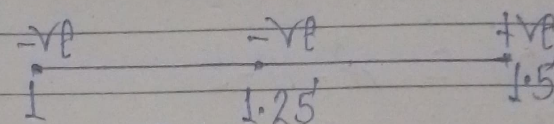
Iteration-2

$x_0 = 1$, $x_1 = 1.5$ by bisection method

$$x_2 = \frac{x_0 + x_1}{2}$$

$$\Rightarrow x_2 = \frac{1 + 1.5}{2}$$

$$\therefore x_2 = 1.25$$



$$f(1.25) = (1.25)^3 - 1.25 - 1$$
$$= -0.296875$$

$f(1.25) \cdot f(1.5) < 0$

hence root lies between 1.25 & 1.5

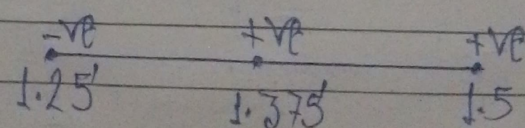
Iteration-3.

$x_0 = 1.25$, $x_1 = 1.5$ by bisection method.

$$x_2 = \frac{x_0 + x_1}{2}$$

$$\Rightarrow x_2 = \frac{1.25 + 1.5}{2}$$

$$\therefore x_2 = 1.375$$



$$f(1.375) = (1.375)^3 - 1.375 - 1$$
$$= 0.224609 3$$

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$f(1.25) \cdot f(1.375) < 0$

Iteration-4 hence root lies between 1.25 & 1.375

$x_0 = 1.25$, $x_1 = 1.375$ by bisection method

$$x_2 = \frac{x_0 + x_1}{2}$$

$$\Rightarrow x_2 = \frac{1.25 + 1.375}{2}$$

$$\therefore x_2 = 1.3125$$

-ve	-ve	+ve
1.25	1.3125	1.375

$$f(1.3125) = (1.3125)^3 - 1.3125 - 1$$
$$= -0.05$$

$f(1.3125) \cdot f(1.375) < 0$

Iteration-5 hence root lies between 1.3125 & 1.375

$x_0 = 1.3125$, $x_1 = 1.375$ by bisection method

$$x_2 = \frac{x_0 + x_1}{2} = \frac{1.3125 + 1.375}{2}$$

$$\therefore x_2 = 1.34375$$

-ve	+ve	+ve
1.3125	1.34375	1.375

$$f(1.34375) = (1.34375)^3 - 1.34375 - 1$$
$$= 0.082611$$

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The real root of given equation is 1.3.

2. Regula falsi method or Method of falsi position :-

$$x_2 = \frac{x_0 f_1 - x_1 f_0}{f_1 - f_0}$$

$$x_2 = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$$

Q.) Find the real root of the given correct to 1 decimal place $f(x) = x^3 - x - 1$

$f(x) = x^3 - x - 1$
$f(1) = 1 - 1 - 1 = -1$
$f(2) = 8 - 2 - 1 = 5$

$f(1) \cdot f(2) < 0$
Hence root lies between 1 & 2.

Iteration 1

$x_0 = 1, \ x_1 = 2$
$f_0 = -1 ; \ f_1 = 5$

by regula falsi method.

$$x_2 = \frac{x_0 f_1 - x_1 f_0}{f_1 - f_0}$$

$$\Rightarrow x_2 = \frac{1 \times 5 + 2 \times 1}{5 + 6}$$

$$\Rightarrow x_2 = \frac{7}{6} = 1.167$$

$$\underset{1}{-ve} \qquad \underset{1.167}{-ve} \qquad \underset{2}{+ve}$$

$f(1.167) = (1.167)^3 - 1.167 - 1$
$= -0.578$

$f(1.167) \cdot f(2) < 0$
hence root lies between 1.167 & 2.

Iteration-2.

$x_0 = 1.167 \quad , \quad x_1 = 2$
$f_0 = -0.578 \quad , \quad f_1 = 5$

$$x_2 = \frac{x_0 f_1 - x_1 f_0}{f_1 - f_0}$$

$$\Rightarrow x_2 = \frac{1.167 \times 5 + 2 \times 0.578}{5 + 0.578}$$

$$\Rightarrow x_2 = \frac{6.991}{5.578} = 1.253.$$

$$\underset{1.167}{-ve} \qquad \underset{1.253}{-ve} \qquad \underset{2}{+ve}$$

$f(1.253) = (1.253)^3 - 1.253 - 1$
$= -0.286$

$f(1.253) \cdot f(2) < 0$
hence root lies between 1.253 & 2.

Iteration-3

$x_0 = 1.253 \quad , \quad x_1 = 2$
$f_0 = -0.286 \quad , \quad f_1 = 5$

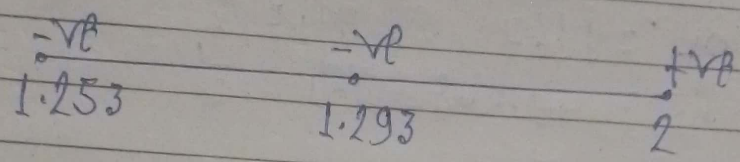
by regula falsi method

$$x_2 = \frac{x_0 f_1 - x_1 f_0}{f_1 - f_0}$$

$$\Rightarrow x_2 = \frac{1.253 \times 5 + 2 \times 0.286}{5 + 0.286}$$

$$\Rightarrow x_2 = \frac{6.837}{5.286}$$

$$\therefore x_2 = 1.293.$$



$$f(1.293) = (1.293)^3 - 1.293 - 1$$
$$= -0.131$$

$f(1.293) \cdot f(2) < 0$

hence root lies between 1.293 & 2

Iteration 4.

$x_0 = 1.293$, $x_1 = 2$
$f_0 = -0.131$, $f_1 = 5$

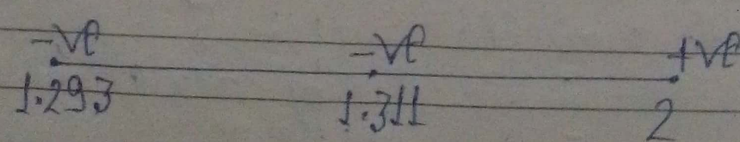
by regula falsi method.

$$x_2 = \frac{x_0 f_1 - x_1 f_0}{f_1 - f_0}$$

$$\Rightarrow x_2 = \frac{1.293 \times 5 + 2 \times 0.131}{5 + 0.131.}$$

$$\Rightarrow x_2 = \frac{6.727}{5.131}$$

$$\therefore x_2 = 1.311$$



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$f(1.311) = (1.311)^3 - 1.311 - 1$
$= -0.058$

Iterations 5

$f(1.311) \cdot f(2) < 0$

hence root lies between 1.311 & 2.

$x_0 = 1.311$, $x_1 = 2$
$f_0 = -0.058$, $f_1 = 5$

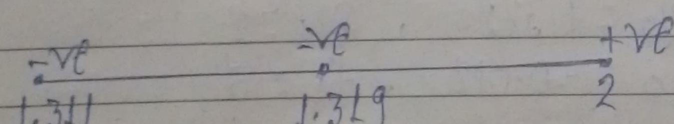
by regula falsi method.

$$x_2 = \frac{x_0 f_1 - x_1 f_0}{f_1 - f_0}$$

$$\Rightarrow x_2 = \frac{1.311 \times 5 + 2 \times 0.058}{5 + 0.058}$$

$$\Rightarrow x_2 = \frac{6.671}{5.058}$$

$$\therefore x_2 = 1.319$$



$f(1.319) = (1.319)^3 - 1.319 - 1$
$= -0.024$

$f(1.319) \cdot f(2) < 0$

hence root lies between 1.319 & 2.

Iteration-6.

$x_0 = 1.319$, $x_1 = 2$
$f_0 = -0.024$, $f_1 = 5$

by regula falsi method.

$$x_2 = \frac{x_0 f_1 - x_1 f_0}{f_1 - f_0}$$

$$\Rightarrow x_2 = \frac{1.319 \times 5 + 2 \times 0.024}{5 + 0.024}$$

$$\Rightarrow x_2 = \frac{6.643}{5.024}$$

$$\therefore x_2 = 1.322$$

-ve ——— -ve ——— +ve

1.319 ——— 1.322 ——— 2

$$f(1.322) = (1.322)^3 - 1.322 - 1$$
$$= -0.012$$

$f(1.322) \cdot f(2) < 0$

hence root lies between 1.322 & 2

Iteration-7

$x_0 = 1.322$, $x_1 = 2$

$f_0 = -0.012$, $f_1 = 5$

$$x_2 = \frac{x_0 f_1 - x_1 f_0}{f_1 - f_0}$$

$$\Rightarrow x_2 = \frac{1.322 \times 5 + 2 \times 0.012}{5 + 0.012}$$

$$\Rightarrow x_2 = \frac{6.634}{5.012}$$

$$\therefore x_2 = 1.324$$

-ve ——— -ve ——— +ve

1.322 ——— 1.324 ——— 2

$f(1.324) = (1.324)^3 - 1.324 - 1$
$= -0.003$

$f(1.324) \cdot f(2) < 0$

Iteration-8

hence root lies between 1.324 & 2

$x_0 = 1.324 \quad , \quad x_1 = 2$
$f_0 = -0.003 \quad , \quad f_1 = 5$

by regula falsi method

$$x_2 = \frac{x_0 f_1 - x_1 f_0}{f_1 - f_0}$$

$$\Rightarrow x_2 = \frac{1.324 \times 5 + 2 \times 0.03}{5 + 0.003}$$

$$\Rightarrow x_2 = \frac{6.626}{5.003}$$

$\therefore x_2 = 1.3244$

$$\begin{array}{ccc} -ve & -ve & +ve \\ 1.324 & 1.3244 & 2 \end{array}$$

$f(1.3244) = (1.3244)^3 - 1.3244 - 1$
$= -.001$

$f(1.3244) \cdot f(2) < 0$

Iteration-9

hence root lies between 1.3244 & 2.

$x_0 = 1.3244 \quad , \quad x_1 = 2$
$f_0 = -0.001 \quad , \quad f_1 = 5$

by regula falsi method

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$$x_2 = \frac{x_0 f_1 - x_1 f_0}{f_1 - f_0}$$

$$\Rightarrow x_2 = \frac{1.3244 \times 5 + 2 \times 0.001}{5 + 0.001}$$

$$\Rightarrow x_2 = \frac{6.624}{5.001}$$

$$\therefore x_2 = 1.3245$$

-ve	-ve	+ve
1.3244	1.3245	2

$$f(1.3245) = (1.3245)^3 - 1.3245 - 1$$
$$= -0.0009$$

$f(1.3245) \cdot f(2)$

Hence root lies between 1.3245 & 2.

Iteration-10

$x_0 = 1.3245$ $\quad x_1 = 2$

$f_0 = -0.0009$ $\quad f_1 = 5$

$$x_2 = \frac{x_0 f_1 - x_1 f_0}{f_1 - f_0}$$

$$\Rightarrow x_2 = \frac{1.3245 \times 5 + 2 \times 0.0009}{5 + 0.0009}$$

$$\Rightarrow x_2 = 1.325$$

-ve	+ve	+ve
1.3245	1.325	2

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$$f(1.325) = (1.325)^3 - 1.325 - 1$$
$$= 0.001$$

The real root of given equation is 1.3

Ans

3. Newton Raphson Method :-

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

or

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

Q.) Find the real root of the given correct to 1 decimal place $f(x) = x^3 - x - 1$

$$f(x) = x^3 - x - 1$$
$$f'(x) = 3x^2 - 1$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$\Rightarrow x_{n+1} = x_n - \frac{x_n^3 - x_n - 1}{3x_n^2 - 1}$$

$$\Rightarrow x_{n+1} = \frac{3x_n^3 - \cancel{x_n} - x_n^3 + \cancel{x_n} + 1}{3x_n^2 - 1}$$

$$\Rightarrow x_{n+1} = \frac{2x_n^3 + 1}{3x_n^2 - 1} \quad \text{—— (i)}$$

$$f(x) = x^3 - x - 1$$
$$f(0) = -1$$
$$f(1) = -1$$
$$f(2) = 5$$

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Root lies between 1 & 2

$$x_0 = \frac{1+2}{2} = \frac{3}{2} = 1.5$$

Root $x=0$ in eqn (i)

Iteration 1

$$x_1 = \frac{2x_0^3+1}{3x_0^2-1}$$

$$\Rightarrow x_1 = \frac{2\times(1.5)^3+1}{3\times(1.5)^2-1}$$

$$\Rightarrow x_1 = \frac{6.750+1}{6.75-1}$$

$$\therefore x_1 = 1.348$$

Iteration-2.

$$x_2 = \frac{2x_1^3+1}{3x_0^2-1}$$

$$\Rightarrow x_2 = \frac{2\times(1.348)^3+1}{3\times(1.348)^2-1}$$

$$\Rightarrow x_2 = \frac{5.899}{4.451}$$

$$\therefore x_2 = 1.325$$

The real root of given eqn is 1.3