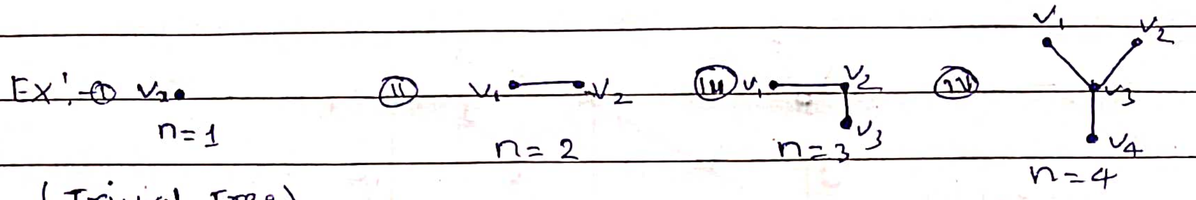
Unit :- 04

Sr. No.________ Date:________

Trees and cut-sets.

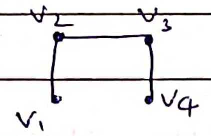
⇒ Tree :- A ~~graph~~ connected graph without any circuit is called tree.

Ex :- (i)



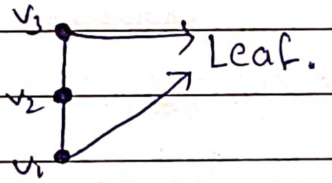
(Trivial Tree)

(v)



* The edge of tree is called branches.

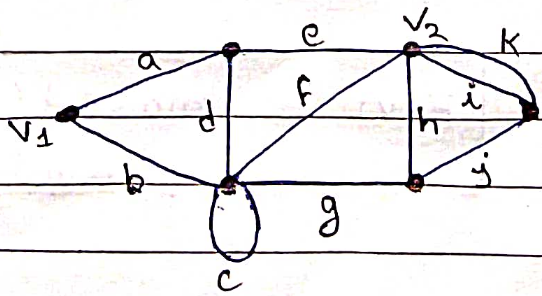
* The vertex of degree 1 is called **leaf**



Distance and ~~Centre~~ Centre in a ~~tree~~ Graph:

⇒ Let G be a connected graph and v_i, v_j are two vertices in a graph G. then, The distance ~~b~~ $d(v_i, v_j)$ between the vertices v_i and v_j is the length of the shortest path between v_i and v_j

$d(v_1, v_2) = 2$



$(b, f) = 2$

$(a, e) = 2$

$(b, c, f) = 3$

$(b, g, h) = 3$

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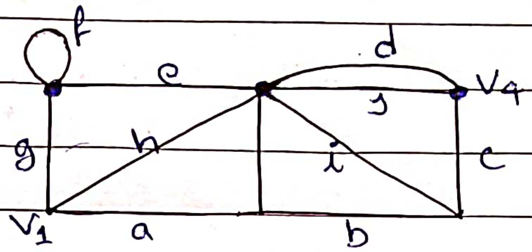
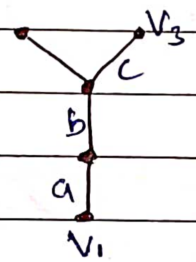
* Distance in connected graph may not be unique.

Ex:-

$d(v_1, v_4) = 2$



* In a tree distance between two vertex is unique.



$d(v_1, v_3) = 3$

Very - very Important.

Ecentricity of a vertex :-

This ~~ecen~~ Ecentricity $E(v)$ of a vertex v in a graph G is the distance between v and the vertex v_i farthest from v in G.

$$\boxed{E(v) = \max_{v_i \in G} d(v, v_i)}$$

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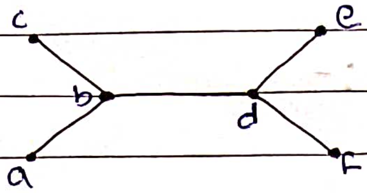
Ex:-



$E(c) = 3$

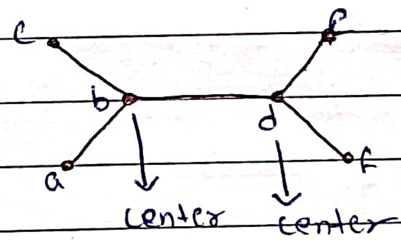
$E(d) = 2$

$E(e) = 3$

$E(f) = 3$

Center of a graph :-

⇒ A center of a graph G is a vertex whose Ecentricity in G is minimum.



$E(a) = 3$

$\boxed{E(b) = 2}$

$E(c) = 3$

$\boxed{E(d) = 2}$

$E(e) = 3$

$E(f) = 3$

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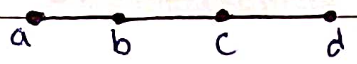
Redius of a tree :-

⇒ The ecentricity of a center in a tree is called Redius of a tree.

Ex:-



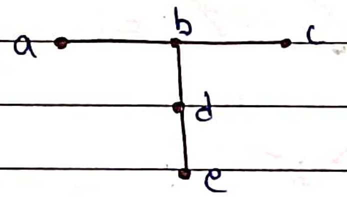
$E(a) = 3$

$E(b) = 2$ $\qquad C(T_1) = b, c$

$E(c) = 2$

$E(d) = 3$ $\qquad \boxed{R(T_1) = 2}$ Ans.

Ex:-



$E(a) = 3$

$E(b) = 2$ $\qquad C(T_2) = b, d$

$E(c) = 3$

$E(d) = 2$ $\qquad \boxed{R(T_2) = 2}$ Ans.

$E(e) = 3$

Diameter of a tree :-

⇒ The diameter of a tree is defined as the length of longest path in a tree.

$E(a) = 3$

$E(b) = 2$

$E(c) = 2$

$E(d) = 3$

$C(T_1) = a, d$

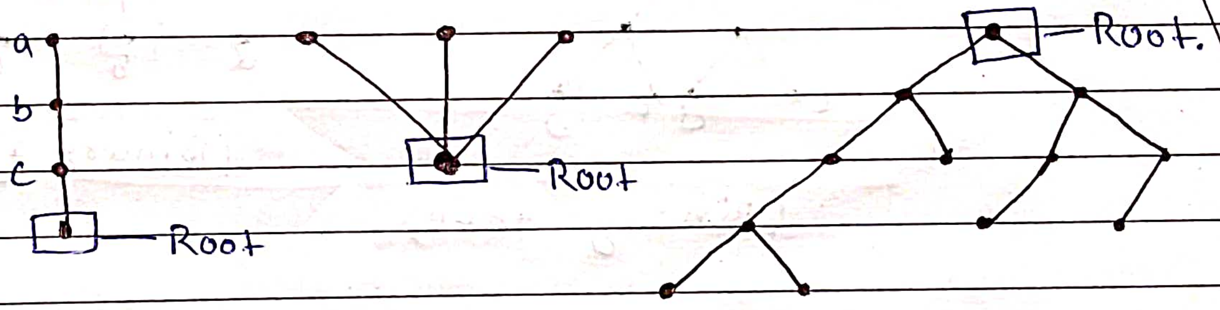
$R(T_1) = 2$

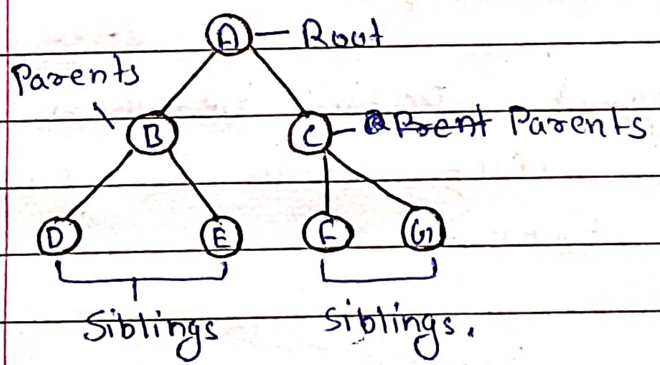
$\therefore \boxed{d(T_1) = 3}$ Ans.

Rooted Tree:-

⇒ A ~~tree~~ rooted tree is a tree with a distinguisted vertex called the root (Noode).

Ex:-





Leaf Node :- A vertex which do not have any child is called leaf Node.

Sr. No.

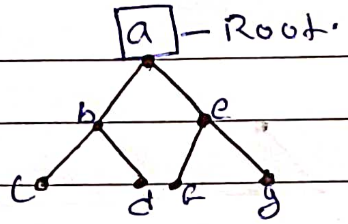
Binary tree:-

⇒ A tree in which tree is exactly one vertex of degree two and each of remaning verties is of degree one or three is called binary tree.

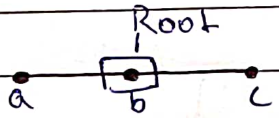
Note:- In binary tree, vertex of degree two is called root.

Ex:-

(1)



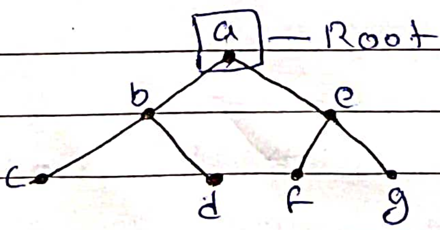
(Binary tree)



Binary tree

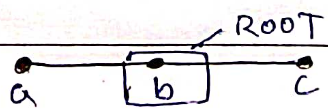
Height of Binary tree:-

→ The length of largest path between root and the leaf node is called hight of Binary tree.



(Binary tree)

$\boxed{H(B.T) = 2}$ Ans



(Binary tree)

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$\boxed{H(B.T) = 1}$

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Th-1. There is one and only one path between every pair of vertices in a tree T.

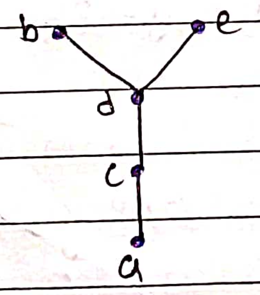
Proof. Let T be a tree.
⇒ T is connected. Since T is connected there must exist at least one Path between every Pair of vertices in T Now Suppose, That between two vertices a and b of T, there are two distinct Paths, The union of these two paths will contain a circuit.

⇒ T is not a tree

So we get contradiction.

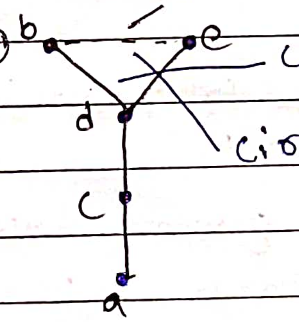
⇒ There is one and only Path between every Pair of verties in a tree T.

ex!- (i)



Tree

(ii)



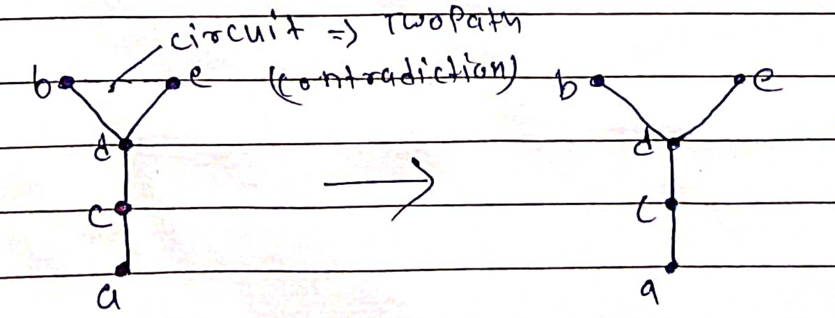
Not tree.

Th-2 If in a graph G, there is one and only one Path between every pair of vertices, then G is a tree.

Proof:— Let G be a connected graph. A circuit in a graph G implies that there is at least one Pair of vertices a, b such that there are two distinct Path between a and b.
Since G has one and only Path between every Pair of vertices.
⇒ G has no circuit.
⇒ G is connected without any circuit.
⇒ G is a tree.

$\boxed{\text{Tree = Connected + No Circuit}}$

Let G be a connected graph and G contain circuit, then there is two Path from a to b but given that there is one and only Graph-G Path b/w a and b ⇒ contraction
⇒ G has no circuit ⇒ G is a tree.



Graph-G.

Connected graph with no circuit. G is tree.

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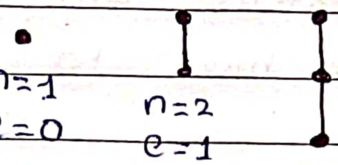
Th-3. A tree of n vertices has $n-1$ edges.

Proof. We Prove this theorem by Induction theorem is true for $n = 1, 2, 3$.

Let us assume that the theorem is true for k vertices and $k < n$.



$n=1$, $e=0$ $n=2$, $e=1$ $n=3$, $e=2$

Now,

Let us suppose T is tree with n vertices. Let e_k be an edge with end vertices v_i and v_j. There is no other Path between v_i and v_j except e_k. Therefore detection of e_k from T will disconnect the graph. Further more $T - e_k$ consist of exantly two Components and since we have no circuit in T to begin with each of these Components is a tree. Both these trees t_1 and t_2 have fewer vertices each.

Therefore by induction each contain one less edge than the no. of vertices in it. These $T - e_k$ have $n-2$ edges.

$\Rightarrow$ T has exactly $n-1$ edges.

Ex:-

Let the theorem is true for k vertex.

$k < n$

< 10

$k-1$.

$n = k + l$

$10 = 4 + 6$

$e = 8$

$= 10 - 2 = n - 2$

$= n - 2 + 1$.

$\boxed{e = n - 1}$

C_1 $k=4$ $e=4-1$

Evergreen $k=6$ C_2 $e=6-1$

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Date:

Th-4 Every connected graph with n vertices and (n-1) edges is a tree.

Proof. Let G be a connected graph with n vertices and (n-1) edges.
we have to show that G has no circuit.
Let us consider G has at least one circuit, since removing an edge from a circuit does not disconnecte a graph so we may remove edges, but no verties from circuits in G unitil the resulting graph G* is circuit free.
Now G* is a connected graph with n vertices and contains no ~~vertices~~ circuit. This G* has (n-1) edge, But now the graph G has more than (n-1) edges.
⇒ Contraditition.
⇒ G has no circuit
⇒ G is a tree.

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Th-5. A graph G with n vertices, $(n-1)$ edges and no circuit is tree.

Proof. Let G be a graph with n vertices, $(n-1)$ edges and has no circuit. we have to show that G is Connected, If Possible, suppose that G is disconnected Then G will consists of two or more circuitless components.

Let G consists of two components G_1 and G_2 we add an edge e between a vertex v_1 in G_1 and v_2 in G_2.

since v_1 and v_2 are in different components of G.

$\Rightarrow$ There is no path between v_1 and v_2 in G.

$\Rightarrow$ Addition of edge e will not create a circuit.

This G is a circuitless, connected graph of n vertices and n edges which is not possible because a tree with n vertices has $(n-1)$ edges.

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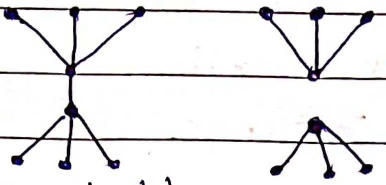
Ex:-



minimally connected Graph:-

⇒ A connected graph G is said to be minimally connected if removal of any edge from G disconnects the graph.

V.V.I

Th:- 06 Every tree has either one or two centres.

Proof [Note:- In any tree, there are at least two Pendant vertices.]

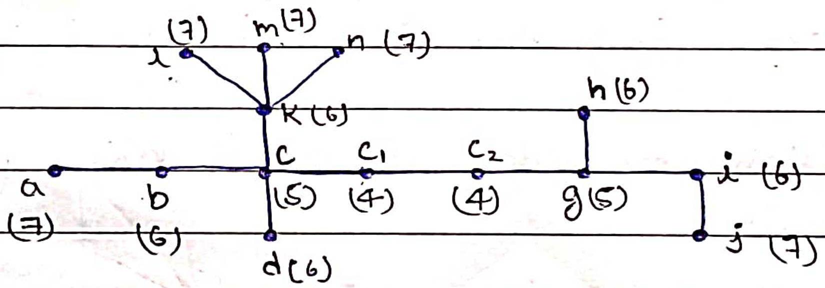


figure - 1 (Graph-T)

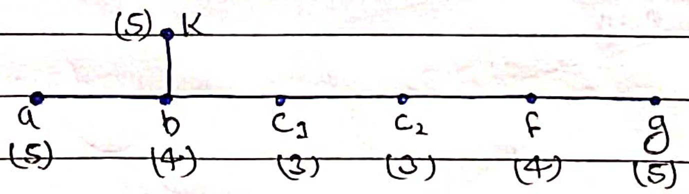


figure - 2 (Graph - T')

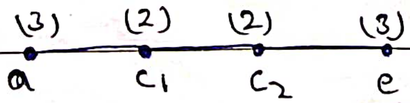


figure - 3 (Graph - T'')

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figure - 4 (graph T''')

Let T be a tree having more than two vertices.

⇒ T Must have at least two Pendant vertices (By Note)

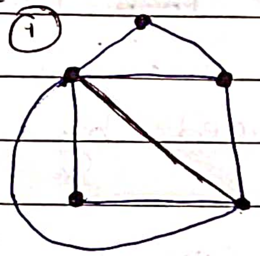
Remove all the pendent vertices from T to obtain another graph T'. Then T' is a tree we observe that the ecentricities of the vertices in T' get reduced by one from T.

⇒ All vertices that are centree in T will still remain centres in T' Remove again all Pendent vertices from T' to obtain another tree T'' contices until we are left with either a vertex or an edge whose end vertices are the centre of T.

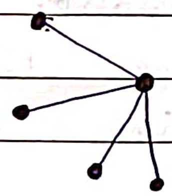
Spanning tree:- / skeleton :-

⇒ A subgraph T of a connected graph G is said to be spanning tree of G if the sub graph T is a Tree and contains all the vertics of G.

Ex:- (1)

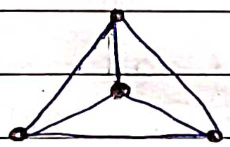


(Connected graph)

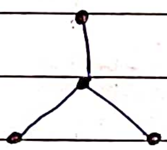


(Spanning tree)

(2)



(Connected graph)



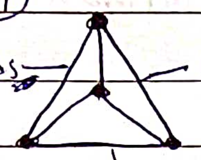
(Spanning tree)

Branches and Chords :-

⇒ Let G be a connected graph with n vertices and e edges let T be a spanning tree of G.

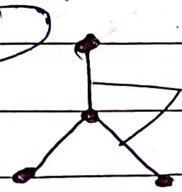
" The edgs of spanning tree are called branches of T " and, "The edge of G which are not in the given spanning tree is called chord or link ~~on~~ or tie".

EX:- (I)



(Connected chords graph)

(II)

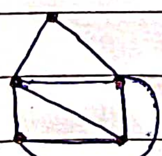


(Spanning Tree)

Th:- 1:- Every connected graph has at least one spanning tree.

Proof:- L G be a connected graph. if G has no circuit then it is its own spanning tree. If G has a circuit then delete an edge from the circuit The graph obtained by removing an edge from a circuit in G will remain connected. If there are more circuits, repeat the process until we get a connected, circuit-free graph that contains all the vertices of G. This graph will then be a spanning tree of G.

(1)



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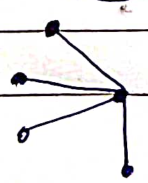
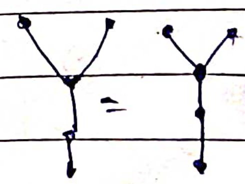


Connected graph

Circuit — non circuit (Tree)



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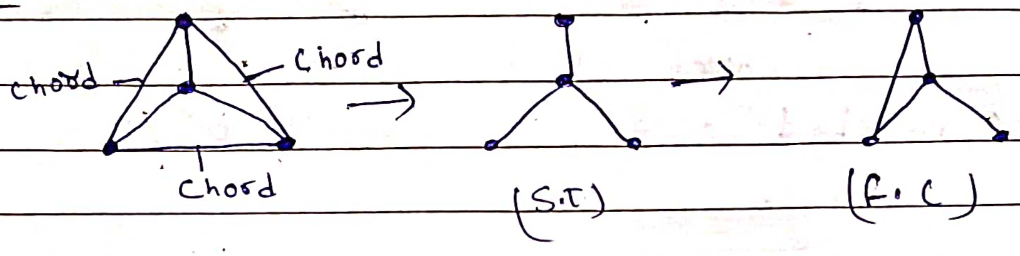
Let G be connected Graph

Fundamental circuits :-

⇒ A circuit fromed by adding a ~~code~~ chord in to spanning treee of a connected graph G, is called fundamental circuits.

Ex:-



* No. of fundamental circuit in G = e-n+1

Where, e = no. of edge.

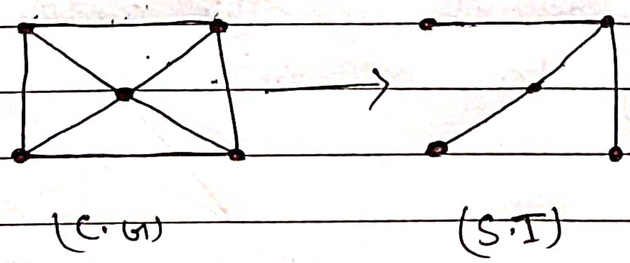
n = no. of vertices.

Solu,

= 6-4+1

= 3 —— fundamental circuits बनेंगे |

Q. Find all fundamental circuits.

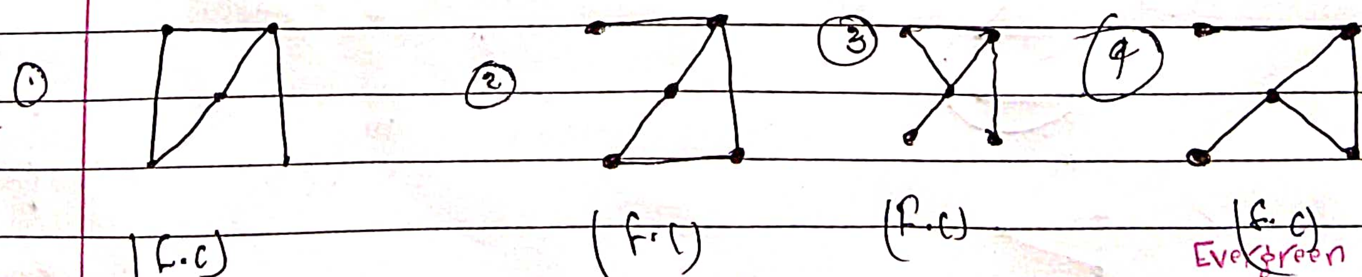


= e-n+1

= 8-5+1

= 3+1

= 4



Rank and nullity :-

⇒ let G be a graph with n vertices, e edge and k – components, then Rank of graph G.

$$\boxed{r = n - k}$$

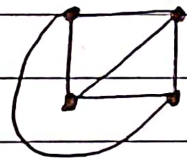
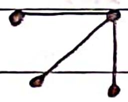
and nullity (cyclomatik no. or betti no) = $\boxed{e-(n-k)}$

$\mu = \boxed{e - r}$

* for connected graph $\boxed{\text{rank } (r) = \text{no. of branches of graph } G.}$

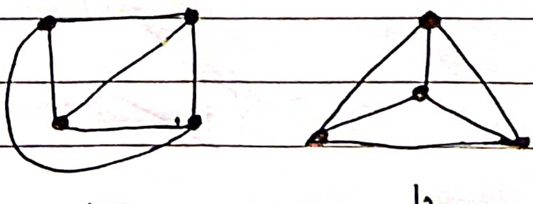
* $\boxed{\text{Nullity } (\mu) = \text{No. of chords of graph } G.}$

Ex:-

⇒  ⇒ 

$r = 3$
$\mu = 3$ } Ans

Ex:- disconnected graph.



K_1 K_2.

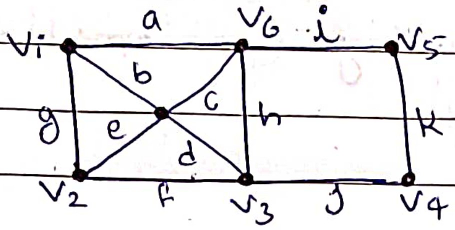
$n = 8$
$e = 12$
$K = 2$

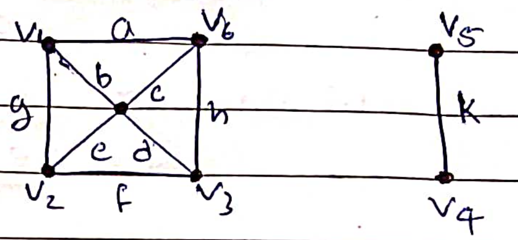
$r = n - k = 8 - 2 = 6$
and nullity $= e - r = 12 - 6 = 6$ } Ans.

Cut-set :-

⇒ A cut set S of a connected graph G is a minimal set of edges of G whose removeal from G disconnect the graph G.

(1)

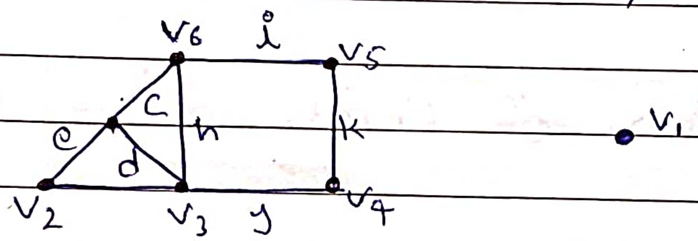




Cut-set

$S_1 = \{i, j\}$

(2)



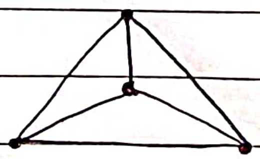
$S_2 = \{a, b, g\}$

Cut set.

$S_3 = \{a, c, d, f\}$

Cut set.

H.W.



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Properties of Cut-set :-

⇒ ① Every cut-set in a connected graph G must contain at least one branch of every spanning tree of G.

② In a connected graph G any minimal set of edges containing at least one ~~brain~~ branch of every spanning tree of G is a cut-set.

③ Every circuit has an even no. of edges in common with any cut set.

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Matrices and Graph :-

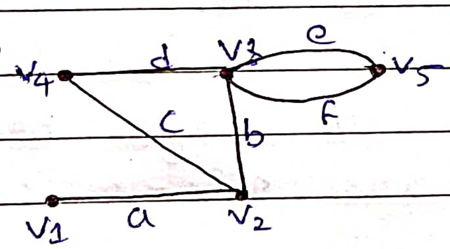
① Incidence Matrix :-

⇒ Let G be a graph with n vertices v_1, v_2, v_3 --- v_n and m edges e_1, e_2, e_3 --- e_n and no Self-loop then incidence matrix defined as follows.

⇒ $a_{ij} = 1$. if j^{th} edge in incidence on i^{th} vertex v_i and $a_{ij} = 0$

* Incidance matrix is denoted by A (G).
* Incidance Matrix is also known as "bit matrix"
* n rows represent n verties m columns represent m edges.

Ex:- write Incidence matrix.

① 

	a	b	c	d	e	f
v_1	1	0	0	0	0	0
v_2	1	1	1	0	0	0
v_3	0	1	0	1	1	1
v_4	0	0	1	1	0	0
v_5	0	0	0	0	1	1

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Properties of Incidence.

1.) Each column of I.M of graph G without any self loops has exactly ~~3~~ two.

2) The sum of each row gives the degree of the corresponding vertex.

3) A row with all zero represent an isolated vertex.

② Adjacency matrix :-

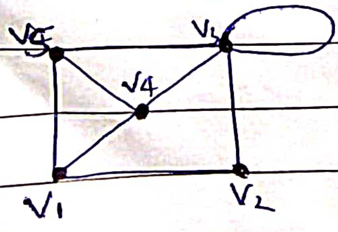
⇒ Adjacency Matrix of a graph with n vertices and no Parallel edges (self-loops are allowed) is $n \times n$ matrix which is defined as.

⇒ $x_{ij} = 1$ If there is an edge between i^{th} and j^{th} vertices and $x_{ij} = 0$ otherwise.

* A.M is denoted by X.

(Adjacency Matrix)

Ex:- ① Write A.M of this graph.



$$
\begin{array}{c}
 \\ V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5
\end{array}
\begin{array}{c}
\begin{array}{ccccc} V_1 & V_2 & V_3 & V_4 & V_5 \end{array} \\
\begin{bmatrix}
0 & 1 & 0 & 1 & 1 \\
1 & 0 & 1 & 0 & 0 \\
0 & 1 & 1 & 1 & 1 \\
1 & 0 & 1 & 0 & 1 \\
1 & 0 & 1 & 1 & 0
\end{bmatrix}
\end{array}
$$

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Properties of Adjacency Matrix.

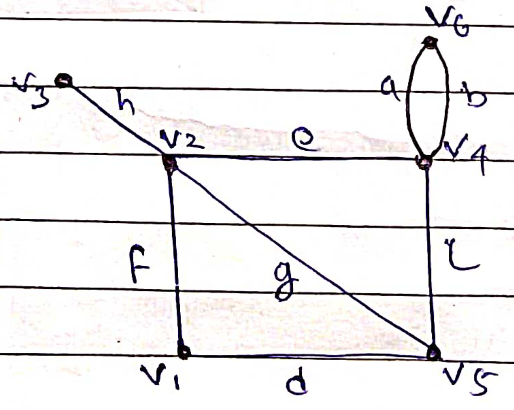
1. A.M of graph G is symmetric
2. If graph has no self-loop then the diagonal entrices of A.M are zero and vice versa.
3. If the graph has no self-loops then sum of i^{th} row (or column) is the degree of v_i.

3) Path - Matrix.

⇒ A Path Matrix is defined for a given Pair of vertices in a graph, say (u,v) and is written as $P(u,v)$ and defined as.

$P_{ij} = 1$ if j^{th} edge lies in i^{th} Path and
$P_{ij} = 0$.

Ex:- find Path matrix between v_3 and v_4



$P_1 = \{H, e\}$
$P_2 = \{H, G, C\}$
$P_3 = \{H, F, D, C\}$

~~$v_3 h v_2 e v_4$~~ $= \{h, e\}$

$$P(v_3, v_4) = \begin{array}{c} \\ P_1 \\ P_2 \\ P_3 \end{array} \begin{array}{c} \begin{array}{cccccccc} a & b & c & d & e & f & G & h \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 \end{bmatrix} \end{array}$$

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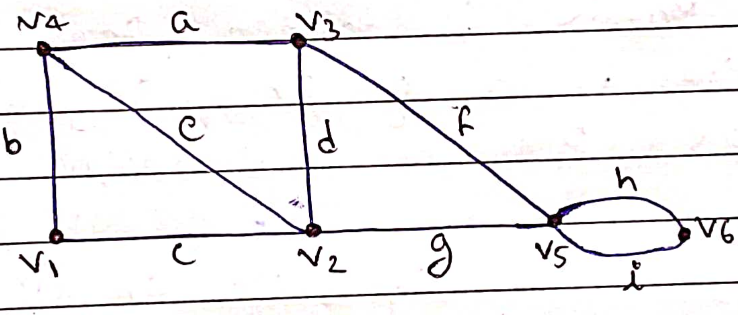
(4) # Circuit Matrix :-

⇒ Let G be a graph with e edge and q different circuits then circuit Matrix B is defiened as,

$b_{ij} =$ 1 if i^{th} circuit include j^{th} edgs.

$b_{ij} = 0$

ex:- Find circuit matrix.



$c_1 = \{b, c, e\}$

$c_2 = \{a, d, e\}$

$c_3 = \{d, g, f\}$

$c_4 = \{h, i\}$

$c_5 = \{a, b, c, d\}$

$c_6 = \{a, b, c, g, f\}$

$c_7 = \{a, e, g, f\}$

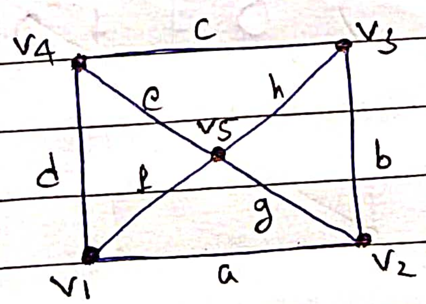
$$C(G) = \begin{array}{c} \\ c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \end{array} \begin{array}{c} \begin{array}{ccccccccc} a & b & c & d & e & f & g & h & i \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

(5) Cut Set Matrix :-

=> A Matrix c is set to be cut-set Matrix if.

$c_{ij} = 1$ if ith cut-set j^{th} edges.
$c_{ij} = 0$ other wise.

* Row corrospond to cut-set and columans are corres pond to edge.



$S_1 = \{d, e, c\}$

$S_2 = \{e, h, g, f\}$

$S_3 = \{a, f, d\}$

$S_4 = \{a, g, b\}$

$S_5 = \{b, h, c\}$

$S_6 = \{a, f, e, c\}$

$S_7 = \{a, g, h, c\}$

$S_8 = \{b, h, e, d\}$

$S_9 = \{b, g, f, d\}$

	a	b	c	d	e	f	g	h
S_1	0	0	1	1	1	0	0	0
S_2	0	0	0	0	1	1	1	1
S_3	1	0	0	1	0	1	0	0
S_4	1	1	0	0	0	0	1	0
S_5	0	1	1	0	0	0	0	1
S_6	1	0	1	0	1	1	0	0
S_7	1	0	1	0	0	0	1	1
S_8	0	1	0	1	1	0	0	1
S_9	0	1	0	1	0	1	1	0

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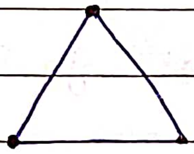
Planar graph :-

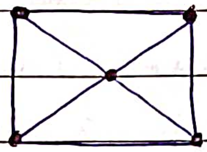
⇒ A graph G is said to be planar if The graph can be drown in the plane so that no edges crosses to each other. ~~at~~ axcept at the vertices.

Ex:- ①

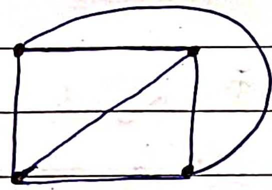


Planar graph

②

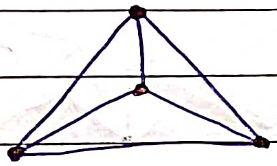


graph-G.



Planar graph.

③

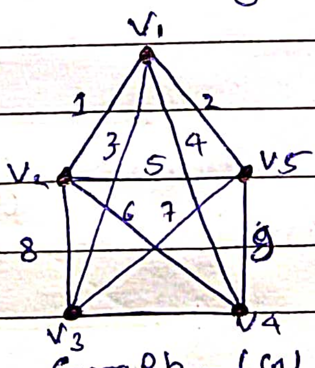


Planar graph.

non-planar Graph :-

⇒ A graph G that can not be drawn on a Plane without Crossing of its edges is called non-Planar graph.

ex:- ①



(non-Planar Graph)

Graph-(G)

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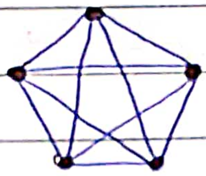
Kuratowski's two graphs
(कुराटोस्की)

→ A complete graph with 5 vertices is known as first Kuratowski's graph and the second graph of Kuratowski's is $K_{3,3}$

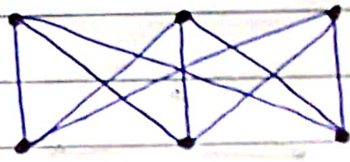
* $K_{3,3}$ is regular connected graph with 6 vertices and 9 edges.

Ex:- ①



first graph.

②



$K_{3,3}$ (second graph)

* Both graph are non-Planar.
$K_{3,3}$ graph is known as utility graph.

Region:-

A Region of a Planar graph is an area of the Plane that is bainded by edges.

A region is finite if the area it enclose is finite, otherwise it is called infinite.

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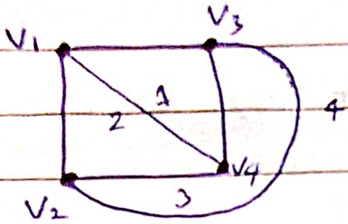
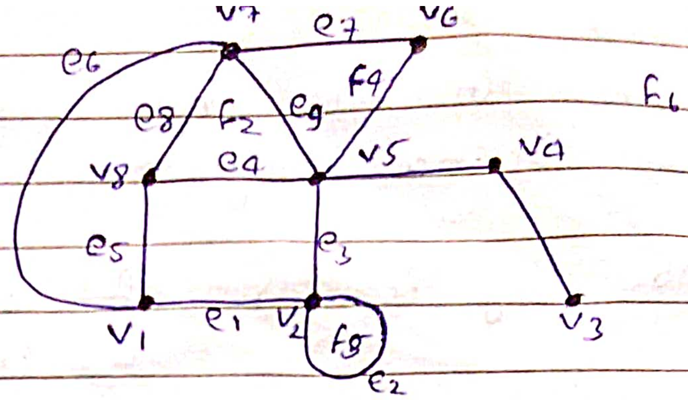




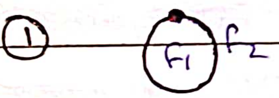
V. Imp (15)

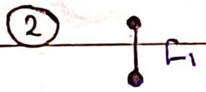
Euler's formula :-

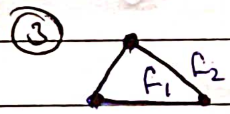
Th-1 :- A connected planar graph with n vertices and e edges has e-n+2 regions.

Proof :- Let G be a connected ~~Graph~~ Planar Graph with n vertices and e edge. This thorem is proop by Induction method.

For n = 1 | n = 2 | n = 3







$f = e-n+2$
$= \cancel{1} - \cancel{1} + 2$
$= 2$ Ans

$f = e-n+2$
$= 1 - \cancel{2} + \cancel{2}$
$= 1$ Ans

$f = e-n+2$
$= \cancel{3} - \cancel{3} + 2$
$= 2$ Ans

This imply the theorm is true for $n=1, e=1, n=2, e=1, n=3, e=3$. Now let us consider the theorm is true for $e-1$ edges.

Now Let G be a connected graph with n vertices, e edges and f- regions.

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Case - I When G has no circuit ⇒ G is a tree.
⇒ G has only one region.

now by formula $f = e - n + 2$

$f = (\cancel{n}-1) - \cancel{n} + 2$ $\{e = n-1$ G is a tree,

$= -1 + 2$

$\boxed{f = 1}$

Case:-II:- when G has a circuit. Then let circuit has an edge 'a'

now removed of an edge 'a' will merge 2 regions into 1 region.

now in this case no. of edges = $e-1$

and no of regions = $f-1$

By using formula ⇒ $f = e - n + 2$

$f - 1 = e - 1 - n + 2$

$f = e - \cancel{1} - n + 2 + \cancel{1}$

$\boxed{f = e - n + 2}$

proved

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V. Imp.

NOTE:- if a connected simple planar graph G has $n \geq 3$ vertices e-edes and f - regions then.

$$\boxed{3f \leq 2e} \quad \text{— (i)}$$

$$3(e-n+2) \leq 2e$$

$$3e - 3n + 6 \leq 2e$$

$$\boxed{e \leq 3n-6} \quad \text{— (ii)}$$

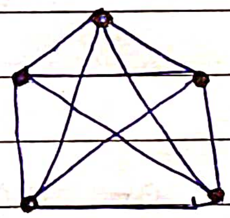
Imp.

(I) ~~K_5 is non - planar.~~

Q. Proved that K_5 is non - Planar.

Solⁿ

$n = 5, \; e = 10$



Apply

$$3f \leq 2e$$

$$3(e-n+2) \leq 2e$$

$$3(10-5+2) \leq 2 \times 10$$

$$21 \leq 20 \quad \text{(condition is failing)}$$

Hence:- K_5 is non - Planar.

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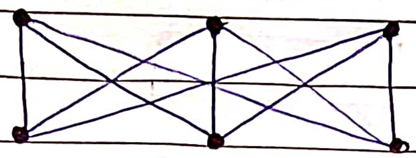
Q. Prove that $K_{3,3}$ is non Planar.

Sol^n



$K_{3,3}$

$4f \leq 2e$ $\quad n=6$

$e=9$.

$4(e-n+2) \leq 2e$

$4(9-6+2) \leq 2\times 9$

$20 \leq 18$ (condition failing)

$\Rightarrow K_{3,3}$ is non Planar.

Detection of Planarity :-

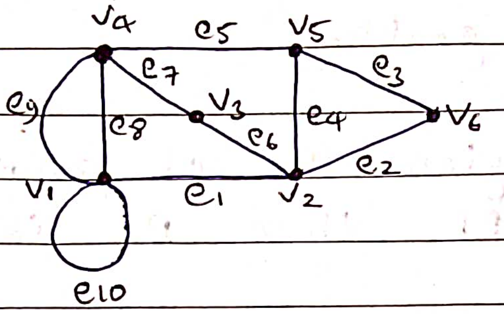
Rule :- ① Remove all Self-loop

② Remove Parallel edges.

③ Removal of a vertex of degree 2 by merging 2 edge in series.

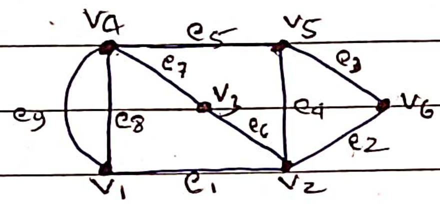
④ RePeat Step 2nd and 3rd again & again untill graph become simply.

Q1. Reduce or (simplify) the graph without affacting its Planarity.

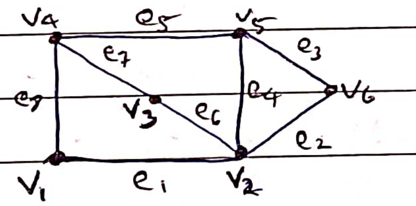


Solv.

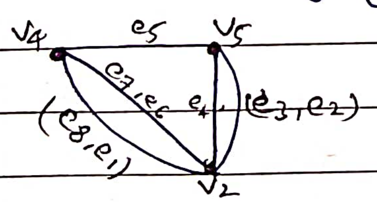
Step :- ① Remove Self - loop.



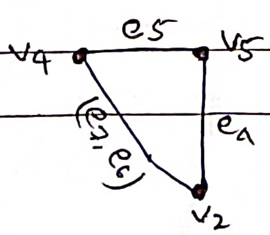
Step :- ② Remove Parallel edge.



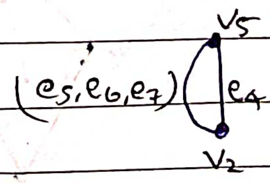
Step :- ③ Merging of an edge')



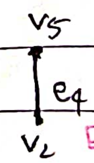
Step :- ④ repeat Step - 2



Step :- ⑤ repeat. Step - 3



Step :- 6 :- Repeat Step - 2



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Dual Graph :-

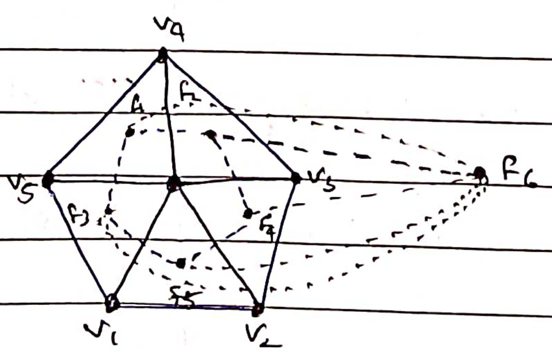
=) Given a Plane represuntatin of Plannar graph we construct another graph G^* is called dual graph.

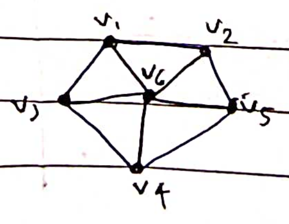
Rules:- ① Inside each region f_i of grap G, we take a Point P_i these P_i is vertices of G^*

② if two regions f_i and f_j are adjacent we draw a line to join the point P_i and P_j of f_i and f_j.

③ For an edge e lying entirely in one region f_k, draw a self loop at Point P_k of f_k.

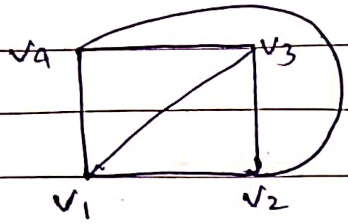
Q. Find the dual of the graph G.

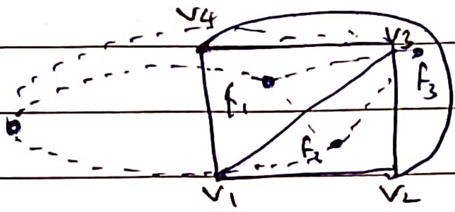


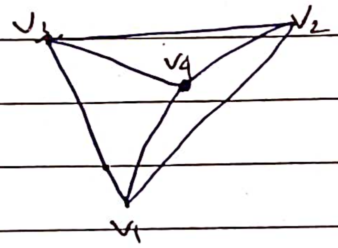


Q. Find the dual of graph.

Sol:

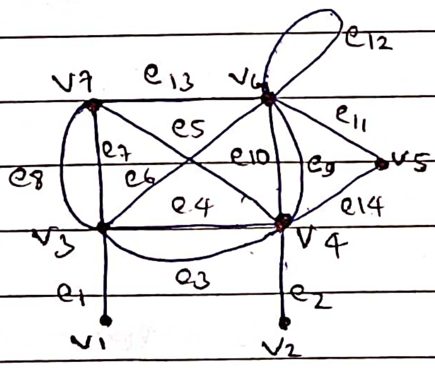






Extra

Th-6. Every tree has either one or two centre.



① Degeree each vertex.
② even vertex.
③ odd vertex.
④ Pendent vertex.

①	② even vertex.	odd vertex:-	③ Pendent vertex
$d(v_1) = 1$			
② $d(v_2) = 1$			
$d(v_3) = 6$	$d(v_3) = 6$	$d(v_1) = 1$	$d(v_1) : d(v_2)$
$d(v_4) = 7$	$d(v_5) = 2$	$d(v_2) = 1$	
$d(v_5) = 2$	$d(v_7) = 4$	$d(v_4) = 7$	
$d(v_6) = 7$		$d(v_6) = 7$	
$d(v_7) = 4$			