UNIT-II. INTERPOLATION AND EXTRAPOLATION

Finite difference :-

$$\Delta f(x) = f(x+h) - f(x)$$

$$\Delta f(x+h) = f(x+2h) - f(x+h)$$
$$\Rightarrow \Delta f(x+2h) = f(x+3h) - f(x+2h)$$

$$\Delta^2 f(x) = \Delta[\Delta f(x)]$$
$$= \Delta[f(x+h) - f(x)]$$
$$= \Delta f(x+h) - \Delta f(x)$$

$$\Delta^2 f(x) = [f(x+2h) - f(x+h)] - [f(x+h) - f(x)]$$
$$\Delta^2 f(x) = f(x+2h) - 2f(x+h) + f(x)$$

Forward operator. (Δ)

$$\Delta f(x) = f(x+h) - f(x)$$

1. x
2. $x+h$
3. $x+2h$
4. $x+3h$.

$$\Delta f(x) = f(x+h) - f(x)$$
$$\Rightarrow \Delta f(x+h) = f(x+2h) - f(x+h)$$
$$\Rightarrow \Delta^2 f(x) = \Delta[\Delta f(x)]$$

$$\Rightarrow \Delta^2 f(x) = \Delta[f(x+h) - f(x)]$$

$$\Rightarrow \Delta^2 f(x) = \Delta f(x+h) - \Delta f(x)$$

$$\Rightarrow \Delta^2 f(x) = [f(x+2h) - f(x+h)] - [f(x+h) - f(x)]$$

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$$\Rightarrow \Delta^2 f(x) = f(x+2h) - 2f(x+h) + f(x)$$

$$\Rightarrow \Delta^3 f(x) = \Delta[\Delta^2 f(x)]$$

$$\Rightarrow \Delta^3 f(x) = \Delta[f(x+2h) - 2f(x+h) + f(x)]$$

$$\Rightarrow \Delta^3 f(x) = \Delta f(x+2h) - 2\Delta f(x+h) + \Delta f(x)$$

$$\Rightarrow \Delta^3 f(x) = [f(x+3h) - f(x+2h)] - 2[f(x+2h) - f(x+h)] + f(x+h) - f(x)$$

$$\Rightarrow \Delta^3 f(x) = f(x+3h) - 3f(x+2h) + 3f(x+h) - f(x)$$

Q.) $\Delta(x^2+1)$ where h is unity $(h=1)$

$$f(x) = x^2+1$$
$$\Rightarrow f(x+h) = x^2+1$$
$$\Rightarrow f(x+1) = (x+1)^2+1$$
$$\Rightarrow f(x+1) = x^2+2x+1+1$$
$$\therefore f(x+1) = x^2+2x+2$$

Now,

$$\Delta(x^2+1) = f(x+1) - f(x)$$
$$\Rightarrow \Delta(x^2+1) = x^2+2x+2-x^2-1$$
$$\therefore \Delta(x^2+1) = 2x+1$$

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Q.) $\Delta^2(x+4)$, where h is unity $(h=1)$

$$f(x) = x+4$$
$$\Rightarrow f(x+h) = x+4$$
$$\Rightarrow f(x+1) = x+1+4$$
$$\therefore f(x+1) = x+5$$

$$f(x+2h) = x+4$$
$$\Rightarrow f(x+2) = x+2+4$$
$$\therefore f(x+2) = x+6$$

Now,

$$\Delta^2 f(x) = f(x+2h) - 2f(x+h) + f(x)$$
$$\Rightarrow \Delta^2 f(x) = x+6 - 2(x+5) + x+4$$
$$\Rightarrow \Delta^2 f(x) = x+6 - 2x - 10 + x + 4$$
$$\therefore \Delta^2 f(x) = 0$$

3. Backward difference Operator (∇)

$$\boxed{\nabla f(x) = f(x) - f(x-h)}$$

4. Central difference Operator (δ) :-

$$\boxed{\delta f(x) = f\left(x+\frac{h}{2}\right) - f\left(x-\frac{h}{2}\right)}$$

5. Shift Operator (E) :-

$$\boxed{Ef(x) = f(x+h)}$$

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6. Identity Operator (I) :-

$$If(x) = f(x)$$

7. $\Delta = E - I$

Here,

$$\Delta f(x) = f(x+h) - f(x)$$

$$\Rightarrow \Delta f(x) = Ef(x) - If(x)$$

$$\Rightarrow \Delta f(x) = f(x)[E-I]$$

$$\therefore \Delta = E - I$$

Proved

8. $\nabla = I - E^{-1}$

Here

$$\nabla f(x) = f(x) - f(x-h)$$

$$\Rightarrow \nabla f(x) = If(x) - E^{-1}f(x)$$

$$\Rightarrow \nabla f(x) = f(x)[I - E^{-1}]$$

$$\therefore \nabla = I - E^{-1}$$

Proved

9. $\Delta^2 = E^2 - 2E + I$

Here

$$\Delta^2 = f(x+2h) - 2f(x+h) + f(x)$$

$$\Rightarrow \Delta^2 f(x) = E^2 f(x) - 2Ef(x) + If(x)$$

$$\Rightarrow \Delta^2 f(x) = (E^2 - 2E + I) f(x)$$

$$\therefore \Delta^2 = E^2 - 2E + I$$

Proved

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Q.) $\left[\frac{\Delta^2}{E}\right] x^2$

$\Rightarrow \left[\frac{E^2-2E+I}{E}\right] x^2$

$\Rightarrow (E^2-2E+E^{-1})\, x^2$

$\Rightarrow Ex^2-2Ix^2+E^{-1}x^2$

$\Rightarrow (x+h)^2-2x^2+(x-h)^2$

$\Rightarrow x^2+h^2+2xh-2x^2+x^2+h^2-2xh$

$\Rightarrow 2h^2$

6. Difference Table for equal interval.

Q.) Find the difference table for the given data

x : 0 1 2 3

$f(x)$: 1 2 5 9.

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$
0	1			
		1		
1	2		2	
		3		-1
2	5		1	
		4		
3	9			

7.) Newton Forward Interpolation Difference.

$$f(x_0+nh) = f(x_0) + \frac{n\Delta f(x_0)}{1!} + \frac{n(n-1)}{2!}\Delta^2 f(x_0) + \frac{n(n-1)(n-2)}{3!}\Delta^3 f(x_0) + \frac{n(n-1)(n-2)(n-3)}{4!}\Delta^4 f(x_0)$$

8.) Find the value of $f(9)$ where.

x	2	4	6	8	10
$f(x)$	15	20	30	35	50

← Difference Table →

	x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$
x_0	2	15				
			5			
	4	20		5		
			10		-10	
	6	30		-5		25
			5		15	
	8	35		10		
			15			
	10	50				

$x_n = x_0 + nh$

$\Rightarrow 9 = 2 + n \times 2$

$\Rightarrow 9-2 = 2n$

$\Rightarrow 2n = 7$

$\therefore n = 3.5$

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$$f(9) = f(x_0) + \frac{n\Delta f(x_0)}{1!} + \frac{n(n-1)}{2!}\Delta^2 f x_0$$

$$+ \frac{n(n-1)(n-2)}{3!}\Delta^3 f(x_0) + \frac{n(n-1)(n-2)(n-3)}{4!}\Delta^4 f(x_0)$$

$$\Rightarrow f(9) = 15 + (3.5 \times 5) + \frac{3.5(3.5-1)}{2} \times 5$$

$$+ \frac{3.5(3.5-1)(3.5-2)}{6} \times (-10) + \frac{3.5(3.5-1)(3.5-2)(3.5-3)}{24}$$

$$\times 25$$

$$\Rightarrow f(9) = 15 + 17.5 + 21.875 - 21.875 + 6.836$$

$$\therefore f(9) = 39.336 \text{ Ans}$$

8.) Newton Backward Interpolation Difference :-

$$f(x_n) = f(x_0) + n\nabla f(x_0) + \frac{n(n+1)}{2!}\nabla^2 f(x_0)$$

$$+ \frac{n(n+1)(n+2)}{3!}\nabla^3 f(x_0) + \frac{n(n+1)(n+2)(n+3)}{4!}$$

$$\times \nabla^4 f(x_0) + \dots\dots$$

Q.) Find the value of $f(9)$ where.

x	2	4	6	8	10
$f(x)$	15	20	30	35	50

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← Difference Table →

	x	$f(x)$	$\nabla f(x)$	$\nabla^2 f(x)$	$\nabla^3 f(x)$	$\nabla^4 f(x)$
	2	15				
			5			
	4	20		5		
			10		-10	
	6	30		-5		25
			5		15	
	8	35		10		
			15			
x_0	10	50				

$$x_n = x_0 + nh$$
$$\Rightarrow 9 = 10 + n \times 2$$
$$\Rightarrow -1 = 2n$$
$$\therefore n = -0.5$$

$$f(x_n) = f(x_0) + n\nabla f(x_0) + \frac{n(n+1)}{2!}\nabla^2 f(x_0) + \frac{n(n+1)(n+2)}{3!} \times \nabla^3 f(x_0) + \frac{n(n+1)(n+2)(n+3)}{4!} \times \nabla^4 f(x_0)$$

$$\Rightarrow f(9) = 50 - 0.5 \times 15 - \frac{0.5(-0.5+1)}{2} \times 10 - \frac{0.5(-0.5+1)(-0.5+2)}{6} \times 15 - \frac{0.5(-0.5+1)(-0.5+2)(-0.5+3)}{24} \times 25$$

$$\Rightarrow f(9) = 50 - 7.5 - 1.25 - 0.9375 - 0.9766$$
$$\Rightarrow f(9) = 39.3359$$
$$\therefore f(9) = 39.336$$

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9. CENTRAL DIFFERENCE FORMULA :-

i.) Gauss's forward Interpolation formula :-

$$y_n = y_0 + n\Delta y_0 + \frac{n(n-1)}{2!}\Delta^2 y_{-1} + \frac{(n+1)n(n-1)}{3!}\Delta^3 y_{-1} + \frac{(n+1)n(n-1)(n-2)}{4!}\Delta^4 y_{-2} + \frac{(n+2)(n+1)n(n-1)(n-2)}{5!}\Delta^5 y_{-2} + \dots$$

Q. find the value of function at x = 5 where.

x	0	2	4	6	8
$f(x)$	1	10	25	60	70

⟵ difference Table ⟶

	x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$
x_{-2}	0	$1_{y_{-2}}$				
			$9_{y_{-2}}$			
x_{-1}	2	$10_{y_{-1}}$		$6_{y_{-2}}$		
			$15_{y_{-1}}$		$14_{y_{-2}}$	
x_0	4	25_{y_0}		$20_{y_{-1}}$		$-59_{y_{-2}}$
			35_{y_0}		$-45_{y_{-1}}$	
x_1	6	60_{y_1}		-25_{y_0}		
			10_{y_1}			
x_2	8	70_{y_2}				

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$x_n = x_0 + nh$
$\Rightarrow 5 = 4 + n \times 2$
$\Rightarrow 1 = 2n$
$\therefore n = 0.5$

$$y_n = y_0 + n\Delta y_0 + \frac{n(n-1)}{2!}\Delta^2 y_{-1} + \frac{(n+1)n(n-1)}{3!} \times \Delta^3 y_{-1} + \frac{(n+1)n(n-1)(n-2)}{4!}\Delta^4 y_{-2}$$

$$\Rightarrow y_n = 25 + 0.5 \times 35 + \frac{0.5(0.5-1)}{2} \times 20 + \frac{(0.5+1)0.5(0.5-1)}{6} \times (-45) + \frac{(0.5+1)0.5(0.5-1)(0.5-2)}{24} \times (-59)$$

$$\Rightarrow y_n = 25 + 17.5 - 2.5 + 2.8125 - 1.3828$$

$$\therefore y_n = 41.4297$$

ii.) Gauss's Backward Interpolation Formula :-

$$y_n = y_0 + n\Delta y_{-1} + \frac{n(n+1)}{2!}\Delta^2 y_{-1} + \frac{(n+1)n(n-1)}{3!}\Delta^3 y_{-1}$$
$$+ \frac{(n+2)(n+1)n(n-1)}{4!} \times \Delta^4 y_{-2}$$
$$+ \frac{(n+2)(n+1)n(n-1)(n-2)}{5!} \times \Delta^5 y_{-3}.$$

Q.) Find the value of y when y : 3.75 where

x	2.5	3	3.5	4	4.5	5
$f(x)$	24.145	22.043	20.225	18.644	17.262	16.047

← Difference Table →

	x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
x_{-2}	2.5	24.145					
		y_{-2}	-2.102				
x_{-1}	3	22.043	y_{-2}	0.284			
		y_{-1}	-1.818	y_{-2}	-0.047		
x_0	3.5	20.225	y_{-1}	0.237	y_{-2}	0.009	
		y_0	-1.581	y_{-1}	-0.038	y_{-2}	-0.003
x_1	4	18.644	y_0	0.199	y_{-1}	0.006	y_{-2}
		y_1	-1.382	y_0	-0.032	y_{-1}	
x_2	4.5	17.262	y_1	0.167	y_0		
		y_2	-1.215	y_1			
x_3	5	16.047	y_2				
		y_3					

$$x_n = x_0 + nh$$
$$\Rightarrow 3.75 = 3.5 + n \times 0.5$$
$$\Rightarrow 0.25 = n \times 0.5$$
$$\therefore n = 0.5$$

$$y_n = y_0 + n\Delta y_{-1} + \frac{n(n+1)}{2!}\Delta^2 y_{-1} + \frac{(n+1)n(n-1)}{3!}\Delta^3 y_{-2} + \frac{(n+2)(n+1)n(n-1)}{4!}\Delta^4 y_{-2} + \frac{(n+2)(n+1)n(n-1)(n-2)}{5!}\Delta^5 y_{-3}$$

$$\Rightarrow y_{3.75} = 20.225 + 0.5\times(-1.818) + \frac{0.5(0.5+1)}{2}\times 0.237 + \frac{(0.5+1)0.5(0.5-1)}{6}\times(-0.047) + \frac{(0.5+2)(0.5+1)0.5(0.5-1)}{24}\times 0.009$$

$$\Rightarrow y_{3.75} = 20.225 - 0.909 + 0.08888 + 0.00294 - 0.00035$$

$$\therefore y_{3.75} = 19.40747$$

iii.) Stirling formula :-

$$y_n = y_0 + n\left[\frac{\Delta y_0 + \Delta y_{-1}}{2}\right] + \frac{n^2}{2!}\Delta^2 y_{-1} + \frac{n(n^2-1)}{3!}\left[\frac{\Delta^3 y_{-1} + \Delta^3 y_{-2}}{2}\right] + \frac{n^2(n^2-1)}{4!}\Delta^4 y_{-2} + \cdots$$

Q.) Compute $10^5 y_{12.2}$ from the following table.

x:	10	11	12	13	14
y:	23967	28060	31788	35209	38368

← Difference Table →

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
10	23967_{-2}				
		4093_{-2}			
11	28060_{-1}		-365_{-2}		
		3728_{-1}		58_{-2}	
12	31788_{0}		-307_{-1}		-13_{-2}
		3421_{0}		45_{-1}	
13	35209_{1}		-262_{0}		
		3159_{1}			
14	38368_{2}				

$$x_n = x_0 + nh$$
$$\Rightarrow 12.2 = 12 + n \times 1$$
$$\therefore n = 0.2$$

$$10^5 y_{12.2} = y_0 + n\left[\frac{\Delta y_0 + \Delta y_{-1}}{2}\right] + \frac{n^2}{2!}\Delta^2 y_{-1} + \frac{n(n^2-1)}{3!}\left[\frac{\Delta^3 y_{-1} + \Delta^3 y_{-2}}{2}\right] + \frac{n^2(n^2-1)}{4!}\Delta^4 y_{-2}$$

$$\Rightarrow 10^5 y_{12.2} = 31788 + 0.2\left[\frac{3421 + 3728}{2}\right] + \frac{(0.2)^2}{2} \times (-307) + \frac{0.2[(0.2)^2 - 1]}{6}\left[\frac{45 + 58}{2}\right] + \frac{(0.2)^2[(0.2)^2 - 1]}{24} \times (-13)$$

$$\Rightarrow 10^5 y_{12.2} = 31788 + 714.9 - 6.14 + 1.648 + 0.0208$$
$$\Rightarrow 10^5 y_{12.2} = 32498.4288$$
$$\therefore y_{12.2} = 0.32498$$

iv.) Bessel's formula :-

$$y_n = \left[\frac{y_0 + y_1}{2}\right] + \left(n - \frac{1}{2}\right)\Delta y_0 + \frac{n(n-1)}{2!}\left[\frac{\Delta^2 y_0 + \Delta^2 y_{-1}}{2}\right] + \frac{n\left(n - \frac{1}{2}\right)(n-1)}{3!}\Delta^3 y_{-1}$$

$$+ \frac{n(n+1)(n-1)(n-2)}{4!}\left[\frac{\Delta^4 y_{-1} + \Delta^4 y_{-2}}{2}\right] + \frac{n(n+1)\left(n - \frac{1}{2}\right)(n-1)(n-2)}{5!}\Delta^5 y_{-2} + \ldots$$

Q.) Compute $10^5 y_{12.2}$ from the following table.

x : 10 11 12 13 14

y : 23967 28060 31788 35209 38368

← Difference Table. →

	x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
x_{-2}	10	23967				
		x_{-2}	4096			
x_{-1}	11	28060	x_{-2}	-365		
		x_{-1}	3728	x_{-2}	58	
x_0	12	31788	x_{-1}	-307	x_{-2}	-13
		x_0	3421	x_{-1}	45	x_{-2}
	13	35209	x_0	-262	x_{-1}	
			3159	x_0		
	14	38368				

$$x_n = x_0 + nh$$
$$\Rightarrow 12.2 = 12 + n \times 1$$
$$\therefore n = 0.2$$

$$10^5 y_{12.2} = \left[\frac{35209 + 31788}{2}\right] + (0.2 - 0.5) \times 3421$$
$$+ \frac{0.2(0.2-1)}{2}\left[\frac{-262-307}{2}\right] +$$
$$\frac{0.2(0.2-0.5)(0.2-1)}{6} \times 45$$

$$\Rightarrow 10^5 y_{12.2} = 33498.5 - 1026.3 + (-0.08)(-284.5)$$
$$+ 0.36$$

$$\Rightarrow 10^5 y_{12.2} = 33498.5 - 1026.3 + 22.76 + 0.36$$
$$\Rightarrow 10^5 y_{12.2} = 32495.32$$
$$\therefore y_{12.2} = 0.32495$$

v.) Laplace - Everett formula :-

$$y_n = u y_1 + \frac{u(u^2-1)}{3!} \Delta^2 y_0 + \frac{u(u^2-1)(u^2-2^2)}{5!}$$
$$\times \Delta^4 y_{-1} + \dots\dots$$
$$+ v y_0 + \frac{v(v^2-1)}{3!} \Delta^2 y_{-1} + \frac{v(v^2-1)(v^2-2^2)}{5!}$$
$$\times \Delta^4 y_{-2} + \dots$$

Where

$$v = 1 - u$$

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Q.) Apply Everett's formula to obtain y_{25}, given $y_{20} = 2854$, $y_{24} = 3162$, $y_{28} = 3544$, $y_{32} = 3992$.

← Difference Table →

	x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
x_{-1}	20	2854_{-1}			
			308_{-1}		
x_0	24	3162_0		74_{-1}	
			382_0		-8_{-1}
x_1	28	3544_1		66_0	
			448_1		
x_2	32	3992_2			

$x_n = x_0 + uh$

$\Rightarrow 25 = 24 + 4u$

$\therefore u = 0.25$

$v = 1 - u$

$\Rightarrow v = 1 - 0.25$

$\therefore v = 0.75$

$y_{25} = uy_1 + \frac{u(u^2-1)}{3!}\Delta^2 y_0 + vy_0 + \frac{v(v^2-1)}{3!}\Delta^2 y_{-1}$

$\Rightarrow y_{25} = 0.25 \times 3544 + \frac{0.25[(0.25)^2-1]}{6} \times 66 +$

$0.75 \times 3162 + \frac{0.75[(0.75)^2-1]}{6} \times 74$

$\Rightarrow y_{25} = 886 - 2.578125 + 2371.5 - 4.046875$

$\therefore y_{25} = 3250.875$ Ans

10. Interpolation with unequal interval
→ Divided difference Table:-

Q.) Construct a divided difference Table:-

$x = 4 \quad 5 \quad 7 \quad 10 \quad 11 \quad 13$
$y = 48 \quad 100 \quad 294 \quad 900 \quad 1210 \quad 2028$

← Divided difference Table →

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
4	48					
		52				
5	100		15			
		97		1		
7	294		21		0	
		202		1		0
10	900		27		0	
		310		1		
11	1210		33			
		409				
13	2080					

11.) Newton Divided Difference Table for unequal Interval:-

$$f(x) = f(x_0) + (x-x_0) f(x_0, x_1) + (x-x_0)(x-x_1) f(x_0, x_1, x_2) + (x-x_0)(x-x_1)(x-x_2) f(x_0, x_1, x_2, x_3) + (x-x_0)(x-x_1)(x-x_2)(x-x_3) f(x_0, x_1, x_2, x_3, x_4) + \dots$$

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Q.) find the value of y when $x = 10$.

$x = 5 \quad 6 \quad 9 \quad 11$

$y = 12 \quad 13 \quad 14 \quad 16$

← Divided Difference Table →

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
5	12			
		1		
6	13		-0.167	
		0.333		0.05
9	14		0.133	
		1		
11	16			

$$f(x) = f(x_0) + (x-x_0) f(x_0, x_1) + (x-x_0)(x-x_1) f(x_0, x_1, x_2) + (x-x_0)(x-x_1)(x-x_2) + f(x_0, x_1, x_2, x_3)$$

$$\Rightarrow f(10) = 12 + (10-5) \times 1 + (10-5)(10-6) \times (-0. + (10-5)(10-6)(10-9) \times (0.05)$$

$$\Rightarrow f(10) = 12 + 5 + 20 \times (-0.167) + 20 \times (0.0$$

$$\Rightarrow f(10) = 12 + 5 - 3.34 + 1$$

$$\therefore f(10) = 14.66$$

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Q.) Find the polynomial of the lowest possible degree which assumes the values 3, 12, 15, −21 when x has the values 3, 2, 1, −1 respectively by using Newton divided difference table.

← Divided Difference Table →

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$
−1	−21			
		18		
			−7	
1	15			1
		−3		
			−3	
2	12			
		−9		
3	3			

$f(x) = -21 + (x+1)\,18 + (x+1)(x-1) \times (-7)$
$+ (x+1)(x-1)(x-2)$

$\Rightarrow f(x) = -21 + 18 + 18 + (x^2-1)(-7) + (x^2-1)(x-2)$
$\Rightarrow f(x) = -3 + 18x - 7x^2 + 7 + x^3 - x - 2x^2 + 2$
$\therefore f(x) = x^3 - 9x^2 + 11x + 6$

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12. Lagrange's Interpolation formula for unequal Intervals.

$$f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)\cdots(x-x_n)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)\cdots(x_0-x_n)} \times f(x_0)$$
$$+ \frac{(x-x_0)(x-x_2)(x-x_3)\cdots(x-x_n)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)\cdots(x_1-x_n)} \times f(x_1)$$
$$+ \cdots \frac{(x-x_0)(x-x_1)(x-x_2)\cdots(x-x_{n-1})}{(x_n-x_0)(x_n-x_1)(x_n-x_2)\cdots(x_n-x_{n-1})} \times f(x_n)$$

Q.) find the value $\log_{10}656$ when
$\log_{10}654 = 2.8156$, $\log_{10}658 = 2.8182$
$\log_{10}659 = 2.8189$, $\log_{10}661 = 2.8202$.

Here,

$x_0 = 654$ $\quad$ $f(x_0) = 2.8156$
$x_1 = 658$ $\quad$ $f(x_1) = 2.8182$
$x_2 = 659$ $\quad$ $f(x_2) = 2.8189$
$x_3 = 661$ $\quad$ $f(x_3) = 2.8202$

$x = 656$.

Now

$$y = \frac{(656-658)(656-659)(656-661)}{(654-658)(654-659)(654-661)} \times 2.8156$$
$$+ \frac{(656-654)(656-659)(656-661)}{(658-654)(658-659)(658-661)} \times 2.8182$$
$$+ \frac{(656-654)(656-658)(656-661)}{(659-654)(659-658)(659-661)} \times 2.8189$$
$$+ \frac{(656-654)(656-658)(656-659)}{(661-654)(661-658)(661-659)} \times 2.8202$$

$$\Rightarrow y = \frac{(-2)\times(-3)\times(-5)}{(-4)\times(-5)\times(-7)} \times 2.8156 + \frac{2\times(-3)\times(-5)}{4\times(-1)\times(-3)} \times 2.818$$

$$+ \frac{(-2)\times 2\times(-5)}{5\times 1\times(-2)} \times 2.8189 + \frac{2\times(-2)\times(-3)}{7\times 3\times 2} \times 2.8202$$

$$\Rightarrow y = 0.603 + 7.0455 - 5.6378 + 0.806$$

$$\therefore y = 2.8167$$

Ans