Mathematical Logic

Statement or proposition:
A sentence which is either true or false is called statement.

Eg:-
1. Ram is good boy.
2. Hari is rich man.
3. It is raining.
4. what is your name?
5. Oh my God!
6. Shut up
7. Blood is green.

4,5,6 are not a statements

Compound statement:
A statement obtain from the combination of two or more statement is called compound statement.

Eg:- P: Blood is green
2: I am hungry

$P \vee 2 =$ Blood is green or I am hungry

P: Ram is good boy

२: shyam is good boy

P∧२ = Ram and shyam are good boys.

Atomic or simple statement:

A statement which is not a combination of other statement is called simple statement.

eg:- P: Blood is green

२: $2+6 = 11$

Note

* Generally statements are denoted by P, २, २, etc.

Connective:

Two statements can be combine by "and", "or", "not" etc. called connectives. There are following connectives with their symbols.

English word	connectives Name	Symbols
Not	Negation	$\sim$, $\neg$
AND	Conjunction	$\wedge$
OR	Disjunction	$\vee$
One way implication	conditional	$\rightarrow$, $\Rightarrow$
Iff or if and only if	Biconditional	$\leftrightarrow$, $\Leftrightarrow$

1. Negation of statement –

Let P be Statement them negation of P difined as:

(i) If P is true then negation of P is false.
(ii) If P is false then negation of P is True

* we can use " not " or " It is false that " to negate the statement.

P: Riya is poor
$\sim$P: Riya is not poor
$\sim$P: It is false that Riya is poor

P	$\sim$P
T	F
F	T

2. Conjunction of statement -

The conjunction of two statements p and q is defind as p∧q (read as p and q) Statement is true when both p and q are true and false when either one of them of both are false.

P: Ansh is a bad boy
q: shubham is a bad boy

P∧q: Ansh and shubham are bad boys

Truth Table of p∧q

P	q	P∧q
T	T	T
T	F	F
F	T	F
F	F	F

3. Disjunction of statement -

Let p and q be two statements, then the disjunction of P and q is defined as p∨q (read as p or q) is True when one or both statements are True and false it both statements are false.

Eg:- P: I am hungry
 q: I like to eat pizza.

PVq: I am hungry or I like to
 eat pizza.

Truth Table of PVq

P	q	PVq
T	T	T
T	F	T
F	T	T
F	F	F

4. Conditional statement:

Let p and q be two statement then the statement "If P then q" is called conditional statement.

* P is called antecedent or hypothesis
* q is called consequent or conclusion
* denoted by P → q or P ⇒ q (implication)

Truth Table of P → q or P ⇒ q

P	q	P → q
T	T	T
T	F	F
F	T	T
F	F	T

eg :- P : I am hungry
     2 : I will eat

5. Biconditional statement :
                              Let p and 2
be two statements then the
statement "p if and only if 2" is
called Biconditional statement.

eg :- p : I am 18 year old.
      2 : I can vote

Truth Table for P ↔ 2 or P ⇔ 2

P	2	P ↔ 2
T	T	T
T	F	F
F	T	F
F	F	T

Toutology :-
                A statement which is
always True is called Toutology.

Eg :- P → P

P	P	P → P
T	T	T
F	F	T

Contradiction:

A statement which is always False is called contradiction.

Eg :- $P \wedge \sim P$

P	$\sim P$	$P \wedge \sim P$
T	F	F
F	T	F

Some important formulas

(1). Commutative Law

(a) $P \wedge q = q \wedge P$

(b) $P \vee q = q \vee P$

(2) Associative Law

(a) $P \vee (q \vee r) = (P \vee q) \vee r$

(b) $P \wedge (q \wedge r) = (P \wedge q) \wedge r$

* (3) Distributive Law

(a) $P \wedge (q \vee r) = (P \wedge q) \vee (P \wedge r)$

(b) $P \vee (q \wedge r) = (P \vee q) \wedge (P \vee r)$

* (4) Idempotent Law

(a) $P \wedge P = P$

(b) $P \vee P = P$

(5) Law of absouption
    (a) $P \wedge (P \vee Q) = P$
    (b) $P \vee (P \wedge Q) = P$

* (6) Involution Law
    $\sim(\sim P) = P$

(7) Complement law
    (a) $P \vee \sim P = T$
    (b) $P \wedge \sim P = F$

(8) Operation with T
    (a) $P \vee T = T$
    (b) $P \wedge T = P$

(9) Operation with F
    (a) $P \vee F = P$
    (b) $P \wedge F = F$

* (10) De-Morgan's Law
    (a) $\sim(P \wedge Q) = \sim P \vee \sim Q$
    (b) $\sim(P \vee Q) = \sim P \wedge \sim Q$

* (11) $P \rightarrow Q = \sim P \vee Q$

* (12) $P \leftrightarrow Q = (P \rightarrow Q) \wedge (Q \rightarrow P)$

Normal Form

(1) Disjunctive Normal Form (DNF):-

A Formula which has a sum of elementary product and is equivalent to the given formula is called DNF.

$$Eg:- (i) \; (P \wedge q) \vee (q \wedge r) \vee (r \wedge p)$$
$$(ii) \; P \vee q \vee r$$
$$(iii) \; (P \wedge q) \vee (P \wedge r)$$

(2) Conjunction Normal Form (CNF):-

A Formula which has a product of elementary sum and is equivalent to the given formula is called CNF.

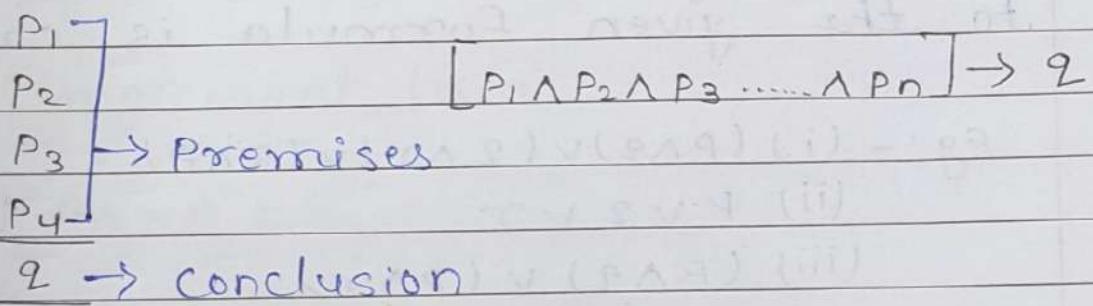$$Eg:- (i) \; (P \vee q) \wedge (q \vee r) \wedge (r \vee P)$$
$$(ii) \; P \wedge q \wedge r$$
$$(iii) \; (P \vee q) \wedge (P \wedge r)$$

Rules to obtain DNF/CNF

1. Remove $\rightarrow, \leftrightarrow$ connectives by using proper formu
2. Eliminate $\sim$ before sum and product by using De-morgan's Law.
3. Apply Distribution Law untill the form DNF or CNF obtained.

Argument:

An argument is a statement which is formed by a given set of statement called premises and gives conclusion.

$$
\left.\begin{array}{l}
P_1 \\
P_2 \\
P_3 \\
P_4
\end{array}\right] \to \text{Premises}
$$

$$
\left[P_1 \wedge P_2 \wedge P_3 \cdots \wedge P_n\right] \to q
$$

$$
\underline{q} \to \text{conclusion}
$$

P_1 - Today is sunday
P_2 - Market is closed today

q - I am not going to market

$$P_1 \wedge P_2 \to q$$

Fallacy (Invalid):

An argument which is not valid is called fallcy.

valid Argument:

An argument is said to be valid if the conclusion is toutology.

check the argument is valid on not?

$$\frac{P \qquad P \to q}{q}$$

$$[P \wedge (P \to q)] \to q$$

P	q	$P \to q$	$P \wedge (P \to q)$	$[P \wedge (P \to q)] \to q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

Law of Detachment (Modus ponens)

$$\frac{P \qquad P \to q}{q}$$

$$[P \wedge (P \to q)] \to q$$

Law of syllogism

$$\frac{P \to q \qquad q \to r}{P \to r}$$

$$[(p \to q) \land (q \to r)] \to p \to r$$

Let

$$[(p \to q) \land (q \to r)] = x$$

P	q	r	$p \to q$	$q \to r$	x	$p \to r$	$x \to p \to r$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	F	T	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	F	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

∵ since conclunsion is toutology
∴ statement is valid

The End