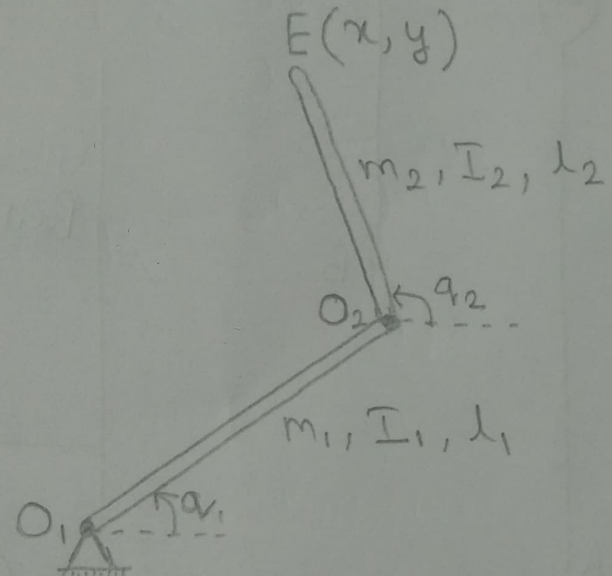
Name :- Yash Meshram

Roll No. :- 18110192

Course :- ME 639 : Introduction to Robotics

Mini - Project

Task 0



E : End effector

(x, y) : End effector position

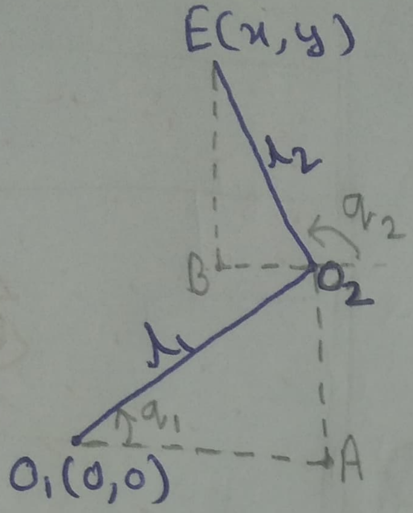
(q_1, q_2) : Joint angles

Let's assume O_1 as origin and position of point E(x, y) is relative to pt. O_1 (origin).

Motors are connected to both joints O_1 & O_2.

We can control either torque τ_1 & τ_2 applied at point O_1 & O_2 respectively or the angles q_1 & q_2

End - effector position



$$x = O_1A - O_2B$$

$$y = AO_2 + BE$$

$$x = l_1 \cos q_1 - l_2 \cos(180 - q_2)$$

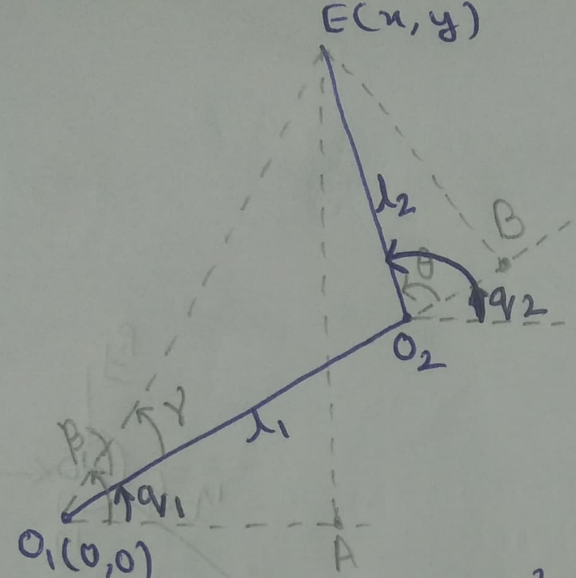
$$\left.\begin{aligned} x &= l_1 \cos q_1 + l_2 \cos q_2 \\ y &= l_1 \sin q_1 + l_2 \sin q_2 \end{aligned}\right\} - ①$$

End effector velocity

Differentiating eqⁿ ①

$$\left.\begin{aligned} \dot{x} &= -l_1 \sin q_1 \cdot \dot{q}_1 - l_2 \sin q_2 \cdot \dot{q}_2 \\ \dot{y} &= l_1 \cos q_1 \cdot \dot{q}_1 + l_2 \cos q_2 \cdot \dot{q}_2 \end{aligned}\right\} - ②$$

Joint angles



In triangle $O_1 O_2 E$

Apply cosine rule

$$(O_1E)^2 = (O_1O_2)^2 + (O_2E)^2 - 2(O_1O_2)(O_2E)\cos(180-\theta)$$

$$x^2+y^2 = l_1^2 + l_2^2 + 2 l_1 l_2 \cos\theta$$

$$\theta = \cos^{-1}\left(\frac{x^2+y^2-l_1^2-l_2^2}{2 l_1 l_2}\right)$$

$$q_1 = \beta - \gamma \qquad \beta = \tan^{-1}\left(\frac{AE}{O_1A}\right) = \tan^{-1}\left(\frac{y}{x}\right)$$

$$\& \; q_2 = q_1 + \theta \qquad \gamma = \tan^{-1}\left(\frac{EB}{O_1B}\right) = \tan^{-1}\left(\frac{l_2\sin\theta}{l_1+l_2\cos\theta}\right)$$

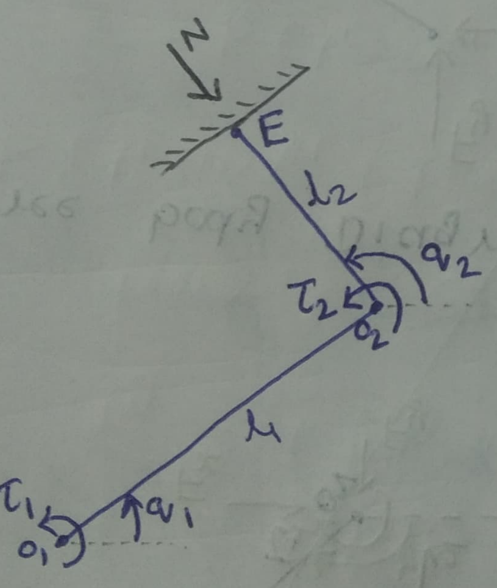
$$q_2 = \beta - \gamma + \theta$$

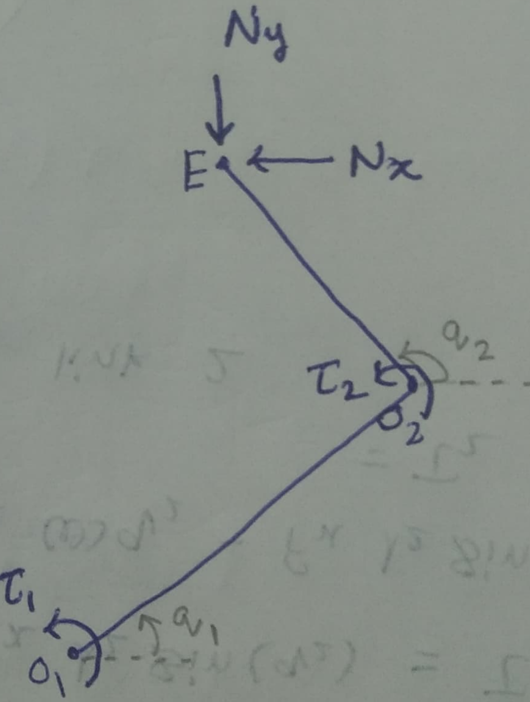
$$(3) - \left\{\begin{aligned} \theta &= \cos^{-1}\left(\frac{x^2+y^2-l_1^2-l_2^2}{2 l_1 l_2}\right) \\ q_1 &= \tan^{-1}\left(\frac{y}{x}\right) - \tan^{-1}\left(\frac{l_2\sin\theta}{l_1+l_2\cos\theta}\right) \\ q_2 &= \tan^{-1}\left(\frac{y}{x}\right) - \tan^{-1}\left(\frac{l_2\sin\theta}{l_1+l_2\cos\theta}\right) + \cos^{-1}\left(\frac{x^2+y^2-l_1^2-l_2^2}{2 l_1 l_2}\right) \end{aligned}\right.$$

Torque

In 2R elbow manipulator the End effector moves freely. When it hit an object (for ex. wall) it applies force on it & by Newtons 3^{rd} law the ~~wall~~ object (wall) applies equal and opposite force on the End effector.

Let's assume that the End effector applies only Normal force on the object (wall) & ~~the~~ it doesn't slip on the surface of the object (wall)





Let's assume F_y & F_x are forces applied by End effector on object (wall) in y & x direction respectively

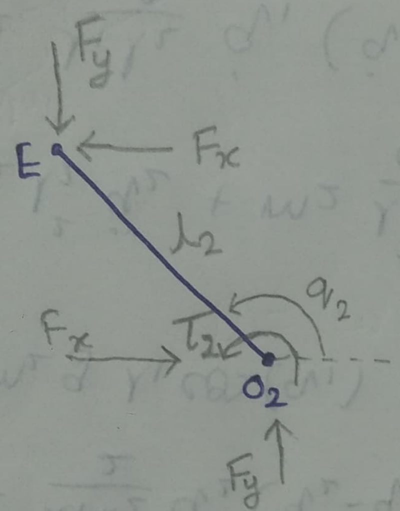
N_y & N_x are the reaction forces of F_y & F_x respectively

Thus,

$$|N_y| = |F_y| \quad \& \quad |N_x| = |F_x|$$

Let's ignore gravity for the time being

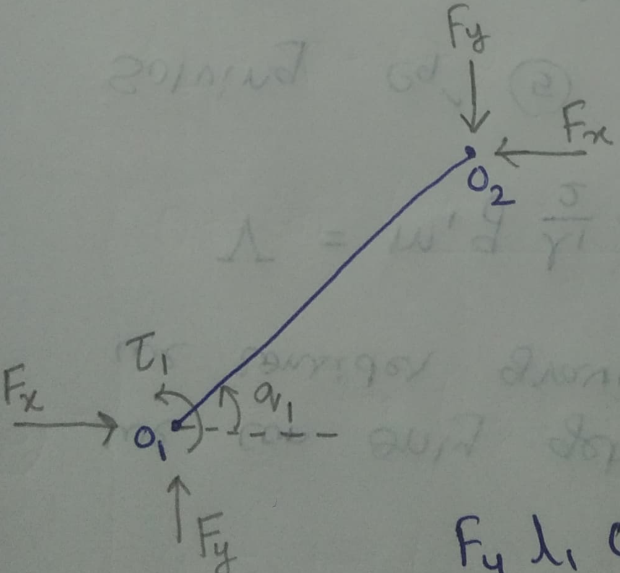
Free body diagram of link 2



$$\Sigma M_{O_2} = 0$$

$$F_y l_2 \cos q_2 - F_x l_2 \sin q_2 = \tau_2$$

Free body diagram of link 1



$$\Sigma M_{O_1} = 0$$

$$F_y l_1 \cos q_1 - F_x l_1 \sin q_1 = \tau_1$$

$$\left.\begin{aligned} F_y l_1 \cos q_1 - F_x l_1 \sin q_1 &= \tau_1 \\ F_y l_2 \cos q_2 - F_x l_2 \sin q_2 &= \tau_2 \end{aligned}\right\} \text{—(4)}$$

Therefore, we need to apply τ_1 ~~torque~~ and τ_2 torque at the motor ① & ② so that the ~~wall~~ force applied by the rod on wall will be F. i.e F_x in x-directn & F_y in y-direction.

Torque

Lagarangian ($\mathcal{L}$) = Kinetic energy (K) - Potential energy (V)

Lagarangian eqn

$$\left.\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}_i}\right) - \frac{\partial \mathcal{L}}{\partial q_i} = \theta'_i \right\} - ⑤$$

θ'_i : General forces derived using principle of virtual work.

$$K = \frac{1}{2} I_1 \omega_1^2 + \frac{1}{2} m_2 V_{c_2}^2 + \frac{1}{2} I_2 \omega_2^2$$

Pure rotatn about O_1

Kinetic energy by translatn

Rotatn of L_2 about center of gravity

V_{c_2} : Velocity of center of mass

$$K = \frac{1}{2}\left(\frac{1}{3} m_1 l_1^2\right)\dot{q}_1^2 + \frac{1}{2} m_2 v_{c_2}^2 + \frac{1}{2}\left(\frac{1}{12} m_2 l_2^2\right)\dot{q}_2^2$$

$$v_{c_2}^2 = (l_1 \dot{q}_1)^2 + \left(\frac{l_2}{2}\dot{q}_2\right)^2 + 2 l_1 \dot{q}_1 \frac{l_2}{2} \dot{q}_2 \cos(q_2 - q_1)$$

we can only get potential energy (V) when we consider gravity (g)

$$V = m_1 g \frac{l_1}{2} \sin q_1 + m_2 g\left(l_1 \sin q_1 + \frac{l_2}{2} \sin q_2\right)$$

Solving eq^n (5) . we get,

$$\frac{1}{3} m_1 l_1^2 \ddot{q}_1 + m_2 l_1^2 \ddot{q}_1 + m_2 \frac{l_1 l_2}{2} \ddot{q}_2 \cos(q_2 - q_1)$$
$$- m_2 \frac{l_1 l_2}{2} \dot{q}_2 (\dot{q}_2 - \dot{q}_1) \sin(q_2 - q_1) + m_1 g \frac{l_1}{2} \cos(q_1)$$
$$+ m_2 g l_1 \cos(q_1) = \tau_1$$

$$\frac{1}{3} m_2 l_2^2 \ddot{q}_2 + m_2 \frac{l_2^2}{4} \ddot{q}_2 + m_2 \frac{l_1 l_2}{2} \ddot{q}_1 \cos(q_2 - q_1)$$
$$- m_2 \frac{l_1 l_2}{2} \dot{q}_1 (\dot{q}_2 - \dot{q}_1) \sin(q_2 - q_1) + m_2 g \frac{l_2}{2} \sin(q_2)$$
$$= \tau_2$$

(6)

Spring torque

2R elbow manipulator should behave as a spring

$$\therefore \quad F_x = Kx \qquad K: \text{Spring constant}$$

$$F_y = Ky$$

from eqn ①

$$F_x = K(l_1 \cos q_1 + l_2 \cos q_2)$$

$$F_y = K(l_1 \sin q_1 + l_2 \sin q_2)$$

from eqn ④

$$\left.\begin{aligned} &K(l_1 \sin q_1 + l_2 \sin q_2)\, l_2 \cos q_2 \\ &\quad - K(l_1 \cos q_1 + l_2 \cos q_2)\, l_2 \sin q_2 = \tau_{2s} \\[1em] &K(l_1 \sin q_1 + l_2 \sin q_2)\, l_1 \cos q_2 \\ &\quad - K(l_1 \cos q_1 + l_2 \cos q_2)\, l_1 \sin q_1 = \tau_{1s} \end{aligned}\right\} \quad (7)$$

Setting the torque of motor ① & ② to $\tau_1 + \tau_{1s}$ & $\tau_2 + \tau_{2s}$ will make the 2R elbow manipulator behave like a spring.