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Course :- ME 639 : Introduction to Robotics

Assignment 2

Q1 →

$$R_0^1 = \begin{bmatrix} \underbrace{\begin{matrix}\hat{i}_1 \cdot \hat{i}_0 \\ \hat{i}_1 \cdot \hat{j}_0 \\ \hat{i}_1 \cdot \hat{k}_0\end{matrix}}_{V_1} & \underbrace{\begin{matrix}\hat{j}_1 \cdot \hat{i}_0 \\ \hat{j}_1 \cdot \hat{j}_0 \\ \hat{j}_1 \cdot \hat{k}_0\end{matrix}}_{V_2} & \underbrace{\begin{matrix}\hat{k}_1 \cdot \hat{i}_0 \\ \hat{k}_1 \cdot \hat{j}_0 \\ \hat{k}_1 \cdot \hat{k}_0\end{matrix}}_{V_3} \end{bmatrix}$$

$$R_0^1 = [V_1 \quad V_2 \quad V_3]$$

Thus $$(R_0^1)^T = \begin{bmatrix} V_1^T \\ V_2^T \\ V_3^T \end{bmatrix}$$

$$(R_0^1)^T (R_0^1) = \begin{bmatrix} V_1^T \\ V_2^T \\ V_3^T \end{bmatrix} [V_1 \quad V_2 \quad V_3]$$

$$(R_0^1)^T (R_0^1) = \begin{bmatrix} v_1^T v_1 & v_1^T v_2 & v_1^T v_3 \\ v_2^T v_1 & v_2^T v_2 & v_2^T v_3 \\ v_3^T v_1 & v_3^T v_2 & v_3^T v_3 \end{bmatrix} \quad \text{— (1)}$$

we know,

$$(R_0^1)^T (R_0^1) = I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{— (2)}$$

from (1) & (2)

$$v_i^T v_j = \begin{cases} 0 & \text{if } i \neq j \\ 1 & \text{if } i = j \end{cases}$$

Since, $v_i^T v_j = 0$ for $i \neq j$

thus, we can say that

~~Rotatn~~ colums of Rotatn matrix are orthogonal.

Q2] $\rightarrow$

$$(R_0^1)^T (R_0^1) = I \quad \text{--- (Property of rotatn matrix)}$$

$$(R_0^1)^T \underbrace{(R_0^1)(R_0^1)^{-1}}_{I} = (R_0^1)^{-1}$$

$$(R_0^1)^T = (R_0^1)^{-1} \quad \text{— (A)}$$

we know,

$$(R_0^1)^{-1} = \frac{\operatorname{adj}(R_0^1)}{\det(R_0^1)}$$

$$\det(R_0^1) = (R_0^1)\operatorname{adj}(R_0^1) \quad \text{— (1)}$$

$$(R_0^1)^T = (R_0^1)^{-1} = \frac{\operatorname{adj}(R_0^1)}{\det(R_0^1)}$$

$\det(R_0^1) \rightarrow$ Scalar quantity

$$(R_0^1) = \left((R_0^1)^T\right)^T = \frac{\left(\operatorname{adj}(R_0^1)\right)^T}{\det(R_0^1)}$$

$$(R_0^1) = \frac{\operatorname{adj}\left((R_0^1)^T\right)}{\det(R_0^1)}$$

$$(R_0^1) = \frac{adj\left((R_0^1)^{-1}\right)}{\det(R_0^1)} \quad \cdots \text{(from A)}$$

$$(R_0^1) = \frac{(adj\,(R_0^1))^{-1}}{\det(R_0^1)}$$

$$\det(R_0^1) = \frac{1}{(R_0^1)\,adj\,(R_0^1)} \quad — (2)$$

from (1) & (2).

$$\det(R_0^1) = \frac{1}{\det(R_0^1)}$$

$$\left[\det(R_0^1)\right]^2 = 1$$

$$\det(R_0^1) = \pm 1$$

$$\boxed{\det(R_0^1) = 1}$$

Q5] $\rightarrow$ Since R is a rotatn matrix

$$R^T = R^{-1}$$

$$R\,S(a)\,R^T = R\,S(a)\,R^{-1}$$

$$= R\,(a \times R^{-1}) \quad \cdots \left(\because S(a)R = a \times R\right)$$

$$= Ra \times \underbrace{RR^{-1}}_{I}$$

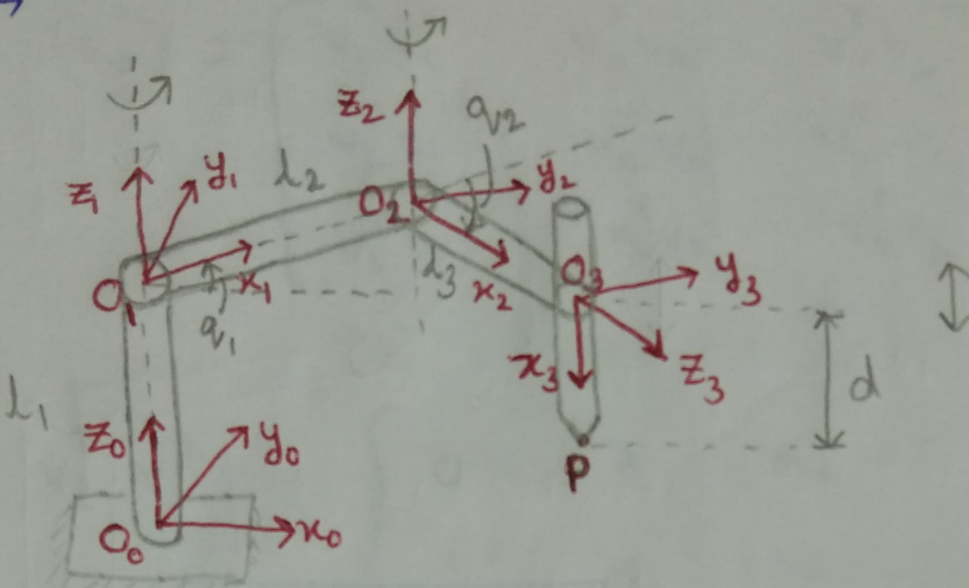
$$= Ra \times I$$

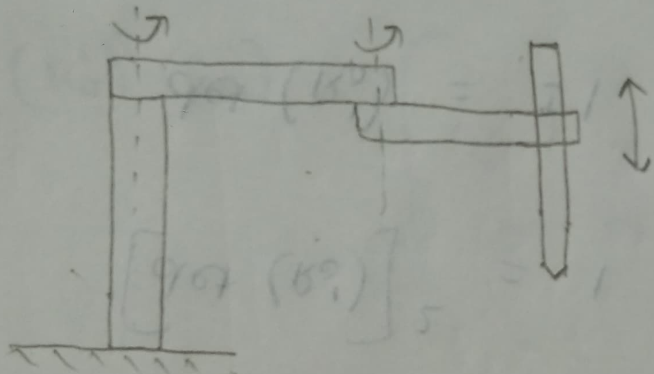
$$= S(Ra)\,I$$

~~RtSa~~ $= S(Ra)$ — ①

Thus $\boxed{R\,S(a)\,R^T = S(Ra)}$

Q6] →





Side view

$$R_0^1 = \begin{bmatrix} \cos q_1 & -\sin q_1 & 0 \\ \sin q_1 & \cos q_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$d_0^1 = \begin{bmatrix} 0 \\ 0 \\ l_1 \end{bmatrix}$$

$$R_1^2 = \begin{bmatrix} \cos q_2 & -\sin q_2 & 0 \\ \sin q_2 & \cos q_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad d_1^2 = \begin{bmatrix} l_2 \\ 0 \\ 0 \end{bmatrix}$$

$$R_2^3 = \begin{bmatrix} \cos\frac{\pi}{2} & 0 & \sin\frac{\pi}{2} \\ 0 & 1 & 0 \\ -\sin\frac{\pi}{2} & 0 & \cos\frac{\pi}{2} \end{bmatrix}, \quad d_2^3 = \begin{bmatrix} l_3 \\ 0 \\ 0 \end{bmatrix}$$

$$P_3 = \begin{bmatrix} d \\ 0 \\ 0 \end{bmatrix}$$

$$\cancel{P_0 = H_0^1 \, H_1^2 \, H_2^3 \, P_3}$$

$$H_0^1 = \begin{bmatrix} R_0^1 & d_0^1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos q_1 & -\sin q_1 & 0 & 0 \\ \sin q_1 & \cos q_1 & 0 & 0 \\ 0 & 0 & 1 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_1^2 = \begin{bmatrix} R_1^2 & d_1^2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos q_2 & -\sin q_2 & 0 & \lambda_2 \\ \sin q_2 & \cos q_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_2^3 = \begin{bmatrix} R_2^3 & d_2^3 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & \lambda_3 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = H_0^1 \; H_1^2 \; H_2^3 \begin{bmatrix} P_3 \\ 1 \end{bmatrix}$$

$$H_2^3 \begin{bmatrix} P_3 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & \lambda_3 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} d \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} \lambda_3 \\ 0 \\ -d \\ 1 \end{bmatrix}$$

$$H_1^2 H_2^3 \begin{bmatrix} P_3 \\ 1 \end{bmatrix} = \begin{bmatrix} \cos q_2 & -\sin q_2 & 0 & l_2 \\ \sin q_2 & \cos q_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} l_3 \\ 0 \\ -d \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} l_3 \cos q_2 + l_2 \\ l_3 \sin q_2 \\ -d \\ 1 \end{bmatrix}$$

$$H_0^1 H_1^2 H_2^3 \begin{bmatrix} P_3 \\ 1 \end{bmatrix} = \begin{bmatrix} \cos q_1 & -\sin q_1 & 0 & 0 \\ \sin q_1 & \cos q_1 & 0 & 0 \\ 0 & 0 & 1 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} l_3 \cos q_2 + l_2 \\ l_3 \sin q_2 \\ -d \\ 1 \end{bmatrix}$$

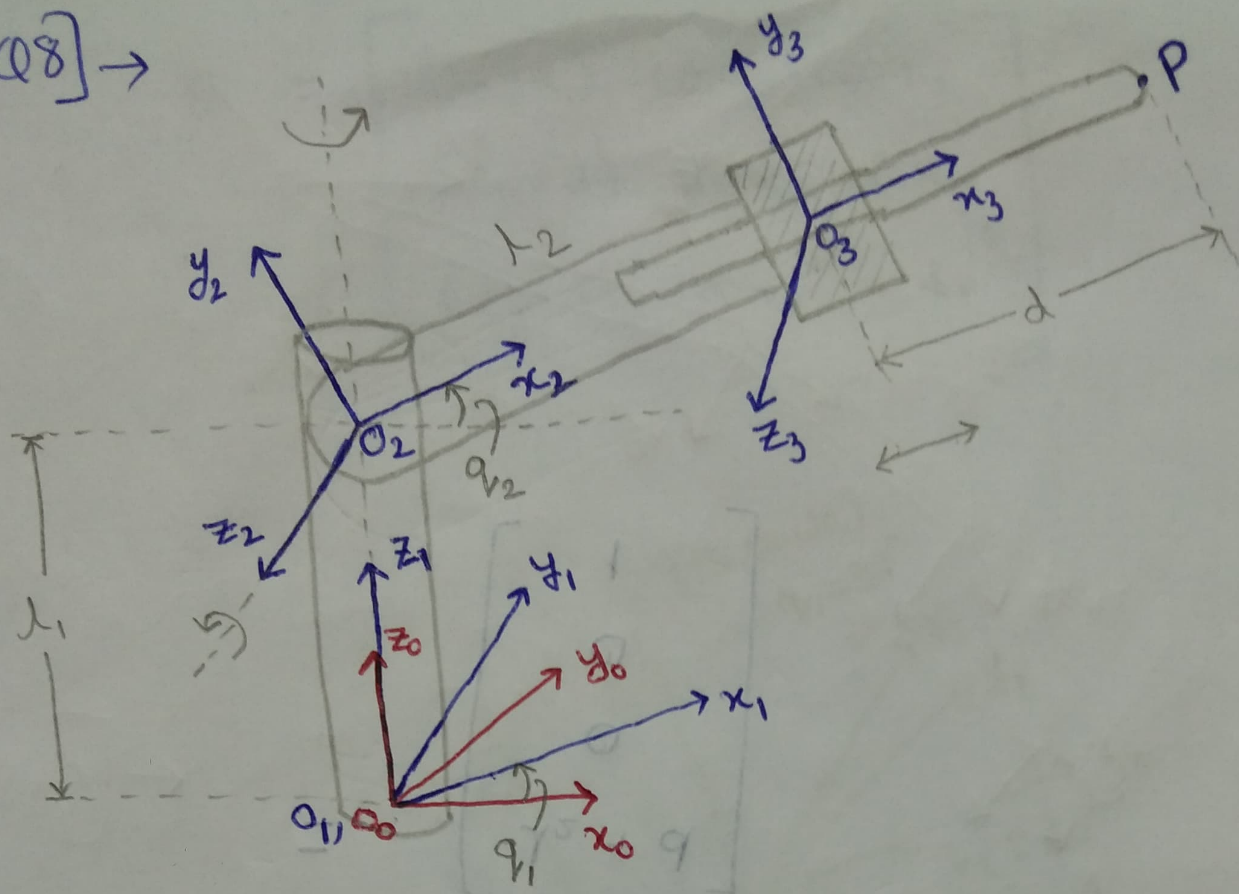
$$= \begin{bmatrix} l_3 \cos q_1 \cos q_2 + l_2 \cos q_1 - l_3 \sin q_1 \sin q_2 \\ l_3 \sin q_1 \cos q_2 + l_2 \sin q_1 + l_3 \cos q_1 \sin q_2 \\ -d + l_1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = H_0^1 \, H_1^2 \, H_2^3 \begin{bmatrix} P_3 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = \begin{bmatrix} \lambda_3 \cos(q_1+q_2) + \lambda_2 \cos q_1 \\ \lambda_3 \sin(q_1+q_2) + \lambda_2 \sin q_1 \\ -d + \lambda_1 \\ 1 \end{bmatrix}$$

$$P_0 = \begin{bmatrix} \lambda_3 \cos(q_1+q_2) + \lambda_2 \cos q_1 \\ \lambda_3 \sin(q_1+q_2) + \lambda_2 \sin q_1 \\ \lambda_1 - d \end{bmatrix}$$

Q8] →



$$R_0^1 = \begin{bmatrix} \cos q_1 & -\sin q_1 & 0 \\ \sin q_1 & \cos q_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad d_0^1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$R_1^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cancel{\cos\frac{\pi}{2}}^{\,0} & \left(-\cancel{\sin\frac{\pi}{2}}^{\,1}\right) \\ 0 & \cancel{\sin\frac{\pi}{2}}^{\,1} & \cancel{\cos\frac{\pi}{2}}^{\,0} \end{bmatrix} \begin{bmatrix} \cos q_2 & -\sin q_2 & 0 \\ \sin q_2 & \cos q_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$d_1^2 = \begin{bmatrix} 0 \\ 0 \\ \lambda_1 \end{bmatrix}, \quad R_1^2 = \begin{bmatrix} \cos q_2 & -\sin q_2 & 0 \\ 0 & 0 & -1 \\ \sin q_2 & \cos q_2 & 0 \end{bmatrix}$$

R_2^3 : Rotation about any axis with $0°$

$$R_2^3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

$$d_2^3 = \begin{bmatrix} \lambda_2 \\ 0 \\ 0 \end{bmatrix}, \quad P_3 = \begin{bmatrix} d \\ 0 \\ 0 \end{bmatrix}$$

$$H_0^1 = \begin{bmatrix} R_0^1 & d_0^1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos q_1 & -\sin q_1 & 0 & 0 \\ \sin q_1 & \cos q_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_1^2 = \begin{bmatrix} R_1^2 & d_1^2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos q_2 & -\sin q_2 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ \sin q_2 & \cos q_2 & 0 & \lambda_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_2^3 = \begin{bmatrix} R_2^3 & d_2^3 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & \lambda_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = H_0^1 \; H_1^2 \; H_2^3 \begin{bmatrix} P_3 \\ 1 \end{bmatrix}$$

$$H_2^3 \begin{bmatrix} P_3 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & \lambda_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} d \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} \lambda_2 + d \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$H_1^2 H_2^3 \begin{bmatrix} P_3 \\ 1 \end{bmatrix} = \begin{bmatrix} \cos q_2 & -\sin q_2 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ \sin q_2 & \cos q_2 & 0 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} l_2 + d \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

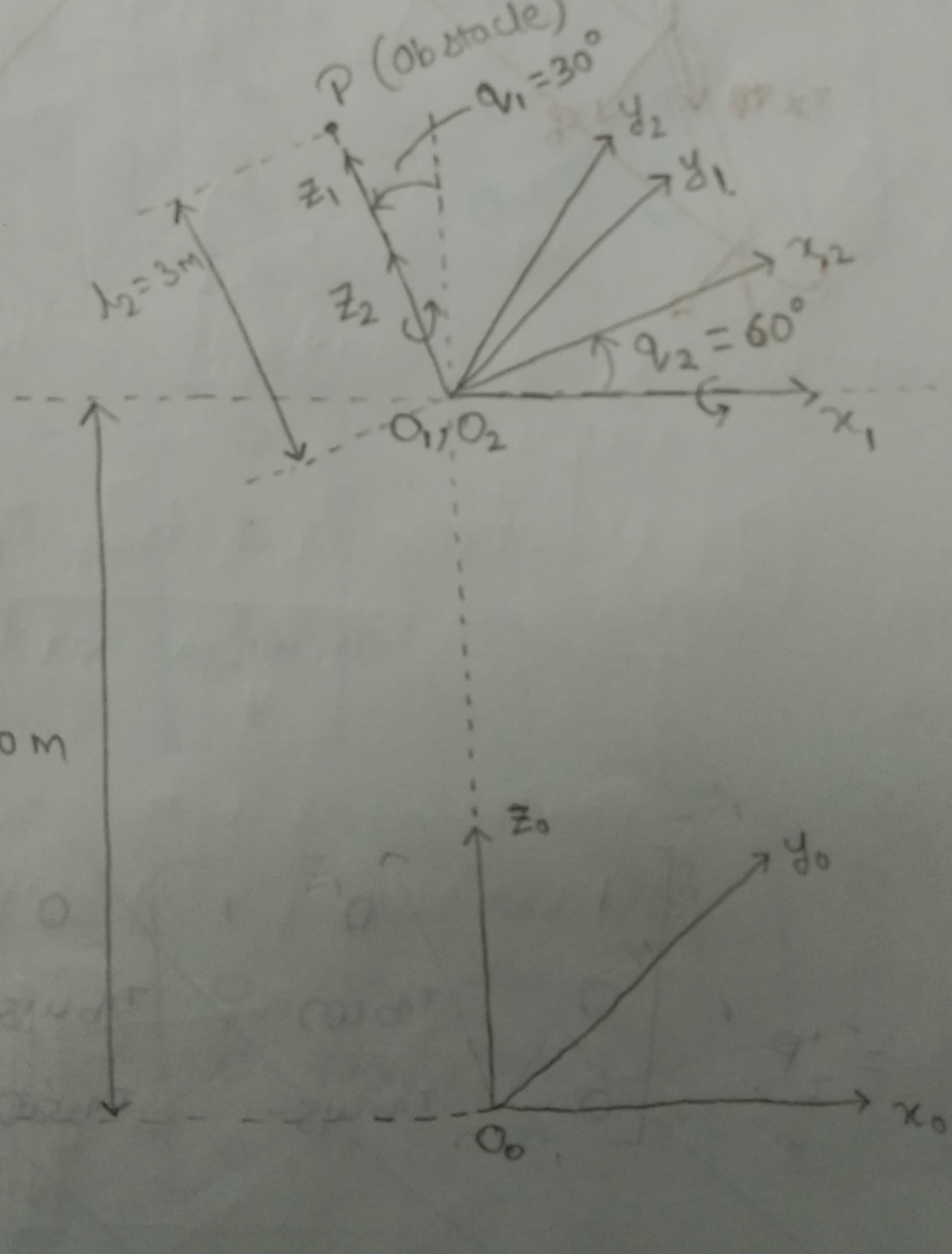
$$= \begin{bmatrix} (l_2+d)\cos q_2 \\ 0 \\ (l_2+d)\sin q_2 + l_1 \\ 1 \end{bmatrix}$$

$$H_0^1 H_1^2 H_2^3 \begin{bmatrix} P_3 \\ 1 \end{bmatrix} = \begin{bmatrix} \cos q_1 & -\sin q_1 & 0 & 0 \\ \sin q_1 & \cos q_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} (l_2+d)\cos q_2 \\ 0 \\ (l_2+d)\sin q_2 + l_1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = \begin{bmatrix} (l_2+d)\cos q_1 \cos q_2 \\ (l_2+d)\sin q_1 \cos q_2 \\ (l_2+d)\sin q_2 + l_1 \\ 1 \end{bmatrix}$$

$$P_0 = \begin{bmatrix} (l_2+d)\cos q_1 \cos q_2 \\ (l_2+d)\sin q_1 \cos q_2 \\ (l_2+d)\sin q_2 + l_1 \end{bmatrix}$$

Q9 →



$$R_0^1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos q_1 & -\sin q_1 \\ 0 & \sin q_1 & \cos q_1 \end{bmatrix}, \quad d_0^1 = \begin{bmatrix} 0 \\ 0 \\ \lambda_1 \end{bmatrix}$$

$$R_1^2 = \begin{bmatrix} \cos q_2 & -\sin q_2 & 0 \\ \sin q_2 & \cos q_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad d_1^2 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$H_0^1 = \begin{bmatrix} R_0^1 & d_0^1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos q_1 & -\sin q_1 & 0 \\ 0 & \sin q_1 & \cos q_1 & \lambda_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_1^2 = \begin{bmatrix} R_1^2 & d_1^2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos q_2 & -\sin q_2 & 0 & 0 \\ \sin q_2 & \cos q_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$P_2 = \begin{bmatrix} 0 \\ 0 \\ \lambda_2 \end{bmatrix}$$

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = H_0^1 \, H_1^2 \begin{bmatrix} P_2 \\ 1 \end{bmatrix}$$

$$H_1^2 \begin{bmatrix} P_2 \\ 1 \end{bmatrix} = \begin{bmatrix} \cos q_2 & -\sin q_2 & 0 & 0 \\ \sin q_2 & \cos q_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ l_2 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 0 \\ l_2 \\ 1 \end{bmatrix}$$

$$H_0^1 \, H_1^2 \begin{bmatrix} P_2 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos q_1 & -\sin q_1 & 0 \\ 0 & \sin q_1 & \cos q_1 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ l_2 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -l_2 \sin q_1 \\ l_2 \cos q_1 + l_1 \\ 1 \end{bmatrix}$$

$$P_0 = \begin{bmatrix} 0 \\ -l_2 \sin q_1 \\ l_2 \cos q_1 + l_1 \end{bmatrix}$$

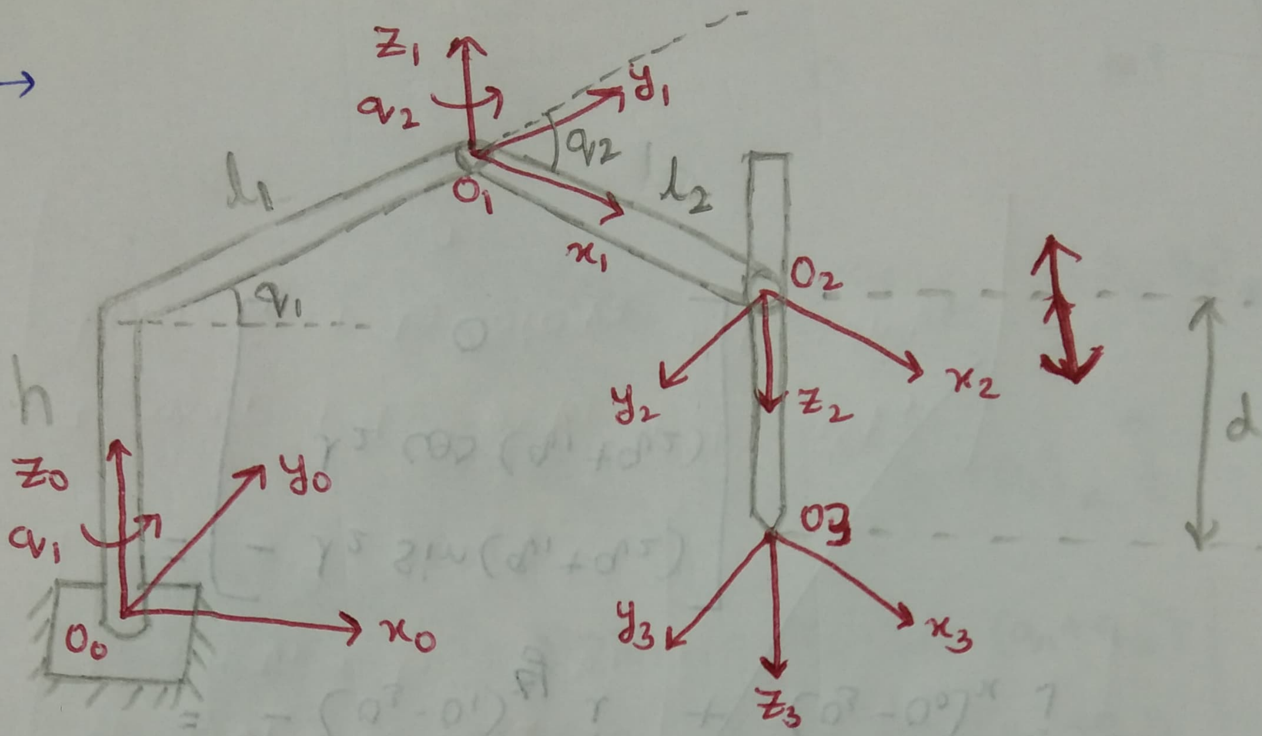
$l_1 = 10\ m$, $l_2 = 3\ m$,

$q_1 = 30^\circ$

$$P_0 = \begin{bmatrix} 0 \\ -3 \times \sin 30^\circ \\ 3 \cos 30^\circ + 10 \end{bmatrix}$$

$$P_0 = \begin{bmatrix} 0 \\ -3/2 \\ \frac{20 + 3\sqrt{3}}{2} \end{bmatrix} \approx \begin{bmatrix} 0 \\ -1.5 \\ 12.6 \end{bmatrix}$$

Q 11 →



For linear velocity

$$J_v = [J_1 \quad J_2 \quad J_3]$$

$$J_i = \begin{cases} z_{i-1} \times (O_n - O_{i-1}) & \text{for Revolute Joint} \\ z_{i-1} & \text{for Prismatic Joint} \end{cases}$$

In our case 1^{st} & 2^{nd} Joints are revolute and 3^{rd} joint is Prismatic

$$J_v = [z_0 \times (O_3 - O_0) \quad z_1 \times (O_3 - O_1) \quad z_2]$$

For angular velocity

$$J_\omega = \begin{bmatrix} J_1 & J_2 & J_3 \end{bmatrix}$$

$$J_1 = \begin{cases} Z_{i-1} & \text{for Revolute joint} \\ 0 & \text{for Prismatic joint} \end{cases}$$

$$J_\omega = \begin{bmatrix} Z_0 & Z_1 & 0 \end{bmatrix}$$

$$z_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad z_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad z_2 = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}, \quad z_3 = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}$$

$$O_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad O_1 = \begin{bmatrix} l_1 \cos q_1 \\ l_1 \sin q_1 \\ h \end{bmatrix}$$

$$O_2 = \begin{bmatrix} l_1 \cos q_1 + l_2 \cos(q_1+q_2) \\ l_1 \sin q_1 + l_2 \sin(q_1+q_2) \\ h \end{bmatrix}$$

$$O_3 = \begin{bmatrix} l_1 \cos q_1 + l_2 \cos(q_1+q_2) \\ l_1 \sin q_1 + l_2 \sin(q_1+q_2) \\ h-d \end{bmatrix}$$

$$O_3 - O_0 = O_3$$

$$O_3 - O_1 = \begin{bmatrix} l_2 \cos(q_1+q_2) \\ l_2 \sin(q_1+q_2) \\ -d \end{bmatrix}$$

$$z_0 \times (O_3 - O_0) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 1 \\ (O_3-O_0)_x & (O_3-O_0)_y & (O_3-O_0)_z \end{vmatrix}$$

$$= -(O_3-O_0)_y\,\hat{i} + (O_3-O_0)_x\,\hat{j}$$

$$= \begin{bmatrix} -l_1 \sin q_1 - l_2 \sin(q_1+q_2) \\ l_1 \cos q_1 + l_2 \cos(q_1+q_2) \\ 0 \end{bmatrix}$$

$$z_1 \times (O_3 - O_1) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 1 \\ (O_3-O_1)_x & (O_3-O_1)_y & (O_3-O_1)_z \end{vmatrix}$$

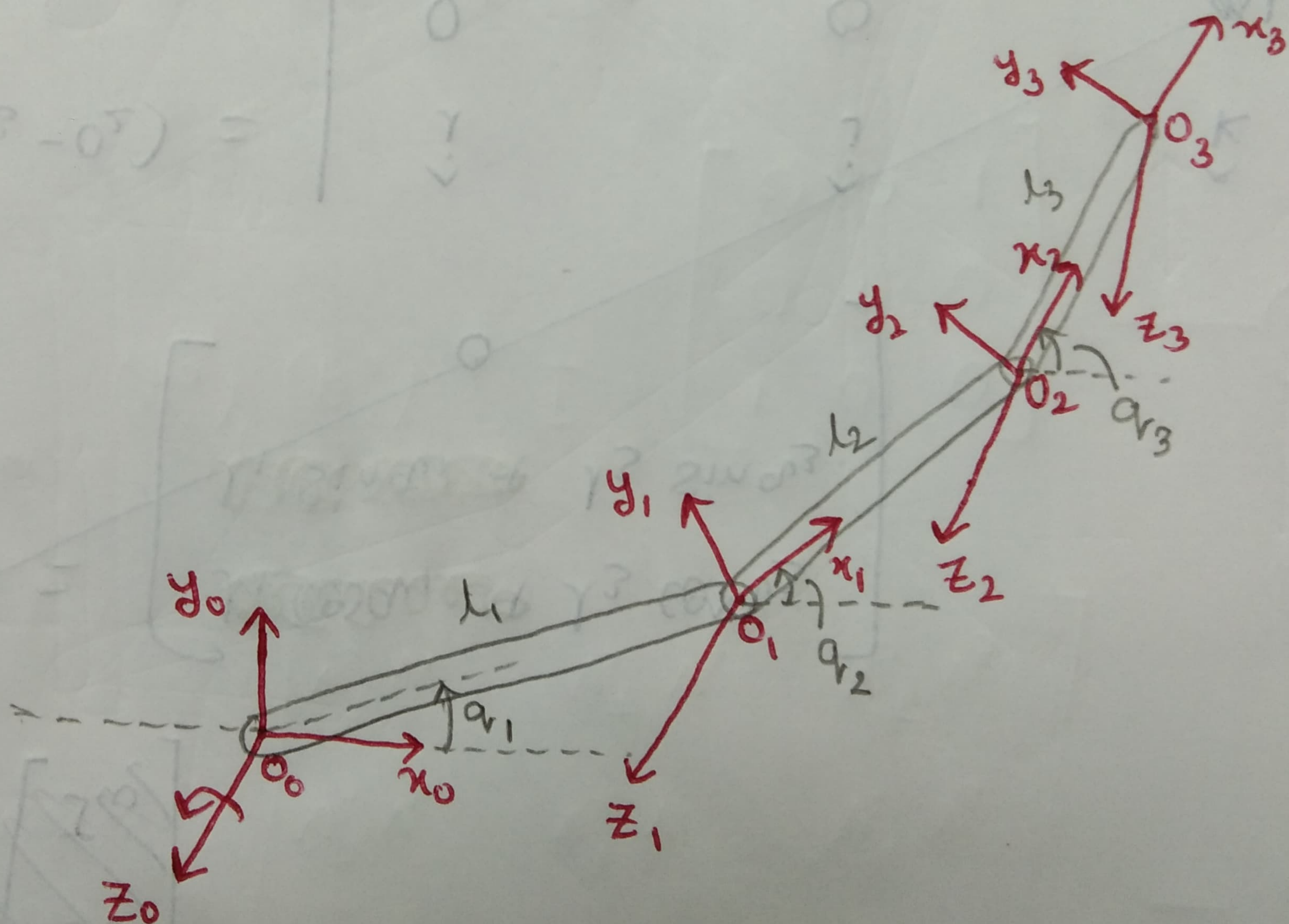
$$= -(O_3-O_1)_y\,\hat{i} + (O_3-O_1)_x\,\hat{j}$$

$$= \begin{bmatrix} -l_2 \sin(q_1+q_2) \\ l_2 \cos(q_1+q_2) \\ 0 \end{bmatrix}$$

$$J = \begin{bmatrix} J_v \\ J_\omega \end{bmatrix} = \begin{bmatrix} z_0 \times (o_3 - o_0) & z_1 \times (o_3 - o_1) & z_2 \\ z_0 & z_1 & 0 \end{bmatrix}$$

$$J = \begin{bmatrix} -l_1 \sin q_1 - l_2 \sin(q_1 + q_2) & -l_2 \sin(q_1 + q_2) & 0 \\ l_1 \cos q_1 + l_2 \cos(q_1 + q_2) & l_2 \cos(q_1 + q_2) & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

Q13] →



In this case since all joints are revolute joints

Therefore

$$J_v = \left[z_0 \times (O_3 - O_0) \quad z_1 \times (O_3 - O_1) \quad z_2 \times (O_3 - O_2) \right]$$

$$J_\omega = \left[z_0 \quad z_1 \quad z_2 \right]$$

$$z_0 = z_1 = z_2 = z_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$O_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad O_1 = \begin{bmatrix} l_1 \cos q_1 \\ l_1 \sin q_1 \\ 0 \end{bmatrix}$$

$$O_2 = \begin{bmatrix} l_1 \cos q_1 + l_2 \cos q_2 \\ l_1 \sin q_1 + l_2 \sin q_2 \\ 0 \end{bmatrix}$$

$$O_3 = \begin{bmatrix} l_1 \cos q_1 + l_2 \cos q_2 + l_3 \cos q_3 \\ l_1 \sin q_1 + l_2 \sin q_2 + l_3 \sin q_3 \\ 0 \end{bmatrix}$$

$$\cancel{O_3} \quad O_3 - O_0 = O_3$$

$$O_3 - O_1 = \begin{bmatrix} l_2 \cos q_2 + l_3 \cos q_3 \\ l_2 \sin q_2 + l_3 \sin q_3 \\ 0 \end{bmatrix}$$

$$Z_0 \times (O_3 - O_0) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 1 \\ \begin{pmatrix} l_1 \cos q_1 \\ + l_2 \cos q_2 \\ + l_3 \cos q_3 \end{pmatrix} & \begin{pmatrix} l_1 \sin q_1 \\ + l_2 \sin q_2 \\ + l_3 \sin q_3 \end{pmatrix} & 0 \end{vmatrix}$$

$$= (-l_1 \sin q_1 - l_2 \sin q_2 - l_3 \sin q_3)\, \hat{i}$$
$$+ (l_1 \cos q_1 + l_2 \cos q_2 + l_3 \cos q_3)\, \hat{j}$$

$$Z_1 \times (O_3 - O_1) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 1 \\ \begin{pmatrix} l_2 \cos q_2 \\ + l_3 \cos q_3 \end{pmatrix} & \begin{pmatrix} l_2 \sin q_2 \\ + l_3 \sin q_3 \end{pmatrix} & 0 \end{vmatrix}$$

$$= (-l_2 \sin q_2 - l_3 \sin q_3)\, \hat{i}$$
$$+ (l_2 \cos q_2 + l_3 \cos q_3)\, \hat{j}$$

$$O_3 - O_2 = \begin{bmatrix} l_3 \cos q_3 \\ l_3 \sin q_3 \\ 0 \end{bmatrix}$$

$$Z_0 \times (O_3 - O_2) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 1 \\ l_3 \cos q_3 & l_3 \sin q_3 & 0 \end{vmatrix}$$

$$= (-l_3 \sin q_3)\,\hat{i} + (l_3 \cos q_3)\,\hat{j}$$

$$J = \begin{bmatrix} J_v \\ J_\omega \end{bmatrix} = \begin{bmatrix} Z_0 \times (O_3 - O_0) & Z_1 \times (O_3 - O_1) & Z_2 \times (O_3 - O_2) \\ Z_0 & Z_1 & Z_2 \end{bmatrix}$$

$$J = \begin{bmatrix} \begin{pmatrix} -l_1 \sin q_1 \\ -l_2 \sin q_2 \\ -l_3 \sin q_3 \end{pmatrix} & \begin{pmatrix} -l_2 \sin q_2 \\ -l_3 \sin q_3 \end{pmatrix} & (-l_3 \sin q_3) \\ \begin{pmatrix} l_1 \cos q_1 \\ +l_2 \cos q_2 \\ +l_3 \cos q_3 \end{pmatrix} & \begin{pmatrix} l_2 \cos q_2 \\ +l_3 \cos q_3 \end{pmatrix} & (+l_3 \cos q_3) \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

Q10] → ## Planetary Gearbox

- Highly versatile
- Can be used when high RPM with low torque is needed and also when low RPM with high torque is expected.

High Power Gearbox

- Used when output requirement is high torque with low RPM
- Used is trackter, tanks, etc
- Can be used when we have to lift a heavy weight

Universal Gearbox

- Have varies applicatn and can be mounted in different ways.
- It have 2 hexagonal shaft and 1 normal (circular) shaft.
- The 2 types of output shafts have different RPM and torque.

Speed Gearbox

- The best thing about this gearbox is that you can customize the gear ratio to some extend. (6 different types of gear ratio are available)
- According to your applicatn you have to select the gear ratio and then assemble it.

Question 7:
https://colab.research.google.com/drive/1rL1CrvBBgwkCUMBjzAp9TyRQznMEHWwn?usp=sharing

Question 8:
https://colab.research.google.com/drive/1LPfkkvNnfiVCUBaCbBTR4j4NNOU3OYYi?usp=sharing

Question 12:
https://colab.research.google.com/drive/1F6kL2mgkEu0Y33S_POSQfjexJxnnOjyZ?usp=sharing

Question 14:
https://colab.research.google.com/drive/1_w-suNruLuUTTlyIm_-fGCYkYECNKJsG?usp=sharing