Name :- Yash Meshram

Roll No :- 18110192

Course :- ME 639 : Introduction to Robotics

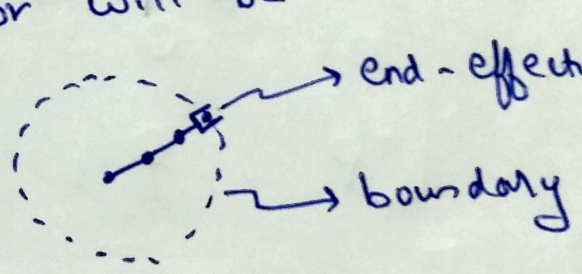
Assignment - 3

Q1] → <u>Singular Configuration</u> :- Configuration at which robot looses the ability to move in one or more direction.

Types of Singularity :

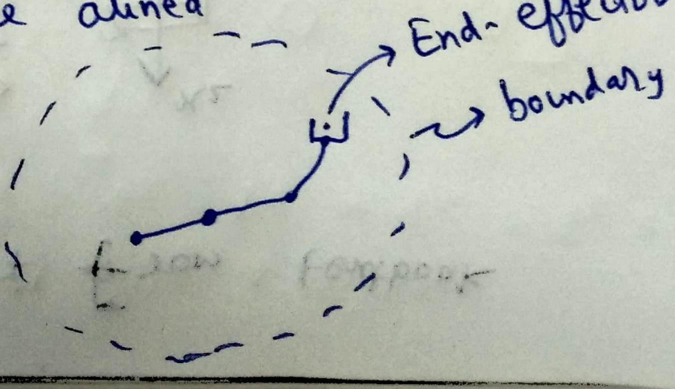
1) Workspace - boundary ~~so~~ singularities :-

- End effector will be at the boundary.



2) Workspace - interior Singularities :-

- End effector is not on the boundary but 2 or more ~~joint~~ links are alined



Condition for singularities to exist.

$$\det(J(\theta)) = 0$$

By solving the above eqn we can find out the angles θ_i at which singularities exist.

We ~~can~~ can programme our robot for not reaching to this θ_i values or closer to it. So, that we don't loss any DOF.

Suppose :- Particular configuration given is 'θ_A'

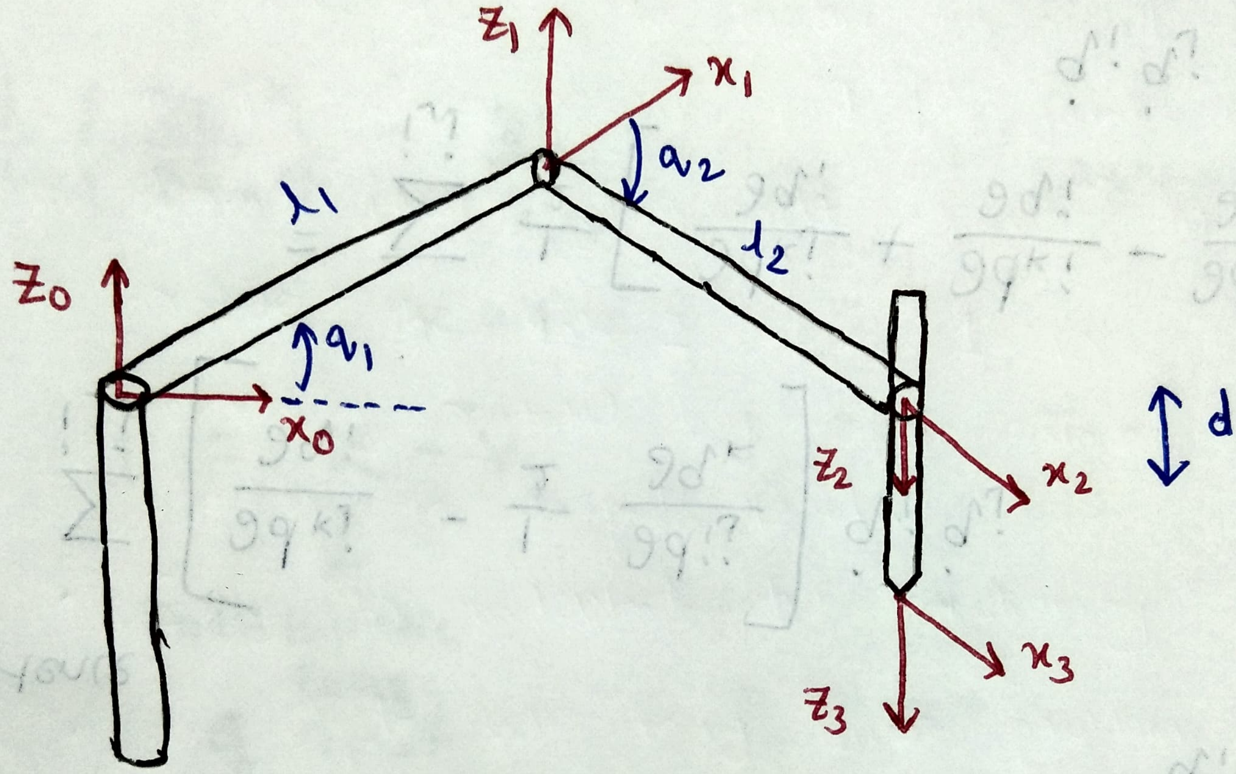
I will see the closest θ_i value to it.

~~I can~~ Since I will know the θ_i

I can also calculate how close θ_A & θ_i are.

Q4]

RRP - SCARA



DH parameters

link	a	α	d	θ
1	l_1	0	0	q_1
2	l_2	180	0	q_2
3	0	0	d	0

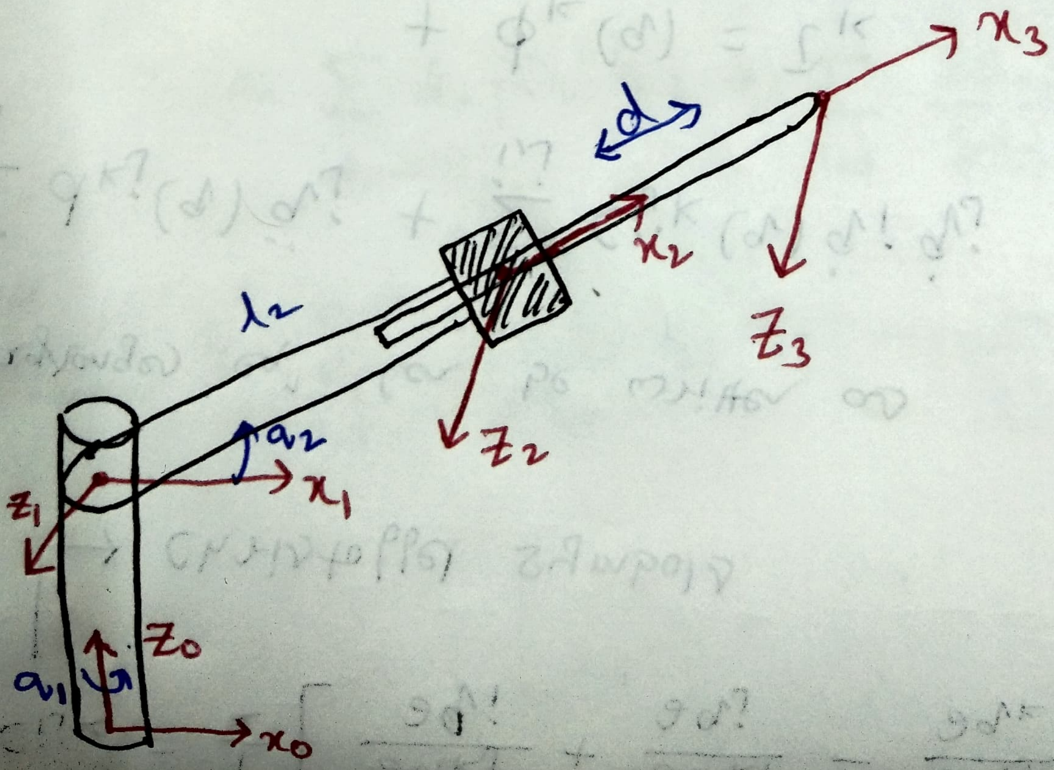
The position of the end-effector matches with the positn from the code of this Assignment & the code of previous Assignment & the textbook

For $l_1 = l_2 = 10$, $d = 5$

$q_1 = 30$, $q_2 = 45$

Position of End-effector $= \begin{bmatrix} 11.248 \\ 14.659 \\ -5 \end{bmatrix}$

RRP - Stend ford



DH Parameters

link	a	α	d	θ
1	0	90	l_1	q_1
2	l_1	0	0	q_2
3	d	0	0	0

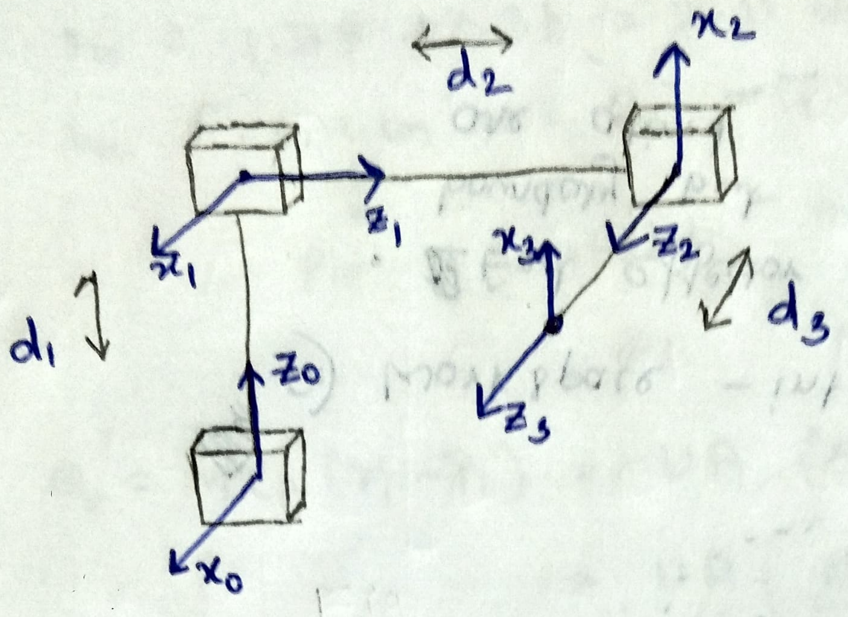
The position of the end-effector matches with the positn from the code of this Assignment & the code of previous Assignment & the textbook

For $l_1 = l_2 = 10$, $d = 5$

$q_1 = 30$, $q_2 = 45$

Positn of End-effector $= \begin{bmatrix} 9.185 \\ 5.303 \\ 20.606 \end{bmatrix}$

Q5] → Problem 3-7 from textbook



DH Parameters

link	a_i	α_i	d_i	θ_i
1	0	-90	d_1	0
2	0	-90	d_2	-90
3	0	0	d_3	0

$$A_i = \begin{bmatrix} C_{\theta_i} & -S_{\theta_i} C_{\alpha_i} & S_{\theta_i} S_{\alpha_i} & a_i C_{\theta_i} \\ S_{\theta_i} & C_{\theta_i} C_{\alpha_i} & -C_{\theta_i} S_{\alpha_i} & a_i S_{\theta_i} \\ 0 & S_{\alpha_i} & C_{\alpha_i} & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

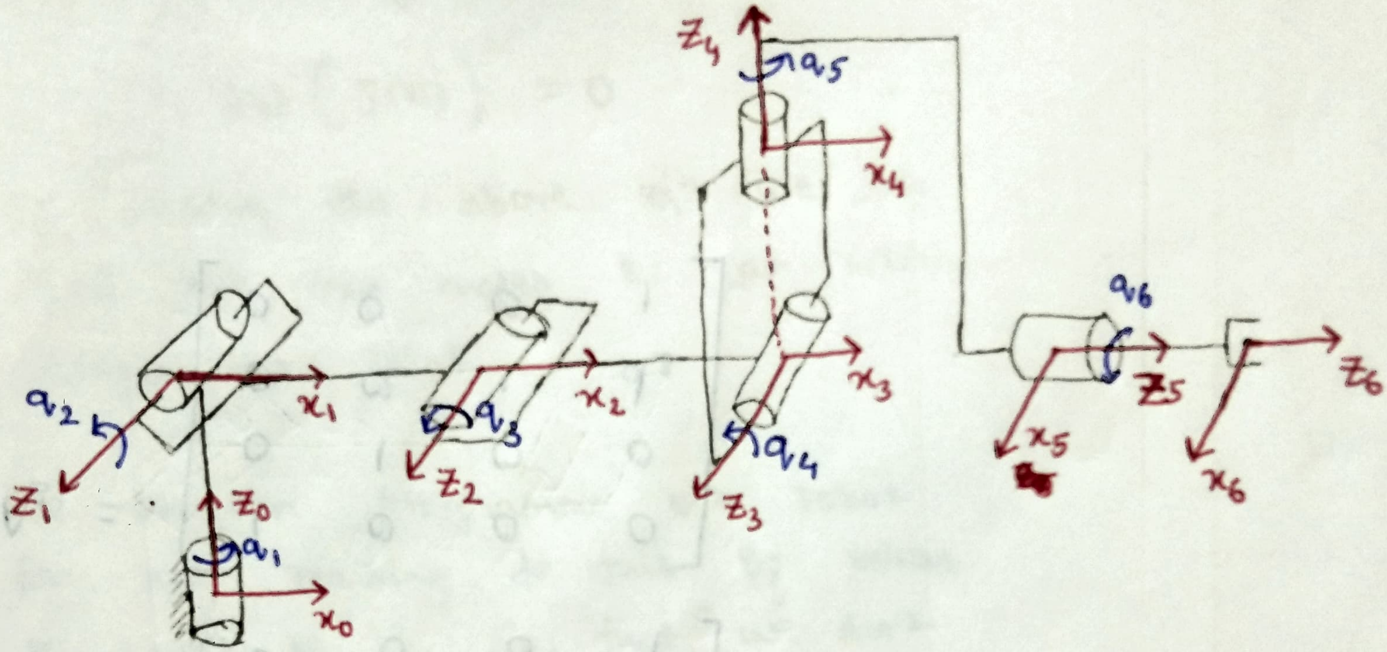
$$A_2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_1 A_2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d_2 \\ 1 & 0 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_1 A_2 A_3 = T = \begin{bmatrix} 0 & 0 & 1 & d_3 \\ 0 & -1 & 0 & d_2 \\ 1 & 0 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Q6 →



DH Parameters

link	a_i	α_i	d_i	θ_i
1	0	90	0	q_1
2	d_2	0	0	q_2
3	d_3	0	0	q_3
4	0	-90	0	q_4
5	0	-90	0	$-90+q_5$
6	0	0	d_6	q_6

Let consider the suitvation

$q_1 = q_2 = q_3 = q_4 = q_5 = q_6 = 0°$

$$A_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 1 & 0 & 0 & d_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 1 & 0 & 0 & d_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_5 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_6 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T = A_1 A_2 A_3 A_4 A_5 A_6$$

$$T = \begin{bmatrix} 0 & 0 & 1 & d_2+d_3+d_6 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Q7] → <u>Direct Drive 2-R Manipulator</u>

- In this type the motor is placed at each joint to control the motion
- Since motor is placed at each joint, the weight of the motor affects the motion of the link.
- If we want ~~pre~~ accurate parameters (positn, velocity, etc) then its recommend to use different type of drive or to use ~~high~~ very high quality of controller. By using high quality (in working/accuracy sense) controller we can overcome the disturbances cause by the weight of motor.
  But adding ~~high quality~~ controller that good may ~~add~~ cost a lot.

- Easy to understand the working and to change parameters like velocity,

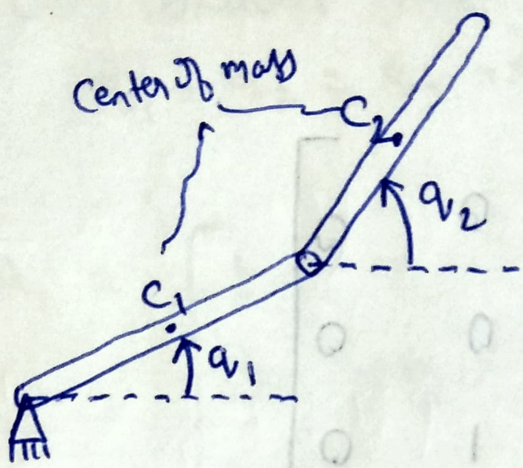
Remotely driven 2R Manipulator

- In this type both the motor is placed in the same place (at 1st joint or ground joint). The belt is use to rotate the link 2. Link 1 is directly rotated by motor, since it is in ground level.
- Easy to understand the functioning
- The belt use may cause problem sometimes. It can happen that the shaft just slip over the belt. This will give us inaccurate ~~valu~~ ~~outp~~ output.

5-bar parallelogram arrangement

- In this type 5-bar are used to work as an 2R manipulator
- Complex structure make it difficult to understand the working and to change the parameters like positn, velocity, etc
- Can have comparatively more number of singular configuration

Q8] →



Velocity of center of mass of link 1 & 2.

$$V_{c_1} = \begin{bmatrix} -\frac{l_1}{2}\sin q_1 \\ \frac{l_2}{2}\cos q_1 \\ 0 \end{bmatrix} \dot{q}_1$$

$$V_{c_2} = \begin{bmatrix} -l_1 \sin q_1 & -\frac{l_2}{2}\sin q_2 \\ l_1 \cos q_1 & \frac{l_2}{2}\cos q_2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}$$

$\omega_1 = \dot{q}_1 \hat{k}$, $\omega_2 = \dot{q}_2 \hat{k}$

No. of links $(n) = 2$

$$D(q)\,\ddot{q} + C(q,\dot{q})\,\dot{q} + g(q) = \tau$$

for 2-R manipulator

$$D(q) = \begin{bmatrix} m_1 \frac{l_1^2}{4} + m_2 l_1^2 + I_1 & m_2 \frac{l_1 l_2}{2} \cos(q_2 - q_1) \\ m_2 \frac{l_1 l_2}{2} \cos(q_2 - q_1) & m_2 \frac{l_2^2}{4} + I_2 \end{bmatrix}$$

$$= \begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix}$$

Computing the christoffel symbols

$$C_{ijk} = \frac{1}{2} \left[\frac{\partial d_{kj}}{\partial q_i} + \frac{\partial d_{ki}}{\partial q_j} - \frac{\partial d_{ij}}{\partial q_k} \right]$$

$$C_{221} = \frac{1}{2} \left[\frac{\partial d_{12}}{\partial q_2} + \frac{\partial d_{12}}{\partial q_2} - \frac{\partial d_{22}}{\partial q_1} \right]$$

$$C_{221} = \frac{1}{\cancel{2}}\left[-\cancel{2} \times m_2 \frac{l_1 l_2}{2} \sin(q_2 - q_1) - 0\right]$$

$$C_{221} = - m_2 \frac{l_1 l_2}{2} \sin(q_2 - q_1)$$

$$C_{112} = \frac{1}{2}\left[\frac{\partial d_{21}}{\partial q_1} + \frac{\partial d_{21}}{\partial q_1} - \frac{\partial d_{11}}{\partial q_2}\right]$$

$$= \frac{1}{\cancel{2}}\left[-\cancel{2} \times \frac{m_2 l_1 l_2}{2} \sin(q_2 - q_1) - 0\right]$$

$$C_{112} = - m_2 \frac{l_1 l_2}{2} \sin(q_2 - q_1)$$

$$V = m_1 g \frac{l_1}{2} \sin q_1 + m_2 g \left(l_1 \sin q_1 + \frac{l_2}{2} \sin q_2\right)$$

$$\phi_1 = \frac{\partial V}{\partial q_1} = m_1 g \frac{l_1}{2} \cos q_1 + m_2 g \left(l_1 \cos q_1\right)$$

$$\phi_2 = \frac{\partial V}{\partial q_2} = m_2 g \frac{l_2}{2} \cos q_2$$

Final eqⁿs are

$$d_{11}\ddot{q}_1 + d_{12}\ddot{q}_2 + C_{221}\dot{q}_2^2 + \phi_1 = \tau_1 \quad \text{— (1)}$$

$$d_{21}\ddot{q}_1 + d_{22}\ddot{q}_2 + C_{112}\dot{q}_1^2 + \phi_2 = \tau_2 \quad \text{— (2)}$$

$$d_{11} = \frac{m_1 l_1^2}{4} + m_2 l_1^2 + \frac{1}{12} m_1 l_1^2 = \frac{1}{3} m_1 l_1^2 + m_2 l_1^2$$

$$d_{22} = m_2 \frac{l_2^2}{4} + \frac{1}{12} m_2 l_2^2 = \frac{1}{3} m_2 l_2^2$$

eqⁿ (1)

$$\left(\frac{1}{3} m_1 l_1^2 + m_2 l_1^2\right)\ddot{q}_1$$

$$+ \left(m_2 \frac{l_1 l_2}{2} \cos(q_2 - q_1)\right)\ddot{q}_2$$

$$- \left(m_2 \frac{l_1 l_2}{2} \sin(q_2 - q_1)\right)\dot{q}_2^2$$

$$+ m_1 g \frac{l_1}{2} \cos q_1 + m_2 g l_1 \cos q_1 = \tau_1.$$

eq^n ②

$$\left(m_2 \frac{l_1 l_2}{2}\cos(q_2-q_1)\right)\ddot{q}_1$$

$$+\left(m_2 \frac{l_2^2}{3}\right)\ddot{q}_2$$

$$-\left(m_2 \frac{l_1 l_2}{2}\sin(q_2-q_1)\right)\dot{q}_1^2$$

$$+\; m_2 g \frac{l_2}{2}\cos q_2 = \tau_2$$

Comparing this eq^n, with the eq^{n}s from min- project

I found that there is one extra term in the eq^n of min-project for eq^n ① & 2 extra term in eq^n of min-project for eq^n ②

$m_2 \frac{l_1 l_2}{2} \dot{q}_1 \dot{q}_2 \Rightarrow$ Extra term in eq^n 1 from mini project

$m_2 \frac{l_2^2}{4} \ddot{q}_2 - m_2 \frac{l_1 l_2}{2} \dot{q}_1 \dot{q}_2 \Rightarrow$ Extra terms in eq^n 2 from mini project

Q10) → Let 'n' be the number of links

$$D(q) = \begin{bmatrix} d_{11} & d_{12} & d_{13} & \cdots & d_{1n} \\ d_{21} & d_{22} & d_{23} & \cdots & d_{2n} \\ d_{31} & d_{32} & d_{33} & \cdots & d_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ d_{n1} & d_{n2} & d_{n3} & \cdots & d_{nn} \end{bmatrix}$$

$$C(q, \dot{q}) = [c_{kj}]_{k \times j}$$

$$k, j = 1, 2, 3, \ldots, n$$

$$c_{kj} = \sum_{i=1}^{n} c_{ijk}(q)\,\dot{q}$$

$$= \sum_{i=1}^{n} \frac{1}{2}\left[\frac{\partial d_{kj}}{\partial q_i} + \frac{\partial d_{ki}}{\partial q_i} - \frac{\partial d_{ij}}{\partial q_k}\right]\dot{q}$$

If you know $D(q)$ then you can find d_{kj}, d_{ki} and d_{ij} & ~~thus~~ can ~~~~ calculate c_{kj} which ultimately lets you to $C(q,\dot{q})$

$V(q)$ ---(known)

$\phi_k = \frac{\partial V}{\partial q_k}$ ---- (This can be easily calculated if $V(q)$ is known)

Derivatn

$q_i \quad (i = 1, 2, \ldots, n)$ ---- (joint variable for revolute)

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_k}\right) - \frac{\partial L}{\partial q_k} = \tau_k \quad \text{---} \text{(Lagranges eq}^n\text{)}$$

$k = 1, 2, \ldots, n$

$L = K - V$

K: Total kinetic energy of robot

V: Total Potential energy of robot

In general for rigid body,

$$K = \frac{1}{2} m V_c^T V_c + \frac{1}{2} \omega^T I \omega$$

V_c : velocity of center of mass

I : moment of inertia @ center of mass

Assume - all n links are rigid

$$K = \sum_{i=1}^{n} \left[\frac{1}{2} m v_c^T v_c + \frac{1}{2} \omega^T I \omega \right]$$

$$K = \frac{1}{2} \sum_{i,j}^{n} d_{ij}(q)\, \dot{q}_i \dot{q}_j$$

$$= \frac{1}{2} \dot{q}^T D \dot{q}$$

$$L = K - V = \frac{1}{2} \sum_{i,j}^{n} d_{ij}(q)\, \dot{q}_i \dot{q}_j - V(q)$$

we have,

$$\frac{\partial L}{\partial \dot{q}_k} = \sum_j d_{kj}(q)\, \dot{q}_j$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_k}\right) = \sum_j d_{kj}(q)\, \ddot{q}_j + \sum_j \frac{d}{dt}\left(d_{kj}(q)\right) \dot{q}_j$$

$$= \sum_j d_{kj}(q)\, \ddot{q}_j + \sum_{i,j} \frac{\partial d_{kj}}{\partial q_i} \dot{q}_i \dot{q}_j$$

$$\frac{\partial d}{\partial q_k} = \frac{1}{2} \sum_{i,j} \frac{\partial d_{ij}}{\partial q_k} \dot{q}_i \dot{q}_j - \frac{\partial V}{\partial q_k}$$

Thus Lagranges eq^n becomes

$$\sum_{j} d_{kj}\ddot{q}_j + \sum_{i,j}\left[\frac{\partial d_{kj}}{\partial q_i} - \frac{1}{2}\frac{\partial d_{ij}}{\partial q_k}\right]\dot{q}_i\dot{q}_j$$

$$+ \frac{\partial V}{\partial q_k} = \tau_k$$

$(k = 1, 2, \ldots, n)$

by Symmetry

$$\sum_{i,j}\left(\frac{\partial d_{kj}}{\partial q_i}\right)\dot{q}_i\dot{q}_j = \frac{1}{2}\sum_{i,j}\left[\frac{\partial d_{kj}}{\partial q_i} + \frac{\partial d_{ki}}{\partial q_j}\right]\dot{q}_i\dot{q}_j$$

Hence,

$$\sum_{i,j}\left[\frac{\partial d_{kj}}{\partial q_i} - \frac{1}{2}\frac{\partial d_{ij}}{\partial q_k}\right]\dot{q}_i\dot{q}_j$$

$$= \sum_{i,j}\frac{1}{2}\left[\frac{\partial d_{kj}}{\partial q_i} + \frac{\partial d_{ki}}{\partial q_j} - \frac{\partial d_{ij}}{\partial q_k}\right]\dot{q}_i\dot{q}_j$$

Lets define

$$c_{ijk} = \frac{1}{2}\left[\frac{\partial d_{kj}}{\partial q_i} + \frac{\partial d_{ki}}{\partial q_j} - \frac{\partial d_{ij}}{\partial q_k}\right]$$

$\hookrightarrow$ Christoffel symbols

Lagranges eq^n can be written as

$$\sum_j d_{kj}(q)\,\ddot{q}_j + \sum_{i,j} c_{ijk}(q)\,\dot{q}_i\,\dot{q}_j + \phi_k(q) = \tau_k$$

$$(k = 1, 2, \ldots, n)$$

$$\boxed{D(q)\,\ddot{q} + C(q,\dot{q})\,\dot{q} + g(q) = \tau}$$

where,
$$g(q) = \begin{bmatrix} \phi_1(q) \\ \phi_2(q) \\ \vdots \\ \phi_n(q) \end{bmatrix}, \quad \tau = \begin{bmatrix} \tau_1 \\ \tau_2 \\ \vdots \\ \tau_n \end{bmatrix}$$

$D(q)$ & $C(q,\dot{q})$ has been discus earlier.

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Question 3, 4, 5, 6:
https://colab.research.google.com/drive/1dDsw6iEWGr_SDsUYh53GNTGoEGyO3-ZQ

Question 11
https://colab.research.google.com/drive/1EvAM1h4yrEMfKtdsNnk6UphKg-QZkilJ