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Loss function in deep learning.

- Start with loss function?

- Loss function is a model of evaluating how your algorithm is modelling your dataset.
- high value ⇒ Algorithm performance is poor
  low value ⇒ Algorithm performing great.
- Loss function is just mathematical function.
- Loss function is function of parameter in machin learning.
  e.g. for linear regression ⇒ L(m, b)
- How we change the loss function value?
  - By changing parameter value.

Why is loss function important?

- [You can't improve what you can't measure]
  - Peter Drucker
- This line by Peter Drucker perfectly explain importance of loss function in machine learning/ deep learning.

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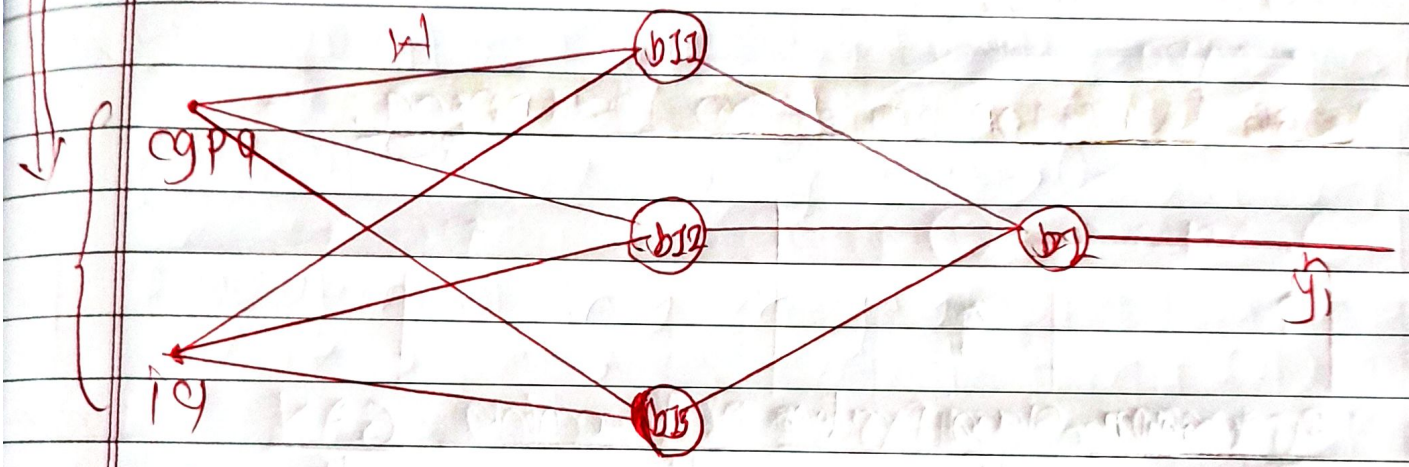
2) What is role of loss function in deep learning?

cgpa	iq	package(lpa)
7.1	83	3.2
8.5	91	4.5
6.3	102	6.1
5.1	87	2.7



known,

- To role we will discuss backpropgation for now,
- We put first student 'cgpa and iq' here and we will put this in neural network by using forward propogation it will predict for first student.
- Let's say for first student it comming out to be 3.7.
- Since this is regression problem, we will

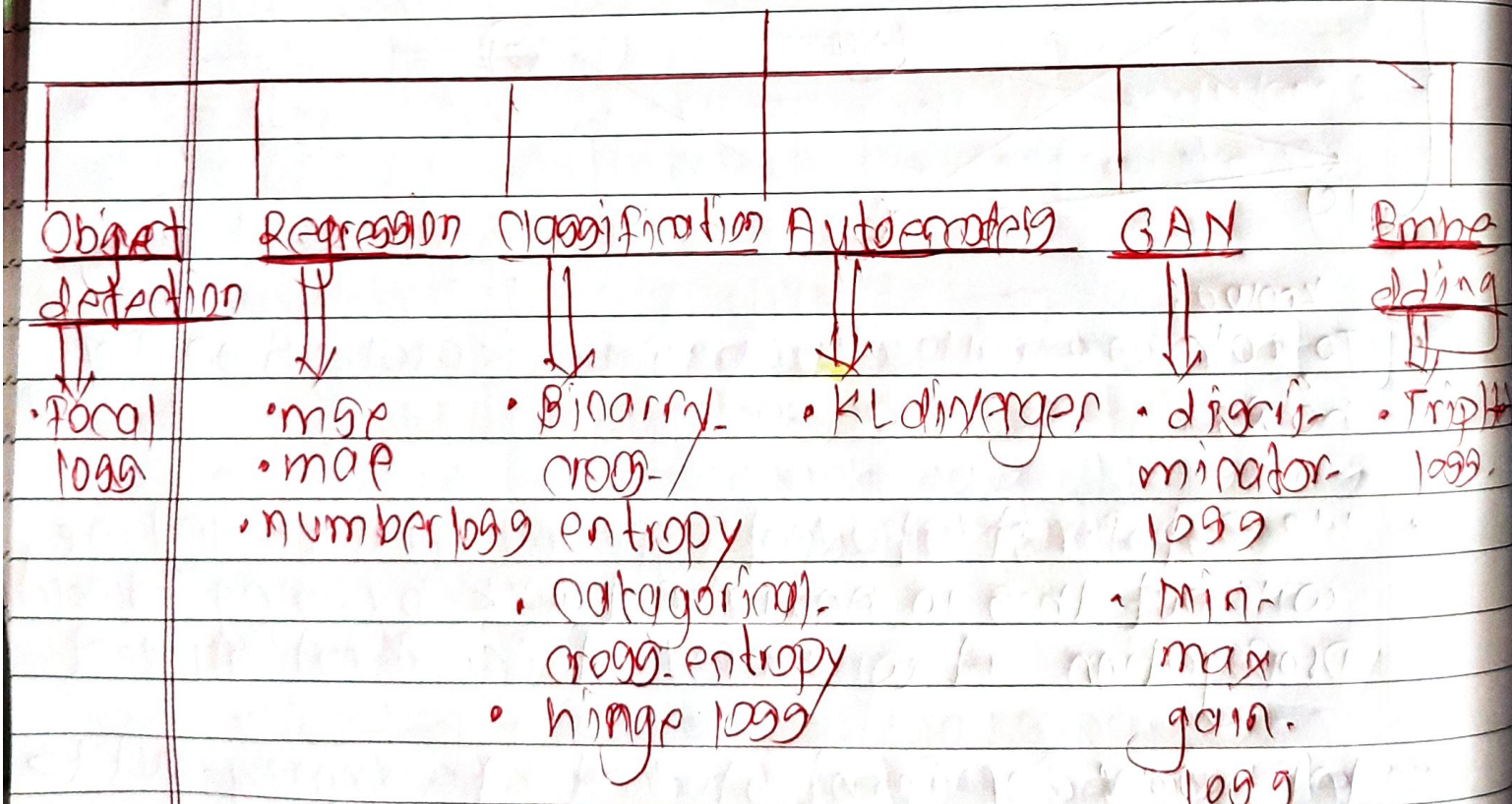
use, mean squared error.

$(3.2 - 3.7)^2 = (0.5)^2 = 0.25$

- Apply gradient descent on it and then update weight and biases
- We will do this for every point, ~~data~~ this is backpropogation.

Idea of updating weight and biases is to reduce loss.

Loss function in deep learning.

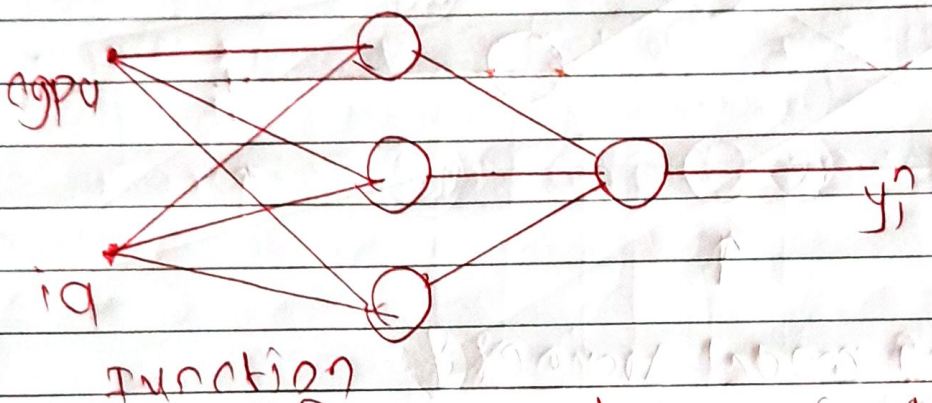


- We can create our custom loss function too, Keras allow to do that.

Loss function and cost function difference

- This is our data set,

cgpa	iq	package	Prediction
6.3	100	6.3	6.1
7.1	91	4.1	4
8.5	89	3.5	3.7
9.2	102	7.2	7



Function

- Loss ~~(student)~~ calculate on single training example

⇒ For first row loss function would be,

$$(y_i - \hat{y_i})^2 = (6.3 - 6.1)^2 \Rightarrow \text{loss function}$$

- When we calculate for single row, this is known as loss function

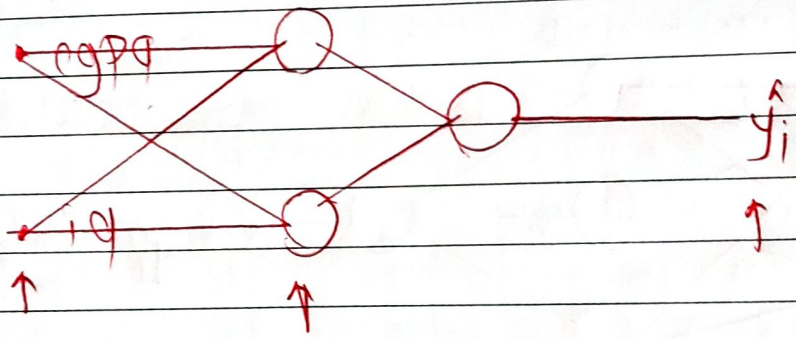
- When we calculate on batch it will become cost function

- **Cost function**

$$\frac{1}{n}\sum(y_i - \hat{y}_i)^2$$

Mean squared error

Also known as, squared Loss/ L2 loss, Let's study in context of deep learning.



- **How this model trained?**
  - We insert rows one by one and we get $\hat{y}_i$ (forward propagation)
  - After that we check how off is this prediction, By using loss function

$$(y_i - \hat{y}_i)^2$$

  - Based on this loss function we update weights and biases
  - We will do this for every point, Idea is to minimize value of loss function.

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- We will discuss advantages of MSE also disadvantages of it to know which dataset it could apply on.

- ~~Why just $(y_i - \hat{y_i})$ instead of $(y_i - \hat{y_i})^2$~~

  - Because of squaring negative values get down
  - Total error reduced because of minus value, we want all error to be added not subtract.

- After squaring, $(y_i - \hat{y_i})^2$ becomes quadratic,

  $(\text{true}(y_i) - \text{predictive}(\hat{y_i})) \Rightarrow 1$ unit

  After squaring → 1 unit.

  2 unit → After squaring → 4 unit

  3 unit → After squaring → 9 unit

- Those points making more error/which are very far from actual line leads to change more weight and bias drastically.

  ~~Wrongly classified~~ / ~~$(y_i - \hat{y_i})^2$~~ too much } ⇒ makes more changes in W, b

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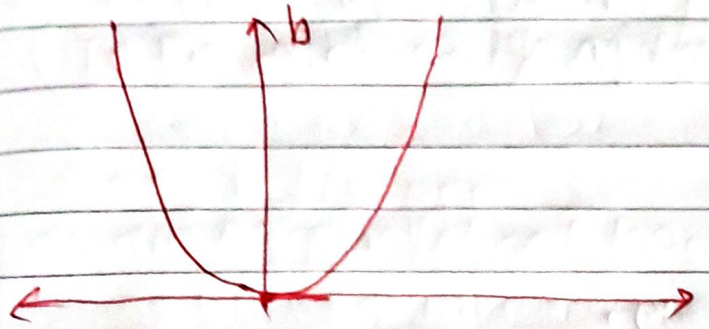
Advantages

- Easy to interpreate
  Because of quadratic nature this function is always differentiable.



- There is only 'I' local minima.

Disadvantages

- Error unit is square.
- Not Robust to outlier.

Condition to apply MSE in deep learning.

- Last neuron activation function always should be 'linear'.

Mean Absolute error (MAE)

- Also known as 'L1 loss'
- $L = |y_i - \hat{y}_i| \implies$ Loss function
- $L = 1/n \Sigma |y_i - \hat{y}_i| \implies$ cost function.

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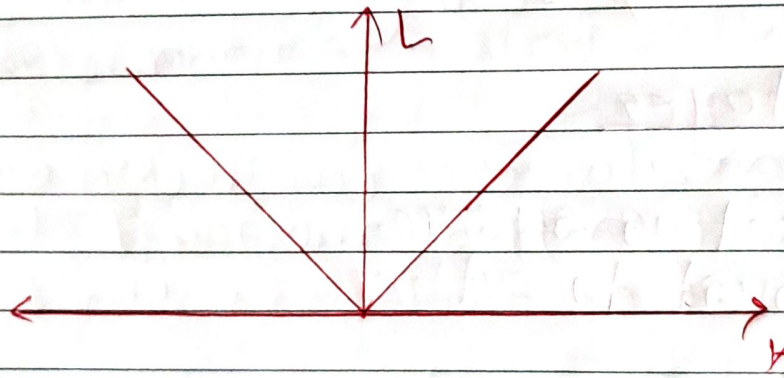
Advantages

- Easy to understand.
- Unit same as unit of y.
- Robust to outlier (so error cannot increase / punish, b drastically as in quadratic loss function i.e. MSE).

Disadvantages

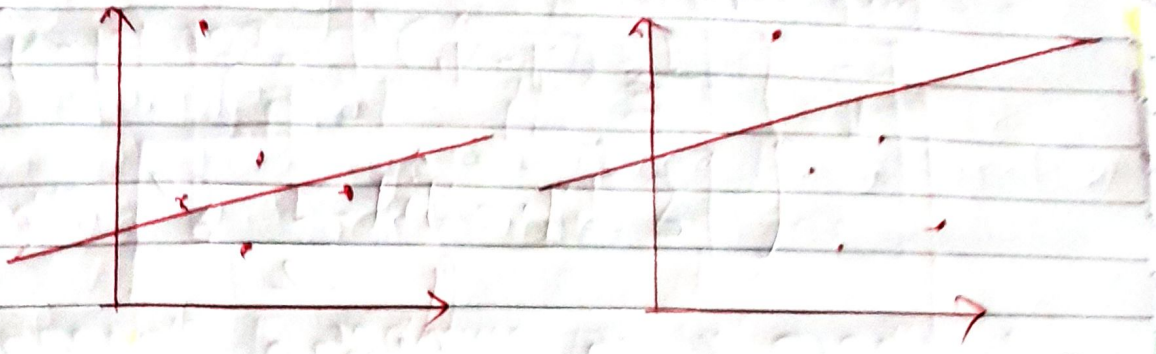
- Graph of MAE is,



- This graph is not differentiable, so we cannot apply gradient directly we have to calculate sub-gradient first.
- Calculating sub-gradient is computationally expensive.

Huber loss

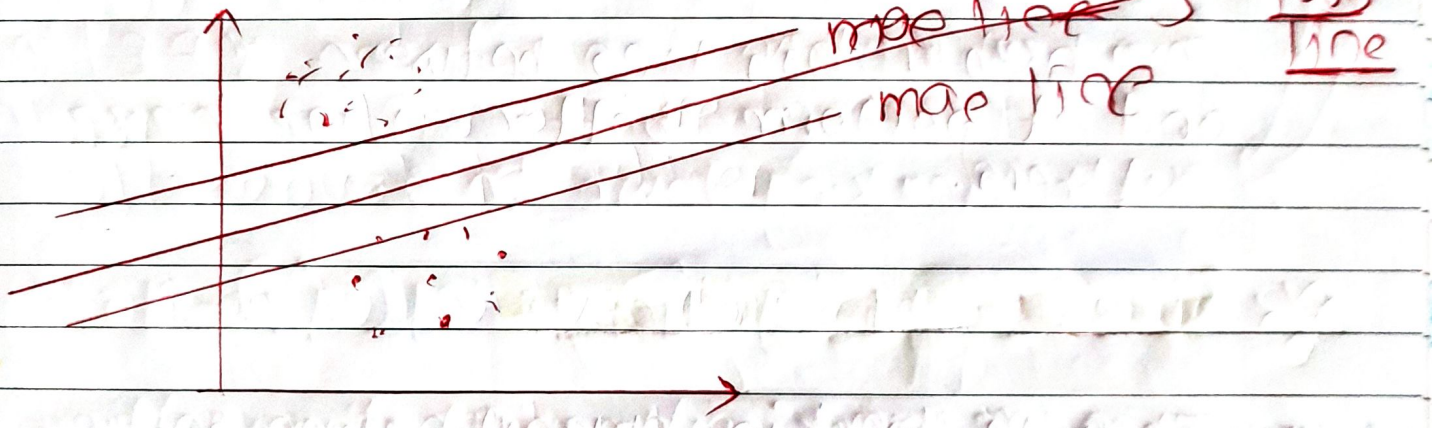
- mse cause drastic change in weights & biases and mae cause less changes in weight & biases



mae to outlier

mse to outlier

- Lets we have this kind of data that it have 25% of outlier (so that 25% is not a outlier firstely).



- In this case we want middle line which is we get after updating weight and biases by using Huber loss as Loss function.

- If outlier not present ⟹ behave like MSE.
  If outlier present ⟹ behave like MAE.

- mathamatically above is,

$$L = \begin{cases} \frac{1}{2}(y_i - \hat{y}_i)^2 & \text{for } |y - \hat{y}_i| \le \delta \\ \delta |y_i - \hat{y}_i| - \frac{1}{2}\delta^2 & \text{otherwise} \end{cases}$$

(first branch → mse; second branch → mae)

- 'δ' parameter used to determine which point is outlier or not.
  'δ' by adjusting this parameter we can check performance.
- This huber loss lies between MAE / MSE and it perform better if we have more outliers.

Binary Cross Entropy

- This is used in classification setting
- It will be only used when you have only two classes.
- Loss function $\Rightarrow -y \log(\hat{y}) - (1-y)\log(1-\hat{y})$

  $\hat{y} \Rightarrow$ neural net architecture prediction
  $y \Rightarrow$ Actual value / taget.
- Activation function for last neuron is sigmoid

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cost function ⟹ ~~=y log~~

$$-\frac{1}{n}\left[\sum_{i=1} y_i \log \hat{y}_i + (1-y_i)\log(1-\hat{y}_i)\right]$$

- After calculating loss we updates weight and biases by using gradient descent.

Advantages

- This loss function is differentiable (we can apply gradient descent).

Disadvantages

- Multiple minima could present.
- This is not intuitive.
- Some people called it log-loss too.

Categorical cross Entropy

- Used in classification scenarios, only when you have multi class classification problem.
- ~~This our set up,~~ This same loss function used in softmax regression too.
- To calculate above for single student, this is

Our formula,

$$L = -\sum_{j=1}^{k} y_j \log(\hat{y}_j)$$

- Where 'k' is number of classes in data
- This is our set up,

capital	city	placed
8	80	yes
6	60	no
7	70	maybe

(Classes after one-hot encoding)

yes	no	maybe
1	0	0
0	1	0
0	0	1

- After breaking down this,

$$L = -y_1 \log(\hat{y}_1) - y_2 \log(\hat{y}_2) - y_3 \log(\hat{y}_3)$$

- When we work with multi class classification some changes occurs, in output layers we use no. of neurons = no. of class.
- Activation function for output layer is soft max, this is formula.

$$f(z_1) = \frac{e^{z_1}}{e^{z_1} + e^{z_2} + e^{z_3}}$$

$z_1 \Longrightarrow$ yes
$z_2 \Longrightarrow$ no
$z_3 \Longrightarrow$ maybe

- $f(z_1) + f(z_2) + f(z_3) = 1$

- This is loss function for single row,

$$L = -y_1 \log(\hat{y_1}) - y_2 \log(\hat{y_2}) - y_3 \log(\hat{y_3})$$

- For first row, all neuron's which basically three neuron output is,

$$[\underset{\downarrow}{0.2} \quad \underset{\downarrow}{0.3} \quad \underset{\downarrow}{0.5}]$$
$$\text{yes} \quad \text{no} \quad \text{maybe}$$

$$[\underset{\downarrow}{1} \quad \underset{\downarrow}{0} \quad \underset{\downarrow}{0}] \Longrightarrow \text{true lable}$$
$$y_1 \quad y_2 \quad y_3$$

- We apply this loss function,

$$L = -y_1 \log(\hat{y_1}) - 0 - 0$$

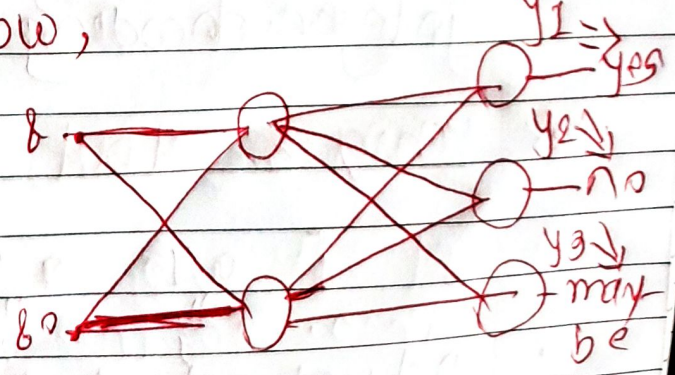
$$L = -1 \log(0.2) \Longrightarrow$$ We update's weight & biases accordingly.

- This is loss for first row.

- Similarly for second row,

prediction $[0.3 \quad 0.6 \quad 0.1]$

$$[\underset{\downarrow}{0} \quad \underset{\downarrow}{1} \quad \underset{\downarrow}{0}]$$
$$y_1 \quad y_2 \quad y_3$$



- Loss becomes,

$$L = -y_2 \log(\hat{y}_2)$$
$$= -1 \log(0.6)$$

- similarly for third point/row,

$$L = -y_3 \log(\hat{y}_3)$$

After that we update weight biases accordingly

Sparse categorical cross Entropy

cgpa	iq	placed		After integer encoding
7	70	yes	⇒	1
8	80	no	⇒	2
6	60	maybe	⇒	3

- If we use categorical cross entropy we use one-hot encoding
- For sparse categorical cross entropy we use integer encoding
- This is output three neurons for first row,

$$[\underset{\downarrow}{0.1} \quad \underset{\downarrow}{0.4} \quad \underset{\downarrow}{0.5}]$$

- Actual output for first row is (yes = 1)

- so loss becomes,

Forgot about them.

$$L_1 = -y_1 \log(\hat{y}_1)$$
$$= -1 \log(0.2) \Rightarrow$$

$-y_2 \log(\hat{y}_2)$ & $-y_3 \log(\hat{y}_3)$

After this weights are updated accordingly.

- Similarly for second and third row,

$$L_2 = -y_2 \log(\hat{y}_2) = -1 \log(\hat{y}_2)$$
$$L_3 = -y_3 \log(\hat{y}_3) = -1 \log(\hat{y}_3)$$

- If we have too much classes we should use sparse categorical entropy, since other calculation not needed and we don't have to one-hot encode it.

$-y_3 \log(\hat{y}_3)$ & $y_2 \log(\hat{y}_2)$

$= -y_2 \log(\hat{y}_2)$ & $y_1 \log \hat{y}_1)$

} Forgot about them.

- Cost function,

$$C = \frac{-1}{n} \sum_{i=1}^{n} \sum_{j=1}^{k} y_{ij} \log(\hat{y}_{ij})$$

Summary

- Regression

- use mae
- use mae if outlier present. } for combinat outlier and normal use number loss.

- **classification**
  - Binary classification ⟹ Binary cross entropy.
  - Multi-class classification ⟹ categorical cross entropy. (less category).
    - If we have more category/classes then use sparse categorical cross entropy.