School of Engineering and Applied Science (SEAS), Ahmedabad University

Probability and Stochastic Processes (MAT 277)

Tutorial 3

March 14, 2024

Practice Problems

1. Let X, Y have joint PDF

$$f(x,y) = \begin{cases} ce^{-x}e^{-2y}, & 0 < x < \infty, 0 < y < \infty \\ 0, & \text{otherwise} \end{cases}.$$

   (a) Find c that makes this a valid PDF;
   (b) Find $\mathbb{P}(X < Y)$;
   (c) Set up the double integral for $\mathbb{P}(X > 1, Y < 1)$;
   (d) Find the marginal $f_X(x)$.

2. **C**onsider X and Y given by the joint density

$$f(x,y) = \begin{cases} 10x^2y, & 0 \le y \le x \le 1 \\ 0, & \text{otherwise} \end{cases}.$$

   (A) Find the marginal PDFs, $f_X(x)$ and $f_Y(y)$.
   (B) Are X and Y independent random variables?
   (C) Find $\mathbb{P}(Y \le \frac{1}{2})$.
   (D) Find $\mathbb{P}\left(\frac{Y}{4} \le \frac{X}{2}\right)$.
   (E) Find $\mathbb{E}[X]$.

3. Suppose the joint density function of X and Y is $f(x,y) = \frac{1}{4}$ for $0 < x < 2$ and $0 < y < 2$.
   (A) Find $\mathbb{P}\left(\frac{1}{2} < X < 1, \frac{2}{3} < Y < \frac{4}{3}\right)$.
   (B) Find $\mathbb{P}(XY < 2)$.
   (C) Find the marginal distributions $f_X(x)$ and $f_Y(y)$.

4. The joint probability density function of X and Y is given by

$$f(x,y) = e^{-(x+y)}, 0 \leqslant x < \infty, 0 \leqslant y < \infty.$$

   Find $\mathbb{P}(X < Y)$.

5. Let X_1 and X_2 be continuous random variables with joint density function

$$f(x_1, x_2) = \begin{cases} cx_1x_2 & \text{for } 0 < x_1 < x_2 < 1 \\ 0 & \text{otherwise} \end{cases}$$

(A) Find c.
(B) Find $P(X_1 + X_2 < 1)$.
(C) Find marginal probability density function of X_1 and X_2.

6. Let's say we have two independent random Poisson variables for requests received at a web server in a day: $X = \#$ requests from humans/day, $X \sim \text{Poi}(\lambda_1)$ and $Y = \#$ requests from bots/day, $Y \sim \text{Poi}(\lambda_2)$. Since the convolution of Poisson random variables is also a Poisson we know that the total number of requests $(X + Y)$ is also a Poisson $(X + Y) \sim \text{Poi}(\lambda_1 + \lambda_2)$. What is the probability of having k human requests on a particular day given that there were n total requests?