Unmet Breakdown Models

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Contents

Note: Adding sub-region to hierarchy?
Note: Check survey/time variance
Note: fixed intercept model? talk benefits
Note: ratio is flat even if we stratify

Note: bayse change point model, using indicator?

1 Notation

notation in rscript is not up to date

satisfied contraceptive limiting = cl

satisfied contraceptive spacing = cs

unmet limiting = ul

unmet spacing = us

unmet = ul + us

total limiting = cl + ul

total space = cs + us

total demand = cl + cs + ul + us

limiting out of demand(PDL, formerly luup) = $\frac{cl+ul}{cl+cs+ul+us} = \frac{totalLimiting}{totalDemand}$

limiting out of Unmet (LUP, formerly lup) = $\frac{ul}{ul+us} = \frac{unmetLimiting}{unmet}$

ratio of satisfied needs (ratio, formerly ratio) = $\frac{\frac{cl}{cl+ul}}{\frac{cs}{cs+us}} = \frac{\frac{satisfiedLimiting}{totalLimiting}}{\frac{satisfiedSpacing}{totalSpacing}}$

2 PDL Varying Intercept Models And Slope(without shrinkage on slope

2.1 No AR

$y_i|\mu_{ct}, \sigma_y \sim N(\mu_{ct}, \sigma_y)$
$logit(\mu_{ct}) = \pi_{ct}$
$\pi_{ct} = \alpha_c + \beta * demand_{ct}$
$\alpha_c|\alpha_r, \sigma_{c\alpha} \sim N(\alpha_r, \sigma_{c\alpha})$
$\alpha_r|\alpha_w, \sigma_{r\alpha} \sim N(\alpha_w, \sigma_{r\alpha})$
$\alpha_w \sim N(0, 100)$

$\sigma_y \sim U(0, 2)$
$\sigma_{c\alpha} \sim U(0, 2)$
$\sigma_{r\alpha} \sim U(0, 2)$
$\beta \sim N(0, 100)$

2.2 AR

$y_i|\mu_{ct}, \sigma_y \sim N(\mu_{ct}, \sigma_y)$
$logit(\mu_{ct}) = \pi_{ct} + \delta_{ct}$
$\pi_{ct} = \alpha_c + \beta * demand_{ct}$
$\alpha_c|\alpha_r, \sigma_{c\alpha} \sim N(\alpha_r, \sigma_{c\alpha})$
$\alpha_r|\alpha_w, \sigma_{r\alpha} \sim N(\alpha_w, \sigma_{r\alpha})$
$\alpha_w \sim N(0, 100)$

$\sigma_y \sim U(0, 2)$
$\sigma_{c\alpha} \sim U(0, 2)$
$\sigma_{r\alpha} \sim U(0, 2)$
$\beta \sim N(0, 100)$

$\delta_{ct} = \rho * \delta_{c,t-1} + \epsilon_{c,t}$
$\epsilon_{c,t} \sim N(0, \sigma_\delta^2)$

3 PDL Varying Intercept and Slope Models

3.1 Varying Country Slope

$y_i|\mu_{ct}, \sigma_y \sim N(\mu_{ct}, \sigma_y)$
$logit(\mu_{ct}) = \pi_{ct} + \delta_{ct}$
$\pi_{ct} = \alpha_c + \beta_c * demand_{ct}$

$$\begin{pmatrix} \alpha_c \\ \beta_c \end{pmatrix} \sim N_2 \left(\begin{pmatrix} \alpha_{r1} \\ \alpha_{r2} \end{pmatrix} \begin{pmatrix} \sigma_{c\alpha}^2 & \rho_{\alpha\beta} * \sigma_{c\alpha} * \sigma_{c\beta} \\ \rho_{\alpha\beta} * \sigma_{c\alpha} * \sigma_{c\beta} & \sigma_{c\beta}^2 \end{pmatrix} \right)$$

$\alpha_{r1}|\alpha_{w1}, \sigma_{r1} \sim N(\alpha_{w1}, \sigma_{r1})$
$\alpha_{r2}|\alpha_{w2}, \sigma_{r2} \sim N(\alpha_{w2}, \sigma_{r2})$
$\alpha_{w1} \sim N(0, 100)$
$\alpha_{w2} \sim N(0, 100)$

$\sigma_y \sim U(0, 2)$
$\sigma_{c\alpha} \sim U(0, 3)$
$\sigma_{c\beta} \sim U(0, 3)$
$\rho_{\alpha\beta} \sim U(-.95, 1)$
$\sigma_{r1} \sim U(0, 3)$
$\sigma_{r2} \sim U(0, 3)$

$\delta_{ct} = \rho * \delta_{c,t-1} + \epsilon_{c,t}$
$\epsilon_{c,t} \sim N(0, \sigma_\delta^2)$

3.2 Varying Region Slope

$y_i|\mu_{ct},\sigma_y \sim N(\mu_{ct},\sigma_y)$
$logit(\mu_{ct}) = \pi_{ct} + \delta_{ct}$
$\pi_{ct} = \alpha_c + \beta_r * demand_{ct}$
$\alpha_c|\alpha_r,\sigma_{c\alpha} \sim N(\alpha_r,\sigma_{c\alpha})$

$$\begin{pmatrix}\alpha_r\\ \beta_r\end{pmatrix} \sim N_2\left(\begin{pmatrix}\alpha_{w1}\\ \alpha_{w2}\end{pmatrix}\begin{pmatrix}\sigma^2_{r\alpha} & \rho_{\alpha\beta} * \sigma_{r\alpha} * \sigma_{r\beta}\\ \rho_{\alpha\beta} * \sigma_{r\alpha} * \sigma_{r\beta} & \sigma^2_{r\beta}\end{pmatrix}\right)$$

$\alpha_{w1} \sim N(0,100)$
$\alpha_{w2} \sim N(0,100)$

$\sigma_y \sim U(0,2)$
$\sigma_{c\alpha} \sim U(0,3)$
$\sigma_{r\alpha} \sim U(0,3)$
$\sigma_{r\beta} \sim U(0,3)$
$\rho_{\alpha\beta} \sim U(-.95,1)$

$\delta_{ct} = \rho * \delta_{c,t-1} + \epsilon_{c,t}$
$\epsilon_{c,t} \sim N(0,\sigma^2_\delta)$

3.3 Time Change Model

$y_i|\mu_{ct}, \tau_y \sim N(\mu_{ct}, \tau_y)$
$\tau_y = \frac{1}{seluup_i^2 + nonsamplsd}$
$nonsamplesd = .025$
$logit(\mu_{ct}) = \pi_{ct} + \delta_{ct}$
$\pi_{ct} = \alpha_c + \beta_c * demand_{ct} + \rho_\pi * \gamma * (t > \Delta) * \log(\max(t, \Delta + 1) - \Delta)$
$\Delta \sim U(0, 26)$

$$\begin{pmatrix} \alpha_c \\ \beta_c \end{pmatrix} \sim N_2 \left(\begin{pmatrix} \alpha_{r1} \\ \alpha_{r2} \end{pmatrix} \begin{pmatrix} \sigma_{c\alpha}^2 & \rho_{\alpha\beta} * \sigma_{c\alpha} * \sigma_{c\beta} \\ \rho_{\alpha\beta} * \sigma_{c\alpha} * \sigma_{c\beta} & \sigma_{c\beta}^2 \end{pmatrix} \right)$$

$\alpha_{r1}|\alpha_{w1}, \sigma_{r1} \sim N(\alpha_{w1}, \sigma_{r1})$
$\alpha_{r2}|\alpha_{w2}, \sigma_{r2} \sim N(\alpha_{w2}, \sigma_{r2})$
$\alpha_{w1} \sim N(0, 100)$
$\alpha_{w2} \sim N(0, 100)$

$\sigma_{c\alpha} \sim U(0, 3)$
$\sigma_{c\beta} \sim U(0, 3)$
$\rho_{\alpha\beta} \sim U(-.95, 1)$
$\sigma_{r1} \sim U(0, 3)$
$\sigma_{r2} \sim U(0, 3)$

$\delta_{ct} = \rho * \delta_{c,t-1} + \epsilon_{c,t}$
$\epsilon_{c,t} \sim N(0, \sigma_\delta^2)$

4 Combined Model (PDL, LUP, ratio)

4.1 data model

$PDL_i|\mu_{ct}, \tau_y \sim N(\mu_{ct}, \tau_y)$
$LUP_i|\mu_{lup_{ct}}, \tau_y \sim N(\mu_{lup_{ct}}, \tau lup_y)$
$\tau_y = \frac{1}{sePDL_i^2 + nonsamplsd}$
$\tau lup_y = \frac{1}{seLUP_i^2 + nonsamplsd}$
$nonsamplesd = .025$

4.2 PDL

$logit(\mu_{ct}) = \pi_{ct} + \delta_{ct}$
$\pi_{ct} = \alpha_c + \beta_c * demand_{ct} + \rho_\pi * \gamma * (t > \Delta) * \log(\max(t, \Delta + 1) - \Delta)$
$\Delta \sim U(0, 26)$

$$\begin{pmatrix} \alpha_c \\ \beta_c \end{pmatrix} \sim N_2 \left(\begin{pmatrix} \alpha_{r1} \\ \alpha_{r2} \end{pmatrix} \begin{pmatrix} \sigma_{c\alpha}^2 & \rho_{\alpha\beta} * \sigma_{c\alpha} * \sigma_{c\beta} \\ \rho_{\alpha\beta} * \sigma_{c\alpha} * \sigma_{c\beta} & \sigma_{c\beta}^2 \end{pmatrix} \right)$$

$\alpha_{r1}|\alpha_{w1}, \sigma_{r1} \sim N(\alpha_{w1}, \sigma_{r1})$
$\alpha_{r2}|\alpha_{w2}, \sigma_{r2} \sim N(\alpha_{w2}, \sigma_{r2})$
$\alpha_{w1} \sim N(0, 100)$
$\alpha_{w2} \sim N(0, 100)$

$\sigma_{c\alpha} \sim U(0, 3)$
$\sigma_{c\beta} \sim U(0, 3)$
$\rho_{\alpha\beta} \sim U(-.95, 1)$
$\sigma_{r1} \sim U(0, 3)$
$\sigma_{r2} \sim U(0, 3)$

4.3 ratio

$q = demand_{ct}/cpr_{ct}$
$\mu_{logratio_{c,t}} = exp(\mu_{ratio_{c,t}})$
$\mu_{ratio_{c,t}} = \pi_{ratio_{c,t}} + \delta_{ratio_{c,t}}$
$\pi_{ratio_{ct}} = \alpha_{ratio_c} + \beta_{ratio_c} * q + (beta_{ratio_c} - alpha_{ratio_c}) * q^2$

$$\begin{pmatrix}\alpha_c\\ \beta_c\end{pmatrix} \sim N_2\left(\begin{pmatrix}\alpha_{r1}\\ \alpha_{r2}\end{pmatrix}\begin{pmatrix}\sigma^2_{c\alpha} & \rho_{\alpha\beta}*\sigma_{c\alpha}*\sigma_{c\beta}\\ \rho_{\alpha\beta}*\sigma_{c\alpha}*\sigma_{c\beta} & \sigma^2_{c\beta}\end{pmatrix}\right)$$

$\alpha_{r1}|\alpha_{w1},\sigma_{r1} \sim N(\alpha_{w1},\sigma_{r1})$
$\alpha_{r2}|\alpha_{w2},\sigma_{r2} \sim N(\alpha_{w2},\sigma_{r2})$
$\alpha_{w1} \sim N(0,100)$
$\alpha_{w2} \sim N(0,100)$

$\sigma_{c\alpha} \sim U(0,3)$
$\sigma_{c\beta} \sim U(0,3)$
$\rho_{\alpha\beta} \sim U(-.95,1)$
$\sigma_{r1} \sim U(0,3)$
$\sigma_{r2} \sim U(0,3)$

4.4 LUP, Combine PDL and ratio

$w = \mu_{c,t}$
$z = \mu_{ratio_{c,t}}$
$q = demand_{ct}/cpr_{ct}$
$x1_{c,t} = \frac{z*q*w}{1-w+(z*w)}$
$x2_{c,t} = w - x1_{c,t}$
$x3_{c,t} = q - x1_{c,t}$
$x4_{c,t} = 1 - x1_{c,t} - x2_{c,t} - x3_{c,t} = 1 - w - q + x1_{c,t}$
$\mu_{lup+c.t} = \frac{x2_{c,t}}{x2_{c,t}+x4_{c,t}}$

4.5 AR1

$\delta_{ct} = \rho * \delta_{c,t-1} + \epsilon_{c,t}$
$\epsilon_{c,t} \sim N(0,\sigma^2_{\delta})$