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Course: CS600

Homework 10

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Chapter 19

Exercise 19.8.3

Suppose a certain birth defect occurs independently at random with probability p 0.02 in any live birth. Use a Chernoff bound to bound the probability that more than 4% of the 1 million children born in a given large city have this birth defect.

Answer:

Let us assume X, an independent random variable that represents any live birth defects. The probability of birth defect p = 0.02 in any birth

$$E[X] = E[X1] + E[X2] + \cdots + E[X100]$$

$$E[X] = \frac{2}{100} + \frac{2}{100} + \cdots + \frac{2}{100}$$

$$E[X] = 2$$

For n = 100000 births, $E[X] = \frac{2}{n}$

Chernoff bound to bound, the probability of more than 4% of 1 million children born in each major city having this birth defect is as follows:

$$\Pr\left[X \geq \frac{4}{n}\right] \leq \frac{E[X]}{\frac{4}{n}}$$

$$= \frac{\frac{2}{n}}{\frac{4}{n}}$$

$$= \frac{1}{2}$$

Thus, the solution for this question is $\Pr\left[X \geq \frac{4}{n}\right] \leq \frac{1}{2}$.

Exercise 19.8.18

Answer:

In the Fisher-Yates random shuffling algorithm, we use random (k+1) and take k for the loop down to 0 from n-1; the last element is swapped with an equal chance of 1/n, the second last element by 1/n-1, and so on.

$$\frac{1}{n}\left(\frac{1}{n-1}\right)\cdots\frac{1}{2}.\frac{1}{1}=\frac{1}{n!}$$

For each permutation, such that every permutation has the same chance of occurring.

In this issue statement, the call to random (k + 1) is substituted with random(n), and the likelihood of finding the last element's position is 1/n. Similarly, for the second to last slot, it will be 1/n, and so on.

Therefore, the modified Fischer-Yates algorithm would look like:

$$\frac{1}{n}.\frac{1}{n}.\frac{1}{n}\cdots\frac{1}{n}=1/n^{n-1}$$

As a result, all n! permutations have an equal shot of getting selected.

Exercise 19.8.35

Answer:

We employ probabilistic packet marketing to track down denial-of-service attacks (PPM). Furthermore, if a router R has some equal probability p = 12, it is possible to trace the attack back to the sender.

If an attacker launches a denial-of-service assault against a recipient and each of the d routers along the attack path performs probabilistic packet marking, the probability of getting a packet marked by the nth router along the attack path is $p(1-p)^{d-n}$.

Similarly, the number of packets required by the nth router along the attack path is at least $\frac{1}{p(1-p)^{d-n}}$, which represents the predicted number of packets that the recipient must collect to identify all routers along the path from sender to recipient.