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<u>Course: CS600</u>

<u>Homework 12</u>

<u>CWID: 10460520</u>

<u>Chapter 26</u>

Exercise 26.6.7

Answer:

The given LP problem is

Minimize:$3y_1+2y_2+y_3$

Subject to: $-3y_1+y_2+y_3>=1$,

$2y_1+y_2-y_3>=2$,

And y, y, y>=0

The linear program is then transformed to standard form

1. In standard form, all variables involved must be non-negative.

2.All constraints are equalities, with constraints after the right-hand side constant being non-negative.

So, the standard form of the given LP problem is

Minimize: z = 3y+2y+y123

Subject to: -3y+y+y-S=1

2y+y-y- S=2

And y_1, y_2, y_3, S_1, S_2>=0,

Where S_1, S_2 are surplus variable.

We will need to separate the upper hands from the lower bonds and add two slack and two surplus variables. We have the following inequalities:

$-3y_1 + y_2 + y_3 = 1$

$2y_1 + y_2 - y_3 >= 2$

$y_1, y_2, y_3 >= 0$

Thus, the standard form of the given LP problem is:

$z = 3y_1 + 2y_2 + y_3$

Subject to: $-3y_1 + y_2 + y_3 - S_1 = 1$

$2y_1 + y_2 - y_3 - S_2 = 2$

And $y_1, y_2, y_3, S_1, S_2 >= 0$

Where S_1 and S_2 are surplus variable.

(b) Nonzero lower bounds on variables:

Min $y_1 + y_2 + y_3$

Subject to

$-3y_1 + 2y_2 + y_3 = 1$

$Y_1 >= 1, y_2 >= 2, y_3 >= 1$

There are a few ways to do this

$y_1 = y_1 - 1$

$y_2 = y_2 - 2$

$y_3 = y_3 - 1$

Then,

$y_1 + y_2 + y_3 = y_1 - 1 + y_2 - 2 + y_3 - 1$

$= y_1 + y_2 + y_3 + 4$

And

$3y_1 + 2y_2 + y_3 = 3y_1 + 1 + 2(y_2 + 2) + y_3 + 1$

$= 3y_1 + 2y_2 + y_3 + 8$

Then the problem standard form is

Min $y_1 + y_2 + y_3$

Subject to $3y_1 + 2y_2 + y_3$

$y_1 >= 0$, $y_2 >= 0$, $y_3 >= 0$.

Exercise 26.6.26

Answer:

Let G be an undirected graph, together with a (possibly negative) edge cost function c. A spanning tree T on G is a connected subset of edges containing no cycles (a tree) such that every vertex is covered by an edge in the tree Note that this definition is equivalent to T having no cycles, and n−1 edges (a fact proven most easily by induction, by removing vertexes and edges from T which form the leaves of T). T is a spanning tree if it has n − 1 edges, and |E(S)| ≤ |S| − 1 for all vertex sets S (where E(S) = {uv: u, v ∈ S, uv ∈ T}). Given G, find a spanning tree containing the fewest edges. We can formulate the problem as an LP in exponentially many constraints in functions x: E → R,

$$\sum_e c(e)x(e)$$

$$\text{s.t } x\big(E(S)\big) \geq |S| - 1 \ \forall S \subset V$$

$$x(E(V)) = n\text{-}1$$

$$0 \leq x$$

By considering S to be a pair of vertices corresponding to an edge, it should be noted that the restriction x 1 is already encoded in the problem. Any integral solution to this technique corresponds to a spanning tree in G, according to the debate we just had. Although it can be demonstrated that all extreme points are integral, we will only demonstrate how to select an ideal extreme point that is integral. The minimal spanning tree linear program is now unworkable due to the excessive number of constraints. If we use the dual of this program instead, we have a much better optimization problem.

Exercise 26.6.31

Answer:

1)
Let there be 'x' ads of radio, 'y' ads of print and 'z' ads of tv
The total impact would be

impact = (x*a + y*b + z*c)
which is to be maximized
2)
Let total budget be 'B'
Then the total cost for all the ads would be
cost = (10000*x + 70000*y + 110000*z)
which should be less than of equal to total budget
(10000*x + 70000*y + 110000*z) <= B
where B is the total budge
3)
There is a bound to maximum number of each type of ads which gives
x <= 25
y <= 7
z <= 15
4)
The Linear Program would become
MAXIMIZE: impact = (x*a + y*b + z*c)
SUBJECT TO: (10000*x + 70000*y + 110000*z) <= B
BOUNDS:
x <= 25
y <= 7
z <= 15

5) This LP would be solvable if numerical values of (a, b ,c, b) are given