मोतीलाल नेहरू राष्ट्रीय प्रौद्योगिकी संस्थान इलाहाबाद
प्रयागराज-211004 [भारत]
Motilal Nehru National Institute of Technology Allahabad
Prayagraj-211004 [India]

Department of Mathematics
Mid Semester (Odd) Examination 2022-23

Programme Name: B. Tech.
Course Code: MA11101
Branch:.......
Duration: 90 Minutes

Semester: I
Course Name: Mathematics I
Student Reg. No.: | | | | | | | |
Max. Marks.: 20

Instructions: (i) Attempt all the questions. Marks are indicated on write side.
(ii) Attempt each part of a question in continuation.

			Marks
Q1	a	State and prove Lagrange's Mean Value theorem. Discuss the geometrical interpretation of it.	2
	b	Using Cauchy Mean Value theorem, show that $\frac{2\log_e x}{2\sin^{-1}x-\pi} < \frac{\sqrt{1-x^2}}{x}$.	2
Q2	a	Define ε- δ definition of continuity of a function of two variables $f : D \subset R^2 \to R$. Using ε- δ definition, discuss the continuity of the function $f(x,y) = \begin{cases} \frac{x^2+y^2}{3+\sin x}, & if\ (x,y) \neq (0,0) \\ 0, & if\ (x,y) = (0,0) \end{cases}$ at (0,0).	2
	b	Define the differentiability of function of two variables $f : D \subset R^2 \to R$ at the point (x,y). Check the differentiability of the function $f(x,y) = \begin{cases} (x^2+y^2)\sin\left(\frac{1}{x^2+y^2}\right), & if\ (x,y) \neq (0,0) \\ 0, & if\ (x,y) = (0,0), \end{cases}$ at (0,0).	2
Q3	a	Show that for the function $f(x,y) = \begin{cases} \frac{y(x^2-y^2)}{x^2+y^2}, & if\ (x,y) \neq (0,0) \\ 0, & if\ (x,y) = (0,0), \end{cases}$ $f_{xy}(0,0) \neq f_{yx}(0,0)$.	2
	b	If $u = sin^{-1}\sqrt{\frac{x^{1/3}+y^{1/3}}{x^{1/2}+y^{1/2}}}$, then show that $x^2u_{xx} + 2xyu_{xy} + y^2u_{yy} = \frac{\tan u}{12}\left(\frac{13}{12} + \frac{tan^2 u}{12}\right)$.	2
Q4	a	If $xyz - u = 0, x^2 + y^2 + z^2 - v = 0$, and $x + y + z - w = 0$, find $\frac{\partial(x,y,z)}{\partial(u,v,w)}$.	2
	b	Obtain a quadratic Taylor's series approximation to the function $f(x,y) = cosx\ cosy$ about the point (0,0). Also, estimate the maximum absolute error in the region $\|x\| < 0.2, \|y\| < 0.1$.	3
Q5	a	The temperature T at any point (x,y,z) in space is given by $T(x,y,z) = kxyz^2$, where k is a positive constant. Find the highest temperature on the surface of the sphere $x^2 + y^2 + z^2 = a^2$.	3

END

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Department of Mathematics

End Semester (Odd) Examination 2022-23

Programme Name: B.Tech **Semester:** I

Course Code: MA11101 Course Name: Mathematics I

Branch: All Student Reg. No.: | | | | | | | | |

Duration: 150 Minutes **Max. Marks: 50**

Instructions: (i) Attempt all the questions. Marks are indicated on right side.
(ii) Attempt each part of a question in continuation.

			Marks
Q1 3	a	Show that the functions $u = 3x + 2y - z$, $v = x - 2y + z$ and $w = x(x + 2y - z)$ are functionally related.	2
	b	If $u = \ln(x^3 + y^3 - x^2y - xy^2)$, then prove that $u_x + u_y = \frac{2}{x+y}$. Hence, show that $u_{xx} + 2u_{xy} + u_{yy} = -\frac{4}{(x+y)^2}$.	4
	c	State and prove the sufficient condition for the continuity of a function of two variables $f: D \subset \mathbb{R}^2 \to \mathbb{R}$ at a point $(x_0, y_0) \in D$.	4
Q2 4	a	Calculate the area lying inside the cardioid $r = 2(1 + \cos\theta)$ and outside the circle $r = 2$.	2
	b	Find the volume of the spindle-shaped solid generated by revolving the astroid $x = a\sin^3 t$, $y = a\cos^3 t$ about x-axis.	4
	c	Find the volume of the region bounded by the cylinder $x^2 + y^2 = 4$ and the planes $y + z = 4$, and $z = 0$.	4
Q3	a	Evaluate the integral $\int_0^{\pi/2}\int_x^{\pi/2} \frac{\sin y}{y}\,dy\,dx$ by changing the order of the integration.	2

	b	Using the Beta and Gamma functions, evaluate the following integrals: (i) $\int_{-1}^{1}(1-x^2)^n dx \quad (n \in \mathbb{N})$, (ii) $\int_0^\infty \frac{x^c}{c^x} dx$.	4
	c	Find the unit normal vector to the surface $xy^2 + 2yz = 8$ at the point $(3, -2, 1)$. Also, find the equation of the tangent plane at this point.	4
Q4 8	a	Show that the vector field $\vec{F}(x, y, z) = 2x(y^2 + z^3)\hat{\imath} + 2x^2y\hat{\jmath} + 3x^2z^2\,\hat{k}$ is conservative. Find its scalar potential and work done in moving a particle from $(-1,2,1)$ to $(2,3,4)$.	2
	b	State and prove the Green's theorem. Hence, evaluate $\oint_C x^2y dx + x^2 dy$, where C is the boundary of the triangle with vertices $(0,0)$, $(0,1)$ and $(1,1)$, described counter-clockwise.	4
	c	Verify Gauss's divergence theorem for the vector field $\vec{F}(x, y, z) = 4xz\hat{\imath} - y^2\hat{\jmath} + yz\,\hat{k}$ over the surface S. Here, S is the surface of the cube bounded by the planes $x = 0$, $x = 1$, $y = 0$, $y = 1$, $z = 0$ and $z = 1$.	4
Q5 10	a	Solve the first order differential equation $(1 + y^2)dx = (\tan^{-1} y - x)dy$.	2
	b	Find the general solution of the differential equation $x^2y'' - 3xy' + 5y = x \log x$.	4
	c	Using the method of undetermined coefficient, solve the following differential equation $y'' - 4y' + y = x^2 - 2x + 2$.	4

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