मोतीलाल नेहरू राष्ट्रीय प्रौद्योगिकी संस्थान इलाहाबाद
प्रयागराज-211004 [भारत]
Motilal Nehru National Institute of Technology Allahabad
Prayagraj-211004 [India]

Mid Semester Examination Even Semester (Session 2022-23)

Programme Name: B.Tech. **Semester: II**
Course Code: MA12102 **Course Name: Mathematics-II**
Branch: Chemical Engineering **Student Reg. No.:**

Duration: $1\frac{1}{2}$ Hours **Max. Marks: 25**

Instructions:
1. *Figures to the right indicate the full marks.*
2. *All questions are compulsory.*
3. *Solve each part in continuation.*

			Marks
Q 1	a)	Find $Ker(T)$ and $ran(T)$ and their dimensions for Linear Transformation $T:\mathbb{R}^3 \to \mathbb{R}^2$ defined by $T\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}2x-y\\3x+z\end{pmatrix}$.	3
	b)	Let $T:\mathbb{R}^3 \to \mathbb{R}^2$ be a Linear Transformation defined by $T\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}x+y\\x-z\end{pmatrix}$. Find the matrix representation of T with respect to the ordered basis $X = \left\{\begin{pmatrix}1\\0\\1\end{pmatrix}, \begin{pmatrix}1\\1\\0\end{pmatrix}, \begin{pmatrix}0\\1\\1\end{pmatrix}\right\}$ in $\mathbb{R}^3$ and $Y = \left\{\begin{pmatrix}1\\0\end{pmatrix}, \begin{pmatrix}0\\1\end{pmatrix}\right\}$ in $\mathbb{R}^2$.	2
Q 2	a)	Verify the Cayley-Hamilton theorem for the matrix A. Find A^{-1}, if it exists where $A = \begin{bmatrix}1 & 1 & 2\\1 & 3 & -1\\-2 & -1 & 1\end{bmatrix}$	2
	b)	Show that the matrix $A = \begin{bmatrix}0 & 2 & 1\\2 & 0 & 3\\1 & -3 & 0\end{bmatrix}$ is diagonalizable. Find the matrix P such that $P^{-1}AP$ is a diagonal matrix.	3
Q 3	a)	Find all the eigenvalues and corresponding eigen vectors of the matrix $\begin{bmatrix}1 & 2 & 3\\3 & 1 & 0\\-2 & 0 & 1\end{bmatrix}$.	4
	b)	For what values of k do the following set of vectors form a basis in $\mathbb{R}^3$ $\{(k, 1-k, k), (0, 3k-1, 2), (-k, 1, 0)\}$	1

Q 4 a) Find the Laplace Transform of the given functions **2**

(i) $\sin t \, u_\pi(t)$

(ii) $f(t) = \begin{cases} k, & 0 \le t < 2 \\ 0, & 2 \le t < 4 \\ k, & t \ge 4 \end{cases}$

b) State and Prove Convolution theorem for Laplace Transform. **3**

Q 5 Solve the following initial value problems $4y'' - 8y' + 3y = sint,\ y(0) = 0, y'(0) = 2.$ **5**

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Department of Mathematics,
End Semester Examination, Session 2022-23 (Even)

Programme:	*B.Tech.*	**Branch:**	*Chemical Engineering*	**Semester:**	*II*
Course Name:	*Mathematics-II*	**Max. Marks:**	*50*		
Course Code:	*MAN12102*	**Registration No.:**	20222038		
Time:	$2\frac{1}{2}$ *HRS*				

Instructions (related to question paper):
1. *All questions are compulsory.*
2. *Solve each part in continuation.*

			Marks	Corresponding course outcome with weightage (if any)
Q1			(10)	CO1
	a	$T:\mathbb{R}^2 \to \mathbb{R}^3$ be a Linear Transformation. Let $A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \\ 3 & 4 \end{bmatrix}$ be the matrix representation of Linear Transformation T with respect to the ordered basis $v_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, v_2 = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$ in $\mathbb{R}^2$ and $w_1 = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}, w_2 = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}, w_3 = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$ in $\mathbb{R}^3$, then determine the Linear Transformation T.	5	
	b	Show that the matrix $A = \begin{bmatrix} 3 & 1 & -1 \\ -2 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}$ is diagonalizable. Hence, find P such that $P^{-1}AP$ is a diagonal matrix. Then obtain the matrix $B = A^2 + 5A + 3I$.	5	
Q2			(10)	CO2
	a	Use Laplace Transform to evaluate the following integrals (i) $\int_0^\infty \frac{e^{-t}\sin\sqrt{3}\,t}{t}\,dt$ (ii) $\int_0^\infty \frac{e^{-2t}-e^{-4t}}{t}\,dt$	5	
	b	Solve by Laplace Transform first order initial-boundary value problem $$\frac{\partial u}{\partial x} + x\frac{\partial u}{\partial t} = xt^2, \quad u(x,0) = 0, u(0,t) = t.$$	5	

Q3

a Find the particular solution of Lagrange's equation $(2y^2 + z)p + (y + 2x)q = 4xy - z$, which passes through the straight line $z = 1, y = x$. **(10)** **5** **CO4**

a Solve the P.D.E. **5**

$$\frac{\partial^2 z}{\partial x^2} - 3\frac{\partial^2 z}{\partial x \partial y} + 2\frac{\partial^2 z}{\partial y^2} = e^{2x+3y} + sin(x - 2y).$$

Q4

a Find the Fourier Series for the function $f(x) = x + x^2, \quad -\pi < x < \pi$. Hence show that **(10)** **5** **CO3**

(i) $\frac{\pi^2}{6} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots\ldots$ (ii) $\frac{\pi^2}{12} = 1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots\ldots$

b (i) A continuous random variable X has probability density function $f(x) = \frac{3}{4}(x^2 + 1)$, $0 \le x \le 1$. Find a such that $P(X \le a) = P(X > a)$. **5** **CO6**

(ii) If the probability density function of a continuous random variable is given by $(x) = e^{-x}$, $0 \le x < \infty$. Find the mean and variance.

Q5 **(10)**

a A tightly stretched flexible string has its ends fixed at $x = 0$ and $x = l$. At time $t = 0$, the string is given a shape defined by $F(x) = \mu x(l - x)$, μ is a constant and then released. Find the displacement $y(x, t)$ of any point x of string at any time $t > 0$. **8** **CO5**

b Find $L^{-1}\left[\frac{1}{s^2(s^2+4)}\right]$ **2** **CO2**

Course Outcomes:

CO1: ***This unit is designed to make students familiar with the basic concepts of linear algebra, such as vector spaces, basis, dimension, linear transformation. Students will learn basic concepts like eigenvalues, eigenvector and its application, diagonalization, which are fundamental concepts in many engineering problems.***

CO2: ***The course provides a basic understanding of Laplace transformation to address the engineering problems governed by ordinary and partial differential equations.***

CO3: ***This unit provides fundamental knowledge about the Fourier series and Fourier transforms, which are fundamental concepts for solving boundary value problems and signal processing.***

CO4: ***Development of the basic understanding and solution methods for the linear/nonlinear partial differential equations which arises in the modeling of engineering/physical problems.***

CO5: ***The student will be able to classify and solve the PDE's of second order which arises in the modeling of many engineering/physical problems. Also, the students will be able to apply the technique to solve, heat, wave and Laplace equations.***

CO6: ***The students will have the basic knowledge of random variables, Probability distributions of***