The depth for all the test cases is taken as 3.

MiniMaxOpening:

board1.txt M

board1.txt

```
WxWxBBWWxBBxxxxxxx
```

board2.txt M

board2.txt

```
Board position is: WxWxBBWWWxBxxxxxxx

Positions evaluated by static estimation: 1692

MINIMAX estimate: 2
```

board1.txt M

board1.txt

```
xWxBxWxxxxxxxWxBxB
```

board2.txt M

board2.txt

```
Board position is: xWxBxWxxxxxxxWxBWB

Positions evaluated by static estimation: 1862

MINIMAX estimate: 1
```

MiniMaxGame:

board1.txt M

board1.txt

```
WxWxBBWWxBBxxxxxxx
```

board2.txt M

board2.txt

```
Board position is: xWWxBBWWxBBxxxxxxx

Positions evaluated by static estimation: 274

MINIMAX estimate: 967
```

board1.txt M

board1.txt

```
xWxBxWxxxxxxxWxBxB
```

board2.txt M

board2.txt

```
Board position is: xxxBxWxxxxxxxWxBWB

Positions evaluated by static estimation: 46657

MINIMAX estimate: -36
```

ABOpening:

board1.txt M

board1.txt

```
WxWxBBWWxBBxxxxxxx
```

board2.txt M

board2.txt

```
Board position is: WxWxBBWWWxBxxxxxxx

Positions evaluated by static estimation: 274

MINIMAX estimate: 2
```

board1.txt M

board1.txt

```
xWxBxWxxxxxxxWxBxB
```

board2.txt M

board2.txt

```
Board position is: xWxBxWxxxxxxxWxBWB

Positions evaluated by static estimation: 759

MINIMAX estimate: 1
```

ABGame:

board1.txt

```
WxWxBBWWxBBxxxxxxx
```

board2.txt

```
Board position is: xWWxBBWWxBBxxxxxxx

Positions evaluated by static estimation: 76

MINIMAX estimate: 967
```

board1.txt

```
xWxBxWxxxxxxxWxBxB
```

board2.txt

```
Board position is: xxxBxWxxxxxxxWxBWB

Positions evaluated by static estimation: 5432

MINIMAX estimate: -36
```

<u>MiniMaxOpeningBlack:</u>

board1.txt

```
WxWxBBWWxBBxxxxxxx
```

board2.txt

```
Board position is: WxWxBBxWxBBBxxxxxx

Positions evaluated by static estimation: 1819

MINIMAX estimate: 2
```

board1.txt

```
xWxBxWxxxxxxxWxBxB
```

board2.txt

```
Board position is: xWxBxWxBxxxxxWxBxB

Positions evaluated by static estimation: 2116

MINIMAX estimate: 2
```

MiniMaxGameBlack:

board1.txt

```
WxWxBBWWxBBxxxxxxx
```

board2.txt

```
Board position is: WxWBBxWWxBBxxxxxxx

Positions evaluated by static estimation: 183

MINIMAX estimate: -10
```

board1.txt

```
xWxBxWxxxxxxxWxBxB
```

board2.txt

```
Board position is: xxxxxWxxxxxxxWxBBB

Positions evaluated by static estimation: 46668

MINIMAX estimate: 10000
```

MiniMaxOpeningImproved:

board1.txt

```
WxWxBBWWxBBxxxxxxx
```

board2.txt

```
Board position is: WxWxBBWWWxBxxxxxxx

Positions evaluated by static estimation: 1692

MINIMAX estimate: 33
```

board1.txt

```
xWxBxWxxxxxxxWxBxB
```

board2.txt

```
Board position is: xWxBxWxxxxxxxWxBWB

Positions evaluated by static estimation: 1862

MINIMAX estimate: 13
```

MiniMaxGameImproved:

board1.txt M

board1.txt

```
WxWxBBWWxBBxxxxxxx
```

board2.txt M

board2.txt

```
Board position is: xWWxBBWWxBBxxxxxxx

Positions evaluated by static estimation: 274

MINIMAX estimate: 230
```

board1.txt M

board1.txt

```
xWxBxWxxxxxxxWxBxB
```

board2.txt M

board2.txt

```
Board position is: xxxBxWxxxxxxxWxBWB

Positions evaluated by static estimation: 46657

MINIMAX estimate: -330
```

<u>Cases where Alpha-Beta produces savings over MiniMax</u>:

1. MiniMaxOpening vs ABOpening:

<u>Case 1:</u>

Input: WBWWBBWBxxxxxxxxxx
Depth: 5

MinMaxOpening:

```
board1.txt M ×
board1.txt
WBWWBBWBxxxxxxxxxx
```

```
board2.txt M ×
board2.txt
Board position is: WBWWBBWBWxxxxxxxxx

Positions evaluated by static estimation: 41976

MINIMAX estimate: 1
```

ABOpening:

```
board1.txt M ×
board1.txt
WBWWBBWBxxxxxxxxxx
```

```
board2.txt M ×
board2.txt
Board position is: WBWWBBWBWxxxxxxxxx

Positions evaluated by static estimation: 1238

MINIMAX estimate: 1
```

Case 2:

Input: xWxxxWxxBxxxxxBxxx
Depth: 6

MinMaxOpening:

```
board1.txt M
board1.txt
xWxxxWxxBxxxxxBxxx
```

```
board2.txt M
board2.txt
Board position is: xWxWxWxxxxxxxxBxxx

Positions evaluated by static estimation: 4963580

MINIMAX estimate: 1
```

ABOpening:

```
board1.txt M
board1.txt
xWxxxWxxBxxxxxBxxx
```

```
board2.txt M
board2.txt
Board position is: xWxWxWxxxxxxxxBxxx

Positions evaluated by static estimation: 35865

MINIMAX estimate: 1
```

Case 3:

Input: xWxBxWxBxWxBxWxBWB
Depth: 7

MiniMaxOpening:

board1.txt M

board1.txt

```
xWxBxWxBxWxBxWxBWB
```

board2.txt M

board2.txt

```
Board position is: xWxxxWxBxWWBxWxBWB

Positions evaluated by static estimation: 1717343

MINIMAX estimate: 1
```

ABOpening:

board1.txt M

board1.txt

```
xWxBxWxBxWxBxWxBWB
```

board2.txt M

board2.txt

```
Board position is: xWxxxWxBxWWBxWxBWB

Positions evaluated by static estimation: 36833

MINIMAX estimate: 1
```

2. MiniMaxGame vs ABGame:

<u>Case 1:</u>

Input: WBWWBBWBxxxxxxxxxx
Depth: 5

MinMaxGame:

```
board1.txt M ×
board1.txt
1    WBWWBBWBxxxxxxxxxx
```

```
board2.txt M ×
board2.txt
1    Board position is: xBWWBBWBxxxxxxxWxx
2
3    Positions evaluated by static estimation: 6556
4
5    MINIMAX estimate: -7
```

ABGame:

```
board1.txt M ×
board1.txt
1    WBWWBBWBxxxxxxxxxx
```

```
board2.txt M ×
board2.txt
1    Board position is: xBWWBBWBxxxxxxxWxx
2
3    Positions evaluated by static estimation: 719
4
5    MINIMAX estimate: -7
```

Case 2:

Input: xBBxxxWxxWxxBxBxxW
Depth: 4

MinMaxGame:

```
board1.txt M ×
board1.txt
xBBxxxWxxWxxBxBxxW
```

```
board2.txt M ×
board2.txt
Board position is: xBBxxxxxxWxWBxBxxW

Positions evaluated by static estimation: 143704

MINIMAX estimate: -36
```

ABGame:

```
board1.txt M ×
board1.txt
xBBxxxWxxWxxBxBxxW
```

```
board2.txt M ×
board2.txt
Board position is: xBBxxxxxxWxWBxBxxW

Positions evaluated by static estimation: 3887

MINIMAX estimate: -36
```

Case 3:

Input: xWxBxWxBxWxBxWxBWB
Depth: 5

MiniMaxGame:

board1.txt M

board1.txt

```
xWxBxWxBxWxBxWxBWB
```

board2.txt M

board2.txt

```
Board position is: xWxBxWxxxxWBxWxBWB

Positions evaluated by static estimation: 169681

MINIMAX estimate: 995
```

ABGame:

board1.txt M

board1.txt

```
xWxBxWxBxWxBxWxBWB
```

board2.txt M

board2.txt

```
Board position is: xWxBxWxxxxWBxWxBWB

Positions evaluated by static estimation: 13489

MINIMAX estimate: 995
```

In all the cases, the board positions and the MINIMAX estimate were the same for both MiniMax and AlphaBeta Pruning.

The difference is in the positions evaluated, and alpha beta reduces the positions to be evaluated significantly when compared to MiniMax and this savings improves much better as the depth increases.

For instance, in the case 3 of the MiniMaxOpening vs ABOpening the positions evaluated by minimax are 1717343, and the positions evaluated by alpha-beta are 36833.
<u>This case has a saving of approximately 98%.</u>

Examples where improved evaluation function produced different moves than the standard evaluation function:

1. MiniMaxOpening vs MiniMaxOpeningImproved:

Case 1:

Input: xWBxWWBBxxBWxxWWBx
Depth: 5

MinMaxOpening:

```
board1.txt M ×
board1.txt
xWBxWWBBxxBWxxWWBx
```

```
board2.txt M ×
board2.txt
Board position is: xWBWWWxBxxBWxxWWBx

Positions evaluated by static estimation: 88729

MINIMAX estimate: 3
```

MiniMaxOpeningImproved:

```
board1.txt M ×
board1.txt
xWBxWWBBxxBWxxWWBx
```

```
board2.txt M ×
board2.txt
Board position is: xWBxWWxBxxBWxxWWBW

Positions evaluated by static estimation: 88729

MINIMAX estimate: 53
```

Case 2:

Input: WBxBBWxBxWxxxWWWBB
Depth: 5

MinMaxOpening:

board1.txt M

```
board1.txt
WBxBBWxBxWxxxWWWBB
```

board2.txt M

```
board2.txt
Board position is: WBxBBWWBxWxxxWWWBB

Positions evaluated by static estimation: 18595

MINIMAX estimate: 1
```

MiniMaxOpeningImproved:

board1.txt M

```
board1.txt
WBxBBWxBxWxxxWWWBB
```

board2.txt M

```
board2.txt
Board position is: WxxBBWxBxWxxWWWWBB

Positions evaluated by static estimation: 18595

MINIMAX estimate: 30
```

2. MiniMaxGame vs MiniMaxGameImproved:

<u>Case 1:</u>

Input: BWBxWWWBBxWBBBWBWW
Depth: 5

MinMaxGame:

board1.txt M ×

board1.txt

```
BWBxWWWBBxWBBBWBWW
```

board2.txt M ×

board2.txt

```
Board position is: BWBWWxWBBxWBBBWBWW

Positions evaluated by static estimation: 2614

MINIMAX estimate: 996
```

MiniMaxGameImproved:

board1.txt M ×

board1.txt

```
BWBxWWWBBxWBBBWBWW
```

board2.txt M ×

board2.txt

```
Board position is: BWBxWWWBBWxBBBWBWW

Positions evaluated by static estimation: 2614

MINIMAX estimate: 940
```

Case 2:

Input: BWBxxWxxWxBxxxBxxB
Depth: 3

MinMaxGame:

board1.txt M

board1.txt
```
BWBxxWxxWxBxxxBxxB
```

board2.txt M

board2.txt
```
Board position is: BWxWxWxxxxBxxxBxxB

Positions evaluated by static estimation: 10806

MINIMAX estimate: -1009
```

MiniMaxGameImproved:

board1.txt M

board1.txt
```
BWBxxWxxWxBxxxBxxB
```

board2.txt M

board2.txt
```
Board position is: BxBxxWxxWxBWxxBxxB

Positions evaluated by static estimation: 10806

MINIMAX estimate: -40
```

Explanation for the improvement for the evaluation function:

We can see that the standard evaluation function is very basic and naive as it checks only the difference between the number of white pieces and black pieces in the opening and additionally the number of black moves in the midgame/endgame. I think there are a very vast number of parameters to consider to make the algorithm perform better. I think my algorithm will perform better than the standard evaluation function that was given to us as I have considered additional parameters like

1. The difference between the number of mills of our's and the opponent's.
2. If a board position made a mill in its last move.
3. The number of 2 piece configurations of our pieces as it can possibly result in a mill in the future.
4. The number of blocked pieces of the opponent as this will make that piece not to move in the midgame.

I have considered these parameters in addition to the parameter in the standard evaluation function. So, this will make my evaluation function perform at least as good as the standard evaluation function in the worst case.