Practical no :- 1 :

Q1

2)

$$\lim_{x\to a}\left[\frac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}}\right]$$

$$\lim_{x\to a}\frac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}}\times\frac{\sqrt{a+2x}+\sqrt{3x}}{\sqrt{a+2x}+\sqrt{3x}}\times\frac{\sqrt{3a+x}+2\sqrt{x}}{\sqrt{3a+x}+2\sqrt{x}}$$

$$\lim_{x\to a}\frac{(a+2x-3x)}{(3a+x-4x)}\frac{(\sqrt{3a+x}+2\sqrt{x})}{\sqrt{a+2x}+\sqrt{3x}}$$

$$\lim_{x\to a}\frac{(a-x)}{(3a-3x)}\quad\frac{\sqrt{3a+x}+2\sqrt{x}}{\sqrt{a+2x}+\sqrt{3x}}$$

$$\frac{1}{3}\lim_{x\to a}\frac{(a-x)}{(a-x)}\quad\frac{\sqrt{3a+2a}+2\sqrt{a}}{\sqrt{a+2a}+\sqrt{3a}}$$

$$\frac{1}{3}\quad\frac{\sqrt{4a}}{3a}+\frac{2\sqrt{a}}{\sqrt{3a}}$$

$$\frac{1}{3}\quad\frac{2\sqrt{a}}{\sqrt{3a}}+\frac{2\sqrt{a}}{\sqrt{3a}}$$

$$\frac{1}{3}\times\frac{4\sqrt{a}}{2\sqrt{3a}}$$

$$=\frac{2}{3\sqrt{3}}$$

2) $\lim_{y \to 0} \left[\frac{\sqrt{a+x} - \sqrt{a}}{y \sqrt{a+x}} \right]$

$\lim_{y \to 0} \frac{\sqrt{a+y} - \sqrt{a}}{y \sqrt{a+y}} \times \frac{\sqrt{a+x} + \sqrt{a}}{\sqrt{a+x} + \sqrt{a}}$

$\lim_{y \to 0} \frac{a+y-a}{y \sqrt{a+y} (\sqrt{a+y} + \sqrt{a})}$

$\lim_{y \to 0} \frac{y}{y \sqrt{a+y} (\sqrt{a+x} + \sqrt{a})}$

$\frac{1}{\sqrt{a+0} (\sqrt{a+0} + \sqrt{a}}$

$\frac{1}{\sqrt{a} (\sqrt{a} + \sqrt{a})}$

$\frac{1}{\sqrt{a} (2\sqrt{a})}$

$= \frac{1}{2a}$

$\lim_{x \to \pi/6} \frac{\cos x - \sqrt{3} \sin x}{\pi - 6x}$

by subsituting $x - \pi/6 = h$

$x = h + \pi/6$

where $h \to 0$

$$\lim_{h\to 0} \frac{\cos(h+\pi/6) - \sqrt{3}\sin\left(h+\frac{\pi}{6}\right)}{\pi - 6\left(h+\frac{\pi}{6}\right)}$$

$$\lim_{h\to 0} \frac{\cos h \cos\frac{\pi}{6} - \sin h \sin\frac{\pi}{6} - \sqrt{3}\sin h \cos\frac{\pi}{6} + \cos h \sin\frac{\pi}{6}}{\pi - 6\left(\frac{6h+\pi}{6}\right)}$$

$$\lim_{x\to 0} \frac{\cos h \frac{\sqrt{3}}{2} - \sin h \frac{1}{\sqrt{2}} - \frac{2}{\sqrt{3}}\left(\sin h \frac{\sqrt{3}}{2} + \cos h \frac{1}{\sqrt{2}}\right)}{\pi - 6h + \pi}$$

$$\lim_{h\to 0} \frac{\cos\frac{\sqrt{3}h}{2} - \sin\frac{h}{2} - \sin\frac{3h}{2} - \cos\frac{\sqrt{3}h}{2}}{-6h}$$

$$\lim_{h\to 0} \frac{\sin 4h}{2}$$

$$\lim_{h\to 0} \frac{\sin 4h}{3+2h}$$

$$\frac{1}{3} \lim_{h\to 0} \frac{\sin h}{h} \quad \frac{1}{3}\times 1 = \frac{1}{3}$$

4) $\lim_{x\to\infty} \frac{\sqrt{x^2+5}}{\sqrt{x^2+3}} - \frac{\sqrt{x^2-3}}{\sqrt{x^2+1}}$

by rationalizing numetor and Denominator both

$$\lim_{x\to\infty} \frac{\sqrt{x^2+5}}{\sqrt{x^2+3}} - \frac{\sqrt{x^2-3}}{\sqrt{x^2+1}} \times \frac{\sqrt{x^2+5}+\sqrt{x^2+3}}{\sqrt{x^2+5}+\sqrt{x^2-3}} \times \frac{\sqrt{x^2+3}+\sqrt{x^2+1}}{\sqrt{x^2+3}+\sqrt{x^2+1}}$$

$$\lim_{x\to\infty} \frac{(x^2+5-x^2+3)}{(x^2+3-x^2-1)} \frac{\sqrt{x^2+3}+\sqrt{x^2+1}}{\sqrt{x^2+5}+\sqrt{x^2+-3}}$$

$$\lim_{x\to\infty} \frac{8}{2} \frac{\sqrt{x^2+3}}{\sqrt{x^2+5}} + \frac{\sqrt{x^2+1}}{\sqrt{x^2-3}}$$

$$4 \lim_{x\to\infty} \frac{\sqrt{x^2\left(1+\frac{3}{x^2}\right)}+\sqrt{x^2\left(1+\frac{1}{x^2}\right)}}{\sqrt{x^2\left(1+\frac{5}{x^2}\right)}\ \sqrt{x^2\left(1-\frac{3}{x^2}\right)}}$$

after applying limit
we get
$=4$

5) i) $f(x) = \frac{\sin 2x}{\sqrt{1-\cos 2x}}$ for $0 < x \le \pi/2$

$= \frac{\cos x}{\pi - 2x}$ for $\frac{\pi}{2} < x < \pi$

$f(\pi/2) = \frac{\sin 2\left(\frac{\pi}{2}\right)}{\sqrt{1-\cos^2(\pi/2)}}$ $\therefore f(\pi/2) = 0$

f at $x = \pi/2$ defined

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$$\lim_{x\to\pi/2} f(x) = \lim_{x\to\pi/2} \frac{\cos x}{\pi - 2x}$$

by subsituting method

$$x - \frac{\pi}{2} = h$$

$$x = h + \frac{\pi}{2}$$

where $h \to 0$

$$\lim_{x\to 0} \frac{\cos\left(h+\frac{\pi}{2}\right)}{\pi - 2\left(h+\frac{\pi}{2}\right)}$$

$$\lim_{x\to 0} \frac{\cos\left(h+\frac{\pi}{2}\right)}{\pi - 2\left(\frac{2h+\pi}{2}\right)}$$

$$\lim_{x\to 0} \frac{\cos\left[h+\frac{\pi}{2}\right]}{-2h}$$

$$\lim_{x\to 0} \frac{\cos h \cdot \cos\frac{\pi}{2} - \sin h \cdot \sin\frac{\pi}{2}}{-2h}$$

$$\lim_{h\to 0} \frac{\cos h \cdot 0 - \sin h}{-2h}$$

$$\lim_{h\to 0} \frac{\sin h}{-2h}$$

$$\frac{1}{2} \lim_{h\to 0} \frac{\sin h}{h}$$

$$= \frac{1}{2}$$

$\lim_{x\to\pi/2} f(x) = \lim_{x\to\pi/2} \frac{\sin 2x}{\sqrt{1-\cos 2x}}$

$\lim_{x\to\pi/2} \frac{2\sin x \cdot \cos x}{\sqrt{2\sin^2 x}}$

$\lim_{x\to\pi/2} \frac{2\sin x \cdot \cos x}{\sqrt{2}\sin x}$

$\lim_{x\to\pi/2} \frac{2\cos x}{\sqrt{2}}$

$\frac{2}{\sqrt{2}} \lim_{x\to\pi/2} \cos x$

$\therefore$ LHL = R.H.L

$\therefore$ f is not continous at $x = \pi/2$

$f(x) = \frac{x^2-4}{x-3}$ $\quad 0 < x < 3$

$= x+3$ $\quad 3 < x \le 6$ $\quad$ at $x=3$ and $x=6$

$= \frac{x^2-9}{x+3}$ $\quad 6 \le x < 9$

at $x=3$

$f(x) = \frac{x^3-9}{x-3} = 0$

f at $x=3$ define

(ii) $\lim_{x \to 3} f(x) = \lim_{x \to 3} x+3$

$f(3) = x+3 = 3+3 = 6$

f is define at $x=3$

$\lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} (x+3) = 6$

$\lim_{x \to 3^+} f(x) = \lim_{x \to 3^-} \frac{x^2-9}{x-3}$ $\frac{(x-3)(x+3)}{(x-3)}$

L.H.L = R.H.L

f is continous at $x=3$

for $x = 6$

$f(6) = \frac{x^2-9}{x+3} = \frac{36-9}{6+3}$ $\frac{27}{9} = 3$

$\lim_{x \to 6^+} \frac{x^2-9}{x+3}$

$\lim_{x \to 6^+} \frac{(x-3)(x+3)}{(x+3)}$

$\lim_{x \to 6} (x-3) = 6-3 = 3$

$\lim_{x \to 6} x+3 = 3+6 = 9$

L.H.L ≠ R.H.L

not continous

$$f(x) = \frac{1-\cos 4x}{x^2} \quad x < 0$$

$$= k \quad x = 0 \qquad \text{at } x = 0$$

Soln f is continous at $x = 0$

$$\lim_{x \to 0} f(x) = f(0)$$

$$\lim_{x \to 0} \frac{1-\cos 8x}{x^2} = k$$

$$\lim_{x \to 0} \frac{2 \sin 2x}{x^2} = k$$

$$2 \lim_{x \to 0} \frac{\sin^2 2x}{x^2} = k$$

$$2 \lim_{x \to 0} \left(\frac{\sin 2x}{x}\right)^2 = k$$

$$2(2)^2 = k$$

$$k = 8$$

(ii) $f(x) = (\sec^2 x)^{\cot^2 x} \quad x \neq 0$

$= k \quad x = 0 \qquad$ at $x = 0$

Soln $f(x) = (\sec^2 x)^{\cot^2 x}$

$$\lim_{x \to 0} (\sec^2 x)^{\cot^2 x}$$

$$\lim_{x \to 0} (1 + \tan^2 x)^{\frac{1}{\tan^2 x}}$$

we know that

$$\lim_{x \to 0} (1 + px)^{\frac{1}{px}} = e$$

$$= e$$

$$k = e$$

iii) $f(x) = \frac{\sqrt{3} - \tan x}{\pi - 3x}$ $\quad x \neq \pi/3$ $\quad a+x = \pi/3$

$= k$ $\quad x = \pi/3$

$x - \frac{\pi}{3} = h$

$x = h + \frac{\pi}{3}$

where $h \to 0$

$$f(\pi/3 + h) = \frac{\sqrt{3} \tan(\pi/3 + h)}{\pi - 3(\pi/3 + h)}$$

$$\lim_{x \to 0} \frac{\sqrt{3} - \tan(\pi/3 + h)}{\pi - 3(\pi/3 + h)}$$

$$\lim_{h \to 0} \frac{\frac{\sqrt{3} - \tan\frac{\pi}{3} - \tan h}{1 - \tan\frac{\pi}{3} - \tan h}}{\pi - \pi - 3h}$$

$$\lim_{h \to 0} \frac{\frac{\sqrt{3}(1 - \tan\frac{\pi}{3} \cdot \tan h) - (\tan\frac{\pi}{3} + \tan h)}{1 - \tan\frac{\pi}{3} \cdot \tan h}}{3h}$$

$$\lim_{h \to 0} \frac{\frac{\cancel{\sqrt{3}} - 3\tan h - \sqrt{3} - \tan h}{1 - \sqrt{3}\tan h}}{-3h}$$

$$\lim_{h \to 0} \frac{4\tan h}{3h\,(1 - \sqrt{3}\tan h}$$

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$$\lim_{h\to 0} \frac{4 - 4\tan h}{3h(1-\sqrt{3}\tan h)}$$

$$\frac{4}{3} \lim_{h\to 0} \frac{\tan h}{h} \lim_{h\to 0} \frac{1}{(1-\sqrt{3}\tan h)}$$

$$= \frac{4}{3} \frac{1}{1-\sqrt{3}(0)}$$

$$= \frac{4}{3}\left(\frac{1}{1}\right)$$

$$= \frac{4}{3}$$

$$f(x) = \frac{1-\cos 3x}{x\tan x} \quad x \neq 0$$

$$= 9 \quad x = 0$$

at $x=0$

$$f(x) = \frac{1-\cos 3x}{x\tan x}$$

$$\lim_{x\to 0} \frac{2\sin^2 \frac{3}{2}x}{x\tan x}$$

$$\lim_{x\to 0} = \frac{\frac{2\sin^2 \frac{3x}{2}}{x^2}}{\frac{x \cdot \tan x}{x}} \times x^2 \quad \times x^2$$

$$= 2 \lim_{x\to 0} \frac{\left(\frac{3}{2}\right)^2}{1} = \frac{9}{2}$$

$\lim_{x \to 0} f(x) \quad \frac{9}{2} \qquad 9 = f(0)$

$\therefore f$ is not continous at $x=0$

Redefine function

$f(x) = \frac{1-\cos 3x}{x \tan x} \qquad x \neq 0$

$\frac{9}{2} \qquad x = 0$

Now $\lim_{x \to 0} f(x) = f(0)$

f has removable discontinous at $x=0$

$f(x) = \frac{(e^{3x}-1)\sin x^{\circ}}{x^2} \qquad x \neq 0$

$= \frac{\pi}{6} \qquad x = 0$

$\lim_{x \to 0} \frac{e^{3x}-1}{x} \quad \lim_{x \to 0} \frac{\left(\frac{\pi x}{180}\right)}{x}$

$\lim_{x \to 0} 3 \frac{e^{3x}-1}{3x} \quad \lim_{x \to 0} \frac{\sin\left(\frac{\pi x}{180}\right)}{x}$

$3 \lim_{x \to 0} \frac{e^{3x}-1}{3x} \quad \lim_{x \to 0} \frac{\sin\left(\frac{\pi x}{180}\right)}{x}$

$3 \log e \quad \frac{\pi}{180} = \frac{\pi}{60}$

f is continous at $x=0$

(ii) $f(x) = \frac{e^{x^2} - \cos x}{x^2} \quad x \neq 0$

is continous at $x=0$

$\lim_{x \to 0} f(x) = f(0)$

$\lim_{x \to 0} \frac{e^{x} - \cos x - 1 + 1}{x^2}$

$\lim_{x \to 0} \frac{e^{x^2} - 1 + 1 - \cos x}{x^2}$

$\lim_{x \to 0} \frac{e^{x^2} - 1}{x^2} + \lim_{x \to 0} \frac{2\sin^2 x/2}{x^2}$

$\log e + 2 \lim_{x \to 0} \left(\frac{\sin x/2}{x}\right)^2$

multiplying with 2 on num and donomenuto r

$= \cancel{1+2} \times \frac{\cancel{2}}{\cancel{4}\,2}\cancel{8}$

$= \frac{2}{2} = 1$

a) $f(x) = \dfrac{\sqrt{2} - \sqrt{1+\sin x}}{\cos^2 x}$ $\quad x = \pi/2$

$f(x)$ is continuous at $x = \pi/2$

$$\lim_{x\to\pi/2} \frac{\sqrt{2} - \sqrt{1+\sin x}}{\cos^2 x} \times \frac{\sqrt{2} + \sqrt{1+\sin x}}{\sqrt{2} + \sqrt{1+\sin x}}$$

$$\lim_{x\to\pi/2} \frac{2 - 1+\sin x}{\cos^2 x(\sqrt{2} + \sqrt{1+\sin x}}$$

$$\lim_{x\to\pi/2} \frac{1+\sin x}{1-\sin^2 x(\sqrt{2} + (1+\sin x))}$$

$$\lim_{x\to\pi/2} \frac{1+\sin x}{(1-\sin x)(1+\sin x)(\sqrt{2} + \sqrt{1+\sin x})}$$

$$= \lim_{x\to\pi/2} \frac{1}{(1-\sin x)(\sqrt{2} + 1+\sin 2x)}$$

$$\frac{1}{2(\sqrt{2} + \sqrt{2})}$$

$$= \frac{1}{2(2\sqrt{2})}$$

$$= \frac{1}{4\sqrt{2}}$$

$$f(\pi/2) = \frac{1}{4\sqrt{2}}$$

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practical no :- 2

Topic derivatives

Q1) show that the following function defined from IR to IR are differentiable

1) cot x

$f(x) = \cot x$

$$f(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$= \lim_{x \to a} \frac{\cot x - \cot a}{x - a}$$

$$= \lim_{x \to a} \frac{\frac{1}{\tan x} - \frac{1}{\tan a}}{x - a}$$

$$= \lim_{x \to a} \frac{\tan a - \tan x}{(x-a) \tan x \tan a}$$

Put $x - a = h$

$x = a + h$

as $x \to a$ $h \to 0$

$$Df(h) = \lim_{x \to 0} \frac{\tan a - \tan(a+h)}{(a+h-a) \tan(a+h) \tan a}$$

$$= \lim_{x \to 0} \frac{\tan a - \tan(a+h)}{h \times \tan(a+h) \tan a}$$

formula :- $\tan(a-b) = \dfrac{\tan a - \tan b}{1 + \tan a \tan b}$

$\tan a \cdot \tan b = \tan(a-b)(1 + \tan A \cdot \tan b)$

$$\lim_{h\to 0} \frac{\tan(a-a-h)}{h \times \tan(a+h)\tan a} - \frac{(1+\tan a - \tan(a+h))}{}$$

$$\lim_{h\to 0} -\frac{\tan h}{h} \times \frac{1+\tan a(\tan(a+h))}{\tan(a+h)\tan a}$$

$$= -1 \times \frac{1+\tan^2 a}{\tan^2 a} = \frac{\sec^2 a}{\tan^2 a}$$

$$= \frac{-1}{\cos^2 a} \times \frac{\cos^2 a}{\sin^2 a} = -\operatorname{cosec}^2 a$$

$$Df(a) = \cos^2 a$$

$\therefore f$ is differentiable $\forall a \in R$

ii) $\operatorname{cosec} x$

$$f(x) = \operatorname{cosec} x$$

$$Df(a) = \lim_{x\to a} \frac{f(x)-f(a)}{x-a}$$

$$= \lim_{x\to a} \frac{\operatorname{cosec} x - \operatorname{cosec} a}{x-a}$$

$$= \lim_{x\to a} \frac{\frac{1}{\sin x} - \frac{1}{\sin a}}{(x-a)\cdot \sin a \cdot \sin x}$$

Put $x-a=h$

$x = h+a$

as $x \to a$ $h \to a$

$$(h) = \lim_{h\to 0} \frac{\sin a - \sin(a+h)}{(a+h-a)\ \sin a \cdot \sin(a+h)}$$

formula: $\sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right) \sin\left(\frac{C-D}{2}\right)$

$$= \lim_{h\to 0} \frac{2\cos\left(\frac{a+a+h}{2}\right)\cdot \sin\left(\frac{a-a+h}{2}\right)}{h \times \sin a \cdot \cos(a+h)}$$

$$= \lim_{h\to 0} \frac{-\sin\left(\frac{a+h}{2}\right) \quad \sin\frac{-h}{2} \times -\frac{1}{2}}{\cos a \cdot \cos(a+h) \times h/2}$$

$$= \frac{-\frac{1}{2} \times -2\sin\left(\frac{2a+0}{2}\right)}{\cos a \cdot \cos}$$

$$= \frac{\cos a}{\sin^2 a} = -\cot a \cosec a$$

iii) $\sec x$

$f(x) = \sec x$

$$Df(a) = \lim_{x\to a} \frac{f(x)-f(a)}{x-a}$$

$$= \lim_{x\to a} \frac{\sec x - \sec a}{x-a}$$

$$= \lim_{x\to a} \frac{\frac{1}{\cos x} - \frac{1}{\cos a}}{x-a}$$

$$= \lim_{x\to a} \frac{\cos a - \cos x}{(x-a)\cos a(\cos x)}$$

put $x-a=h$

$x = a+h$ as $x=0$ $h\to 0$

$$Df(h) = \lim_{x\to 0} \frac{\cos a - \cos(a+h)}{h + \cos a \cos(a+h)}$$

formula:- $$\frac{-2\sin\left(\frac{C+D}{2}\right) \cdot \sin\left(\frac{C-D}{2}\right)}{h \times \cos a \cdot \cos(a+h)}$$

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$$\lim_{h\to 0} \frac{-2\sin\left(\frac{2a+h}{2}\right) \cdot \sin\frac{h}{2} \times \frac{1}{2}}{\cos a \cdot \cos(a+h) \times h/2}$$

$$= -\frac{1}{2} \times -2 \frac{\sin\left(\frac{2a+0}{2}\right)}{\cos a \cdot \cos(a+0)}$$

$$= -\frac{1}{2} \times -2 \times \frac{\sin a}{\cos a \times \cos a}$$

$$= \tan a \cdot \sec a$$

Q2 if $f(x) = ax+1 \quad x \leq 2$

$2x^2+5 \quad x>0$ at $x=2$ then find function is differentia-ble or not

Solution L.H.D

$$Df(2) = \lim_{x\to 2^-} \frac{f(x) - f(2)}{x-2}$$

$$= \lim_{x\to 2^-} \frac{4x+1-(4\times 2+1)}{x=2}$$

$$= \lim_{x\to 2^-} \frac{4x+1-9}{x-2}$$

$$= \lim_{x\to 2^-} \frac{4x-8}{x-2}$$

$$= \lim_{x\to 2^-} \frac{4(x-2)}{x-2} = 4$$

$$Df(2-) = 4$$

R.H.D

$$Df(2+) = \lim_{x\to 2^+} \frac{x^2+5+9}{x-2}$$

$$= \lim_{x\to 2^+} \frac{x^2-4}{x-2}$$

$= \lim_{x \to 2^+} \frac{(2+x)(x-2)}{(x-2)}$

$2+2=4$

$Df(2^+) = 4$

R·H·D = L·H·D

f is differentiable at $x=2$

Q3 if $f(x) = 4x+7 \quad x<3$

$= x^2+3x+1 \quad x \geq 3 \quad$ at $x=3$

then find f is differentiable or not

Solution :- R·H·D

$Df(3^+) = \lim_{x \to 3^+} \frac{f(x)-f(a)}{x-3}$

$= \lim_{x \to 3^+} \frac{x^2+3x+1 - (3^2+3\times3+1)}{x-3}$

$= \lim_{x \to 3^+} \frac{x^2+3x+1-19}{-3+x}$

$= \lim_{x \to 3} \frac{x^2+3x-18}{x-3}$

$= \lim_{x \to 3^+} \frac{x(x+6)-3(x+6)}{x-3}$

$\lim_{x \to 3^+} \frac{(x+6)(x-3)}{(x-3)}$

$\therefore 3+6=9$

$Df(3^+) = 9$

L·H·D $= Df(3^-)$

$= \lim_{x \to 3} \frac{f(x)-f(3)}{x-3}$

$= \lim_{x \to 3^-} \frac{4x+7-19}{x-3}$

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$$\lim_{x \to 3} \frac{4x-12}{x-3}$$

$$= \lim_{x \to 3} \frac{4(x-3)}{x-3}$$

$Df(3^+) = 4$ R.H.D $\neq$ L.H.D

$\therefore$ f is not differentiable at $x=3$

Q7 if $f(x) = 8x - 5$ $x \leq 5$

$= 3x^2 - 4x + 7$ $x > 2$ at $x=2$ than

find f is differentiable or not

Solution

$f(2) = 8 \times 2 - 5 = 16 - 5 = 11$

RHD

$$Df(2^+) = \lim_{x \to 2^+} \frac{f(x) - f(2)}{x-2}$$

$$= \lim_{x \to 2^+} \frac{3x^2 - 4x + 7 - 11}{x-2}$$

$$= \lim_{x \to 2^+} \frac{3x^2 - 4x - 4}{x-2}$$

$$= \lim_{x \to 2^+} \frac{3x^2 - 6x - 2x - 4}{x-2}$$

$$\lim_{x \to 2^+} \frac{3x(x-2) + 2(x-2)}{x-2}$$

$$= \lim_{x \to 2^+} \frac{(3x+2)(x-2)}{(x-2)}$$

$$= 3 \times 2 + 2 = 8$$

$Df(2^+) = 8$

$$Df(2-) = \lim_{x \to 2^-} \frac{f(x) - f(2)}{x-2}$$

$$= \lim_{x \to 2^-} \frac{8x - 5 - 11}{x - 2}$$

$$\lim_{x \to 2^-} \frac{8x - 16}{x - 2}$$

$$\lim_{x \to 2^-} \frac{8(x-2)}{(x-2)}$$

$$Df(2-) = 8$$

LHD = RHD

f is differentiable at $x = 3$

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28 practical no3

Topic = application of derivative

1) find the interval in which function is increasing or decreasing

i) $f(x) = x^3 - 5x - 11$ ii) $f(x) = x^2 - 9x$

iii) $f(x) = 2x^3 + x^2 - 20x + 4$

iv) $f(x) = x^3 - 27x + 5$

v) $f(x) = 69 - 24x - 9x^2 + 2x^3$

2) find the interval in which function is concave upward and concave downward

i) $y = 3x^2 - 2x^3$

ii) $y = x^4 - 6x^3 + 12x^2 + 5x + 7$

iii) $y = x^3 - 27x + 5$

iv) $y = 69 - 24x - 9x^2 + 2x^3$

v) $y = 2x^3 + x^2 - 20x + 8$

sol^n

i) To find increasing

$p(x) > 0$

$p(x) = 3x^2 - 5$

$= 3x^2 > 5$

$x = \pm\sqrt{\frac{5}{3}}$

To find decreasing

$p(x) > 0$

$= 3x^2 - 5 < 0$

$= 3x^2 < 5$

$x = \pm\sqrt{\frac{5}{3}}$

$-\sqrt{5/3}$ $+\sqrt{5/3}$

$x \in \left(-\infty \; -\sqrt{\frac{5}{3}}\right) \cup +\left(\sqrt{5/3}, +\infty\right)$

$-\sqrt{5/3}$ $+\sqrt{5/3}$

$x \in \left(-\sqrt{5/3} \; + \sqrt{5/3}\right)$

ii) $f(x) = x^2 - 9x$

To find increasing

$f'(x) > 0$

$f'(x) = 2x - 4 > 0$

$2(x-2) > 0$

$x - 2 > 0$

$x = 2$

To find decreasing

$p(x) < 0$

$f'(x) < 0$

$= 2x - 4 > 0$

$= 2(x-2) < 0$

$x = 2$

$-\infty$ 2 $+\infty$

$x \in (2, \infty)$

$-\infty$ 2 $+\infty$

$x \in (-2, -\infty)$

$f(x) = 2x^3 - x^2 - 20x + 4$

$f'(x)\; x > 0$

for increasing

$p(x) > 0$

$f(x) = 6x^2 - 2x - 20 > 0$

$f'(x) < 0$

For decreasing

$f'(x) < 0$

$f(x) = 6x^2 - 2x - 20 <$

$6x(x-2)+10(x-2)>0$

$6x+10>0 \quad (x-2)>0$

$x=-5/3 \quad , x=2$

$+ \quad\quad -5/3 \quad\quad 2 \quad\quad +$

$x \in (-5/3 \quad +\infty) \cup (2, +\infty)$

$6x(x-2)+10(x-2)<0$

$= 6x+10<0 , x-2<0$

$+ \quad -5/3 \quad\quad 2$

$x \in (-5/3, 2)$

iv) $f(x) = x^3 - 27x + 5$

$f'(x) > 0$

for increasing

$f'(x) = 3x^2 - 27 > 0$

$= x^2 - 9 > 0$

$(x-3)(x+3) > 0$

$x = \pm 3$

$+ \quad -3 \quad - \quad +3 \quad +$

$x \in (-3, -\infty) \cup (+3, +\infty)$

$f'(x) < 0$

for decreasing

$f'(x) = 3x^2 - 27 < 0$

$(x^2 - 9) < 0$

$(x-3)(x+3)$

$x \pm 3$

$+ \quad -3 \quad - \quad +3 \quad +$

$x \in (-3, +3)$

$f(x) = 69 - 24x - 9x^2 + 2x^3$

$f'(x) > 0$

for increasing

$f'(x) = 6x^2 - 18x - 24 > 0$

$= x^2 - 3x - 4 > 0$

$= x^2 - 4x + x - 4$

$x(x+4) + 1(x-4) > 0$

$x \in (-1, -\infty) \cup (4, +\infty)$

$f'(x) < 0$

for decreasing

$f'(x) = 6x^2 - 18x^2 - 24 < 0$

$= x^2 - 3x - 4 < 0$

$= x^2 - 3x - 4 < 0$

$= x^2 - 4x + x - 4 < 0$

$x(x-4) + (x-4) <$

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i) $y = 3x^2 - 2x^3$

case 1 f^n is concave upwards iff $f''(y) > 0$

$f'(y) = 6x - 6x^2$

$f''(y) = 6x - 12x$

$f''(y) > 0$

$6 - 12x > 0$

$x = 1/2$

$-\infty \quad 1/2 \quad +\infty$

$x \in (-\infty, 1/2)$

case II f^n is concave downward iff $f''(y) < 0$

$f''(y) < 0$

$6 - 12x < 0$

$x = 1/2$

$\frac{1}{2} \quad \infty$

$x \in (1/2, +\infty)$

ii) $y = x^4 - 6x^3 + 12x^2 + 5x + 7$

case 1 f^n is concave upwards iff $f''(y) > 0$

$f'(y) = \cancel{4x^3}$

$f''(x) < 0$

$6(1 - 2x) < 0$

$-(2x - 1) < 0$

$x \in (\frac{1}{2}, \infty)$

ii) $y = x^4 - 6x^3 + 12x^2 + 5x + 7$

$y = f(x)$

$f(x) = x^4 - 6x^3 + 12x^2 + 5x + 7$

$f'(x) = 4x^3 - 18x^2 + 24x + 5$

$f''(x) = 12x^2 - 36x + 24$

f is concave upward iff

$f''(x) > 0$

$12x^2 - 36x + 24 > 0$

$12(x^2-3x+2)>0$

$x^3-3x-12>0$

88. $(x-2)(x-1)>0$

$x=2,1$

$x \in (-\infty,1) \cup (2,\infty)$

f is concave downward iff

$f''(x) \geq 0$

$12x^2-36x+24<0$

$12(x^2-3x+2)<0$

$x^2-3x+2 \geq 0$

$(x-2)(x-1)<0$

$x=2,1$

$x \in (1,2)$

$y = x^3-27x+5$

$y=f(x)$

$f(x)=x^3-27x+5$

$f'(x)=3x^2-27$

$f''(x)\ 6x$

f is concave upward iff

$f''(x)>0$

$6x>0$

$x>0$

$x=0$

$x \in (-\infty,0]$

$f(x)=69-24x-9x^2+2x^3$

f is concave upward iff

$f(x)=69-24x-9x^2+2x^3$

$f'(x)=-24-18x+6x^2$

$f''(x)=-18+12x$

$f''(x)>0$

$-18+12x>0$

$6(2x-3)$

$x=3/2 \quad x \in (3/2,\infty)$

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f is concave downward iff

$f''(x) < 0$

$-18 + 12x < 0$

$6(2x - 3) < 0$

$2x - 3 < 0$

$x = 3/2$

$x \in (-\infty, 3/2)$

$y = 2x^3 + x^3 - 20x + 8$

$y = f(x)$

$f(x) = 2x^3 + x^2 + -20x + 8$

$f(x) = 6x^2 + 2x - 20$

$f''(x) = 12x + 2$

f is concave upward iff

$f''(x) > 0$

$2(6x + 1) > 0$

$6x + 1 > 0$

$x = -1/6$

$(-\frac{1}{6}, \infty)$

f is ~~concave~~ downward iff

$f''(x) < 0$

$12x + 2 < 0$

$2(6x + 1) < 0$

$x = -1/6$

$x \in (-\infty, -1/6)$

18/12/19

practical -04

Topic :- application of derivative and newton method

find maximum and minimum value of following

Q1 find maximum and minimum value of following function

i) $f(x) = x^2 + \frac{16}{x^2}$

ii) $f(x) = 3 - 5x^3 + 3x^5$

iii) $f(x) = x^3 - 3x^2 + 1$ in $(-1/2, 4)$

iv) $f(x) = 2x^3 - 3x^2 - 12x + 1$ in $[-2, 3]$

Q2 find the root of the following eq^n :-

i) $f(x) = x^3 - 3x^2 - 55x + 9.5$ take $(x=0)$

ii) $f(x) = x^3 - 4x - 9$ in $[2,3]$

iii) $f(x) = x^3 - 1.8x^2 - 10x + 17$ in $[1,2]$

Sol^n

Q1 i) $f(x) = x^2 + \frac{16}{x^2}$

$\therefore f'(x) = 2x - \frac{32}{x^3}$

now consider

$f'(x) = 0$

$2x - \frac{32}{x^3}$

$x^4 = \frac{32}{2}$

$x^4 = 16$

$x = 2$

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$f''(x) = 2 + \frac{96}{x^4}$

$f''(2) = 2 + \frac{96}{16}$

$= 2 + 6$

$= 8 > 0$

$\therefore$ f has minimum value at $x = 2$

$f(2) = 2^2 + \frac{16}{2^2}$

$= 4 + \frac{16}{4}$

$= 4 + 4$

$= 8$

$f(-2) = 2 + \frac{96}{16}$

$= 2 + 6$

$8 > 0$

$\therefore$ f has minimum value at $x = 2$

ii) $f(x) = 3 - 5x^3 + 3x^5$

$f'(x) = 15x^4 - 15x^2$

consider

$f'(x) = 0$

$15x^4 = 15x^2$

$x^2 = 1$

$x \pm 1$

$f''(x) = -30x + 60x^3$

$f(1) = -30 + 60$

$= 30 > 0$

f has minimum value at $x = 1$

$f(1) = 3 - 5(1) + 3(1)$

$= 1$

$f''(-1) = -30(-1) + 60(-1)^3$

$= 30 - 60$

$= -30 < 0$

$\therefore f$ has maximum value at $x=1$

$\therefore f(-1) = 3 - 5(-1)^3 + 3(-1)^5$

$= 3 + 5 - 3$

$= 5$

$\therefore f$ has the maximum value ats at $x=1$ and has minim value 1 at $x=1$

Q.2 i) $f(x) = x^3 - 3x^2 - 55x + 9.5$

$x_0 = 0$

$f'(x) = 3x^2 - 6x - 55$

by Newton method

$$x_1 = x_0 - \frac{f(0)}{f'(x_0)}$$

$$x_1 = 0 + \frac{9.5}{55}$$

$= 0.1727$

$\therefore f(x_1) = f(0.1727) = (0.1727)^3 - 3(0.1727)^2 - 55(0.1727) + 9.8$

$= 0.0051 - 0.0895 - 9.4985 + 9.5$

$= 0.0829$

$f'(x_1) = 3(0.1727)^2 - 6(0.1729) - 55$

$= 0.0895 - 1.032 - 55$

$= -55.9467$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$= 0.1729 - \frac{0.0829}{55.9867}$

$= 0.1712$

$f(x_2) = (0.1712)^3 - (0.1712)^2 - 55(0.1712) + 9.5$

$= 0.005 - 0.0879 - 9.416 + 9.5 = 0.0011$

$x_3 = x_2 \frac{f(x_2)}{f'(x_1)} = 0.1712 + \frac{0.0011}{55.9393} = 0.1712$

$\therefore$ ~~The~~ The root of the equation is 0.1712

ii)

$f(x) = x^3 - 3x^2 + 1$

$f'(x) = 3x^2 - 6x$

consider $f'(x) = 0$

$\therefore 3x^2 - 6x = 0$

$3x(x-2) = 0$

$3x - 0$ or $x - 2 = 0$

$x = 0$ or $x = 2$

$f''(x) = 6x - 6$

$f''(x) = -6 > 0$

$\therefore f$ has maximum value at $x = 0$

$\therefore f(0) = 0 - 0 + 1$

$= 1$

$f''(2) = 6(2) - 6$

$= 6 > 0$

$\therefore f$ has minimum value at $k = 2$

$f(2) = 2^3 - 3(2)^2 + 1$

$= 8 - 12 + 1$

$= 3$

$\therefore$ f has maximum value 1 at $x=0$ and has minimum value 3 at $x=2$

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(iv) $f(x) = 2x^3 - 3x^2 - 12x + 1$

$\therefore f'(x) = 6x^2 - 6x - 12$

Consider $f'(x) = 0$

$\therefore 6x - 6x - 12 = 0$

$6(x^2 - x - 2) = 0$

$x^2 - 2x + x - 2 = 0$

$x(x-2) + 1(x-2) = 0$

$(x-2)(x+1) = 0$

$x = 2$ or $x = -1$

$\therefore f''(x) = 12x - 6$

$f''(2) = 24 - 6$

$= 18 > 0$

$\therefore$ f has minimum value at $x=2$

$f(2) = 2(8) - 3(4) - 12(2) + 1$

$= 16 - 12 - 24 + 1$

$= -19$

$f''(1) = 12(-1) - 6$

$-18 < 0$

f has maximum value at $x=-1$

$f(-1) = 2(-1)^3 - 3(-1)^2 - 12(-1) + 1$

$= -2 - 3 + 12 + 1$

$= 8$

$\therefore$ f has maximum value 8 at $x=-1$ and has minimum value 19 at $x=2$

2)(ii) $f(x) = x^3 - 4x - 9$ $(2, 3)$

$f'(x) = 3x^2 - 4$

$f(2) = 2^3 - 4(2) - 9$
$= 8 - 8 - 9$
$= -9$

$f(3) = 3^3 - 4(3) - 9$

$= 27 - 12 - 9 = 6$

let $x_0 = 3$ has the initial approximate
∴ by newton Method

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$3 - \frac{6}{23}$$

$$= 2.7392$$

$f(x_1) = f(2.7392) = (2.7392)^3 - 4(2.7392) - 9$

$= 20.5528 - 10.9565 - 9 = 0.596$

$f'(x_1) = 3(2.7392)^2 - 4 = 22.5096 - 4 = 18.5096$

$x_2 = x_1 \frac{f(x_1)}{f'(x_1)} = 2.7392 - \frac{0.592}{18.5096} = 2.7071$

$f(x_2) = f(2.7071) = (2.7071)^3 - 4(2.7071) - 9$

$= 19.8386 - 10.8284 - 9 = 0.0102$

$f(x_3) = (2.7015)^3 - 4(2.7015) - \frac{0.0102}{17.9857} = 2.7015$

∴ $f(x_3) = (2.7015)^3 - 4(2.7015) - 9 = 19.7158 - 10.806 - 9 = 0.0901$

$f'(x_3) = 3(2.7015)^2 - 4 = 21.8493 - 4 = 17.8493$

∴ $x_3 = 2.7015 + \frac{0.0901}{17.8493} = 2.7015 + 0.0050 = 2.7005$

SA

iii) $f(x) = x^3 - 1.8x^2 - 10x + 17$

$f'(x) = 3x^2 - 3.6x - 10$

$f(1) = 1^3 - 1.8(1)^2 - 10 + 17$

$= 1 - 1.5 - 10 + 17 = 6.2$

$f(2) = 2^3 - 1.8(4) - 20 + 17$

$= 8 - 7.2 - 3 = 2.2$

let $x_0 = 2$

$\therefore$ by ~~nob~~ newton method

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 2 - \frac{2.2}{5.2} = 2 - 0.4230 = 1.573$

$\therefore f(x_1) = f(1.573) = (1.573)^3 - 1.8(1.577)^2 - 10(1.377) + 17$

$= 3.9219 - 4.4764 - 15.77 + 17 = 0.6755$

$f'(x_1) = 3(1.577)^2 - 3.6(1.577) - 10 = 7.4608 - 5.6778 - 10$

$\therefore x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 1.577 + \frac{0.6755}{8.2164} = 1.577 + 0.0822 = 1.6592$

$f(x_2) = f(1.6592) = (1.6592)^3 - 1.8(1.6592)^2 - 10(1.6592) + 17$

$= 4.5677 - 4.9553 - 16.592 + 17 = 0.0204$

$f'(x_2) = 3(1.6592)^2 - 3.6(1.6592) - 10 = 8.2588 - 5.9731 - 10 = 7.71$

$x_3 = x_1 - \frac{f(x_2)}{f(x_1)} = 1.6592 + \frac{0.0204}{7.7173} = 1.6592 + 0.0026 = 1.6618$

$f(x_3) = f(4.6618) = (1.6618)^3 - 1.8(1.6618)^2 - 10(1.6618) + 17$

$= 4.5892 - 4.9708 - 16.618 + 17 = 0.0004$

$f'(3) = 3(1.6618)^2 - 3.16(1.6618) - 10 = 8.2897 - 5.924 - 10 = 7.694$

$x_4 = x_3 \frac{f(x_3)}{f'(x_3)} = 1.6618 + \frac{0.0004}{7.6972} = 1.6618$

The root of the equation is 1.6618

practical no 5

Q1 solve the following integration

i) $\int \frac{6\,dx}{\sqrt{x^2+2x-3}}$ ii) $\int (7e^{3x}+1)\,dx$ iii) $\int (2x^2-3\sin x+5\sqrt{x})\,dx$

iv) $\int \frac{x^3+3x+4}{\sqrt{x}}\,dx$ v) $\int t^7 \sin(2t^4)\,dt$

vi) $\int \sqrt{x}\,(x^2-1)\,dx$ vii) $\int \frac{1}{x^3} \sin\frac{1}{x^2}\,dx$

viii) $\int \frac{\cos x}{3\sqrt{\sin^2 x}}\,dx$ ix) $\int e^{\cos 2x} \sin 2x\,dx$

x) $\left(\frac{x^2-2x}{x^3-3x^2+1}\right)dx$

1) $\int \frac{dx}{\sqrt{x^2+2x-3}}$

$I = \int \frac{dx}{\sqrt{x^2+2x-3}}$

$= \int \frac{dx}{\sqrt{x^2+2x+1-4}}$

$= \int \frac{dx}{\sqrt{(x+1)^2-(2)^2}}$

comparing with $\int \frac{dx}{\sqrt{x^2-a^2}}$ $x^2 = (x+1)^2$

$a^2 = (2)^2$

$I = \log |x + \sqrt{x^2 - a^2}| + c$

$= \log |x + 1 + \sqrt{(x+1)^2 - (2)^2}| + c$

ii)

ii) $\int (4e^{3x} + 1)\, dx$

$I = \int (4e^{3x} + 1)\, dx$

$= \int 4e^{3x}\, dx + \int 1\, dx$

$= \frac{4e^{3x}}{3} + x + c$

iii) $\int (2x^2 - 3\sin x + 5\sqrt{x})\, dx$

$I = \int (2x^2 - 3\sin x + 5\sqrt{x})\, dx$

$= 2\int x^2\, dx - 3\int \sin x\, dx + 5\int \sqrt{x}\, dx$

$= \frac{2x^3}{3} + 3\cos x + 5 \times \frac{2}{3} x^{3/2} + c$

$= \frac{2}{3}x^3 + 3\cos x + \frac{10}{3}x^{3/2} + c$

) $\int \frac{x^3 + 3x + 4}{\sqrt{x}}\, dx$

$I = \int \frac{x^3 + 3x + 4}{\sqrt{x}}\, dx$

$= \int \left(\frac{x^3}{\sqrt{x}} + \frac{3x}{\sqrt{x}} + \frac{4}{\sqrt{x}} \right) dx$

$\int \left(\frac{x^3}{x^{1/2}} + \frac{3x}{x^{1/2}} + \frac{4}{x^{1/2}} \right) dx$

$= \frac{2}{7} x^{7/2} + 3 \cdot \frac{2}{3} x^{3/2} + 4x^{1/2} \cdot 2 + c$

$= \frac{2}{7} x^{7/2} + 3 \cdot \frac{2}{3} x^{3/2} + 4x^{1/2} \cdot 2 + c$

v) $\int t^7 \sin(2t^4)\, dt$

$I = \int t^7 \sin(2t^4)\, dt$

let $t^4 = x$

$4t^3\, dt = dx$

$I = \frac{1}{4} \int 4t^3 \cdot t^4 \sin(2t^4)\, dt$

$= \frac{1}{4} \int x \cdot \sin(2x)\, dx$

$= \frac{1}{4} \left[x \int \sin 2x - \int \left[\int \sin 2x \cdot \frac{d(x)}{dx} \right] \right]$

$= \frac{1}{4} \left[-\frac{x \cos 2x}{2} + \frac{1}{2} \int \cos 2x \cdot 1 \right]$

$\frac{1}{4} \left[-\frac{x \cos 2x}{2} + \frac{1}{4} \sin 2x \right] + c$

$= -\frac{1}{8} x \cos 2x + \frac{1}{16} \sin 2x + c$

Resubstituting $x = t^4$

$\therefore I = -\frac{1}{8} t^4 \cos(2t^4) + \frac{1}{16} \sin(2t^4) + c$

vi) $\int \sqrt{x} (x^2 - 1)\, dx$

$I = \int \sqrt{x} (x^2 - 1)\, dx$

$I = \int (\sqrt{x} \cdot x^2 - \sqrt{x})\, dx$

$\int (x^{5/2} - \sqrt{x})\, dx$

$\int x^{5/2}\, dx - \int \sqrt{x}\, dx$

$= \frac{2}{7} x^{7/2} - \frac{2}{3} x^{3/2} + c$

vii) $\int \frac{1}{x^3} \sin\left(\frac{1}{x^2}\right) dx$

$I = \int \frac{1}{x^3} \sin\left(\frac{1}{x^2}\right) dx$

Let $\frac{1}{x^2} = t$

$x^{-2} = t$

$\frac{-2}{x^3} dx = dt$

$I = \frac{1}{2} \int \frac{-2}{x^3} \sin\left(\frac{1}{x^2}\right) dx$

$-\frac{1}{2} \int \sin t \, dt$

$= -\frac{1}{2} [-\cos t] + c$

$\frac{1}{2} \cos t + c$

Resubstituting $t = \frac{1}{x^2}$

$I = \frac{1}{2} \cos\left(\frac{1}{x^2}\right) + c$

viii) $\int \frac{\cos x}{\sqrt[3]{\sin^2 x}}$

$I = \frac{\cos x}{\sqrt[3]{\sin^2 x}} dx$

Let $\sin x = t$

$\cos x \, dx = dt$

$I = \int \frac{dt}{\sqrt[3]{t^2}}$

$I = \int \frac{dt}{t^{2/3}}$

$= \int t^{-2/3} dt$

$3t^{1/3} + c$

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$3(\sin x)^{1/3} + c$

$3\sqrt[3]{\sin x} + c$

(x) $\int e^{\cos^2 x} \sin 2x \, dx$

$I = \int e^{\cos^2 x} \sin 2x \, dx$

Let $\cos^2 x = t$

$-2\cos x \sin x \, dx = dt$

$-2\sin 2x \, dx$

$I = -\int -\sin 2x \, e^{\cos^2 x} \, dx$

$= -\int e^t \, dt$

$-e^t + c$

Resubstituting $t = \cos^2 x$

$I = -e^{\cos^2 x} + c$

x) $\int \left(\frac{x^2 - 2x}{x^3 - 3x^2 + 1}\right) dx$

$I = \left(\frac{x^2 - 2x}{x^3 - 3x^2 + 1}\right) dx$

Let

$x^3 - 3x^2 + 1 = t$

$(3x^2 - 6x) \, dx = dt$

$\cancel{3}(x^2 - 3x) \, dx = dt$

$(x^2 - 2x) \, dx = \frac{dt}{3}$

$I = \frac{1}{t} \frac{dt}{3}$

$\frac{1}{3} \int \frac{dt}{t}$

$= \frac{1}{3} \log t + c$

$= \frac{1}{3}$

Resubsituting $t = x^3 - 3x^2 + 1$

$I = \frac{1}{3} \log (x^3 - 3x^2 + 1) + c$

Practical no 6

Topic :- application of integration and numerical integration

Q1 find the length of the following curve

1) $x = \sin t$, $y = 1-\cos t$ $t \in [0, 2\pi]$

2) $y = \sqrt{4-x^2}$ $x \in [-2, 2]$

3) $y = x^{3/2}$ in $[0, 4]$

4) $x = 3\sin t$ $y = 3\cos t$ $t \in [0, 2\pi]$

5) $x = \frac{1}{6}y^3 + \frac{1}{2y}$ on $y \in [1, 2]$

Q2 Using Simpson Rule solve the following

1) $\int_0^2 e^{x^2} dx$ with $n = 4$

2) $\int_0^4 x^2 dx$ with $n = 4$

3) $\int_0^{\pi/3} \sqrt{\sin x}\, dx$ with $n = 6$

Soln Q1 1) $x = \sin t$ $y = 1-\cos t$ $t \in [0, 2\pi]$

$$L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$$= \int_0^{2\pi} \sqrt{(1-\cos t)^2 + (\sin t)^2}\, dt$$

$$\int_0^{2\pi} \sqrt{1 - 2\cos t + \cos^2 t + \sin^2 t}\, dt$$

$$\int_0^{2\pi} \sqrt{1 - 2\cos t + 1}\, dt$$

$$\int_0^{2\pi} \sqrt{2 - 2\cos t}\, dt$$

$$\int_0^{2\pi} 2\left|\sin\frac{t}{2}\right| dt$$

$$\int_0^{2\pi} 2\sin\frac{t}{2}\, dt$$

$$= \left(-4\cos\left(\frac{t}{2}\right)\right)_0^{2\pi}$$

$$= (-4\cos\pi) - (-4\cos 0)$$

$$= 4 + 4$$

$$= 8$$

2) $y = \sqrt{4-x^2}$

$$\therefore \frac{dy}{dx} = \frac{1}{2\sqrt{4-x^2}} \times (-2x)$$

$$= \frac{x}{\sqrt{4-x^2}}$$

$$L = \int_{-2}^{2} \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$$

$$\int_{-2}^{2} \sqrt{1+\frac{x^2}{4-x^2}}\, dx$$

$$\int_{-2}^{2} \sqrt{\frac{4}{4-x^2}}\, dx$$

$$2\int_{-2}^{2} \frac{1}{\sqrt{2^2-x^2}}\, dx$$

$$2\left[\sin^{-1}\left(\frac{x}{2}\right)\right]_{-2}^{2} \quad 2\left[\pi/2 - \left(\frac{-\pi}{2}\right)\right]$$

$$= 2\left[\sin^{-1}(1) - \sin^{-1}(-1)\right]$$

$$L = 2\pi$$

3) $y = x^{3/2} \quad x \in (0, 9)$

$\frac{dy}{dx} = \frac{3}{2} x^{3/2}$

$L = \int_0^1 \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$

$= \int_0^4 \sqrt{1 + \frac{9}{4}x}\, dx$

$= \frac{1}{2} \int_0^4 \sqrt{4 + 9x}\, dx$

$\frac{1}{2} \left[\frac{(4 + 9x)^{3/2}}{3/2} \times \frac{1}{9} \right]$

$\frac{1}{27} \left[(4 + 9x)^{3/2} \right]$

$-\frac{1}{27} \left[(4 + 0)^{3/2} - (4 + 36)^{3/2} \right]$

$= \frac{1}{27} \left(40^{3/8} - 8 \right)$

4) $x = 3x \sin t \qquad y = 3 \cos t$

$\frac{dx}{dt} = 3 \cos t \qquad \frac{dy}{dx} = -3 \sin t$

$l = \int_0^{2\pi} \sqrt{\left(\frac{dy}{dt}\right)^2 + \left(\frac{dx}{dt}\right)^2}\, dt$

$\int_0^{2\pi} \sqrt{(3 \cos t)^2 + (3 \sin t)^2}\, dt$

$\int_0^{2\pi} \sqrt{9 \sin^2 t \;\; 9 \cos^2 t}\, dt$

$$\int_0^{2x} 3 \, dt$$

$$3\int_0^{2x} dt$$

$$3\,[t]_0^{2x}$$

$$3(2x-0)$$

$$L = 6x$$

$$x = \frac{1}{6}y^3 + \frac{1}{2y}$$

$$\frac{dx}{dy} = \frac{y^2}{2} - \frac{1}{2y^2}$$

$$\frac{dx}{dy} \quad \frac{y^4-1}{2y^2}$$

$$L = \int_0^2 \sqrt{1+\left(\frac{dx}{dy}\right)^2}\, dy$$

$$\int_0^2 \sqrt{1+} \quad \frac{(y^2-1)^2}{2y^2} \, dy$$

$$\int_0^2 \frac{\sqrt{(y^6-1) + 4xy^3 \& 1} \, dy}{4y}$$

$$\int_1^2 \sqrt{\frac{y^4+1}{(2y)^2}} \quad dy$$

$$\int_1^2 \frac{y^4+1}{2y^2} \, dy$$

$$\frac{1}{2}\int_1^2 y^2 \, dy + \frac{1}{2}\int_1^2 y^{-2} \, dy$$

$$\frac{1}{2}\left[\frac{4}{3} - \frac{y^{-2}}{1}\right]_1^0$$

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1.

$= \frac{1}{2}[\frac{1}{3} - \frac{1}{2} - \frac{1}{3} + 1]$

$= \frac{1}{2}[\frac{1}{3} + \frac{1}{2}]$

$= \frac{1}{2}[\frac{\pi}{6}]$

$L = \frac{17}{2}$ units

Q2

1. $\int_0^2 e^x dx$ with $n=4$

$h = \frac{b-a}{n} = \frac{2-0}{4} = 0.5$

x	0	0.5	1	1.5	2
y	1	1.284	2.71	9.58	54.59
	y	y_1	y_2	y_3	y_4

$= \frac{0.5}{3}[(1 + (54.59) + 5(1.28 + 9.98) + 2 \times 2.788 3]$

$\frac{0.5}{3}[55.982 + 43.868 + 5.436]$

$\int_0^2 ex^2 dx = 17.8535$

2) $\int x^2 dx$ $\int_0^4 x^2 dx$ $a=4$ $h = \frac{4 - 0}{h} = 1$

x	0	1	2	3	4
y	0	1	4	9	16
	y_1	y_2	y_3	y_4	y_5

$\int_0^1 x^2 dx = \frac{h}{3}(y_0 + y_0) + 4(y_1 + y_3) + 2^{1/2}]$

$= \frac{1}{3}(16 + 4(10) + 8)$

$\frac{64}{3}$

$\int_0^4 x^2 dx = 21.333$

3) $\int_0^{\pi/3} \sqrt{\sin x}\, dx = 0$

$h = \frac{\pi/3 - 0}{6} = \frac{\pi}{18}$

0	$\frac{\pi}{18}$	$\frac{2\pi}{18}$	$\frac{3\pi}{18}$	$\frac{4\pi}{18}$	$\frac{5\pi}{18}$
0	0.416	0.589	0.70	0.875	0.93
y_0	y_1	y_2	y_3	y_4	y_5

$\int^{2} \sqrt{\sin x}\, dx = \frac{1}{8}(y_0 + y_8 + 4(x_1 + y_3 + y_5) + 2(y_2 + y_6))$

$= \frac{\pi/18}{3}(0.916 + 0.93 + 4(0.41167 + 0.707) + 0.9752 + 2\{0.5898 + 0.[illegible]\})$

$= \frac{\pi}{54}[1.3473 + 7.946 + 2.773]$

$= \frac{\pi}{54} \times 12.3163$

$=$

$\int_0^{\pi/2} \sqrt{\sin}\, dx = 0.7044$

[illegible] 10/2[illegible]

practical no 7

topic :- differential equation

Q1 Solve the following differential equation

1) $x \frac{dy}{dx} + y = e^x$

2) $e^x \frac{dy}{dx} + 2e^x y = 1$

3) $x \frac{dy}{dx} = \frac{\cos x}{x} - 2y$

4) $x \frac{dy}{dx} + 3y = \frac{\sin x}{x^2}$

5) $e^{2x} \frac{dy}{dx} + 2 e^{2x} y = 2x$

6) $\sec^2 x \tan y \, dx + \sec^2 y + \tan x \, dy = 0$

7) $\frac{dy}{dx} = \sin^2 (x - y + 1)$

8) $\frac{dy}{dx} = \frac{2x + 3y - 1}{6x + 4y + 6}$

3)

Sol^n 1) $x \frac{dy}{dx} + y = e^x$

$\frac{dy}{dx} + \frac{1}{x}y = \frac{e^x}{x}$

$P(x) = \frac{1}{x}$ $Q(x) = \frac{e^x}{x}$

$IF = \int 1/x \, dx$

$= e^{\log x}$

$= x$

$y(\cdot if) = \int Q(x)(if) dx + c$

$xy = \int \frac{e^x}{x} \times x \, dx + c$

$= \int e^x dx + c$

$xy = e^x + c$

2) $e^x \frac{dy}{dx} + 2e^x y = 1$

$\frac{dy}{dx} + 2y = e^{-x}$

$P(x) = 2$ $Q(x) = e^x$

$if = e^{P(x) dx}$

$= e^{\int 2 dx}$

$= e^{2x}$

$y(I.F) = \int Q(x)(I.F) dx + c$

$y \cdot e^{2x} = \int e^{2x} \times e^{2x/x} + c$

$= \int e^{-x} dx + c$

$y \cdot e^{2x} = e^x + c$

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3) $x \frac{dy}{dx} = \frac{\cos x}{x} - 2y$

$\therefore \frac{dy}{dx} + \frac{2}{x} y = \frac{\cos x}{x^2}$

$P(x) = 2/x \quad Q(x) = \cos x / x^2$

$I.f = e^{\int P(x) dx}$

$= \int 2/x \, dx$

$= e^{2/x}$

$= e^{x} \, x^2$

$I.f = x^2$

$y(I.f) = \int Q(x) (I.f) dx + c$

$= \int \frac{\cos^2}{x^2} x^2 dx + c$

$\int \cos x + c$

$x^2 y = \sin x + c$

4) $x \frac{dy}{dx} + 3y = \frac{\sin x}{x^2}$

$\frac{dy}{dx} + \frac{3}{x} y = \frac{\sin x}{x^2}$

$P(x) = 3/x \quad Q(x) = \sin x / x^2$

$(I.f) = e^{\int P(x) dx}$

$= e^{\int 3/x \, dx}$

$= e^{\int x}$

$= e^{x^3}$

$I.f = x^3$

$y(I.f) = \int Q(x) (I.f) dx + c$

$x^3 y = \int \frac{\sin x}{x^3} \times x^3 dx + c$

$= \int \sin x \, dx + c$

$x^3 y = -\cos x + c$

5) $e^{2x} \frac{dy}{dx} + 2e^{2x} y = 2x$

$\frac{dy}{dx} + 2y = \frac{2x}{e^{2x}}$

$P(x) = 2 \quad Q(x) = 2x e^{-2x}$

$I.f = e^{\int P(x) dx}$

$= e^{\int 2 dx}$

$= e^{2x}$

$y(I.f) = \int Q(x)(I.f)\, dx + c$

$y(e^{2x}) = \int 2x e^{-2x} \times e^{2x} dx + c$

$y e^{2x} = \int 2x \, dx + c$

$\therefore y e^{2x} = x^2 + c$

6) $\sec^2 x + \tan y \, dx + \sec^2 y \tan x \, dy = 0$

$\sec^2 x \tan y \, dx = \sec^2 y \tan x \, dy$

$\frac{\sec^2 x \, dx}{\tan x} \quad \int \frac{\sec^2 y \, dy}{\tan y}$

$\log|\tan x| = \log|\tan y| + 1$

$\log|\tan x - \tan y| = c$

$\tan x \cdot \tan y = c$

$\frac{dy}{dx} = \sin^2 (x - y - 2) = n$

differentiating both side

$x - y + 1 = v$

$1 - \frac{dy}{dx} = \frac{dv}{dx}$

$1 - \frac{dv}{dx} = \frac{dy}{dx}$

$1 - \frac{dv}{dx} = \sin^2 x$

$\frac{dy}{dx} = \cos^2 x$

$\int \sec^2 v \, dv = \int dx$

$\tan v = n + c$

$\tan (n + y - 1) = n + c$

i) $\frac{dy}{dx} = \frac{2x + 3y - 1}{6x + 9y + 6}$

put $2x + 3y = v$

$2 + \frac{3 \cdot y}{dx} = \frac{dv}{dx}$

$\cancel{\text{to} \frac{dy}{dx}} = \frac{1}{3}\left(\frac{dv}{dx} - 2\right)$

$\frac{1}{3}\left(\frac{dv}{dx} - 2\right) \quad \frac{1}{3}\left(\frac{v-1}{v+2}\right)$

$\frac{dv}{dx} = \frac{v-1}{v+2} + 2$

$\frac{dv}{dx} = \frac{v-1}{v+2} + 2$

$$\frac{dv}{dx} \quad \frac{v-1+2+4}{v+2} = \frac{3v+3}{v+2}$$

$$= 3\left(\frac{v+1}{v+2}\right)$$

$$\int \frac{v+1}{v+1}\,dx + \int \frac{1}{v+1}\,dv = 3x + C$$

$$v + \log(v+1) = 3x + C$$

$$2x + 3y + \log(2x + 3y + 1)\ 3x + c$$

$$3y = x\log(2x + 3y + 1) + c$$

A.K.
02/02

Topic :- Euler's method

Q1 $\frac{dy}{dx} = y + e^x - 2$ $y(0) = 2$ $h = 0.5$ find $y(2) = ?$

sol^n $f(x) = y + e^x - 2$ $x_0 = 0$ $y(0) = 2$ $h = 0.5$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	2	1	2.5
1	0.5	2.5	2.1487	3.5743
2	1	3.5743	4.2925	5.7205
3	1.5	5.7205	8.2021	9.8215
4	2	9.8215		

$\therefore y(2) = 9.8215$

Q2 $\frac{dy}{dx} = 1 + y^2$, $y(0) = 1$ $h = 0.2$ find $y(1) = ?$
$y_0 = 0$ $y_0 = 0$ $h = 0.2$

n	x_n	y_n	$f(x_n, y_n)$	$y(n+1)$
0	0	0	1.04	0.2
1	0.2	0.2	1.1664	0.408
2	0.4	0.408	1.411	0.6412
3	0.6	0.6412	1.825	0.9237
4	0.8	0.9234		1.2939
5	1	1.2939		

$y(1) = 1.2939$

$\frac{dy}{dx} = \frac{\sqrt{x}}{\sqrt{y}}$ $y(0) = 1$ $h = 0.2$ find $y(1) = ?$ $x_0 = 0$ $y(0) = 1$ $h = 0.2$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	1	0	1
1	0.2	1	0.4472	1.0894
2	0.4	1.0894	0.6059	1.2105
3	0.6	1.2105	0.7070	1.3513
4	0.8	1.3513	0.7696	1.5051
5	1	1.5051		

$y(1) = 1.5051$

Q4 $\frac{dy}{dx} = 3x^2+1$ $y(1)=2$ find $y(2)$ $h=0.5$ $h=0.25$ $y_0=2$ $x_0=1$ $h=0.5$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	2	4	4
1	1.5	4	7.75	7.875
2	2	7.875		

$y(2) = 7.875$

5) $y_0=2$ $x_0=1$ $h=0.25$

2)

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	2	4	3
1	1.25	3	5.6875	4.4218
2	1.5	4.4218	59.6564	19.3360
3	1.75	19.3360	1122.6426	299.9960
4	2	299.9960		

$y(2) = 299.9960$

Q6 $\frac{dy}{dx} = 5xy+2$ $y(1)=1$ $h=0.2$ $x_0=1$ $y_0=1$ $h=0.2$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	1	3	3.6
1	1.2	3.6		

$y(1.2) = 3.6$

Topic :- Limit and partial - order derivative

1) Evaluate the foll limits

1) $\lim_{(x,y)\to(-4,-1)} \dfrac{x^3-3y+y^2-1}{xy+5}$ ii) $\lim_{(x,y)\to(2,0)} \dfrac{(y+1)(x^2+y^2-4x)}{x+3y}$

3) $\lim_{(x,y,z)\to(1,1,1)} \dfrac{x^2-y^2-z^2}{x^3-x^2y^2}$

2) find f_x f_y for each of the following

i) $f(x,y) = xye^{x^2+y^2}$ ii) $f(x,y) = e^x \cos y$

iii) $f(x,y) = x^3y^2 - 3x^2y + y^3 + 1$

3) Using defination find values of f_x f_y at $(0,0)$ for $f(x,y) = \dfrac{2x}{1+y^2}$

4) find all second order partial derivatives of f also verify whether $f_{xy} = f_{yx}$

i) $f(x,y) = \dfrac{y^2-xy}{x^2}$ ii) $f(x,y) = x^3+3x^2y^2-\log(x^2+1)$

~~iii) f(x,y) = ...~~

iii) $f(x,y) = \sin(xy) + e^{x+y}$

5) find the linearization of $f(x,y)$ at given point :-

i) $f(x,y) = \sqrt{x^2+y^2}$ at $(1,1)$

2) $f(x,y) = 1-x+y\sin x$ at $\left(\frac{\pi}{2},0\right)$

3) $f(x,y) = \log x + \log y$ at $(1,1)$

solution

1)

i) $\lim_{(x,y)\to(-4,-1)} \dfrac{x^3-3y+y^2-1}{xy+5}$

$= \dfrac{-(-4)^3-3(-1)+(-1)^2-1}{(-4)(-1)+5}$

$\dfrac{64+3+1-1}{4+5}$

$\dfrac{67}{9}$

ii) $\lim_{(x,y)\to(2,0)} \dfrac{(x+1)(x^2+y^2-4x)}{x+3y}$

$\dfrac{0+1\ ((2)^2+(0)^2-4(2))}{2+3(0)}$

$\dfrac{1\ (4+0-8)}{2}$

$\dfrac{-4}{2}=-2$

) $\lim_{(x,y,z)\to(1,1,1)} \dfrac{x^2-y^2-z^2}{x^3-x^2yz}$

$\dfrac{(1)^2-(1)^2(1)^2}{(1)-(1)^2(1)(1)}$

$\dfrac{1-1}{1-1}=\dfrac{0}{0}=0$

i) $f(x,y) = xy e^{x^2+y^2}$

$2xy \cdot e^{x^2} \cdot e^{y^2}$

$f(x) = \frac{\partial}{\partial x} x \cdot y \cdot e^{x^2} e^{y^2}$

$= y e^{y^2} \frac{d}{dx} x \cdot e^{x^2}$

$y e^{y^2} [x \frac{d}{dx} e^{x^2} + e^{x^2} \frac{d}{dx} x]$

$= (2x^2+1) y e^{x^2+y^2}$

$f_y \; 2x e^{x^2} \frac{d}{dy} y \cdot e^{y^2}$

$= x e^{x^2} [y \frac{d}{dy} e^{y^2} + e^{y^2} \frac{d}{dy} y]$

$= x e^{x^2} [2y^2 e^{y^2} + e^{y^2}]$

$f_y = x \cdot e^{x^2+y^2} + 2xy^2 e^{x^2+y^2}$

ii) $f(x,y) = e^x \cos y$

$f(x) = \cos y \, e^x$

$f(y) = e^x - \sin y$

$f_y = \sin y \, e^x$

iii) $f(x,y) = x^3 y^3 - 3x^2 y + y^3 + 1$

$f(x) = y^2 3x^2 - 3y 2x + 0 + 0$

$3x^2 y^2 - 6xy$

$f(y) = x^3 2y - 3x^2 + 3y^2$

$2x^3 y - 3x^2 + 3y^2$

Q3 Using defination find values of $f(x)$ f_y at $(0,0)$ for $f(x,y) = \frac{2x}{1-y^2}$

$f(x)(a,b) = \lim_{h \to 0} f(h,0) - f(0,0)$

$= \lim_{h \to 0} \frac{2h - 0}{h} = 2$

$$f_y(0,0) = \lim_{h \to 0} \frac{f(0,h) - f(0,0)}{h}$$

$$= \lim_{h \to 0} \frac{0-0}{h} = \frac{0}{h} = 0$$

$$f(x) = 2 \quad f(y) = 6$$

Q) find all second order partial derivatives of f also verify whether

$$f(x,y) = f(y,x)$$

1) $$f(x,y) = \frac{y^2 - xy}{x^2}$$

$$f(x,x) = \frac{d^2 f}{dx^2} \qquad f(x,y) = \frac{d^2 f}{dy^2}$$

by applying $\frac{u}{v}$ = rule

$$f(x) = \frac{x^2(0-y) - (y^2 - xy)\,2x}{x^4}$$

$$= \frac{-x^2 y - 2xy^2 + 2x^2 y}{x^4}$$

$$\therefore f(x) = \frac{x^2 y - 2xy^2}{x^4}$$

$$f''(x) = \frac{x^4(2xy - 2y^2) - (x^2 y - 2xy^2)(4x^3)}{x^8}$$

$$= \frac{2x^5 y - 2x^4 y^2 - (4x^5 y - 8x^4 y^2)}{x^8}$$

$$= \frac{2x^5 y + 6x^4 y^2}{x^8}$$

$$f(xx) = f''(x) = \frac{6y^2 - 2xy}{x^4}$$

$fy = \frac{1}{x}(2y-x) \therefore fy = \frac{2y-x}{x^2}$

$fyy = \frac{1}{x^2}(2) = \frac{2}{x^2}$

$f(x,y) = \frac{2y-x}{x^2}$

$\frac{x^2(-1)-(2y-x)(2x)}{x^4}$

$\frac{-x^2-4xy+2x^2}{x^4}$

$\frac{x^2-4xy}{x^4}$

$\frac{x-4y}{x^4}$

$f(x,y) = \frac{x-4y}{x^3}$

$f(y,x) = \frac{x^2y-2xy}{x^4}$

$= \frac{x^2-4xy}{x^4} = \frac{x-4y}{x^3}$

$= f(x,y)$

$= f(yx) = f(xy)$

find the derivative of $f(x,y)$ at the given point

1) $f(x,y) = \sqrt{x^2+y^2}$ at $(1,1)$

$f(x) = \frac{1}{2\sqrt{x^2+y^2}} \times 2x$ $\quad fy = \frac{2y}{2\sqrt{y^2+x^2}}$

$= \frac{x}{\sqrt{x^2+y^2}}$ $\quad = \frac{y}{\sqrt{x^2+y^2}}$

$f_x(1,1) = \frac{1}{\sqrt{2}}$ $= \frac{1}{\sqrt{2}}$

$L(x,y) = f(1,1) + \cancel{f_x} f_x(1,1)(x-1) + f_y(1,1)(y-1)$

$\sqrt{2} + \frac{x-1}{\sqrt{2}} + \frac{y-1}{\sqrt{2}}$

$\sqrt{2} + \frac{x+y-2}{\sqrt{2}}$

$= \frac{x+y}{\sqrt{2}}$

ii) $f(x,y) = 1-x+y\sin x$ at $\pi/2$

$f(\pi/2, 0) = 1-\pi/2+0+\sin\pi/2$

$f(\pi/2, 0) = 2-\pi/2$

$f(x) = -1+y\cos x$ $f_y = \sin x$

$f(x)(\pi/2, 0) = -1+0\cos\pi/2 = -1$

$f_y(\pi/2, 0) = \sin\pi/2$

$(x,y) = f(\pi/2, 0) + f_x(\pi/2, 0)(x-\pi/2)$
$+ f_y(\pi/2, 0)(y-0)$
$= 2-\pi/2 + (-1)(x-\pi/2 + (y))$
$L(x,y) = 1-x+y$

$f(x,y) = \log x + \log y$
$f(1,1) = \log 1 + \log 1$
$= 0$

$f(x) = \frac{1}{x}$ $f_y = \frac{1}{y}$

Q1
i)
ii)
iii)
Q2
Q3
Q4
i)
ii)
Q5
Q7
i) Sc

Practical 10

Q1 Find the directional derivative of the given vector at the given points **56**

i) $f(x,y) = x+2y-3$ at $\bar{u} = 3\hat{i}-\hat{j}$, $a(1,-1)$

ii) $f(x,y) = y^2-4x+1$ at $\bar{u} = \hat{i}+5\hat{j}$, $a(3,4)$

iii) $f(x,y) = 2x+3y$ at $\bar{u} = 3\hat{i}+4\hat{j}$, $a(1,2)$

Q2 Find gradient for the following function at the given point

i) $f(x,y) = x^4+y^x$ $a=(1,1)$

ii) $f(x,y) = (\tan^{-1}x)\, y$ $a=(1,-1)$

iii) $f(x,y,z) = xyz - e^{x+y+z}$ $a=(1,-1,0)$

Q3 Find the equation of tangent and normal of each of the following curve

i) $x^2\cos y + e^{xy} = 2$ at $(1,0)$

ii) $x^2+y^2-2x+3y+2=0$ at $(2,-2)$

Q4 Find the equation of tangent and normal line to each of the following

i) $x^2-2yz+3y+xz=7$ at $(2,-1,0)$

ii) $3xyz-x-y+z=-4$ at $(1,-1,2)$

Q5 Find the local maximum and minimum for the following function

i) $f(x,y) = 3x^2+y^2-3xy+6x-4y$

ii) $f(x,y) = 2x^4+3x^2y-y^2$

i) Solution

$\bar{u} = 3\hat{i}-\hat{j}$

$\hat{u} = \frac{\bar{u}}{|u|} = \frac{1}{\sqrt{3^2+(-1)^2}}(3\hat{i}-\hat{j})$

$\hat{u} = \frac{1}{\sqrt{10}}(3\hat{i}-\hat{j})$

$u = \frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}$

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$f_x = (1,1) = 1 \qquad f_y(1,1) = 1$

$L(x,y) = f(1,1) + f_x(1,1)(x-1) + f_y(1,1)(y-1)$
$= 0 + 1(x-1) + 1(y-1)$

$L(x,y) = x + y - 2$

$a = (1,-1)$

$f(a) = 1 + 2(-1) - 3$
$= 1 + (-2) - 3$
$= -4$

$f(a+hu) = f\left((1,-1) + h\left(\frac{1}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)\right)$

$= f\left(\left(1 + \frac{3}{\sqrt{10}}h\right), \left(-1 - \frac{h}{\sqrt{10}}\right)\right)$

$= \left(1 + \frac{3}{\sqrt{10}}h + 2\left(-1 - \frac{h}{\sqrt{10}}\right)\right)$

$= 1 + \frac{3}{\sqrt{10}}h - 2 - \frac{2h}{\sqrt{10}} - 3$

$f(a+hu) = \frac{h}{\sqrt{10}} - 4$

$\therefore D_u f(a) = \lim_{h \to 0} \frac{f(a+hu) - f(a)}{h}$

$\lim_{h \to 0} \frac{h/\sqrt{10} - 4 - (-4)}{h}$

$= \frac{1}{\sqrt{10}} \lim_{h \to 0} \frac{h}{h}$

$= \frac{1}{\sqrt{10}}$

(ii) solution

$f(x,y) = y^2 - 4x + 1$ $\qquad a(3,4)$

$\bar{u} = i + 5j$

$\hat{u} = \dfrac{\bar{u}}{|\bar{u}|} = \dfrac{i+5j}{\sqrt{1^2+5^2}} = \dfrac{1}{\sqrt{26}}(\hat{i} + 5\hat{j})$

$\hat{u} = \left(\dfrac{1}{\sqrt{26}}, \dfrac{5}{\sqrt{26}}\right)$

$f(a) = (4)^2 - 4(3) + 1$

$= 16 - 12 + 1$

$f(a) = 5$

$f(a+hu) = f\left((3,4) + h\left(\dfrac{1}{\sqrt{26}}, \dfrac{5}{\sqrt{26}}\right)\right)$

$= f\left(\left(3 + \dfrac{h}{26}\right), \left(4 + \dfrac{5h}{\sqrt{26}}\right)\right)$

$= \left(4 + \dfrac{5h}{\sqrt{26}}\right)^2 - 4\left(3 + \dfrac{h}{\sqrt{26}}\right) + 1$

$= 16 + \dfrac{40h}{\sqrt{26}} + \dfrac{25h^2}{26} - 12 - \dfrac{4h}{\sqrt{26}} + 1$

$= \dfrac{25h^2}{\sqrt{26}} - \dfrac{36h}{\sqrt{26}} + 5$

$D_{\cdot} f(a) = \lim\limits_{h \to 0} \dfrac{f(a+hu) - f(a)}{h}$

$= \lim\limits_{h \to 0} \dfrac{\dfrac{25h^2}{26} - \dfrac{36h}{\sqrt{26}} + 5 - 5}{h}$

$\lim\limits_{h \to 0} \dfrac{h\left(\dfrac{25h}{26} - \dfrac{36}{\sqrt{26}}\right)}{h}$

$$\lim_{h\to 0} \frac{25h}{26} - \frac{36}{\sqrt{26}}$$

$$D_u f(a) = \frac{36}{\sqrt{36}}$$

(iii) *solution* $f(x,y) = 2x+3y \qquad a(1,2)$

$$\bar{u} = 3\hat{i} + 4\hat{j}$$

$$\hat{u} = \frac{\bar{u}}{|\bar{u}|} = \frac{1}{\sqrt{3^2+4^2}} (3\hat{i} + 4\hat{j})$$

$$= \frac{1}{\sqrt{25}} (3\hat{i} + 4\hat{j})$$

$$\hat{u} = (3/5, 4/5)$$

$$f(a) = 2(1) + 3(2)$$

$$= 2 + 6$$

$$f(a) = 8$$

$$f(a+hu) = f((1,2) + h(3/5, 4/5))$$

$$= f\left(\left(1+\frac{3h}{5}\right), \left(2+\frac{4h}{5}\right)\right)$$

$$= 2\left(1+\frac{3h}{5}\right) + 3\left(2+\frac{4h}{5}\right)$$

$$= 2 + \frac{6h}{5} + 6 + \frac{12h}{5}$$

$$= \frac{18h}{5} + 8$$

$$D_u f(a) = \lim_{h\to 0} \frac{f(a+hu) - f(a)}{h}$$

$$D_u f(a) \lim_{h\to 0} \frac{\frac{18h}{5} + 8 - 8}{h}$$

$$\therefore \lim_{h\to 0} \frac{18h/5}{h}$$

$$b \cup f(a) = \frac{18}{5}$$

Q2
i) $f(x,y) = x^y + y^x$

$$f x = \frac{\partial}{\partial x}(x^y + y^x)$$

$$f(x) = y x^{y-1} + y^x \cdot \log y$$

$$f(y) = \frac{d}{dy} x^y + y^x$$

$$f(y) = x y^{x-1} + x^y \cdot \log x$$

$$\nabla f(x,y) = (fx, fy)$$

$$\nabla f(x,y) = (y x^{y-1} + y^x \log y, x y^{x-1}, x^y \log x)$$

$$\nabla f(1,1) = (1(1)^{1-1} + 1^1 \log 1, f(1)^{1-1} + 1^1 \log 1)$$

$$\nabla f(1,1) = (1,1)$$

ii) solution

$$f(x,y) = (\tan^{-1}(x))\, y^2$$

$$f(x) = \frac{d}{dx}(\tan^{-1} x)\, y^2$$

$$f(x) = \frac{y^2}{1+x^2}$$

$$fy = \frac{d}{dy}(\tan^{-1}(x))\, y^2$$

$$= 2y \tan^{-1} x$$

$$\nabla f(x,y) = f(fx, fy)$$

$$= \left(\frac{y^2}{1+x^2}, 2y \tan^{-1} x\right)$$

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$$= \left(\frac{1^2}{1+(1)^2}, 2(-1)\tan^{-1}(-1)\right)$$

$$= \left(\frac{1}{1+1}, -2 < \frac{5\pi}{4}\right)$$

$$\nabla f(1,-1) = \left(\frac{1}{2}, \frac{-5\pi}{2}\right)$$

iii) solution

$$f(x,y,z) = xyz - e^{x+y+z}$$

$$f(x) = yz - e^{x+y+z}$$

$$fy = xz - e^{x+y+z}$$

$$fz = xy - e^{x+y+z}$$

$$\nabla f(x,y,z) = (fx, fy, fz)$$

$$= (yz - e^{x+y+z}, xz - e^{x+y+z}, xy - e^{x+y+z})$$

$$\nabla f(1,-1,0) = (-1\times 0 - e^{-1+1+0}, 1(0) - e^{1-1+0}, 1(-1)\, e^{1-1,0})$$

$$\nabla f(1,-1,0) = (-1, -1, -2)$$

i) $x^2 \cos y + e^{xy} = 2$ at $(1,0)$

$$f(x) = \cos y\, 2x + e^{xy} y$$

$$dy = x^2(-\sin y) + e^{xy} x$$

$(x_0, y_0) = (1,0)$ $\therefore x_0 = 1$ $y_0 = 0$

eqn of tanget

$$f(x)(x - x_0) + dy(y - y_0) = 0$$

$$dx(x_0, y_0) = \cos 0\, 2(1) + e^0\, 0$$

$$= 1(2) + 0$$

$$= 2$$

$$dy(x_0 y_0) = (1)^2(-\sin 0) + e^0\, 1$$

$$= 0 + 1 \cdot 1$$

$$= 1$$

$2(x-1) + 1(y-0) = 0$

$2x - 2 + y = 0$

$2x + y - 2 = 0$ it is the required eqn of tanget

eqn of normal

$= ax + by + c = 0$

$= bx + ay + d = 0$

$1(1) + 2(y) + d = 0$

$1 + 2y + d = 0$

$1 + 2(0) + d = 0$

$d + 1 = 0$

$d = -1$

ii) $x^2 + y^2 - 2x + 3y + 2 = 0$

$dx = 2x + 0 + 0 + 0$

$= 2x - 2$

$dy = 0 + 2y - 0 + 3 + 0$

$= 2y + 3$

$(x_0 y_0) = (2, -2)$ $x_0 = 2$ $y_0 = -2$

$dx(x_0 y_0) = 2(2) - 2 = 2$

$dy(x_0 y_0) = 2(-2) + 3 = -1$

eqn of tanget

$dx(x - x_0) + dy(y - y_0) = 0$

$2(x-2) + (-1(y+2) = 0$

$2x - 2 - y - 2 = 0$

$2x - y - 4 = 0$ it is required eqn of tanget

eqn of normal

$= ax + by + c = 0$

$bx + ay + d = 0$

$-1(x) + 2(y) + d = 0$

$-x + 2y + d = 0$ at $(2, -2)$

$-2 + 2(-2) + d = 0$

$-6 + d = 0$

$d = 6$

Q8 Find the eqn of target and normal line to each of the following surface

i) $x^2 - 2yz + 3y + xz = 7$ at $(2, 1, 0)$

$dx = 2x - 0 + 0 + z$

$dx = 2x + z$

$dy = 0 - 2z + 3 + 0$

$= 2z + 3$

$dz = 0 - 2y + 0 + 0x$

$= 2 2y + x$

$(x_0 y_0 z_0) = (2, 1, 0)$ $\therefore x_0 = 2$ $y_0 = 1$ $z_0 = 0$

$dx(x_0, y_0, z_0) = 4(2) + 0 = 8$

$dy(x_0, y_0 z_0) = 2(0) + 3 = 3$

$dz(x_0 y_0 z_0) = -2(1) + 2 = 0$

eqn of tanget

$dx(x_0 - x_0) + dy(y_0 - y_0) + dz(z_0 - z_0) = 0$

$8(x-2) + 3(y-1) + 0(z-0) = 0$

$8x + -8 + 3y - 3 = 0$

$8x + 3y - 11 = 0$ This is reqrerred equation

Eqn at normal at $(8, 3, -11)$

$$\frac{x - x_0}{dx} = \frac{y - y_0}{dy} = \frac{z - z_0}{dz}$$

$$\frac{x-2}{8} = \frac{y-1}{3} = \frac{z+11}{0}$$

ii) $3xyz - xy + z = -4$ at $(1,-1,2)$

$3xyz - x - y + z + 4 = 0$ at $(1,-1,2)$

$dx = 3xyz - 1 - 0 + 0 + 0$

$= 3yz - 1$

$dy = 3xz - 0 - 1 + 0 + 0$

$= 3xz - 1$

$dz = 3xy - 0 - 0 + 1 + 0$

$= 3xy + 1$

$(x_0, y_0, z_0) = (1,-1,2)$ $x_0 = 1$ $y_0 = -1$ $z_0 = 2$

$dx(x_0\ y_0\ z_0) = 3(-1)(2) - 1 = -7$

$dy(x_0\ y_0\ z_0) = 3(+1)(2) - 1 = 5$

$dz(x_0\ y_0\ z_0) = 3(1)(-1) + 1 = -2$

eq^n of tanget

$-7(x-1) + 5(y+1) - 2(z-2) = 0$

$-7x + 7 + 5y + 5 - 2z + 4 = 0$

$-7x + 5y - 2z + 16 = 0$ this is required eq^n of tanget

eq^n of normal at $(-7, 5, -2)$

$$\frac{x-x_0}{dx} = \frac{y-y_0}{dy} = \frac{z-z_0}{dz}$$

$$= \frac{x-1}{7} = \frac{y+1}{5} = \frac{z-2}{2}$$

Q5 Find the local maxima and minima for the following

$f(x,y) = 3x^2 + y^2 - 3xy + 6x - 4y$

$dx = 6x + 0 - 3y + 6 - 0$

$= 6x - 3y + 6$

$dy = 0 + 2y - 3x + 0 - 4$

$= 2y - 3x - 4$

$dx = 0$

Q2

$3(2x-y+3)=0$

$2x-y+3=0$

$2x-y=-2$

$dy=0$

$2y-3x-4=0$

$2y-3x=4$

multiply eqn 1 with 2

$4x-2y=-4$

$2y-3x=4$

$x=0$

substitute value of x in eqn

$2(0)-y=-2$

$-y=-2$

$\therefore$ critical points are $(0,2)$

$r=f(xx)=6$

$t=fyy=2$

$s=fxy=-3$

here $r>0$

$rt-s^2$

$=6(2)-(3)^2$

$=12-9$

$=3>0$

$\therefore$ f has maximum at $(0,2)$

$3x^2+y^2-3xy+6x-4y$ at $(0,2)$

$3(0)^2+(2)^2-3(0)(2)+6(0)-4(2)$

$0+4-0+0-8=-8$

$f(x,y)=2x^4+3x^2y-y^2$

$dx=8x^3+6xy$

$dy = 3x^2 - 2y$
$dx = 0$

$8x^3 + 6xy = 0$

$2x \cdot (4x^2 + 3y) = 0$

$dy = 0$

multiply $3x^2 - 2y = 0$

eqn (1) with 3

(2) with 4

$12x^2 + 9y = 0$

$12x^2 - 8y = 0$

$12y = 0$

substitute value of y in eqn $y = 0$

$4x^2 + 3(0) = 0$

$4(x^2) = 0$

$x = 0$

Critical point is (0,0)

$r = f(xx) = 24x^2 + 6x$

$t = fyy = 0 - 2 = -2$

$s = fxy = 6x - 0 = 6(0) = 0$

r at (0,0)

$= 24(0) + 6(0) = 0$

$r = 0$

$rt - s^2 = 0(-2) - (s)^2$

$= 0 - 0 = 0$

$r = 0$ and $rt - s^2 = 0$

nothing to say

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(iii) $f(x,y) = x^2 - y^2 + 2x + 8y - 70$

$f(x) = 2x + 2$

$bfy = 2y + 8$

$f(x) = 0 \qquad 2x + 2 = 0$

$x = -1$

$dy = 0 \qquad -2y + 8 = 0$

$y = \frac{-8}{-2} \quad 4$

$y = 8$

Critical point is $(0\ -1, 8)$

$r = f(x\,x) =$

$t = fyy = -2$

$s = fxy = 0$

$r > 0$

$rt - s^2 = 2(-2) - (0)^2$

$= -4 - 0$

$= -4$

$f\ x\ d\ (x, y)\ z$ at $(-1, 8)$

$(-1)^2 - (8)^2 + 2(-1) + 8(9) = 70$

$1 + 16 - 2 + 32 - 70$

$17 + 30 - 70$

$= 37 - 70 \quad = 33$

Ali
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