Computational Fluid Dynamics (CFD)

Example:

$$\frac{\partial u}{\partial t} + c\frac{\partial u}{\partial x} = 0$$

$$\frac{\partial u}{\partial x} \approx \frac{\Delta u}{\Delta x} \quad \& \quad \frac{\partial u}{\partial t} \approx \frac{\Delta u}{\Delta t} \text{ (Approximation)}$$

Now for 1D convection we use the backward scheme,
i.e the next value of u depends upon its previous val/node

$$\frac{\partial u}{\partial t} = \frac{U' - U}{\Delta t}$$

Where U' is present node and U is previous node

We get this equation,

$$\frac{u_i^{n+1} - u_i^n}{\Delta t} + c\frac{u_i^{n+1} - u_{i-1}^{n+1}}{\Delta x} = 0$$

Rearrangine we get

$$u_i^{n+1} = u_i^n - c\frac{\Delta t}{\Delta x}(u_i^n - u_{i-1}^n)$$

We obtained 1D Linear Convection equation for calculating next node

We can replace 'c' with u (Current) for Nonlinear Convection

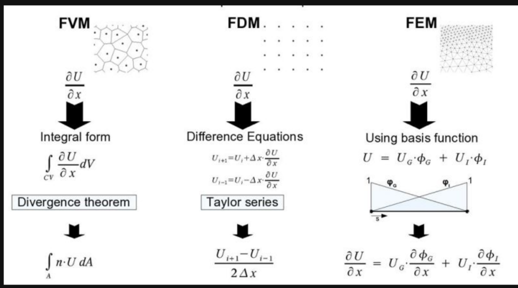
Partial Differential Equations
↓
Discretization
↓
Algebraic Equations (Simplified eqns)
↓
Mesh (Initialization of Array)
↓
Post Processing

Python Libraries

- numpy
- matplotlib

Numerical methods

- Finite Difference Method FDM
- Finit Volume Method FVM
- Finite Element Method FEM



Recommended Books by Anirudh Sir

- Numerical Heat Transfer and Fluid Flow
By Patankar
- CFD the Basics of Application
By John Anderson

```
We use numpy and pyplot to create mesh and visualize this, You can refer the github repository
```

```
Similarly we discretize PDE's for Diffusion, Convection-Diffusion, 2D (Another term added to PDE's we used for 1D)
```

1D Linear Convection

$$u_i^{n+1} = u_i^n - c\frac{\Delta t}{\Delta x}(u_i^n - u_{i-1}^n)$$

1D Non Linear Convection

$$u_i^{n+1} = u_i^n - u_i^n\frac{\Delta t}{\Delta x}(u_i^n - u_{i-1}^n)$$

1D Diffusion

$$u_i^{n+1} = u_i^n + \frac{\nu \Delta t}{\Delta x^2}(u_{i+1}^n - 2u_i^n + u_{i-1}^n)$$

ν - Kinematic Viscocity/ Diffusion Coefficient

Courant number CFL

$$\sigma = \frac{u \Delta t}{\Delta x} \leq \sigma_{\max}$$

1D Burgers (Convection and Diffusion)

$$u_i^{n+1} = u_i^n - u_i^n\frac{\Delta t}{\Delta x}(u_i^n - u_{i-1}^n) + \nu\frac{\Delta t}{\Delta x^2}(u_{i+1}^n - 2u_i^n + u_{i-1}^n)$$

2D Linear Convection

$$u_{i,j}^{n+1} = u_{i,j}^n - c\frac{\Delta t}{\Delta x}(u_{i,j}^n - u_{i-1,j}^n) - c\frac{\Delta t}{\Delta y}(u_{i,j}^n - u_{i,j-1}^n)$$

2D Convection

$$u_{i,j}^{n+1} = u_{i,j}^n - u_{i,j}\frac{\Delta t}{\Delta x}(u_{i,j}^n - u_{i-1,j}^n) - v_{i,j}^n\frac{\Delta t}{\Delta y}(u_{i,j}^n - u_{i,j-1}^n)$$

$$v_{i,j}^{n+1} = v_{i,j}^n - u_{i,j}\frac{\Delta t}{\Delta x}(v_{i,j}^n - v_{i-1,j}^n) - v_{i,j}^n\frac{\Delta t}{\Delta y}(v_{i,j}^n - v_{i,j-1}^n)$$

u,v Since 2D and Convection will occur in 2 directions, x and y

2D Diffusion

$$u_{i,j}^{n+1} = u_{i,j}^n + \frac{\nu \Delta t}{\Delta x^2}(u_{i+1,j}^n - 2u_{i,j}^n + u_{i-1,j}^n) + \frac{\nu \Delta t}{\Delta y^2}(u_{i,j+1}^n - 2u_{i,j}^n + u_{i,j-1}^n)$$

2D Burgers (Convection and Diffusion)

$$u_{i,j}^{n+1} = u_{i,j}^n - \frac{\Delta t}{\Delta x}u_{i,j}^n(u_{i,j}^n - u_{i-1,j}^n) - \frac{\Delta t}{\Delta y}v_{i,j}^n(u_{i,j}^n - u_{i,j-1}^n) + \frac{\nu \Delta t}{\Delta x^2}(u_{i+1,j}^n - 2u_{i,j}^n + u_{i-1,j}^n) + \frac{\nu \Delta t}{\Delta y^2}(u_{i,j+1}^n - 2u_{i,j}^n + u_{i,j-1}^n)$$

$$v_{i,j}^{n+1} = v_{i,j}^n - \frac{\Delta t}{\Delta x}u_{i,j}^n(v_{i,j}^n - v_{i-1,j}^n) - \frac{\Delta t}{\Delta y}v_{i,j}^n(v_{i,j}^n - v_{i,j-1}^n) + \frac{\nu \Delta t}{\Delta x^2}(v_{i+1,j}^n - 2v_{i,j}^n + v_{i-1,j}^n) + \frac{\nu \Delta t}{\Delta y^2}(v_{i,j+1}^n - 2v_{i,j}^n + v_{i,j-1}^n)$$

Laplace Equation

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} = 0 \qquad \nabla^2 \phi = 0$$

Discretizing we get

$$p_{i,j}^n = \frac{\Delta y^2(p_{i+1,j}^n + p_{i-1,j}^n) + \Delta x^2(p_{i,j+1}^n + p_{i,j-1}^n)}{2(\Delta x^2 + \Delta y^2)}$$

Boundary Conditions

- Nuemann Bounday Conditions
- Dirichlet Boundary Condition
- Robin Boundary Condition

Poisson Equation

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} = b \qquad \nabla^2 \phi = b$$

Discretizing we get

$$p_{i,j}^n = \frac{(p_{i+1,j}^n + p_{i-1,j}^n)\Delta y^2 + (p_{i,j+1}^n + p_{i,j-1}^n)\Delta x^2 - b_{i,j}^n \Delta x^2 \Delta y^2}{2(\Delta x^2 + \Delta y^2)}$$

Navier-Stokes Equation for Cavity Flow

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right)$$

We obtain this by d/dx and d/dy of equation and then adding them

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} = -\rho\left(\frac{\partial u}{\partial x}\frac{\partial u}{\partial x} + 2\frac{\partial u}{\partial y}\frac{\partial v}{\partial x} + \frac{\partial v}{\partial y}\frac{\partial v}{\partial y}\right)$$

$$p_{i,j}^n = \frac{\left(p_{i+1,j}^n + p_{i-1,j}^n\right)\Delta y^2 + \left(p_{i,j+1}^n + p_{i,j-1}^n\right)\Delta x^2}{2\left(\Delta x^2 + \Delta y^2\right)} - \frac{\rho \Delta x^2 \Delta y^2}{2\left(\Delta x^2 + \Delta y^2\right)} \times \left[\frac{1}{\Delta t}\left(\frac{u_{i+1,j} - u_{i-1,j}}{2\Delta x} + \frac{v_{i,j+1} - v_{i,j-1}}{2\Delta y}\right) - \frac{u_{i+1,j} - u_{i-1,j}}{2\Delta x}\frac{u_{i+1,j} - u_{i-1,j}}{2\Delta x} - 2\frac{u_{i,j+1} - u_{i,j-1}}{2\Delta y}\frac{v_{i+1,j} - v_{i-1,j}}{2\Delta x} - \frac{v_{i,j+1} - v_{i,j-1}}{2\Delta y}\frac{v_{i,j+1} - v_{i,j-1}}{2\Delta y}\right]$$

We obtain this by rearranging the pressure-Poisson Equation

Discretizing the equation to obtain momentum equation for u, v

$$u_{i,j}^{n+1} = u_{i,j}^n - u_{i,j}^n\frac{\Delta t}{\Delta x}(u_{i,j}^n - u_{i-1,j}^n) - v_{i,j}^n\frac{\Delta t}{\Delta y}(u_{i,j}^n - u_{i,j-1}^n) - \frac{\Delta t}{\rho 2\Delta x}(p_{i+1,j}^n - p_{i-1,j}^n) + \nu\left(\frac{\Delta t}{\Delta x^2}(u_{i+1,j}^n - 2u_{i,j}^n + u_{i-1,j}^n) + \frac{\Delta t}{\Delta y^2}(u_{i,j+1}^n - 2u_{i,j}^n + u_{i,j-1}^n)\right)$$

$$v_{i,j}^{n+1} = v_{i,j}^n - u_{i,j}^n\frac{\Delta t}{\Delta x}(v_{i,j}^n - v_{i-1,j}^n) - v_{i,j}^n\frac{\Delta t}{\Delta y}(v_{i,j}^n - v_{i,j-1}^n)) - \frac{\Delta t}{\rho 2\Delta y}(p_{i,j+1}^n - p_{i,j-1}^n) + \nu\left(\frac{\Delta t}{\Delta x^2}(v_{i+1,j}^n - 2v_{i,j}^n + v_{i-1,j}^n) + \frac{\Delta t}{\Delta y^2}(v_{i,j+1}^n - 2v_{i,j}^n + v_{i,j-1}^n)\right)$$

Navier Stokes Equation for Channel Flow

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) + F$$

In channel flow a we will just get a extra source term in u momentum equation for achieving the effect of pressure driven flow