HCF and LCM

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Factors

- Number of factors
- Sum of factors
- Product of factors
- Number of odd and even factors

LCM & HCF

- Prime Factorization Method
- Relation b/w LCM & HCF
- Problems related to same and different remainders
- Data Sufficiency on related topic

Factors and Multiples : All the numbers that divide a number completely, i.e., without leaving any remainder, are called factors of that number.

For example, 24 is completely divisible by 1, 2, 3, 4, 6, 8, 12, 24. Each of these numbers is called a factor of 24 and 24 is called a multiple of each of these numbers.

What are Factors of a number?
Factors of a number N refers to all the numbers which divide N completely. These are also called **divisors of a number.**

These are certain basic formulas pertaining to factors of a number N, such that,
N= $p^a q^b r^c$

Where, p, q and r are prime factors of the number n.
a, b and c are non-negative powers/ exponents

Number of factors of N = $(a+1)(b+1)(c+1)$

Sum of factors: $(p^0+p^1+...+p^a) (q^0+ q^1+....+q^b) (r^0+r^1+...+r^c)$

Product of factors of N , if N is not a perfect square = $N^{\text{No. of factors}/2}$

Product of factors of N , if N is a perfect square = $N^{(\text{No. of factors}-1)/2} * \sqrt{N}$

Number of even Factors and odd factors:

Let N as a number.
N in prime factorization = a^p × b^q × c^r

No. of factors of N = (p+1)(q+1)(r+1)

Now suppose that b and c are odd prime numbers in prime factorization of N

Now to find even no of factors you have to find odd no of factors first.

Odd no of factors = (q+1)(r+1)

Even no of factors = (total no of factors) - (odd no of factors).

1. What is the number of factors of $3^6 x 6^3$?

A. 28   B. 40   C. 36   D. 30

2. What is the number of factors of $4^6 x 5^3$?

A. 28　　　　　　　B. 52　　　　　　　C. 36　　　　　　　D. 56

3. Find the number of factors of x= $2^a \times 3^b \times 6^c \times 12^d$?

A. (a+ 1)(b+ 1)(c+ 1)(d+ 1)   B. (a+ c+ 2d+ 1)(b+ c+ d+ 1)

C. (a+ c+ d+ 1)(b+ c+ d+ 1)   D. (a+ c+ 2d)(b+ c+ d)

4. The number of factors of N= $2^3 \times 3^5 \times 5^7$ which have more than four factors is:

A. 180   B. 179   C. 182   D. 183

5. What is the sum of the factors of 221?

A. 222   B. 251   C. 252   D. 262

6. What is the sum of the factors of 120?

A. 22   B. 51   C. 45   D. 62

7. What is the Product of the factors of 120?

A. 120^6 B. 120^7 C. 120^8 D. 121^8

8. What is the Product of the factors of 300?

A. 300^6 B. 300^8 C. 300^9 D. 300^7

9. Find the number of even factors of 84?

A. 8   B. 7   C. 6   D. 9

10. Find the number of odd factors of $2^2 * 3^1 * 5^2$?

A. 4   B. 5   C. 3   D. 6

LCM : The least number which is exactly divisible by each of the given

numbers is called the least common multiple of those numbers.

For example, consider the numbers 3, 31 and 62 (2 x 31). The LCM of these

numbers would be

2 x 3 x 31 = 186.

HCF : The largest number that divides two or more numbers is the highest common factor (HCF) for those numbers.

For example, consider the numbers 30 (2 x 3 x 5), 36 (2 x 2 x 3 x 3), 42 (2 x 3 x 7), 45 (3 x 3 x 5).

3 is the largest number that divides each of these numbers, and hence, is the HCF for these numbers.

HCF is also known as Greatest Common Divisor (GCD).

Find HCF and LCM: By Using Prime factorization method

Prime Factorization for HCF:

Take an example of finding the highest common factor of 144, 104 and 160.
Now let us write the prime factors of 144, 104 and 160.
144 = 2 × 2 × 2 × 2 × 3 × 3
104 = 2 × 2 × 2 × 13
160 = 2 × 2 × 2 × 2 × 2 × 5
The common factors of 144, 104 and 160 are 2 × 2 × 2 = 8
Therefore, HCF (144, 104, 160) = 8

LCM By Prime Factorisation:

To calculate the LCM of two numbers 60 and 45. Out of other ways, one way to find the LCM of given numbers is as below:
•List the **prime factors** of each number first.
60 = 2 × 2 x 3 × 5
45 = 3 × 3 × 5

•Then multiply each factor the **most number of times** it occurs in any number.
If the same multiple occurs more than once in both the given numbers, then multiply the factor the most number of times it occurs.
The occurrence of Numbers in the above example:

2: two times
3: two times
5: one time

LCM = 2 × 2 x 3 × 3 × 5 = 180

Relationship between HCF & LCM

The product of HCF (Highest common factor) and LCM(lowest common multiple) of two numbers is equal to the product of two numbers.

HCF X LCM = Product of two given numbers

HCF & LCM OF A FRACTION

$$\text{HCF of Fraction} = \frac{\text{HCF of Numerator}}{\text{LCM of Denominator}}$$

$$\text{LCM of Fraction} = \frac{\text{LCM of Numerator}}{\text{HCF of Denominator}}$$

11. Find the L.C.M. of 2^4 x 3 x 11, 2^5 x 3 and 2^3 x 11?

A. 96   B. 88   C. 132   D. 1056

12. Find the H.C.F. of 2^4 x 3 x 11, 2^5 x 3 and 2^3 x 11?

A. 96   B. 8   C. 132   D. 1056

13. The HCF and LCM of two numbers is 78 and 2340 respectively. If the first number is 390, find the second one?

A. 420   B. 362   C. 312   D. 468

14. The sum of two numbers is 231 and their H.C.F is 33. The number of pairs of numbers satisfying the above conditions is?

A. 3   B. 6   C. 8   D. 12

15. Find the HCF of 5/12, 7/18 and 19/24?

A. 1/72   B. 1/36   C. 25/57   D. 5/48

16. Find the least number exactly divisible by 9, 10, 15, 18 and 30?

A. 85   B. 90   C. 88   D. 93

17. Find the smallest number of 4 digits which is exactly divisible by 12, 30, 42 and 60?

A. 1580   B. 1420   C. 1260   D. 1056

18. Find the least number which when divided by 30, 42, 48 and 32 leaves the same remainder 2 in each case?

A. 3362   B. 3360   C. 3456   D. 3262

19. Find the least number which when divided by 4, 10, 12 and 18 leaves a remainder 3 but leaves no remainder when same number is divided by 11?

A. 281   B. 357   C. 360   D. 363

20. Find the least number which when divided by 9, 15, 30 and 45 leaves remainders 5, 11, 26 and 41 respectively?

A. 72   B. 80   C. 85   D. 86

21. Find the least number which when divided by 12, 15, 30 and 40 leaves remainder 10, 13, 28 and 38 respectively?

A. 124   B. 116   C. 120   D. 118

22. Find the greatest number that will divide 44, 92, and 184 so as to leave the same remainder in each case?

A. 5   B. 7   C. 4   D. 9

23. A milk vendor has three kinds of milk: 85 litres, 136 litres and 51 litres. Find the least number of casks of equal size required to store all the milk without mixing?

A. 15   B. 16   C. 25   D. 30

Directions (24-25) In each of the questions below consists of a question and two statements numbered I and II given below it. You have to decide whether the data provided in the statements are sufficient to answer the question. Read both the statements and;

Give answer:

A. If the data in statement I alone are sufficient to answer the question, while the data in statement II alone are not sufficient to answer the question.

B. If the data in statement II alone are sufficient to answer the question, while the data in statement I alone are not sufficient to answer the question.

C. If the data either in statement I alone or in statement II alone are sufficient to answer the question.

D. If the data given in both statements I and II together are not sufficient to answer the question and;

E. If the data in both statements I and II together are necessary to answer the question.

24. Find the Ratio of two numbers?

Statement I: LCM of two numbers is 210.
Statement II: HCF of two numbers is 5.

25. Find the largest of the two numbers?

Statement I: The HCF of two numbers is 10 and their product is 600.

Statement II: The sum of the two numbers is 50.

Any Doubts???