REMAINDER, FACTORIAL UNIT DIGIT & LAST TWO DIGITS

Remainders

- Concept of positive and negative remainders
- Concept of remainders of higher powers
- Fermat's theorem

Factorials

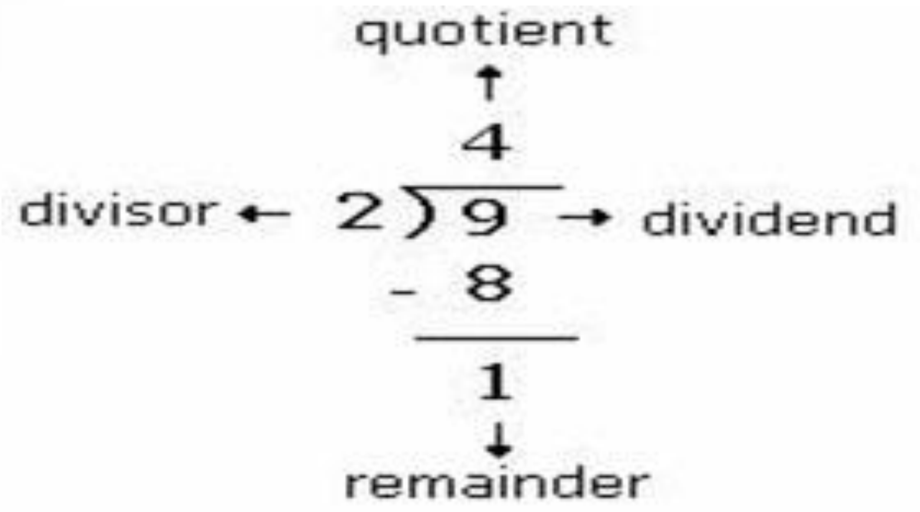
- Highest power of a number in a factorial & Practice from PPT
- Number of zeroes in a factorial & Practice from PPT

Unit Digit & Last Two Digit

- Rules to find the unit digit of a number with higher powers and practice from PPT
- Rules to find last two digit of a number with higher power & Practice from PPT
- Data Sufficiency on related topic

REMAINDER

Introduction of remainder:



REMAINDER

Concept of Negative Remainder:

Example: What is the remainder when 123 × 124 × 125 is divided by 9.

Solution

Remainder obtained when 123 is divided by 9 = -3
Remainder obtained when 124 is divided by 9 = -2
Remainder obtained when 123 is divided by 9 = -1
Final remainder = (-3)(-2)(-1) = -6. The required positive remainder = 9-6 = 3.

REMAINDER

Remainder of higher power terms:

We can find out the remainder of higher power term by using Binomial expansion.
Let us suppose we have to find remainder of X^n when divided by 'a'.

For example:
Example 1: What will be remainder if 10^20 is divided by 9.
Solution: using binomial expansion
{(9+1)^20}/9
So remainder will be 1^20 = 1

REMAINDER

Special case:
when divisor is prime:

Consider we need to find the remainder in case of X^n/Y. where Y is a prime number.
X^{Y-1}/Y gives remainder 1.

Example 1: Find the remainder when 23^6 is divided by 7.

Solution:
7 is prime number.
So $23^{7-1}/7$ or $23^6/7$ gives remainder 1.

REMAINDER

ALTERNATIVE METHOD

Totient number: The number of co-prime pair less than given number is called totient number of that number.

For example :Find the totient number of 6.
We will check how many number less than 6 which are co-prime with 6. Since 1,3,5 are less than 6 and co-prime with 6. So totient number of 6 will be 3.

Example 2.Find the totient no. of 5 .
1,2,3,4 all are co-prime with 5. So totient number of 5 is 4.
In case of **Prime number** , the totient number of any prime number is (Prime no. – 1)

REMAINDER

In case of **Composite number** ,-

Let the no. is $n=a^p b^q c^r$

Then the totient number of n = n(1-1/a)(1-1/b)(1-1/c)

Let 36 = $2^2 * 3^2$

Totient number of 36 = 36(1-1/2) (1-1/3)

= 36 * 1/2 *2/3 =12

(it means there are 12 numbers which are less than 12 and are co-prime with 12)

REMAINDER

Remainder of higher power : whether the divisor is prime or composite.
To find the remainder In case of ,X^n/Y

If X and Y are co-prime and n is the totient number of y. then the remainder will be 1. OR $X^{Y(ø)}/Y$ Gives remainder 1.
[Where Y(ø) is the totient number of Y]

Example 1: Find the remainder when 23^{16} is divided by 8.
Solution:
Divisor is 8 (composite number) and 23 & 8 are co-prime so we will find the totient number of divisor 8.
8=2^3
So totient number of 8 = 8(1-1/2) = 4
Now Rem[$23^4/8$] = 1
$(23^4)^4/8 = 1^4/8 = 1$

REMAINDER

1. Find the Remainder when 81*27 is divided by 8?

A] 9 B] 2

C] 3 D] 1

REMAINDER

2.Find the Remainder when 25*73*528 is divided by 13?

A]9 B]1

C]3 D]6

REMAINDER

3.Find the remainder when 2^{32} divided by 5?

A]1 B]0
C]4 D]3

REMAINDER

4. Find the remainder when 26^{75} is divided by 7?

A]5 B]6
C]0 D]3

5.Find the remainder when 23^{721} is divided by 61?

A]65 B]26
C]23 D]0

REMAINDER

6.Find the remainder when 8888….100times divided by 13?

A]1 B]0
C]9 D]3

7.Find the remainder when 256256256…… upto 93 times is divided by 11?

A] 5 B] 3
C] 0 D] 1

FACTORIALS

Number of zeroes:

It is very easy to find the number of zero at the end , all you have to do is count how many times did 2 and 5 occurred in the question as factor. Number of zeros is equal to the one (2 or 5)which occurred less times.

i.e. 2*5 = 10

2*2*5*5 = 100

So the number of zeros depends upon the number of pairs of 2 and 5.

Example 1. How many numbers of zeros will be there at the trail (end) of the 1*2*3*4*5*6*7*8*9*10?

Solution:

In given expression number of 2's = 8

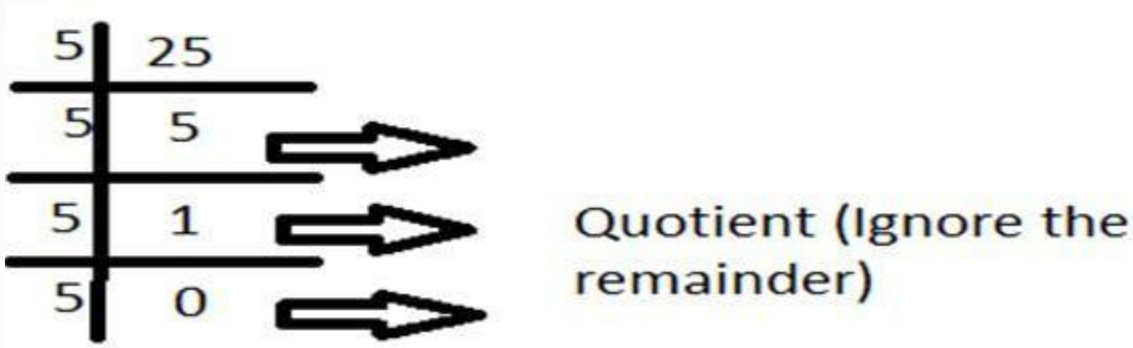
Number of 5's = 2

So total number of pairs = 2

Two zeroes will be there at the end of the calculation.

FACTORIALS

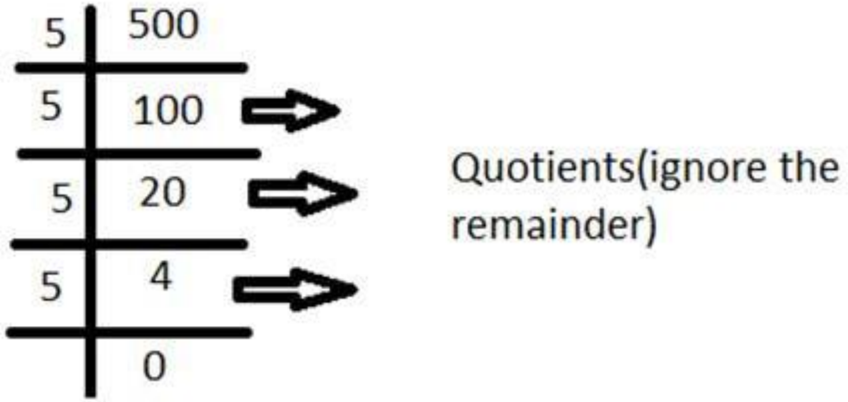
Number of zeroes in a factorial:



There will be 5+1+0 = 6 zeroes at the end of 25!

FACTORIALS

Example . Find the number of zeros at the end of 500!
Solution:



Total number of 5’s = 100+20+4 = 124

FACTORIALS

8. What is the highest power of 33 in the expression of 10000! ?

A] 105 B] 102 C] 103 D] None of these

FACTORIALS

9. The number of zeroes in 29! is:

A] 4 B] 3 C] 5 D] 7

10. The number of zeroes in 100! is:

A] 22 B] 23 C] 26 D] 24

UNIT DIGIT

Unit Digit

CASE I:

When 0, 1, 5, 6 are the digits in the unit's place of the base number N, then the number in the unit's place of N^n will also be 0, 1, 5 & 6 respectively, whatever be the value of "n".

Example 1: Number in the unit's place of (370) is "**0**"
Example 2: Number in the unit's place (391) is "**1**"
Example 3: Number in the unit's place of (75) is "**5**"
Example 4: Number in the unit's place (676) is "**6**"

UNIT DIGIT

Unit Digit

CASE II:

When 4 (or) 9 are in the unit's place of N, then

$4^1 = 4$ $4^2 = 16$

$4^3 = 64$ $4^4 = 256$

$4^5 = 1024$ $4^6 = 4096$

From the above figures we observe that..

4n when "**n**" **is odd number** [i.e. 1, 3, 5, 7 etc.], it contains "4" in the unit's place

4n when "**n**" **even number** [i.e. 2, 4, 6, 8 etc.], it contains "6" in the unit's place

In the same way;

9n, **n = odd number**, "9" in the unit's place

9n, **n = even number**, "1" in the unit's place

Example 1: $(74)^{99} \Rightarrow n = 99$ (odd) $\Rightarrow$ 4 in the unit's place

Example 2: $(84)^{78} \Rightarrow n = 78$ (even) $\Rightarrow$ 6 in the unit's place

Example 3: $(79)^{33} \Rightarrow$9 in the unit's place

UNIT DIGIT

CASE III:

When 2 or 3 or 7 or 8 is in the unit's place of N, then

2^1=**2**	3^1=**3**	7^1=**7**	8^1=**8**
2^2=**4**	3^2=**9**	7^2=4**9**	8^2=6**4**
2^3=**8**	3^3=2**7**	7^3=34**3**	8^3=51**2**
2^4=1**6** after that it start repeating	3^4=8**1** after that it start repeating	7^4=240**1** after that it start repeating	8^4=409**6** after that it start repeating

So these four digits i.e. 2, 3, 7& 8 have a unit digit cyclycity of four steps.

Example 1: What is the number in the unit's place of $(743)^{74}$?

Ans. n = 74, it can be written as 74 = (4 × 18) + 2

'2' is remainder The number in the unit's place $(3)^2$ = "9"

Example 2: What is the number in the unit's place of $(72)^{75}$?

Ans. n = 75, 75 = (4 × 18) + 3

Remainder = 3

$(72)^{75}$ (or) $(72)^3 \Rightarrow 2^3 \Rightarrow$ "8"

UNIT DIGIT

11. The unit's digit in the product of (256 × 27 × 159 × 182) is –

A]7 B]5

C]3 D]6

E] None of these

UNIT DIGIT

12. Find the Unit digit of 788^{194}

A]8 B]4
C]2 D]6
E] None of these

13. What is the number in the unit's place of 7727^{7173}
A]9 B]3
C]7 D]1
E] None of these

UNIT DIGIT

14. The unit's digit in the product $(7^{71} \times 6^{59} \times 3^{65})$ is –

A]3 B]4

C]5 D]6

15. Find the remainder when 1076^{98} is divided by 10

A]8 B]2

C]9 D]6

UNIT DIGIT

16. Find the unit digit of (121!)^67

A]2 B]0

C]5 D]8

UNIT DIGIT

17. Find the Unit Digit of 25^53 * 76^31 * 51^231?

A]1 B]2
C]0 D]5

18. Find the unit digit of $126^{126^{126}}$

A]4 B]8
C]6 D]2

LAST TWO DIGIT

Ten’s digit

Last two digits of a number is the tens place and units place digit of that number. So given a number say 1439, the last two digits of this number are 3 and 9, which is straightforward.

LAST TWO DIGIT

Let the number be in the form X^Y. Based on the value of units digit in the base i.e X, we have four cases

CASE I: Unit digit in x is 1
If x ends in 1, then x raised to y, ends in 1 and its tens digit is obtained by multiplying the tens digit in x with the units digit in y.

EXAMPLE 1: Find the last two digits of $191^{346.}$
Ans. Since the base 91 ends in 1, 191^{346} ends in 1 and the tens place digit is obtained from the units digit in 9*6 which is 4.
Hence, the last two digits of 191^{346} are 4 and 1.

LAST TWO DIGIT

CASE II: Units digit in x is 3, 7 or 9

In this case we will convert the base so that it ends in 1, after which we can use Case 1 to calculate units and tens place digits. i.e.
When x ends in 9 $(..9)^{y}$
Raise the base by 2 and divide the exponent by 2; $(..9^{2})^{y/2}$
Number ending in 9 raised to 2 ends in 1; $(..1)^{y/2}$
Since the base now ends in 1, Tens digit and Unit digit is calculated using the steps in Case 1.

LAST TWO DIGIT

EXAMPLE 1: Find the last two digits of $(79)^{142}$
Ans. Now write it as $(79^2)^{71}$
$=(..41)^{71}$
Unit digit will be 1 and Tens digit will be given by 4*1= 4
Hence, last two digits are 4 and 1.

EXAMPLE 2: Find last two digits of $(17)^{256}$
Ans. Now write it as $(17^4)^{64}$
$=(..21)^{64}$
Unit digit will be 1 and Tens digit will be given by 2*4=8
Hence, last two digits are 8 and 1.

LAST TWO DIGIT

CASE III: Units digit in x is 2, 4, 6 or 8

If x ends in 2, 4, 6 or 8, we can find the last two digits of the number raised to power with the help of following points:

$(2)^{10}$ ends in 24

$(2^{10})^{\text{odd number}}$ ends in 24

$(2^{10})^{\text{even number}}$ ends in 76

$(76)^{\text{number}}$ ends in 76

EXAMPLE 1: Find the last two digits of $(2)^{1056}$

Ans. $(2)^{1056}$ can be written as $(2^{10})^{105} \times (2)^{6}$

Here, $(2^{10})^{105}$ ends in 24 and $(2)^{6}$ ends in 64

Product of 24 and 64 will give 3 and 6 as last two digits.

LAST TWO DIGIT

CASE IV: Units digit in x is 5
The digit in the tens place is **odd** and the exponent y is **odd**, then the number ends in **75**.
If the digit in the tens place is **odd** and the exponent y is **even**, then the number ends in **25**.
If the digit in the tens place is **even** and the exponent y is **odd**, then the number ends in **25**.
If the digit in the tens place is **even** and the exponent y is **even**, then the number ends in **25**.
Hence, when the exponent and the digit in the tens place of the base are odd, the number raised to power ends 75, in other cases it ends in 25.

EXAMPLE 1: Find the last two digits of $(65)^{243}$
Ans. Since the digit in the tens place of the base is even and the exponent is odd, last two digits are 2 and 5

EXAMPLE 2: Find the last two digits of $(135)^{1091}$
Ans. Since the digit in the tens place of the base is odd and the exponent is odd, last two digits are 7 and 5.

LAST TWO DIGIT

19. Find the last two digits of 3^{102}

A] 19 B]09

C]12 D] 27

20. Find the last two digits of 8^{58}

A] 49 B]27

C]34 D]84

LAST TWO DIGIT

21. Find the last two digits of $21^{50} - 8$

A]93 B]73

C]53 D]03

22. What is the remainder when 2375^{2359} is divided by 100

A]35 B]25

C]75 D]00

DATA SUFFICIENCY

Directions

Each of the questions below consists of a statement and/or a question that follows with two statements i.e. I and II. Read both the statements and:

Write the answer (a) if the data in Statement I alone are sufficient to answer the question, while the data in Statement II alone are not sufficient to answer the question.

Give the answer (b) if the data in Statement II alone are sufficient to answer the question, while the data in Statement I alone are not sufficient to answer the question.

Write the answer (c) if the data in Statement I or in Statement II alone are not sufficient to answer the question.

Give the answer (d) if the data even in both Statements I and II together are not sufficient to answer the question.

Write the answer (e) if the data in both Statements I and II together are necessary to answer the question.

DS on REMAINDER

23. What is the Remainder when A is divide by B.

I. A is an odd multiple of 5.

II. B is an even prime number.

A. I alone sufficient while II alone not sufficient to answer

B. II alone sufficient while I alone not sufficient to answer

C. Either I or II alone sufficient to answer

D. Both I and II are not sufficient to answer

E. Both I and II are necessary to answer

DS on UNIT DIGIT

24. What is the unit digit of n^{20} ?

I. n is divisible by 10

II. Sum of n and 5 is odd number

A. I alone sufficient while II alone not sufficient to answer

B. II alone sufficient while I alone not sufficient to answer

C. Either I or II alone sufficient to answer

D. Both I and II are not sufficient to answer

E. Both I and II are necessary to answer

DS on UNIT DIGIT

25. Find the unit digit of $A^{B!}$

I. Value of A is odd and divisible by 5

II. B is greater than 10

A. I alone sufficient while II alone not sufficient to answer
B. II alone sufficient while I alone not sufficient to answer
C. Either I or II alone sufficient to answer
D. Both I and II are not sufficient to answer
E. Both I and II are necessary to answer

Any Doubts???