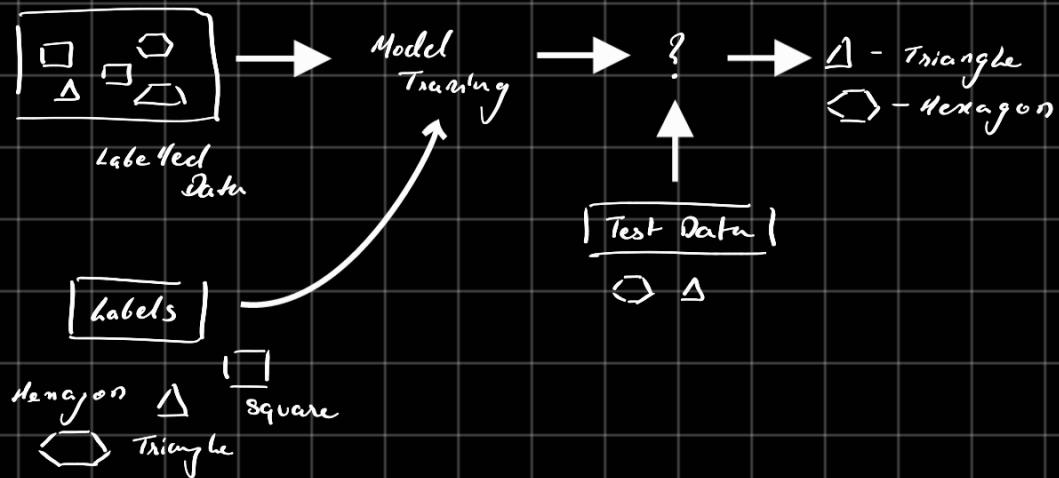


Machine Learning →

It is a branch of AI that enables the computer to self learn and improve over time.

It detects patterns in data and learns from them.



Steps using ML →

1. Collecting Data
2. Preparing the Data
3. Training the model
4. Evaluating the Data
5. Improving the Data

Applications of Machine Learning

1. Health Care
2. Fraud Detection
3. Recommendations
4. Online Search
5. Natural Language Processing
6. Smart Cars
7. Banking and Financial Services
8. Retail.

Machine Learning Life Cycle →

- ① Data gathering
- ② Data Preparation
- ③ Data Wrangling

Exploration

Pre processing

- Repetition of data
- Incomplete/incorrect data
- Outliers

- ④ Analyse

- ⑤ Training the Model

- ⑥ Testing the Model

- ⑦ Deployment

→ Gathering Data

Identify various data source.

Collect data

Integrate the data obtained from different sources.

By performing this we get a coherent set of data, Data Set.

→ Data preparation

Data exploration

To understand the nature, characteristics, format and quality of data.
we find correlation, general trend and outliers.

Data preprocessing

Processing of data to analyse it.

→ Data wrangling

Process of cleaning and converting raw data into useable format.

Missing values

Duplicate data

Invalid data

Noise

Corrupt / Meaning less data.

→ Data Analysis

Selection of analytical techniques

Building Model

Review the result

Classification, Regression, Cluster analysis

Association.

→ Train Model

→ Test Model

→ Deployment



Supervised learning → ex. House price.

In supervised learning, the goal is to learn the mapping between a set of inputs and outputs.

Being able to adapt to new inputs and make prediction is the crucial generalisation part of machine learning.

① Classification →

It is used to group the similar data points into different sections in order to classify them.

Decision boundaries →

Binary Classification →

Those classification tasks that have two class labels.

ex → Email spam detection (spam or not)

Decision Surface →

The entire area that is chosen to define a class

Multi-Class Classification →

Classification tasks having more than two class labels.
ex → face classification.

Multi-Label Classification →

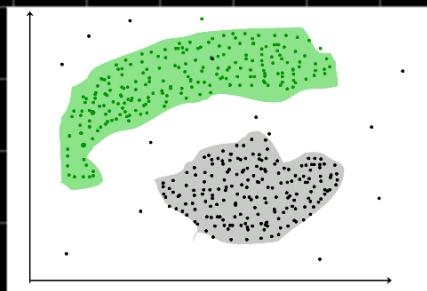
Classification tasks that have two or more class labels

② Regression →

The difference between classification and regression is that regression is that regression outputs a number rather than a class.

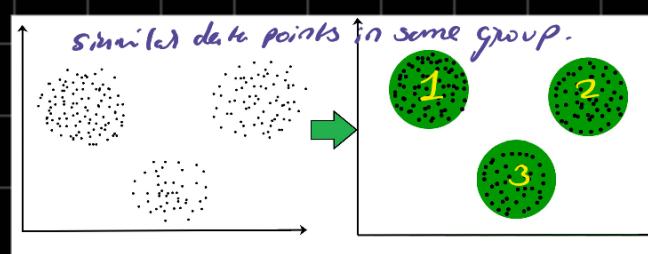
Unsupervised learning →

- Only input data is provided in the examples.
- There are no labelled example output to aim for.



① Clustering →

Is the task of dividing data points into a number of groups such that data points in same group are more similar to data points in the same group.



② Association →

- In this you want to uncover the rules that describe your data.

Semi-Supervised Learning →

- Mix between supervised and un-supervised approaches.
- The learning process isn't closely supervised with example outputs for every single input, also don't let algorithm do its own thing and provide no form of feedback.

- Mix small amount of labelled data with large unlabeled dataset reduces the burden of having enough labelled data.

Reinforcement Learning →

Occasional positive and negative feedback is used to reinforce behaviours.

Reward-motivated behaviour is key in reinforce behaviour.

- Algorithm just aims to maximise its rewards by playing the game over and over again.
- If you can frame a problem with a frequent 'score' as a reward, it is likely to be suited to reinforcement learning.

Quantitative data →

Data which can be put into categories, measured or ranked.

ex:- length, age, weight, cost.

Categorical data →

Data that has been placed into groups. An item cannot belong to more than one group at a time.

Chi-square test is used for analysis used for categorical data.

$$\chi^2 = \sum \frac{(\text{observed} - \text{expected})^2}{\text{expected}}$$

Continuous data →

Numerical data measured on a continuous range or scale.

ex:- Person's height or weight and temperature

Qualitative data →

Data that uses words and descriptions.

- Qualitative data can be observed but is subjective and therefore difficult to use for making comparisons.

Nominal data →

- Is used for naming or labelling variables, without any quantitative value.
- Race have a number of categories, but there is no specific way to order from highest to lowest and vice-versa.

2. Nominal

Nominal data is made of discrete values with no numerical relationship between the different categories — mean and median are meaningless. Animal species is one example. For example, pig is not higher than bird and lower than fish.



Ordinary data →

Categorical data with an order

- The variables in ordinal data are listed in an ordered manner.
- The numbers are not mathematically measured or determined but are merely assigned as labels for opinions.

Exploratory Data Analysis →

Steps of Data exploration and Preparation

- Variable Identification
- Univariate Analysis
- Bivariate Analysis
- Missing Values treatment
- Outlier treatment
- Variable transformation
- Variable creation

Variable Identification →

Types of variable →

Predictor Variable : Gender, Prev_Exam_Marks, Height(cm), weight(kg)

Target variable : Play-Cricket

Data type →

Character : Student_id, Gender

Numeric : Prev_Exam_Marks, Height(cm), weight(kg), Play-Cricket.

Variable Category →

Categorical : Student_id, Gender

Continuous : Prev_Exam_Marks, Height(cm), weight(kg)

Uni-varient →

Method to perform uni-varient analysis will depend on whether the variable type is categorical or continuous.

Bi-varient →

Finds out the relationship between two variables.

Missing Value Treatment

Why missing values treatment is required?

Missing data in the training set can lead to a biased model because we have not analysed the behaviour and relationship with other variables correctly.

Name	Weight	Gender	Play Cricket/ Not
Mr. Amit	58	M	Y
Mr. Anil	61	M	Y
Miss Swati	58	F	N
Miss Richa	55		Y
Mr. Steve	55	M	N
Miss Reena	64	F	Y
Miss Rashmi	57		Y
Mr. Kunal	57	M	N

Gender	#Students	#Play Cricket	%Play Cricket
F	2	1	50%
M	4	2	50%
Missing	2	2	100%

Name	Weight	Gender	Play Cricket/ Not
Mr. Amit	58	M	Y
Mr. Anil	61	M	Y
Miss Swati	58	F	N
Miss Richa	55	F	Y
Mr. Steve	55	M	N
Miss Reena	64	F	Y
Miss Rashmi	57	F	Y
Mr. Kunal	57	M	N

Gender	#Students	#Play Cricket	%Play Cricket
F	4	3	75%
M	4	2	50%

Why data has missing values?

- Data extraction
- Data collection
- Missing completely at random
- Missing at random

How to handle missing or corrupted data in dataset?

- Deletion →

List wise deletion and Pair wise deletion

deleted
{}
F

List wise deletion		
Gender	Manpower	Sales
M	25	343
F	.	280
M	33	332
M	.	272
F	25	.
M	29	326
	26	259
M	32	297

Pair wise deletion		
Gender	Manpower	Sales
M	25	343
F	.	280
M	33	332
M	.	272
F	25	.
M	29	326
	26	259
M	32	297

- Mean / Median / Mode Imputation →

Filling the values with estimate one's.

i) Generalized Imputation →

we calculate the mean or median for all non missing values of that variable then replace missing value with mean or median.

avg. of all non missing values

List wise deletion			Pair wise deletion		
Gender	Manpower	Sales	Gender	Manpower	Sales
M	25	343	M	25	343
F	2	280	F	-	280
M	33	332	M	33	332
M	2	272	M	-	272
F	25	-	F	25	-
M	29	326	M	29	326
	26	259		26	259
M	32	297	M	32	297

avg = 2

ii) Similar case Implementation →

we calculate avg for gender 'Male', 'Female' of non missing values.

List wise deletion			Pair wise deletion		
Gender	Manpower	Sales	Gender	Manpower	Sales
M	25	343	M	25	343
F	-	280	F	-	280
M	33	332	M	33	332
M	-	272	M	-	272
F	25	-	F	25	-
M	29	326	M	29	326
	26	259		26	259
M	32	297	M	32	297

• Prediction Model →

- we divide our data set into (i) without missing data (ii) with missing data .
- (i) data as training data set and the (ii) data set as test data set where the missy values are treated as target variable .

Draw backs →

1. The estimated values are usually more well-behaved than two values.
2. If no relationship with attribute in the data set and the attribute with missing values , then the model will not be precise for estimating missing values.

• KNN Imputation →

The missing values of an attribute are imputed using the given number of attributes that are most similar to the attribute whose values are missing.

- The similarity of two attributes is determined using distance function.

Advantages →

- K-nearest neighbour can predict both qualitative and quantitative attributes.
- Attributes with multiple missing values can be easily treated.

Disadvantages →

KNN algorithm is very time consuming in analysing large dataset.

Regression Analysis →

Is a method to model a relationship between dependent and independent variables with one or more variable.

Regression helps us to understand how the value of dependent variable is changing corresponding to an independent variable.

It predicts temp, age, salary etc.

Dependent → Target
Independent → Predict

Dependent And Independent Variable →

$$a + bL = c$$

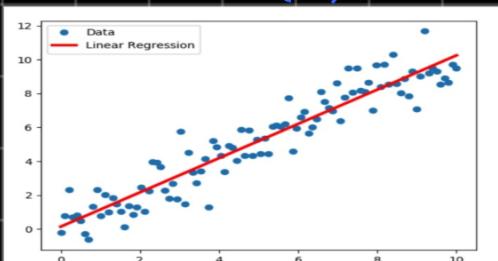
Here 'a' and 'b' are independent and can have any value.
Here 'c' depends on 'a' and 'b'.
does not depend on anything.

ex:- $a = 2$ → Independent
 $b = 10$ → Dependent
 $\therefore c = 22$ → Value depends on 'a' and 'b'.

Types of Regression Analysis →

Linear Regression →

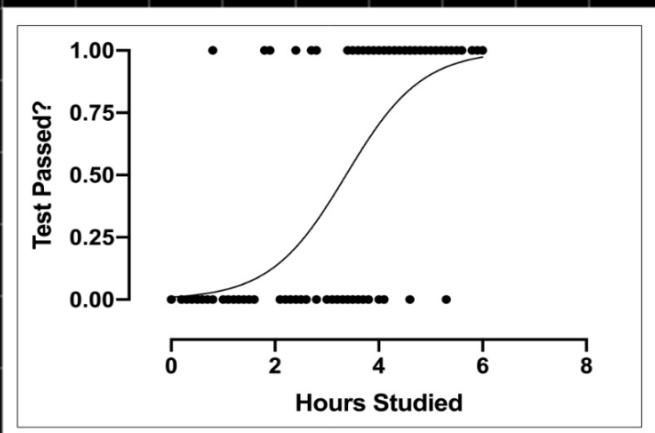
Assumes a linear connection between a dependent variable (y) and independent variable (x). The linear connection is defined as $y = mx + c + e$.



Logistic Regression →

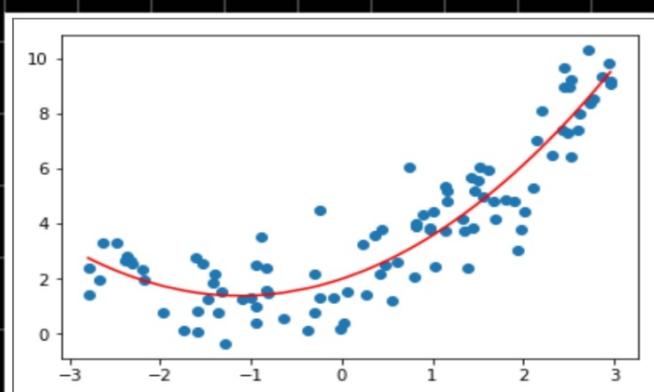
When the dependent variable is discrete. This technique is used to compute the probability of mutually exclusive occurrences such as pass/fail.

The target variable can take on only one of two values, and a sigmoid curve represents its connection to the independent variable, and probability has a value between 0 and 1.



Polynomial Regression →

Is used to represent a non-linear relationship between dependent and independent variables.

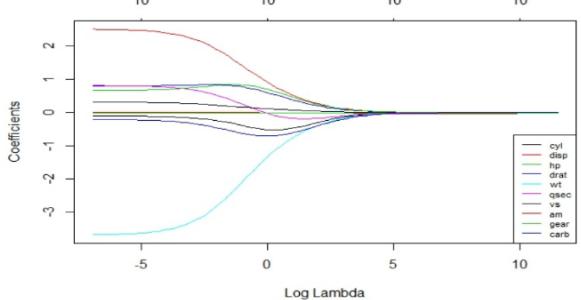


Ridge Regression →

When data exhibits multicollinearity, the ridge regression technique is applied when the independent variables are highly correlated.

The ' λ ' variable in the ridge regression equation resolves the multicollinearity problem.

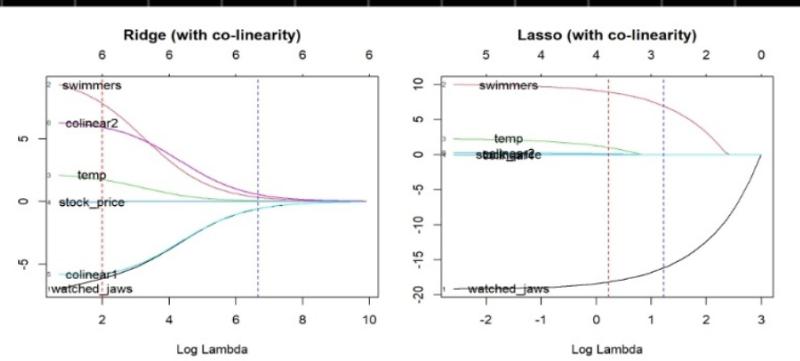
$$= \underset{\beta \in R^p}{\operatorname{arg\,min}} \underbrace{\|y - X\beta\|_2^2}_{\text{Loss}} + \underbrace{\lambda \|\beta\|_2^2}_{\text{Penalty}}$$



Lasso Regression →

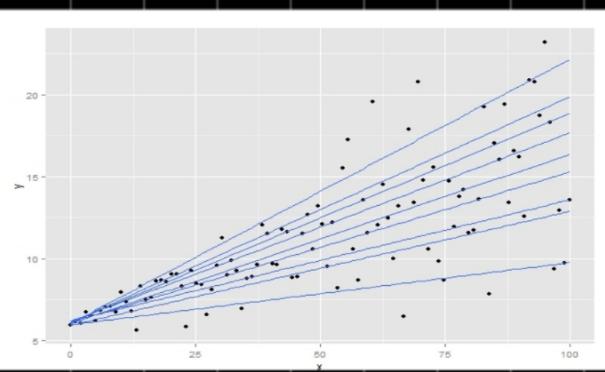
As ridge regression, Lasso technique penalizes the absolute magnitude of the regression coefficient.

Lasso regression technique employs variable selection, which leads to shrinkage of coefficient values to absolute zero.



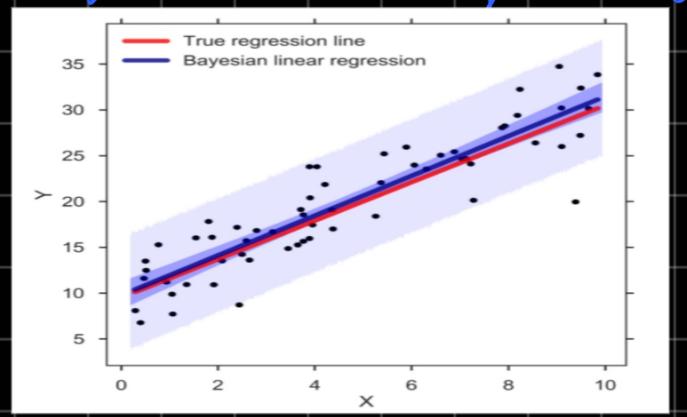
Quantile Regression →

- subset of linear regression technique.
- It is employed when the linear regression requirements are not met or when the data contains outliers.



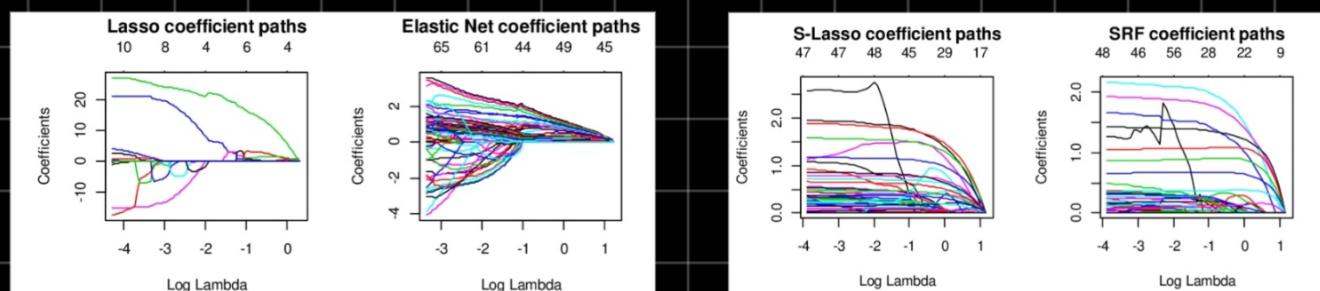
Bayesian Linear Regression →

- Used Bayes theorem to calculate the regression coefficients' values.
- This technique determines the features' posterior distribution. The approach outperforms ordinary linear regression in terms of stability.



Elastic Net Regression →

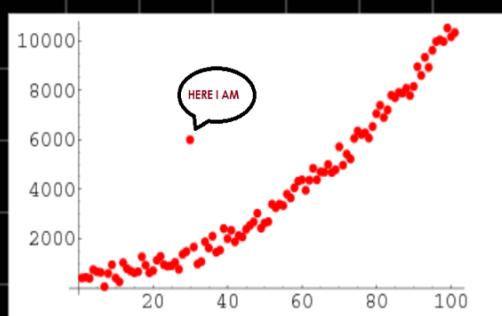
It combines ridge and lasso regression techniques that are partially useful when dealing with strongly correlated data. It regularizes regression models by utilizing the penalties associated with ridge and lasso regression method.



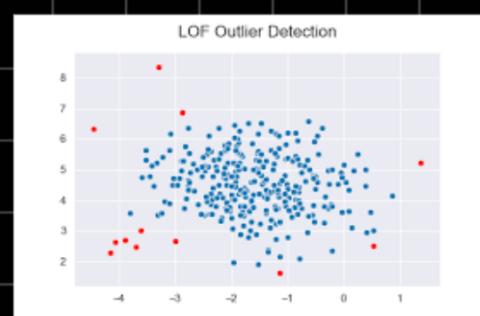
Outliers →

Values that look different from the other values in the data.

Univariate

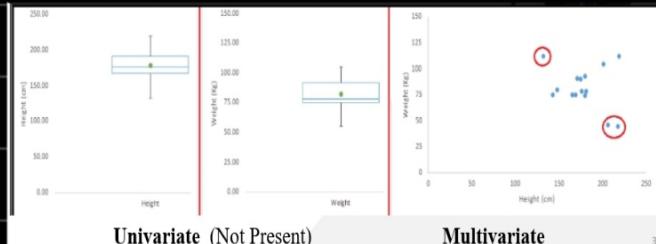


Multivariate



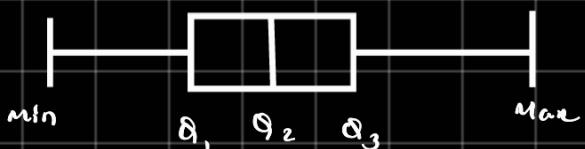
Types of Outliers →

- Univariate
- Multivariate



Outlier Detection →

Box-plot, Histogram, Scatter Plot.



Box plot used
to find the outliers.

$$< Q_1 - 1.5 \text{ IQR}$$

$$> Q_3 + 1.5 \text{ IQR}$$

$$2 \text{ IQR} = Q_3 - Q_1$$

Binning →

It is used to categorize variables.

Outlier's Treatment →

• Deleting outlier values.

• Using mean, median, mode imputation method.

Multicollinearity →

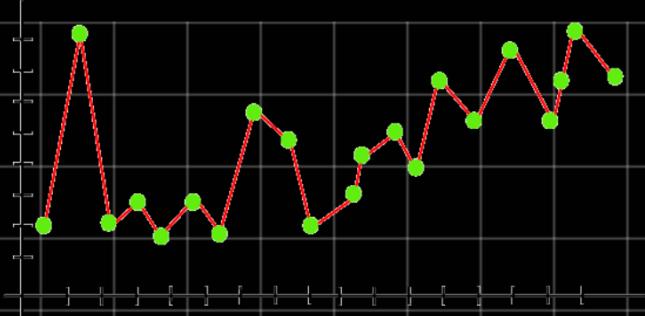
When two or more independent variables are highly correlated with one another in a regression model.

Overfitting →

Ocurs when our machine learning model tries to cover all the data points or more than the required data points present in the given dataset.

Because of this, model starts catching noise and inaccurate values present in the data set, all these factors reduce the efficiency and accuracy of the model.

The overfitted model has low bias and high variance.



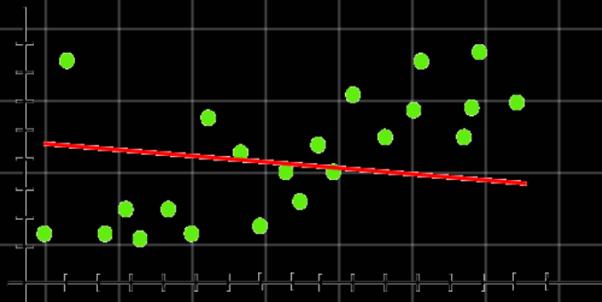
Mean → $\frac{\sum a_i}{n}$
 Median → middle value
 mode → most occurrence

How to avoid overfitting →

- Cross Validation
- Training with more data
- Removing features
- Early stopping training
- Regularization
- Ensembling

Underfitting →

When machine learning model is not able to capture the underlying trend of the data.



How to avoid underfitting →

- By increasing the training time of model.
- By increasing the number of features.

Signal →

Refers to the true underlying pattern of the data that helps the machine learning model to learn from the data.

Noise →

Unnecessary and irrelevant data that reduces the performance of the model.

Bias →

Prediction error that is introduced in the model due to oversimplifying the machine learning algorithms.

Or the difference between the predicted values and the actual values.

Variance →

If machine learning model performs well with the training data set, but does not perform well with the test data set, then variance occurs.

Residual sum of squares →

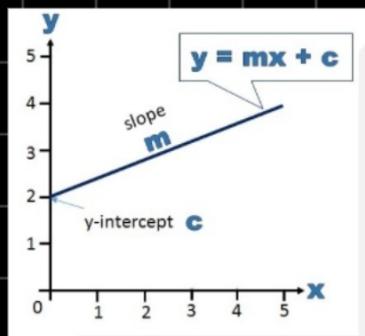
$$RSS = \sum (\text{actual output} - \text{predicted output})^2$$

$$RSS = \sum_{i=1}^n (y_i - (mx_i + c))^2$$

y = dependent variable
 x = independent variable
 m = slope of the line
 c = y-intercept of the line
 n = number of samples.

Formula for linear regression →

$$y = mx + c$$



y = dependent variable
 x = independent variable
 m = slope of the line
 c = y-intercept of the line.

$$m(\text{slope}) = \frac{n \sum xy - (\sum x)(\sum y)}{n \sum x^2 - (\sum x)^2}$$

$$c(\text{intercept}) = \frac{\sum y - n \bar{x} \bar{y}}{n \sum x^2 - (\sum x)^2}$$

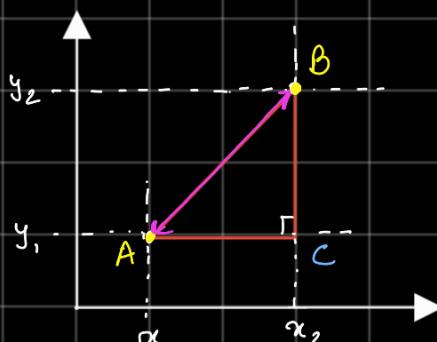
y = dependent variable
 x = independent variable
 m = slope of the line
 c = y-intercept of the line
 n = number of samples

$$\frac{n \sum xy - \bar{x} \bar{y}}{n \sum x^2 - \bar{x}^2}$$

Distance Metrics →

• Euclidean →

- It is the displacement between two points.
- Calculated using "Pythagoras Theorem".



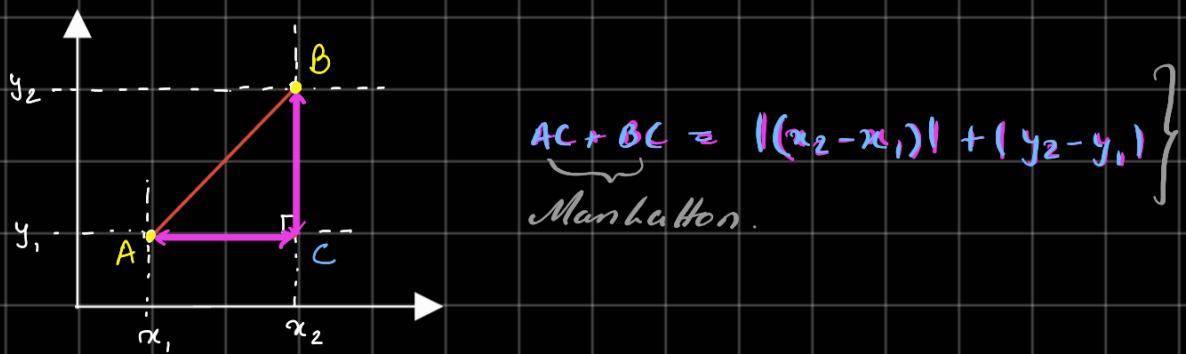
$$\begin{aligned} AB^2 &= AC^2 + BC^2 \\ &= (x_2 - x_1)^2 + (y_2 - y_1)^2 \end{aligned}$$

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad - \text{for 2D plane.}$$

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2} \quad - \text{for 3D plane}$$

• Manhattan →

- The distance between the two points measured along x-axis and y-axis.



Types of Statistics →

• Descriptive Statistics →

Is a way to organise, represent and describe a collection of data using tables, graphs and summary measures.

ex! — The collection of people in a city using the internet or using television.

Types of Descriptive Statistics →

• Measure of frequency

frequency measurement displays the number of times a particular data occurs.

• Measure of dispersion

range, variance, standard deviation are measure of dispersion.

• Measure of central tendency

central tendency are mean, median, mode of the data.

• Measure of position

measure of position describes the percentile and quartile ranks.

Mean →

$$\frac{\text{sum of no. of elements}}{\text{total no. of elements}} = \text{mean}$$

$$9, 3, 1, 8, 6, 3 \rightarrow \frac{30}{6} = 5$$

Median →

median is the middle number.

$$1, 3, \underline{3}, 6, \underline{8}, 9$$

$$\frac{3+6}{2} = \underline{\underline{4.5}}$$

Mode →

The most common number.

$$\underline{\underline{9}}, \underline{\underline{3}}, \underline{\underline{1}}, \underline{\underline{8}}, \underline{\underline{6}}, \underline{\underline{3}}$$

→ 3 is the mode

Standard Deviation →

deviation of values / data from an average mean.

• Population Standard Deviation

$$\sigma = \sqrt{\frac{\sum (x - \mu)^2}{n}}$$

x - Value in data distribution

μ - population mean

\bar{x} - sample mean

n - total number of observation

• Sample Standard Deviation

$$s = \sqrt{\frac{(x - \bar{x})^2}{n-1}}$$

Variance →

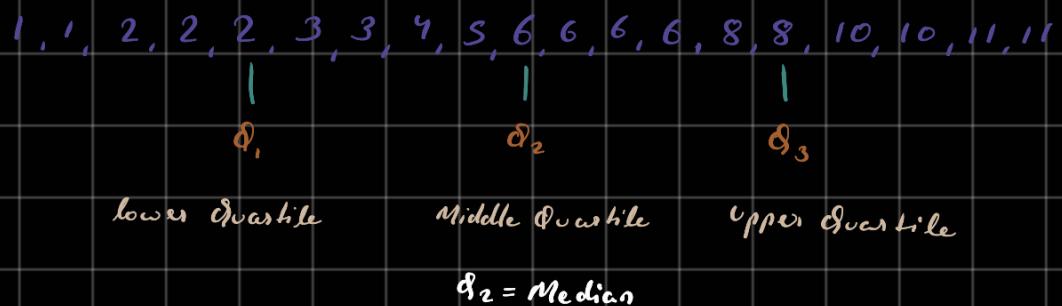
The value of variance = $(\text{standard deviation})^2$

$$= \sigma^2$$

$$= \frac{\sum (x - \mu)^2}{n}$$

Quartile →

It's like median which divides the data in '2' parts, whereas quartile divides the data into '4' parts.

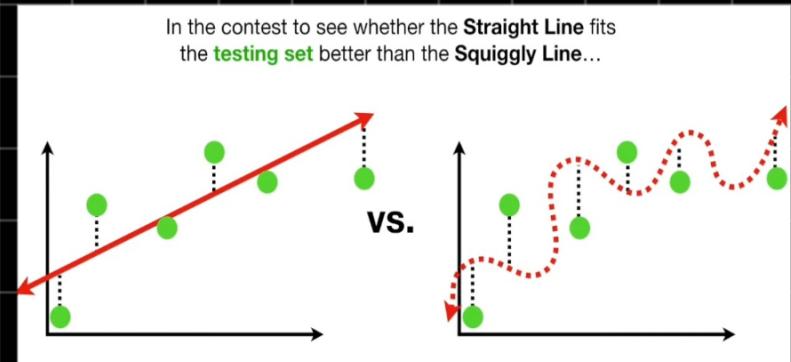
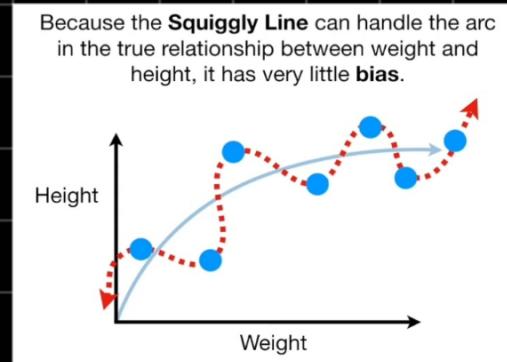
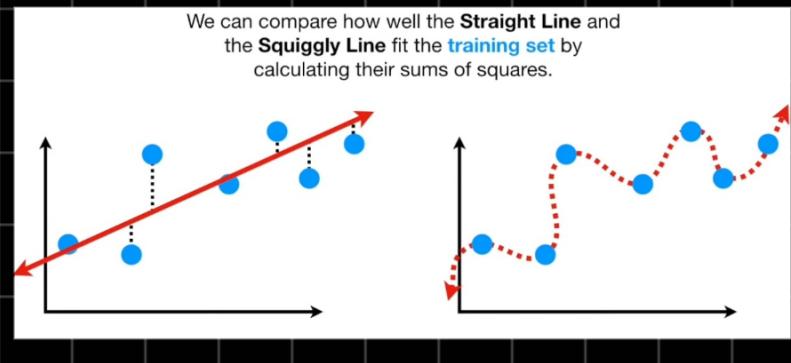
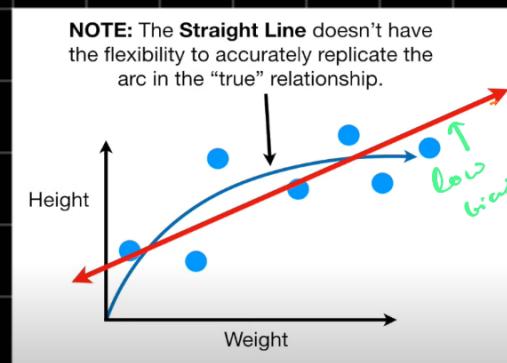
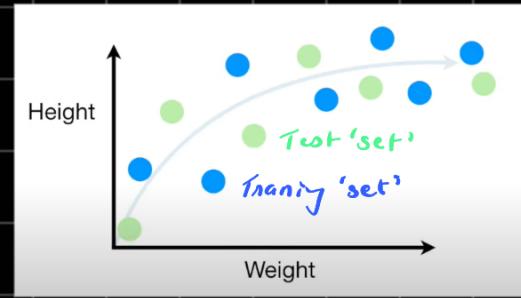
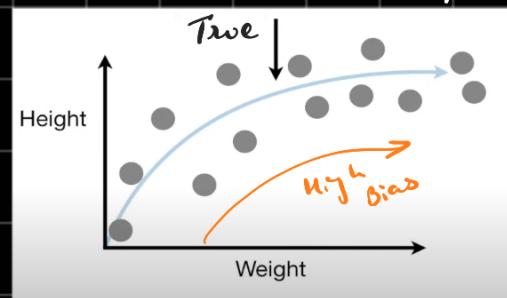


$$\left\{ \begin{array}{l} \text{Interquartile Range} = Q_3 - Q_1 \\ \text{Semi-Interquartile Range} = \frac{Q_3 - Q_1}{2} \end{array} \right. \quad \left. \begin{array}{l} Q_1 = \frac{n+1}{4} \\ Q_3 = \frac{3(n+1)}{4} \end{array} \right\}$$

Bias →

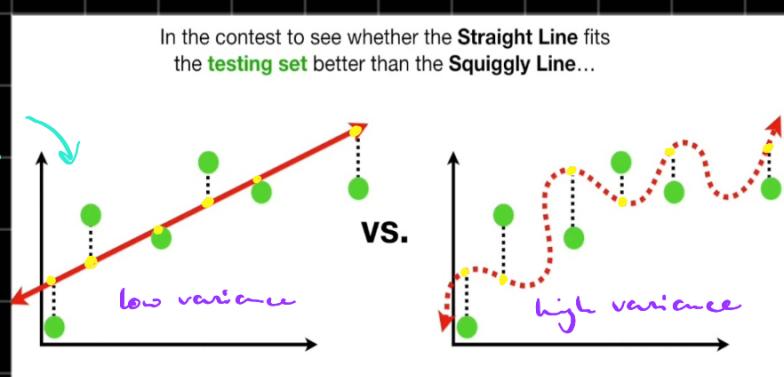
The inability for a machine learning method to capture the true relationship.

Relationship p'



Variance →

Distance between the actual and predicted data points.



Covariance →

✓ direction

✗ degree of proportion

Shows the directional relationship between the two variables.

+ve	0	-ve
both in same direction		both in opp. direction

$$\text{COV} = \frac{\sum (x - \bar{x})(y - \bar{y})}{n-1} \quad \bar{x}, \bar{y} = \text{mean}$$

Correlation → ✓ direction

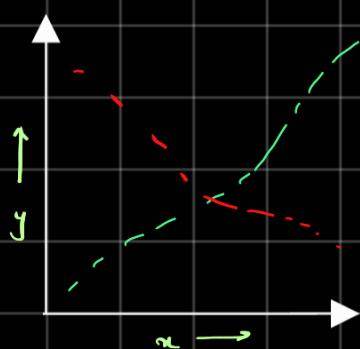
strength /
✓ degree of proportion.

Shows the direction of relation and in what proportion it's changing.

Correlation coefficient → r

$$COR = \frac{\text{COV}(x, y)}{\sigma_x \times \sigma_y} \quad \leftarrow \text{standard deviation.}$$

+ve, -ve correlation →



x inc, y increases "tve"

x inc, y decreases "-ve"

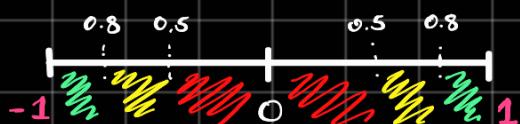
○ No correlation

→ low

→ moderate

→ strong

+,-1 perfect +,-ve correlation.



Bayesian Inference (Intelligent Guess)

3 Techniques →

- i. They do not rely on large data sets.
- ii. common sense, human reasoning
- iii. combining data

Bayes Theorem →

The probability of the occurrence of an event A given that an event B has occurred is called the conditional probability of A given B .

$P(A) = \text{It will rain in our area}$

$P(B) = \text{weather forecasting predicting that it will rain}$

$$P(A) = 0.9$$

$$P(B/A) = \frac{P(A/B) P(B)}{P(A)} = \frac{0.9 \times 0.1}{0.27}$$

$$P(A/B) = \frac{P(B/A) P(A)}{P(B)}$$

$$\left. \begin{aligned} P(A/B) &= \frac{P(A \cap B)}{P(B)} \\ P(B/A) &= \frac{P(B \cap A)}{P(A)} \end{aligned} \right\} \text{Same}$$

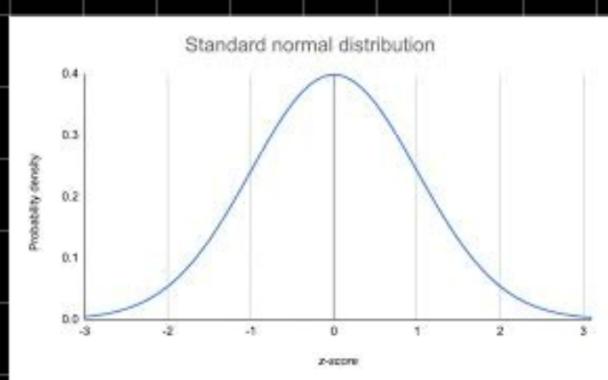
when there are more events

$$\Rightarrow P(A/B) = \frac{P(B/A) \cdot P(A)}{P(B)}$$

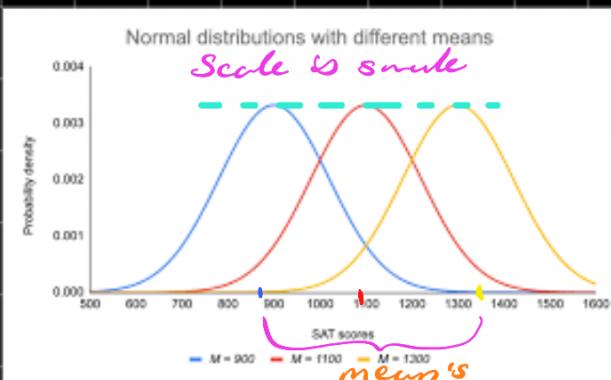
$$P(A) = \sum_{i=0}^n P(E_i) P(A/E_i)$$

$$P(A/B) = \frac{P(B/A) P(A)}{P(E_1) P(A/E_1) + \dots + P(E_n) P(A/E_n)}$$

Normal Distribution →

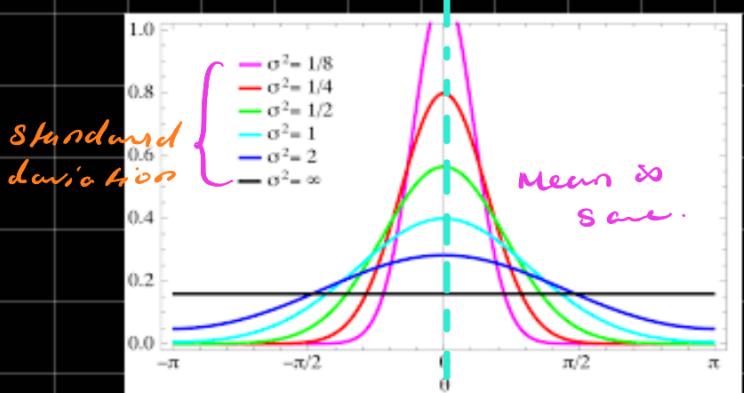


- Mean = Median = Mode
- Mean → location of "bell curve"
- Standard deviation → scale of graph.



location of normal distribution changes as "mean" changes

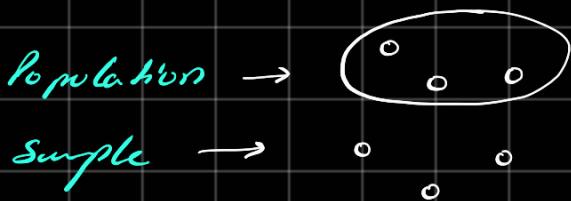
- scale of graph is same



'SD' shows the scale of graph, how broad or thin the area is.

- mean of graph is same

statistical hypothesis → (Hypothesis testing)



Test (statistics) →

1) χ^2 test (chi)

2) t - student

3) Fisher's Z test

Hypothesis →

1) Null hypothesis (H_0)

2) Alternate Hypothesis (H_1)

level of significance (α) (say 1%)

level of confidence (c) (95%, 99%)

n → random sample

Null hypothesis $H_0: \mu = \text{"value"}$

\bar{x} → mean

Alternate hypothesis $H_1: \mu \neq \text{"value"}$

σ → variance

$$z = \left| \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} \right|$$

$$\frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$$

16
 $\sigma_{\bar{x}}$

$$\begin{array}{c}
 \text{Ans} \quad \text{23.6} \quad \rightarrow \quad \text{Ans} \quad 50 \rightarrow \\
 \underline{\underline{1}} = 1.6 \quad \sqrt{50} \quad \sqrt{50} = \underline{\underline{30}}. \\
 \underline{\underline{2}} = 1.9
 \end{array}$$

Q There are two bags e_1 and e_2 . There are 2 white and 3 red in bag e_1 , 4 white and 6 black. Find the probability of black ball drawn from e_2 .

$$\underline{\underline{Ans}} \rightarrow \frac{1}{2}$$

\rightarrow Statistical Hypothesis testing

$$y = mx + c$$

$$m = \frac{n \sum xy - (\sum x)(\sum y)}{n \sum x^2 - (\sum x)^2}$$

$$C = \frac{\sum y \sum x^2 - \sum x \sum xy}{n \sum x^2 - (\sum x)^2}$$

x	y
3	2
7	4
.	6

10 8

$$m \Rightarrow \frac{4(144) - 20(144)}{4(120) - 400}$$

$$\Rightarrow \underline{0.95}$$

$$C = \frac{25(120) - 20(144)}{4(120) - 400}$$

$$y = mx + c$$

$$y = 0.95x + 1.5$$

$$\Rightarrow \underline{1.5}$$

To be followed in hypothesis testing →

1. Determine the tests is null hypothesis or alternative hypothesis.
2. Determine the significant value that be taken α as 0.05.
3. Calculate the probability value
4. Check whether to reject or retain the hypothesis
5. Last you will draw conclusion through

Z-test

1. If we want to know about population mean and variance is given.
2. When the population is in
- 3.

$$z = \frac{x - \mu}{\sigma / \sqrt{n}}$$

$$\text{cov}(x, y) = \frac{(\sum x - \bar{x})(\sum y - \bar{y})}{n}$$

$$\text{cor}(x, y) = \frac{\text{cov}(x, y)}{\sigma_x \sigma_y}$$

$$\text{correlation } 'r' \rightarrow \frac{n \sum xy - \sum x \sum y}{\sqrt{n \sum x^2 - (\sum x)^2} \sqrt{n \sum y^2 - (\sum y)^2}}$$

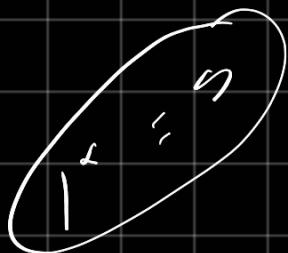
x 1 2 3 4 5
 y 2 3 5 4 5

Scope Vector \rightarrow

KNN \rightarrow

Acid	Durability	Classify	$(7, 3)$	$\underline{k=3}$
7	7	Bad	$\sqrt{0+16} = 4$	
7	4	Bad	$0+1 = 1$	
3	4	Good	$\sqrt{17}$	
1	4	Good	$\sqrt{37}$	
3	4			

$$\frac{\sum (x - \bar{x})(y - \bar{y})}{n \sigma_x \sigma_y}$$



Unit - 2

* Binary Logistic Regression.

$$y = mx + c$$

$$y = \frac{1}{1 + e^{-x}}$$

$$y = \frac{1}{1 + e^{-(mx+c)}}$$

In book

Outlook

1.	S	U	V					
2.	S	U	V					
3.	O	U	V					
4.	R	M	V					
5.	R	C	V	$P(V/Today) = P(S/V) \times P(U/V) \times P(V)$				
6.	R	C	V					
7.	O	C	V	$P(V/14) = \frac{2/9 \times 2/9 \times 2/9}{14} = \frac{8}{63}$				
8.	S	M	V					
9.	S	C	V					
10.	R	M	V	Yes	No	$P(V)$	$P(V)$	
11.	S	M	V	suny	2	3	$2/9$	$3/5$
12.	O	M	V	overcast	4	0	$4/9$	0
13.	O	C	V	Rainy	3	2	$3/9$	$2/5$
14.	R	M	V	Total	9	5		

U	R	I	$2/9$	$2/5$
M	R	I	$4/9$	$2/5$
C	R	I	$4/9$	$1/5$
Total	9	5		

Confusion Matrix →

$$\text{Recall} = \frac{TP}{\text{Actual yes}}$$

$$\text{Accuracy} = \frac{TP + TN}{\text{Total}}$$

$$\text{sensitivity} = \frac{\text{True Positive}}{TP + \text{false negative}}$$

$$\text{specificity} = \frac{\text{True Negatives}}{TN + \text{false positive}}$$

		Predicted	
		No	Yes
Actual	No	TN	FP
	Yes	FN	TP

Type 1 error
Type 2 error

$$\text{Error Rate} = 1 - \text{accuracy}$$

$$= \frac{FP + FN}{\text{Total}}$$

$$\text{precision} = \frac{TP}{FP + TP}$$

Sensitivity → also known as recall or true positive rate is the conditional probability that the predicted class is positive given that actual class is positive.

specificity →

Picking the best splitting Attribute →

① Information Gain

② Gini Index

$$\text{Entropy}(S) = - \sum_{i=1}^n \log_2 i (P_i)$$

$$= \frac{4}{9} \log_2 \frac{4}{9} + \frac{5}{9} \log_2 \frac{5}{9}$$

Information Gain →

$$T \leftarrow (3+, 1-)$$

$$F \leftarrow (1+, 4-)$$

$$IG = \text{Entropy}(S) - \sum_{v \in \{T, F\}} \frac{|S_v|}{|S|} \log_2 (S_v)$$

Instance	a ₁	a ₂	Classification
1	T	T	+
2	T	T	+
3	T	F	-
4	F	F	+
5	F	T	-
6	F	T	-
7	F	F	-
8	T	F	+
9	F	T	-

Q) 2 Red, 2 Yellow, 4 Black. To find out the entropy of this table.

$$-\left[\frac{2}{8} \log_2 \frac{2}{8} + \frac{2}{8} \log_2 \frac{2}{8} + \frac{4}{8} \log_2 \frac{4}{8} \right]$$

$$\log_2 Y_1 = -2 \quad \log_2 Y_2 = \underline{-1}$$

$$-\left[\frac{1}{4}(-2) + \frac{1}{4}(-2) + \frac{1}{2}(-1) \right]$$

$$\Rightarrow -\left[\frac{-1}{2} + \frac{-1}{2} + \frac{-1}{2} \right]$$

$$\Rightarrow \frac{3}{2} \Rightarrow \underline{1.5}$$

 \times \times

$$\left\{ \begin{array}{l} \frac{|S_U|}{|S_T|} = \frac{4}{9} \\ \frac{|S_V|}{|S_T|} = \frac{5}{9} \end{array} \right.$$

$$\text{Entropy } (S) = -\left[\frac{4}{9} \log_2 \frac{4}{9} + \frac{5}{9} \log_2 \frac{5}{9} \right]$$

$$\text{Entropy } (S_T) = -\left[\frac{3}{4} \log_2 \frac{3}{4} + \frac{1}{4} \log_2 \frac{1}{4} \right]$$

$$\text{Entropy } (S_F) = -\left[\frac{1}{5} \log_2 \frac{1}{5} + \frac{4}{5} \log_2 \frac{4}{5} \right]$$

$$\text{Information gain} = \text{Entropy}(S) - \sum_{v \in \{T, F\}} \frac{|S_v|}{|S_T|} \log_2 (S_v)$$

$$\Rightarrow I(S) = \left[\frac{4}{9} \times \text{entropy}(S_T) + \frac{5}{9} \times \text{entropy}(S_F) \right]$$

Cross Validation →

K-fold cross validation

1	2	3	4	5	6	7	8	9
---	---	---	---	---	---	---	---	---

$$\rightarrow K=3$$

$$RMSE =$$

divided into 3 sub sets

↓ Test set with no overlapping

1)

1	2	3	4	5	6	7	8	9
Test set			Train set					

$$E_1$$

2)

1	2	3	4	5	6	7	8	9
Test set			Test set			Set		

$$E = \frac{\sum E_i}{K}$$

3)

1	2	3	4	5	6	7	8	9
Test set			Test set					

ASM →

① Information Gain

② Gini Index

Days	outlook	Temperature	Play Game
1	S	H	No
2	S	H	No
3	O	H	Yes
4	R	M	Yes
5	R	C	Yes
6	R	C	No
7	O	C	Yes
8	S	M	No
9	S	C	Yes
10	R	M	Yes
11	S	M	Yes
12	O	M	Yes
13	O	H	Yes
14	R	M	No

Random Forest

Aggregation of 2 different model to give majority output.
The attributes that we choose in aggregation is different and independent to each other. which gives our model more accurately. As compared to bootstrapping model.

Bootstrapping and Aggregation

the bootstrap.

$$E(S) = -\frac{5}{14} \log_2 \frac{5}{14} - \frac{9}{14} \log_2 \frac{9}{14}$$

Attribute \rightarrow outlook

values (overcast, sunny, rain)

$$E(S_{\text{sunny}}) = -\frac{2}{5} \log_2 \frac{2}{5} - \frac{3}{5} \log_2 \frac{3}{5} = 0.97$$
$$(2+, 3-)$$

$$E(S_{\text{overcast}}) = -\frac{9}{4} \log_2 \frac{9}{4} - \frac{0}{4} \log_2 \frac{0}{4} = 0$$
$$(4+, 0-)$$

$$E(S_{\text{rain}}) = -\frac{3}{5} \log_2 \frac{3}{5} - \frac{2}{5} \log_2 \frac{2}{5} = 0.971$$

$$\text{Gain} = E(S) - \sum \frac{|S_v|}{|S|} \text{Entropy}(S_v)$$

v(overcast, sunny, rain)

$$= 0.97 - \frac{5}{14} (0.97) - 0 - \frac{9}{14} (0.97)$$
$$= 0.24$$

Outlook

Sunny	Overcast	Rain
(2+, 3-)	(4+, 0-)	(3+, 2-)
(D ₁ , D ₂ , D ₃ , D ₄ , D ₁₁)	yes	(D ₅ , D ₆ , D ₇ , D ₈ , D ₉)

Days	Temp	Humidity /	Wind	Play
D ₁	4	high	weak	No
D ₂	4	high	strong	No
D ₃	mild	high	weak	No
D ₄	c	low	weak	Yes
D ₅	mild	low	strong	yes

$\mu \rightarrow$
 $C \rightarrow$
 $\mu \rightarrow$

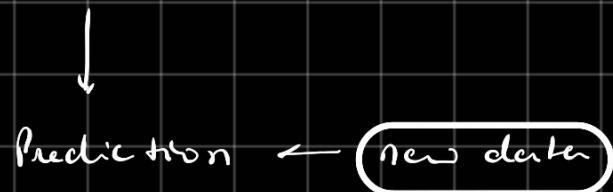
$\left(\begin{array}{c} \mu_{\text{high}} \\ 10^{10} \end{array} \right)$

$+ \mu, \sigma^2$
 $+ \overbrace{\quad}$
 $\text{Gain} = E(S_{\text{sunny}}) - \frac{2}{5}(E_{\text{sk.}}) - \frac{2}{5}(E_{\text{sk.}}) - \frac{1}{5}()$

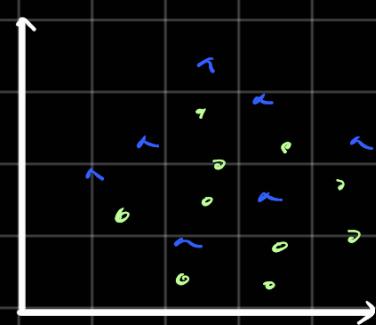
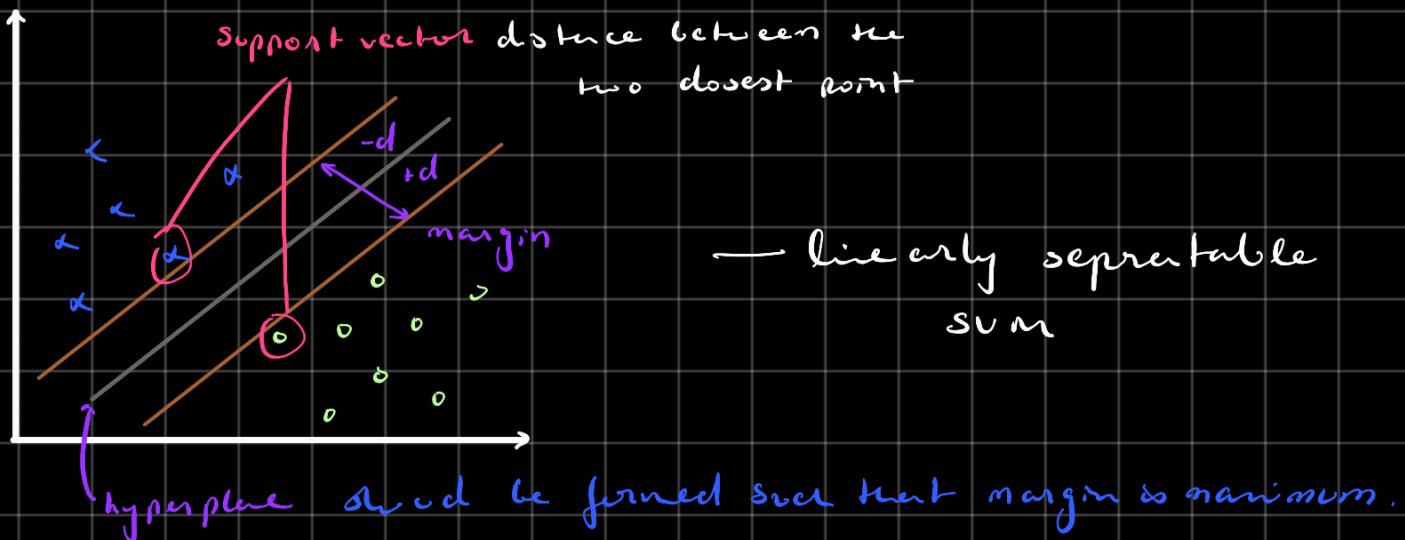
SVM Support Vector Machine →

- Labeled data set.
- Comes under supervised learning.

Labeled data → Machine learning Model



- To check in which class the new data will fall.



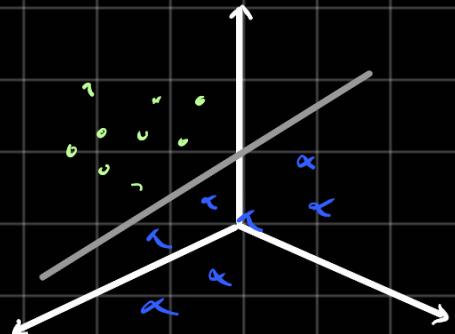
separation under 3D space →

Low dimensional
feature space

Kernel

High dimensional
feature space

leave one
out



Kmean →

• unsupervised learning.

K-means Algorithm		
	Height	Weight
①	185	72
②	170	56
③	168	60
④	179	68
⑤	182	72
⑥	188	77
⑦	180	71
⑧	180	70
⑨	183	84
⑩	180	88
⑪	180	67
⑫	177	76

Centroid
→ K₂

ED for ③ → K₁ → $\sqrt{(168-185)^2 + (60-72)^2}$
 $= 20.80 - K_1$
K₂ → $\sqrt{(168-170)^2 + (60-56)^2}$
 $= 4.48 - K_2$

New Centroid Calculation : after adding point ③

for K₂ = $(\frac{170+168}{2}, \frac{60+56}{2}) = (169, 58)$

ED for K₁ → $\sqrt{(179-185)^2 + (68-72)^2}$
= (6.32)
K₂ → $\sqrt{(179-169)^2 + (68-58)^2}$
= 14.14

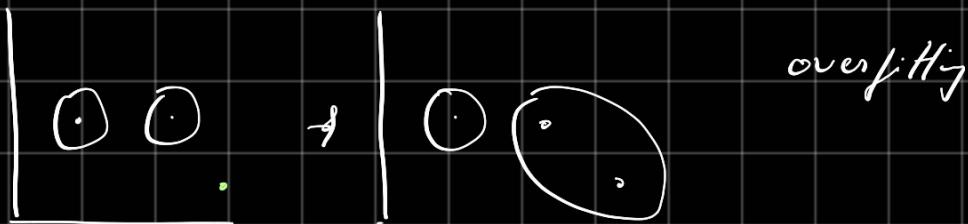
Euclidean Distance

$$\sqrt{(X_0 - X_c)^2 + (Y_0 - Y_c)^2}$$

new point centroid location

$$K_1 \rightarrow \{1, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$$

$$K_2 \rightarrow \{2, 3\}$$



K-Medoid →

K-MEDOID EXAMPLE		
i	x	y
x ₁	2	6
x ₂	3	4
x ₃	3	8
x ₄	4	7
x ₅	6	2
x ₆	6	4
x ₇	7	3
x ₈	7	4
x ₉	8	5
x ₁₀	7	6

Step 1
we select two random representative objects:
C₁(3, 4), C₂(7, 4)

1. $\text{cost}(c_1) = 12+16+4 = 32$

$m = (a, b)$
 $n = (c, d)$

Distance = $|a-c| + |b-d|$

i	x	y	c ₁	Distance/cost	c
x ₁	2	6	3	12+3+16+4 = 32	3
x ₃	3	8	3	0+4	4
x ₄	4	7	3	1+3	4
x ₅	6	2	3	3+2	5
x ₆	6	4	3	3+0	3
x ₇	7	3	3	4+1	5
x ₉	8	5	3	5+1	6
x ₁₀	7	6	3	4+2	6

②

Compare cost of Cost(c₁) and Cost(c₂) for every i & Select the minimum one

i	x	y	c ₁	Distance/cost	c
x ₁	2	6	3	12+3+16+4 = 32	3
x ₃	3	8	3	0+4	4
x ₄	4	7	3	1+3	4
x ₅	6	2	3	3+2	5
x ₆	6	4	3	3+0	3
x ₇	7	3	3	4+1	5
x ₉	8	5	3	5+1	6
x ₁₀	7	6	3	4+2	6

1. $\text{cost}(c_2) = 12+7+16+4 = 37$

$m = (a, b)$
 $n = (c, d)$

Distance = $|a-c| + |b-d|$

Step II) Then cluster are

cluster 1: $\{(2,6), (3,8), (4,7), (3,4)\}$

cluster 2: $\{(7,4), (6,2), (5,4), (7,3), (8,5), (7,6)\}$

Calculate total cost

$$T \text{ cost } (x, c) = \sum_{i=1}^d |x_i - c_i|$$

$$\begin{aligned} \text{Total cost} &= \{ \text{cost } ((3,4), (2,6)), \text{cost } ((3,4), (3,8)), \\ &\quad \text{cost } ((3,4), (4,7)), \text{cost } ((7,4), (8,5)), \\ &\quad \text{cost } ((7,4), (6,2)), \text{cost } ((7,4), (5,4)), \\ &\quad \text{cost } ((7,4), (7,3)), \text{cost } ((7,4), (7,6)) \}, \\ &= (3+4+4) + (3+1+1+2+2) \\ &= 20 \end{aligned}$$

Step 3) Select one of non-medoids O'

$$\text{Let's } O' = (7,3) \text{ i.e. } x_7,$$

So now medoid's are $C(3,4)$ & $O'(7,3)$

i	x	y	O'	Distance/cost	c
x_1	2	6	7	3	(2-7) + (6-3) = 8
x_3	3	8	7	3	4+5
x_4	4	7	7	3	3+4
x_5	6	2	7	3	1+1
x_6	6	4	7	3	1+1
x_8	7	4	7	3	0+1
x_9	8	5	7	3	1+2
x_{10}	7	6	7	3	0+3

Compare the cost of $\text{cost}(c_i)$ and $\text{cost}(O')$ for every i & Select the minimum one

i	x	y	c_i	Distance/cost	c
x_1	2	6	3	4	$12-31 + 16-4 = 3$
x_3	3	8	3	4	$0+4 = 4$
x_4	4	7	3	4	$1+3 = 4$
x_5	6	2	3	4	$3+2 = 5$
x_6	6	4	3	4	$3+0 = 3$
x_8	7	4	3	4	$4+0 = 4$
x_9	8	5	3	4	$5+1 = 6$
x_{10}	7	6	3	4	$4+2 = 6$

⑥

i	c_i	Distance/cost	c	c
x_1	3	4	$12-31 + 16-4 = 3$	8
x_3	3	4	$0+4 = 4$	7
x_4	3	4	$1+3 = 4$	2
x_5	3	4	$3+2 = 5$	2
x_6	3	4	$3+0 = 3$	1
x_8	3	4	$4+0 = 4$	3
x_9	3	4	$5+1 = 6$	3
x_{10}	3	4	$4+2 = 6$	1

⑦

Again create the cluster

cluster 1: $\{(3,4), (2,6), (3,8), (4,7)\}$

cluster 2: $\{(7,3), (6,2), (5,4), (7,4), (8,5), (7,6)\}$

$$\text{current total cost} = (3+4+4) + (2+2+1+3+3)$$

$$= 11 + 11$$

$$= 22$$

⑧

Step 4) So cost of swapping medoid from C_2 to O' 's

$$S = \text{current total cost} - \text{past total cost}$$

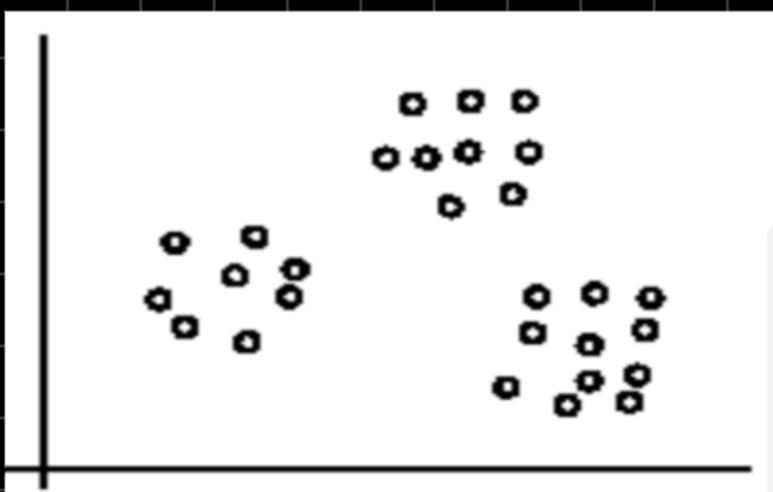
$$= 22 - 20$$

$$= 2 > 0$$

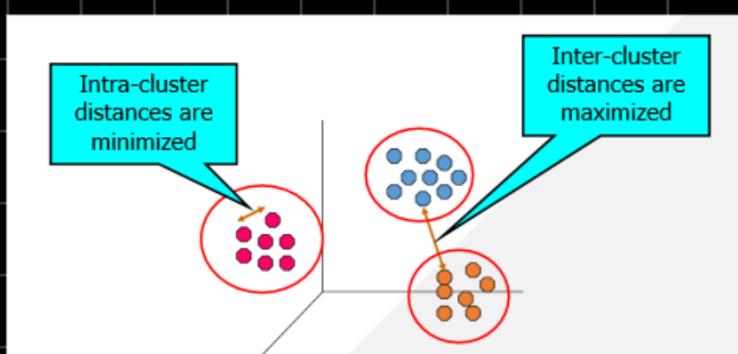
so, moving O' would be a bad idea so previous choice was good

Clusters →

- technique for finding similarity groups in data called clusters.
- It groups data instances that are similar to each other in one cluster as data instances that are very different from each other into different clusters.
 - Clustering is a mode of unsupervised learning
 - Organize data into sensible groups.
 - The data set has 3 natural groups of data points.



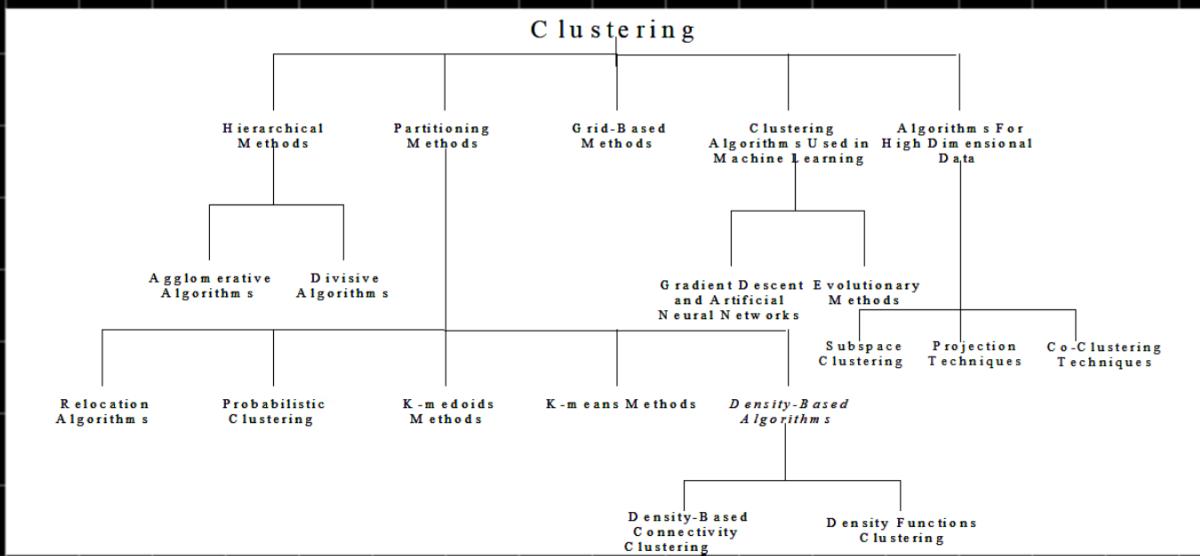
Inter , Intra - cluster



Types of clusters →

- Well-separated clusters
- Center-based clusters
- Contiguous clusters
- Density-based clusters
- Property or Conceptual
- Described by an Objective Function

Types of clustering Algorithm -->



Agglomerative → Bottom-up

Divisive → Top-down

$p_1 \ p_2 \ p_3 \ p_4 \ p_5$

$p_1 \ 0$

$p_2 \ 9 \ 0$

$p_3 \ 3 \ 7 \ 0$

$p_4 \ 6 \ 5 \ 9 \ 0$

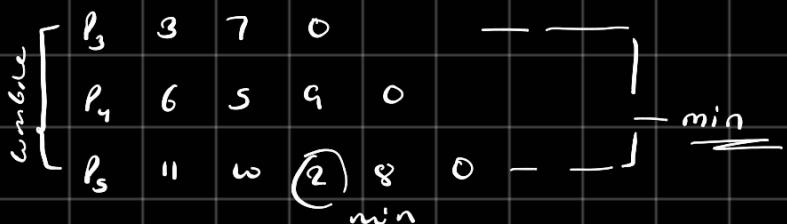
$p_5 \ 11 \ 10 \ 2 \ 8 \ 0$

single Linkage →

$p_1 \ p_2 \ p_3 \ p_4 \ p_5$

$p_1 \ 0$

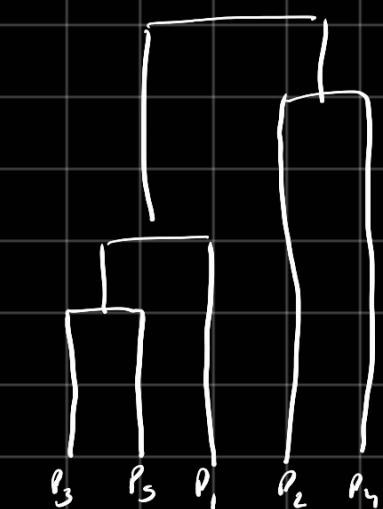
$p_2 \ 9 \ 0$



$$\min(d(p_1, (p_3, p_5))) \rightarrow (3, 11) \Rightarrow 3$$

$$\min(d(p_2, (p_3, p_5))) \rightarrow (7, 10) \Rightarrow 7$$

$$\min(d(p_4, (p_3, p_5))) \rightarrow (9, 8) \Rightarrow 8$$



$p_1 \ p_2 \ p_3 \ p_4 \ p_5$

$p_1 \ 0$

$p_2 \ 9 \ 0$

$p_3 \ 3 \ 7 \ 0$

$p_4 \ 6 \ 5 \ 8 \ 0$

next min

$$\min(d(p_2(p_1, p_3, p_5)) \rightarrow (9, 7, \omega) \rightarrow 7 \quad 1, 3, 5 \quad p_2 \quad p_1$$

$$\min(d(p_4(p_1, p_3, p_5)) \rightarrow (6, 9, 8) \rightarrow 6 \quad 1, 3, 5 \quad 0$$

$$p_2 \quad 7 \quad 0$$

$$p_4 \quad 6 \quad (5) \quad 0 \quad \rightarrow \text{next min}$$

$$\min(d(p_2, p_4), (p_1, p_3, p_5)) \rightarrow 1, 3, 5 \quad p_2, p_4$$

$$(9, 7, \omega), (6, 9, 8) \quad 1, 3, 5 \quad 0$$

$$(7, 6) \rightarrow 6 \quad p_2, p_4 \quad 6 \quad 0$$

Complex Linkage \rightarrow distance between \Rightarrow
max not min

$$p_1 \quad p_2 \quad p_3 \quad p_4 \quad p_5$$

$$p_1 \quad 0$$

$$p_2 \quad 9 \quad 0$$

$$p_3 \quad 3 \quad 7 \quad 0$$

$$p_4 \quad 6 \quad 5 \quad 9 \quad 0$$

$$p_5 \quad 11 \quad \omega \quad (2) \quad 8 \quad 0 \quad -\min$$

$$\max(d(p_1(p_3, p_5)) \rightarrow (3, 11) \rightarrow 11 \quad p_1 \quad p_2 \quad p_3, p_5 \quad p_4$$

$$p_2(p_3, p_5) \rightarrow (7, \omega) = 10 \quad p_1 \quad 0$$

$$p_4(p_3, p_5) \rightarrow (9, 8) = 9 \quad p_2 \quad 9 \quad 0$$

$$p_3, p_5 \quad 11 \quad \omega \quad 0$$

$$p_4 \quad 6 \quad (5) \quad 9 \quad 0 \quad -\min$$

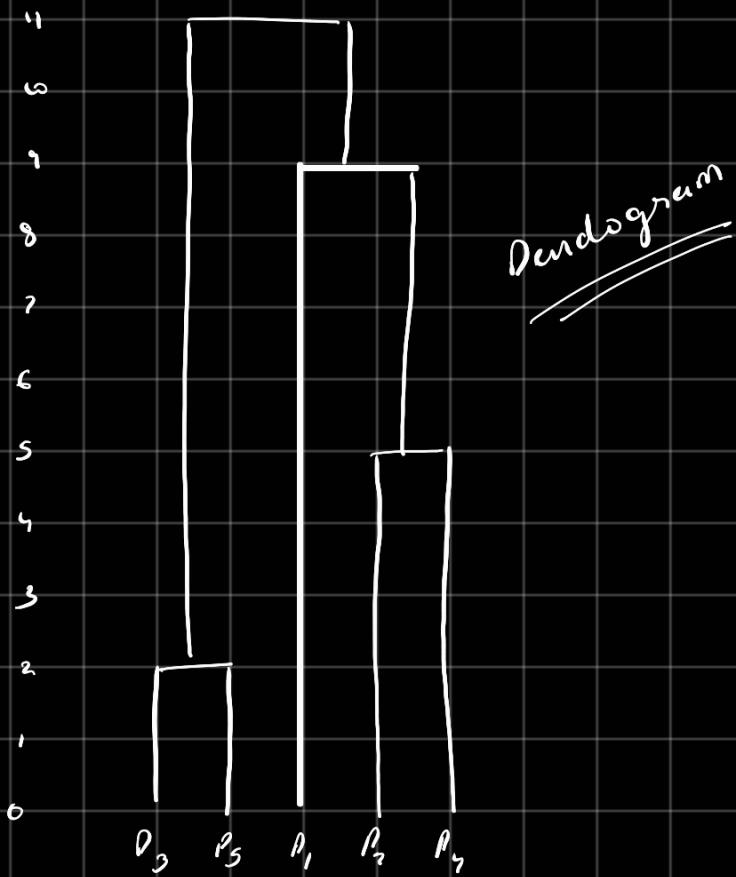
$$\max(d(p_1(p_2, p_4)) \rightarrow (6, 9) = 9 \quad p_1 \quad p_2, p_4 \quad p_3, p_5$$

$$p_1 \quad 0$$

$$p_2, p_4 \quad (9) \quad 0 \quad -\min$$

$$p_3, p_5 \quad 11 \quad \omega \quad 0$$

$\max(d(1, 2, 4), (3, 5))$ 1, 2, 4 3, 5
 $(3, 7, 9), (11, 10, 8) \Rightarrow 11$ 1, 2, 4 0
 $(3, 7, 9), (11, 10, 8) \Rightarrow 11$ 3, 5 11 0



Ansätze →

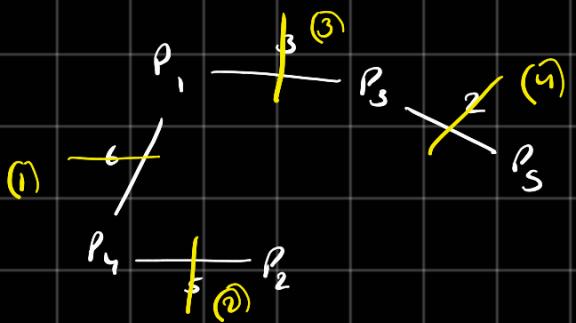
	P_1	P_2	P_3	P_4	P_5
P_1	0				
P_2	9	0			
P_3	3	7	0		
P_4	6	5	9	0	
P_5	11	10	2	8	0

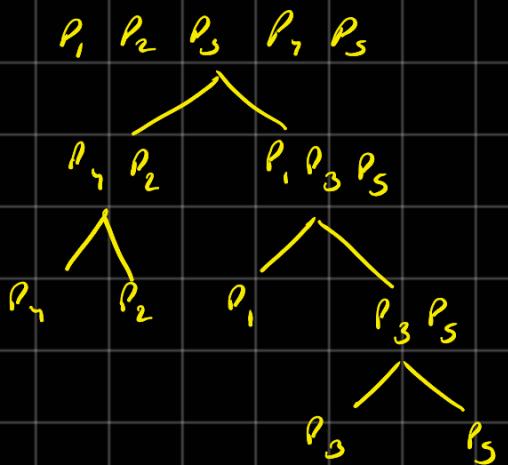
$-\min_2$

$-\min_3$ $-\min_4$

$-\min_5$

$P_5, P_3 - 2$
 $P_3, P_1 - 3$
 $P_4, P_2 - 5$
 $P_4, P_1 - 6$





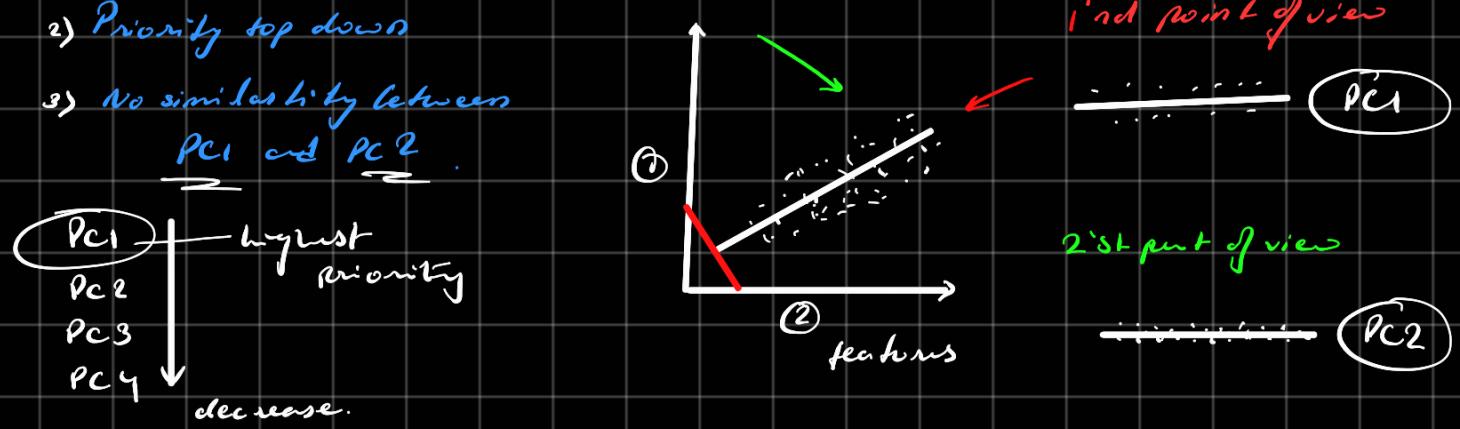
→ Divide Tree top-bottom.

Principal Component Analysis →

To extract important and usefull date from multivariate.

$$nD \rightarrow LD$$

- 1) Point of view
 - 2) Priority top down
 - 3) No similarity between PC1 and PC2.



X		Y		Principle Component Analysis	
2.5	2.4	C = $\begin{bmatrix} \text{Cov}(x,x) & \text{Cov}(x,y) \\ \text{Cov}(y,x) & \text{Cov}(y,y) \end{bmatrix}$		= $\begin{bmatrix} 0.6165 & 0.6154 \\ 0.6154 & 0.7165 \end{bmatrix}$	
0.5	0.7				
2.2	2.9				
1.9	2.2				
3.1	3.0	$\text{Cov}(x,y) = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{n-1}$			
2.3	2.7				
2	1.6	X $x - \bar{x}$ $(x - \bar{x})(x - \bar{x})$	Y $y - \bar{y}$ $(y - \bar{y})(y - \bar{y})$		
1	1.1	2.5 0.69 0.476	2.4 0.49 0.2401		
1.5	1.6	0.5 -1.31 1.7161	0.7 -1.21 1.4641		
1.1	0.9	Sum = 5.5490	Sum = 6.449		
calculate for all		$\frac{9}{9}$			
X		X	Y	$X - \bar{x}$	$Y - \bar{y}$
1.81	1.91	2.5	2.4	0.69	0.49
		0.5	0.7	-1.31	-1.21
mean		Sum = 5.5390			

① Creating the covariance matrix.

$$C - \lambda I = 0$$

$$\begin{bmatrix} 0.6165 & 0.6154 \\ 0.6154 & 0.7165 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 0$$

$$\begin{bmatrix} 0.6165 - \lambda & 0.6154 \\ 0.6154 & 0.7165 - \lambda \end{bmatrix}$$

$$\lambda_1 = 0.0490 \quad \text{eigen value}$$

$$\lambda_2 = 1.2840 \quad \text{eigen value}$$

$$C\gamma = \lambda v$$

$$\begin{bmatrix} 0.6165 & 0.6154 \\ 0.6154 & 0.7165 \end{bmatrix} = 0.0490 \begin{bmatrix} x_1 \\ y_1 \end{bmatrix}$$

$$0.6165x_1 + 0.6154y_1 = 0.0490x_1$$

$$\rightarrow 0.5674 X_1 = -0.6154 Y_1$$

$$X_1 = -1.0845 \quad Y_1 = 1.7614 + 1$$

-1.0845

$\begin{array}{r} 1 \\ 2 \\ \hline 1.7614 \\ 1.7614 \\ \hline 0 \end{array}$

$= 1.7614 - 1$

$\rightarrow \begin{bmatrix} -0.7351 \\ 0.6778 \end{bmatrix}$ eigenvectors for λ_1

$$\rightarrow \begin{bmatrix} x_2 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.92194 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.8499 \\ 1.3601 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0.6778 \\ 0.7351 \end{bmatrix} \text{ eigen vector for } \lambda_2$$

$$\begin{array}{r} \text{calculator} \\ y = 1 \\ n = 7 \end{array}$$

$$\begin{array}{r}
 1 \\
 \overline{14} \\
 14) \overline{100} \\
 - 98 \\
 \hline
 2
 \end{array}$$

Apriori Algorithm →

Apriori Algorithm												
		Support										
o Min Support = 50%	(II)											
o Threshold Confidence = 70%	Itemset											
Eg- <table border="1"> <thead> <tr> <th>TID</th> <th>Items</th> </tr> </thead> <tbody> <tr> <td>100</td> <td>1 3 4</td> </tr> <tr> <td>200</td> <td>2 3 5</td> </tr> <tr> <td>300</td> <td>1 2 3 5</td> </tr> <tr> <td>400</td> <td>2 5</td> </tr> </tbody> </table>	TID	Items	100	1 3 4	200	2 3 5	300	1 2 3 5	400	2 5	{1, 2}	1/4 → 25%. X
TID	Items											
100	1 3 4											
200	2 3 5											
300	1 2 3 5											
400	2 5											
	{1, 3}	2/4 → 50%. ✓										
	{1, 5}	1/4 → 25%. X										
	{2, 3}	2/4 → 50%. ✓										
	{2, 5}	3/4 → 75%. ✓										
	{3, 5}	2/4 → 50%. ✓										
(I) Itemset	Support											
1	2/4 → 50%.											
2	3/4 → 75%.											
3	3/4 → 75%.											
4	1/4 → 25%.											
5	3/4 → 75%.											
(Itemset → 1, 2, 3, 5)												
(III) Itemset	Support											
{1, 3, 5}	1/4 = 25%.											
{2, 3, 5}	2/4 = 50%.											
{1, 2, 3}	1/4 = 25%.											

- To find associations between the two subjects.

- ## • Minimum Support.

Min Support = 50%
Threshold Confidence = 70%

Apriori Algorithm

Rules	Support	Confidence
$(2^3) \rightarrow 5$	2	$2/2 = 100\% \checkmark$
$(3^5) \rightarrow 2$	2	$2/2 = 100\% \checkmark$
$(2^5) \rightarrow 3$	2	$2/3 = 66\%$
$2 \rightarrow (3^5)$	2	$2/3 = 66\%$
$5 \rightarrow (2^3)$	2	$2/3 = 66\%$
$3 \rightarrow (2^5)$	2	$2/3 = 66\%$

$$\text{Confidence} = S(A \cup B) / S(A)$$

$$\text{eg: } - \frac{(2^3) \rightarrow 5}{A} \underset{B}{=} \frac{S((2^3) \cup 5)}{S(2^3)} = 2/2 = 100\%$$

$$(2^3) \rightarrow 5 \quad \& \quad (3^5) \rightarrow 2$$

Itemset	Support
1	2
2	3
3	3
5	3
{1, 3}	2
$\checkmark \{2, 3\}$	2
{2, 5}	3
{3, 5}	2
{2, 3, 5}	2

TID	Items
100	1 3 4
200	2 3 5
300	1 2 3 5
400	2 5

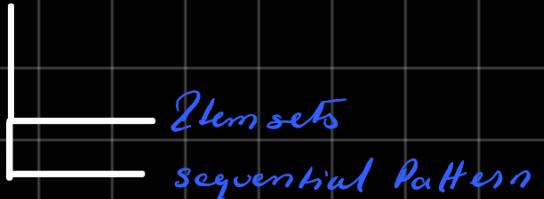
$$\text{confidence} = \frac{f(A, B)}{f(A)}$$

$$\text{support} = \frac{f(A, B)}{n}$$

$$lift = \frac{\text{sup}}{\text{sup}(A) \times \text{sup}(B)}$$

Market Basket Analysis

Frequent Patterns



Itemsets → "Or"

e.g. When one one buys "Milk" he might buy "Bread" too

{ Milk, Bread }

Sequential pattern → "Also"

e.g. When one one buys "Computer" also buys "Antivirus".

{ Computer, Antivirus }

When X is observed, Y is also observed.

$$\begin{array}{cc}
 x & y \\
 4 & 11 \\
 8 & 9 \\
 13 & 5 \\
 7 & 17 \\
 \hline
 8 & 8.5
 \end{array}
 \quad
 \begin{bmatrix}
 c_{00}(x, x) & (x, y) \\
 (y, x) & c_{00}(y, y)
 \end{bmatrix}
 \quad
 \begin{array}{r}
 8.5 \\
 4) \overline{32} \\
 -32 \\
 \hline
 0
 \end{array}$$

$$\sum \frac{(x - \bar{x})(y - \bar{y})}{n-1}$$

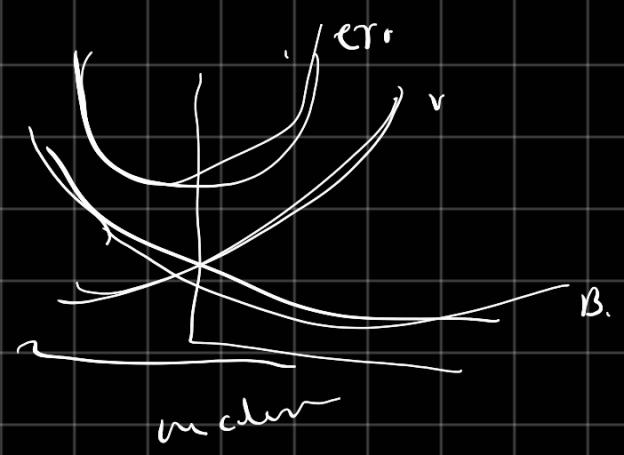
$$\begin{array}{ccccc}
 (x - \bar{x}) & (y - \bar{y}) & (x - \bar{x})(y - \bar{y}) & (y - \bar{y})(y - \bar{y}) & (x \cdot y) \\
 -7 & 2.5 & 46 & 6.05 & -40 \\
 0 & -7.5 & 0 & 20.25 & 0 \\
 5 & -3.5 & 25 & 12.25 & -17.5 \\
 -1 & 5.5 & \hline & 30.35 & -5.5 \\
 & & \hline & 68.90 & - \\
 & & \hline & 3 &
 \end{array}$$

$$\begin{bmatrix}
 c_{00}(x, x) & c_{00}(x, y) \\
 c_{00}(y, x) & c_{00}(y, y)
 \end{bmatrix}$$

$$C - 2Z = 0 \quad \lambda_1 \quad \lambda_2$$

$$C \begin{Bmatrix} x \\ y \end{Bmatrix} = \lambda \begin{Bmatrix} x \\ y \end{Bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix}$$



$$p(A|B) = \frac{p(B|A) p(A)}{p(B)}$$

wel
m.

