- Totology → when the result is always True.

P	Q	P and Q	P or Q	P → Q
F	F	F	F	T
F	T	F	T	T
T	F	F	T	F
T	T	T	T	T

⇒ if - then

Predicate → a symbol that represents a relation.

Quantifiers → variables in predicate.

- Universal
- Existential

$$\boxed{\begin{array}{l}\forall \text{ for all} \\ \exists \text{ their exists}\end{array}}$$

$$\underbrace{x+y+z}_{\text{quantifiers}} \leq a+2bc \rightarrow \text{Predicates}$$

P	Q	$P \wedge Q$	$P \vee Q$	$\sim P$	$\sim P \vee Q$	$P \rightarrow Q$
0	0	0	0	1	1	1
0	1	0	1	1	1	1
1	0	0	1	0	0	0
1	1	1	1	0	1	1

$$\boxed{\sim P \vee Q = P \rightarrow Q}$$

Totology → Always True

Contradiction → Always False

Contigency → Some True & False

$2^3 = 8$

P	Q	R	$P \rightarrow Q$	$(P \rightarrow Q) \rightarrow R$
0	0	0	1	0
0	0	1	1	1
0	1	0	1	0
0	1	1	1	1
1	0	0	0	1
1	0	1	0	1
1	1	0	1	0
1	1	1	1	1

① Good food is not cheap

② Cheap food is not good

x — y

$x \rightarrow \sim y$

$y \rightarrow \sim x$

x	y	$\sim y$	$(x \rightarrow \sim y)$	$\sim x$	$y \rightarrow \sim x$
0	0	1	1	1	
0	1	0	1	1	
1	0	1	1	0	
1	1	0	0	0	

$\boxed{x \leftrightarrows y \text{ if and only if}}$

$P \rightleftharpoons Q$

$(P \rightarrow Q) \wedge (Q \rightarrow P)$

$(\sim P \vee Q) \wedge (\sim Q \vee P)$

$(P \leftrightarrows Q) \rightarrow y$

$((P \rightarrow Q) \wedge (Q \rightarrow P)) \rightarrow y$

$\sim(\underline{(\sim P \vee Q) \wedge (\sim Q \vee P)}) \vee y$

$A \rightarrow B$	A	B	$(A \rightarrow B) \wedge A$	$[(A \rightarrow B) \wedge A] \rightarrow B$
1	0	0	0	1
1	0	1	0	1
0	1	0	0	1
1	1	1	1	1

Totology

$(p \rightarrow q) \rightleftharpoons (\sim q \rightarrow \sim p)$ ⇒ $P \rightleftharpoons Q$ "if and only if"

p	q	$p \rightarrow q$	$\sim p$	$\sim q$	$\sim q \rightarrow \sim p$	$(p \rightarrow q) \rightleftharpoons (\sim q \rightarrow \sim p)$
0	0	1	1	1	1	1
0	1	1	1	0	1	1
1	0	0	0	1	0	1
1	1	1	0	0	1	1

Totology

$((P \rightarrow Q) \rightarrow R) \rightleftarrows \sim P$

Same $\rightarrow$ 1

different $\rightarrow$ 0

P	Q	R	$P \rightarrow Q$	$(P \rightarrow Q) \rightarrow R$	$\sim P$		$((P \rightarrow Q) \rightarrow R) \rightleftarrows \sim P$
0	0	0	1	0	1	$\rightarrow$ diff	0
0	0	1	1	1	1	$\rightarrow$ same	1
0	1	0	1	0	1	$\rightarrow$ diff	0
0	1	1	1	1	1		1
1	0	0	0	1	0		0
1	0	1	0	1	0		0
1	1	0	1	0	0		1
1	1	1	1	1	0		0

Not totology

$\sim A \wedge B \rightarrow \sim (A \vee B)$

A	B	$\sim A$	$\sim A \wedge B$	$A \vee B$	$\sim (A \vee B)$	$\sim A \wedge B \rightarrow \sim (A \vee B)$
0	0	1	0	0	1	1
0	1	1	1	1	0	0
1	0	0	0	1	0	1
1	1	0	0	1	0	1

Not totology

- For $\underbrace{\text{students to do well in descreate}}_{D}$ it is necessary that they $\underbrace{\text{study hard}}_{H}$.
- Student who do well in courses do not skip classes.
- Who study hard they do well in course.
- There fore students who do well in descree course do not skip class.

$D \rightarrow$ students who well in descreate

$H \rightarrow$ working hard

c → To do well in class.

s → Do not skip courses.

1) Prove Totology

2) Equivalent

3) Valid

4) Convert

given → $D \rightarrow H$ — ①

$C \rightarrow S$ — ②

$H \rightarrow C$ — ③

To prove → $D \rightarrow S$

$D \rightarrow H$

$H \rightarrow C$

$D \rightarrow C$ — ④

$C \rightarrow S$

$D \rightarrow S$

Q If there was a ball game (a) then traveling was difficult (b).
If they ride on time (c) then traveling was not difficult (~b).
They ride on time therefore there was no ball game.

$a \rightarrow b$
$c \rightarrow \sim b$ } Given

$c \rightarrow \sim a$ → To prove

$$\boxed{\begin{array}{c} a \rightarrow b \\ \sim b \rightarrow \sim a \end{array}}$$

$\sim b \rightarrow \sim a$

$c \rightarrow \sim b$

$c \rightarrow \sim a$ Proved

Q If I get the job (J) and work hard (W) then I will get promoted (P).
If I get promoted then I am happy (H).
I am not happy (~H) therefore either I will not get job (~J) or not work hard (~W).

$J \wedge W \rightarrow P$
$P \rightarrow H$ } Given

$\sim H \rightarrow \sim J \vee \sim W$ → To prove

$J \wedge W \rightarrow H$

$\sim(J \wedge W) \rightarrow \sim H$

$\sim J \wedge \sim W \rightarrow \sim H \implies \sim H \rightarrow \sim J \vee \sim W$ proved

Q $(p \rightarrow q) \vee \sim q \equiv (p \rightarrow \sim p) \wedge q$

using table →

Q $[(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)] \rightarrow r$

p	q	r	$p \vee q$	$p \rightarrow r$	$\overset{a}{(p \vee q) \wedge (p \rightarrow r)}$	$q \rightarrow r$	$\overset{c}{\overline{q \rightarrow r \wedge a}}$	$c \rightarrow r$
0	0	0	0	1	0	1	0	1
0	0	1	0	1	0	1	0	1
0	1	0	1	1	1	0	0	1
0	1	1	1	1	1	1	1	1
1	0	0	1	0	0	1	0	1
1	0	1	1	1	1	1	1	1
1	1	0	1	0	0	0	0	1
1	1	1	1	1	1	1	1	1

Totology

- Converse → If $p \rightarrow q$ then $q \rightarrow p$
- inverse → $\sim p \rightarrow \sim q$
- contrapositive → $\sim q \rightarrow \sim p$

Q If two angles are congruent then they have same measure. *Converse*

→ If two angles have same measure then they are congruent.

→ If two angles are not congruent then they are not congruent.

Q If a quadrilatral is a rectangle then it has 2 pairs of parallel side.

If a quadrilatral has 2 pairs of parallel sides then it is a rectangle.

Q $(p \to q) \leftrightarrow \sim p \vee q$

p	q	$p \to q$	$\sim p$	$\sim p \vee q$	$(p \to q) \leftrightarrow \sim p \vee q$
0	0	1	1	1	1
0	1	1	1	1	1
1	0	0	0	0	1
1	1	1	1	1	1

$\Big\}$ Totology

∵ Therefore not a contradiction.

Q If a triangle is equilatral then it has equal angles.

- If a triangle has equal angles then it is equilatral.

Q $(p\to q)\wedge(q\to r)\to(p\to r)$

$\wedge$ and
$\vee$ or

p	q	r	a: $p\to q$	b: $q\to r$	c: $a\wedge b$	d: $p\to r$	$c\to d$
0	0	0	1	1	1	1	1
0	0	1	1	1	1	1	1
0	1	0	1	0	0	1	1
0	1	1	1	1	1	1	1
1	0	0	0	1	0	0	1
1	0	1	0	1	0	1	1
1	1	0	1	0	0	0	1
1	1	1	1	1	1	1	1

Totology

Q A solves the problem with probablity 0.2, B 0.3 and chances of question getting solved is 0.4.

Q A box contain 6R, 8G, 10Blue, 12Y, 15W balls. min balls to chose from the box so 9 balls are of same color.

p	q	r	s	a: $p\wedge q$	b: $p\wedge r$	c: $a\vee b$	$c\to s$
0	0	0	0	0	0	0	1
0	0	0	1	0	0	0	1
0	0	1	0	0	0	0	1
0	0	1	1	0	0	0	1
0	1	0	0	0	0	0	1
0	1	0	1	0	0	0	1
0	1	1	0	0	0	0	1
0	1	1	1	0	0	0	1
1	0	0	0	0	0	0	1
1	0	0	1	0	0	0	1
1	0	1	0	0	0	0	1

1	0	1	1	0	0	0	1
1	1	0	0	1	1	1	0
1	1	0	1	1	1	1	1
1	1	1	0	1	1	1	0
1	1	1	1	1	1	1	1

$(\sim p \vee (\sim q \wedge \sim r)) \vee s$

							a	b	
p	q	r	s	$\sim p$	$\sim q$	$\sim r$	$\sim q \wedge \sim r$	$a \vee \sim p$	$b \vee s$
0	0	0	0	1	1	1	1	1	1
0	0	0	1	1	1	1	1	1	1
0	0	1	0	1	1	0	0	1	1
0	0	1	1	1	1	0	0	1	1
0	1	0	0	1	0	1	0	1	1
0	1	0	1	1	0	1	0	1	1
0	1	1	0	1	0	0	0	1	1
0	1	1	1	1	0	0	0	1	1
1	0	0	0	0	1	1	1	1	1
1	0	0	1	0	1	1	1	1	1
1	0	1	0	0	1	0	0	0	0
1	0	1	1	0	1	0	0	0	1
1	1	0	0	0	0	1	0	0	0
1	1	0	1	0	0	1	0	0	1
1	1	1	0	0	0	0	0	0	0
1	1	1	1	0	0	0	0	0	1

- If the races are fixed so the casino are "crooked" then tourist trade will decline.

r = races are fixed, t = tourist trade, t = decline

- If the tourist trade decreases then the police will be happy.

t = tourist trade decreases, p = police will be happy

- The police force is never happy therefore the races are not fixed.

$\sim p$ = police force is never happy; the races are not fixed = To check

$r \to t$
$t \to p$ } Given
$\sim p \to \sim r$ → To prove

$r \to p$ — Transitivity
$\sim p \to \sim r$ — contrapositive
Proved

Q A collection of 10 electrive bulbs contain 3 defective bulb.

1) In how many way a sample of 4 bulbs can be selected
2) In how many ways a sample of 4 bulbs can be selected 2 good 2 bad.
3) In how many way we can select a 4 bulbs such that 1 good, 3 defective or 3 good, 1 bad.

1) $^{10}C_4 \to \frac{10!}{4!\,6!} = \underline{\underline{210}}$

2) $^{7}C_2 \times {}^{3}C_2 \to \underline{\underline{63}}$

3) $^{7}C_3 \times {}^{3}C_1 + {}^{3}C_3 \times {}^{7}C_1 \to \underline{\underline{112}}$

4) At least 1 defective.

1D	3G	$^{3}C_1 \times {}^{7}C_3$
2D	2G	$^{3}C_2 \times {}^{7}C_2$
3D	1G	$^{3}C_3 \times {}^{7}C_1$

} Sum

$^{10}C_4 - {}^{7}C_4 = ?$

Law's $\rightarrow$

1. Idempotent Law $\rightarrow$

Only use a single statement and combine it with $\wedge, \vee$ then statement will be the statement.

$$P \vee P = P$$
$$P \wedge P = P$$

2. Commutative Laws $\rightarrow$

Combine 2 statement with $\wedge, \vee$, then resultant will be same even if change position.

$$P \vee Q = Q \vee P$$
$$Q \wedge P = P \wedge Q$$

3. Associative Law $\rightarrow$

Combine 3 statement with $\wedge, \vee$ resultant will be same even if order change.

$$P \wedge (Q \wedge R) = (P \wedge Q) \wedge R$$
$$P \vee (Q \vee R) = (P \vee Q) \vee R$$

4. Same as Distributive Law $\rightarrow$

$$P \vee (Q \wedge R) = (P \vee Q) \wedge (P \vee R)$$
$$P \wedge (Q \vee R) = (P \wedge Q) \vee (P \wedge R)$$

5. Complement Law $\rightarrow$

$$P \vee \sim P = T$$
$$P \wedge \sim P = F$$

6. De Morgan's Law $\rightarrow$

Combine 2 statements with $\wedge, \vee$ then do the negation of statement should be same as combing negation of both statement seperately.

$$\sim (P \wedge Q) = \sim P \wedge \sim Q$$
$$\sim (P \vee Q) = \sim P \vee \sim Q$$

Normal Forms →

1. **Disjunctive Normal Form (DNF)**

   Only false when both are false.

   $(P \vee Q)$ $\quad F \vee F \Rightarrow F \quad F \vee T \rightarrow T$

2. **Conjunctive Normal Form (CNF)**

   Only true when both are true.

   $(P \wedge Q)$ $\quad T \wedge T \rightarrow T \quad T \wedge F \rightarrow F$

$\exists(x) \longrightarrow$ There exists / some
$\forall(x) \longrightarrow$ For all

→ There exists a man.

Some man are clever.

Some real numbers are rational.

$\exists(x)\ M(x)$

$\exists(x)\ (M(x) \wedge C(x))$ / $\exists(x)\ (M(x) \longrightarrow C(x))$

$\exists(x)\ (R(x) \wedge Ra(x))$

→ Not all graphs are connected.

Some graphs are not connected.

All graphs are conected.

$\sim \forall_x (G(x) \longrightarrow C(x))$

$\exists_x (G(x) \wedge \sim C(x))$

$\forall_x (G(x) \longrightarrow C(x))$

Some ⇒ "$\wedge$" and

All ⇒ "$\longrightarrow$"

→ Some body likes some one.

Every body likes every body.

Every body likes some.

There is some one liked by every body.

$\Rightarrow$ $P(x, y) =$ x likes y

$\exists_x \exists_y P(x,y)$

$\forall_x \forall_y P(x,y)$

$\forall_x \exists_y P(x,y)$

$\exists_x \forall_y P(x,y)$

b

$(((p \to q) \leftrightarrow r) \to \sim(p \vee q))$

p	q	r	$p \to q$ (a)	$a \leftrightarrow r$ (b)	$\sim(p \vee q)$ (c)	$b \to c$
0	0	0	1	0	1	1
0	0	1	1	1	1	1
0	1	0	1	0	0	1
0	1	1	1	1	0	0
1	0	0	0	1	0	0
1	0	1	0	0	0	1
1	1	0	1	0	0	1
1	1	1	1	1	0	0

Contigency.

- In how many ways we can write the word (corporation).

$$\frac{11!}{3!\ 2!}$$

- " " such that all vowels comes together.

$$\frac{5!\ 7!}{3!\ 2!}$$

Rules of Inference →

1. Modus Ponens →

   if P,
   $P \to Q$, then derive Q.

2. Modus Tollens →

   if $P \to Q$,
   $\sim Q$, then derive $\sim P$.

3. Hypothetical Syllogism →

   if $P \to Q$,
   $Q \to R$, then derive $P \to R$.

4. Disjunction Syllogism →

   if $\sim P$,
   $P \vee Q$, then derive Q

5. Addition →

   if P,
   then derive $P \vee Q$.

6. Simplification →

   if $P \wedge Q$,
   then derive P.

7. Conjunction →

   if P,
   Q, then derive $P \wedge Q$.

Modus ponens

P, $P \to Q$ Q?

Modus tollense

$P \to Q$

$\sim Q \to \sim P$?

Ponens

tollense

then

Hypothetical →

$P \to Q$

$Q \to R$ then $P \to R$?

8. Resolution

if $P \vee Q$,
$\sim P \vee R$, then derive $Q \vee R$.

9. Constructive Dilemma

if $(P \rightarrow Q) \wedge (R \rightarrow S)$,
$P \vee R$, then derive $Q \vee S$.

10. Destructive Dilemma

if $(P \rightarrow Q) \wedge (R \rightarrow S)$,
$\sim Q \vee \sim S$, then derive $\sim P \vee \sim R$.

Pigeonhole Principle →

II) We can say as, if n + 1 objects are put into n boxes, then at least one box contains two or more objects.
The abstract formulation of the principle: Let X and Y be finite sets and let be a function.

- If X has more elements than Y, then f is not one-to-one.
- If X and Y have the same number of elements and f is onto, then f is one-to-one.
- If X and Y have the same number of elements and f is one-to-one, then f is onto.

Pigeonhole principle is one of the simplest but most useful ideas in mathematics. We will see more applications that proof of this theorem.

Pigeonhole principle strong form –
Theorem: Let $q_1, q_2, \ldots, q_n$ be positive integers.
If $q_1 + q_2 + \ldots + q_n - n + 1$ objects are put into n boxes, then either the 1st box contains at least q_1 objects, or the 2nd box contains at least q_2 objects, . . ., the nth box contains at least q_n objects.

1) $(p \to q) \leftrightarrow (\sim q \to \sim p)$

2) $[(p \wedge q) \to r] \to (\sim r \to (\sim p \vee \sim q))$

3) $p \wedge q \equiv \sim(\sim p \vee \sim q)$

4) $(p \to \sim q) \equiv \sim(p \wedge (q \vee r))$

p	q	$\sim q$	$\sim p$	$\overset{a}{p \to q}$	$\overset{b}{\sim q \to \sim p}$	$a \leftrightarrow b$
0	0	1	1	1	1	1
0	1	0	1	1	1	1
1	0	1	0	0	0	1
1	1	0	0	1	1	1

Totology

p	q	r	$\overset{a}{p \wedge q}$	$\overset{d}{a \to r}$	$\overset{b}{\sim r}$	$\overset{c}{\sim p \vee \sim q}$	$\overset{e}{b \to c}$	$d \to e$
0	0	0	0	1	1	1	1	1
0	0	1	0	1	0	1	1	1
0	1	0	0	1	1	1	1	1
0	1	1	0	1	0	1	1	1
1	0	0	0	1	1	1	1	1
1	0	1	0	1	0	1	1	1
1	1	0	1	0	1	0	0	1
1	1	1	1	1	0	0	1	1

Totology

→

- If triangle abc isocelus and contain angle 45° then ∠ABC 90° (I, A, R)
- If a quadrilateral abcd is a square then it is a rectangle and a square.
- If a quarilateral abcd, 2 side with equal lengths then its a rectangle and rombus.

- converse
- inverse
- contrapositive

- $(Q \cap A) \to R$

Converse →

$R \to (Q \cap A)$

Inverse →

$\sim(Q \cap A) \to \sim P$

Contrapositive →

$\sim R \to \sim(Q \cap A)$

- How many numbers greater than 1,000,000 can be formed using the digit given 0, 1, 2, 2, 4, 2, 4

① $\frac{6!}{3!\,2!}$ 1 _ _ _ _ _ _ (0 2 2 2 4 4) 6!

② $\frac{6!}{2!\,2!}$ 2 _ _ _ _ _ _ (0 2 1 4 2 4)

③ $\frac{6!}{3!}$ 4 _ _ _ _ _ _ (0 2 1 4 2 2)

① + ② + ③ → Ans

CNF ⟶ reduce such that statements are "$A \wedge B \wedge C \ldots$" form

① $P \cap (P \to q)$

$\sim P \vee q$ — using

$P \cap (\sim P \vee q)$

$(P \wedge \sim P) \vee (P \wedge q)$

$0 \vee (P \wedge q)$

$\Rightarrow (P \wedge q)$ CNF

② $(\sim P \to r) \wedge (q \longleftrightarrow P)$

$(P \vee r) \wedge ((q \to P) \wedge (P \to q))$

$(P \vee r) \wedge ((\sim q \vee P) \wedge (\sim P \vee q))$

$\boxed{\begin{array}{l}\text{Conjunction Normal Form} \\ \text{Disjunction Normal Form}\end{array}}$

(3) $\sim((\sim P \rightarrow \sim q) \wedge \sim r)$

$\sim((P \vee \sim q) \wedge \sim r)$

$\sim(P \vee \sim q) \vee r$

$(\sim P \wedge q) \vee r$

$\Rightarrow (\sim P \vee r) \wedge (q \vee r)$ CNF

(4) $(P \rightarrow q) \rightarrow (\sim r \wedge q)$

$(\sim P \vee q) \rightarrow (\sim r \wedge q)$

$\sim(\sim P \vee q) \vee (\sim r \wedge q)$

$(P \wedge \sim q) \vee (\sim r \wedge q)$

$(P \vee (\sim r \wedge q)) \wedge (\sim q \vee (\sim r \wedge q))$

$(P \vee \sim r) \wedge (P \vee q) \wedge (\sim q \vee \sim r)$

DNF → reduce such that statements are "$A \vee B \vee C$ ..." form

(1) $P \rightarrow ((P \rightarrow Q) \wedge \sim(\sim Q \vee \sim P))$

$\sim P \vee ((P \rightarrow Q) \wedge \sim(\sim Q \vee \sim P))$

$\sim P \vee ((\sim P \vee Q) \wedge \sim(\sim Q \vee \sim P))$

$\sim P \vee ((\sim P \vee Q) \wedge (Q \wedge P))$

$\sim P \vee ((\sim P \wedge \cancel{(Q \wedge Q)}) \vee (Q \wedge (Q \wedge P))$

$\sim P \vee ((\sim P \wedge Q) \vee (Q \wedge P))$

$\sim P \vee$

$P \rightarrow ((\sim P \vee Q) \wedge \sim(\sim Q \vee \sim P))$

$P \rightarrow ((\sim P \vee Q) \wedge (Q \wedge P))$

$P \rightarrow ((\underbrace{\sim P \wedge (Q \wedge P)}_{=0}) \vee (Q \wedge (Q \wedge P)))$

$(Q \wedge Q \wedge Q \wedge P)$

$P \rightarrow (Q \wedge (Q \wedge P))$ → Associative

$\sim P \vee (Q \vee P)$

$(\sim P \vee Q) \vee (\sim P \vee P)$

P	$\sim P$	
0	1	1
1	0	1

(1) $(P \rightarrow (\sim q \wedge r)) \wedge (P \rightarrow \sim q)$

$(\sim P \vee (q \wedge r)) \wedge (\sim P \vee \sim q)$

$(\sim P \vee \sim q) \wedge (\sim P \vee r) \wedge (\sim P \vee \sim q)$

$(\sim P \vee \sim q) \wedge (\sim P \vee r)$ CNF

P	q	r	$\sim q$	$\sim q \wedge r$ (a)	$P \rightarrow a$ (b)	$P \rightarrow \sim q$ (c)	$b \wedge c$
0	0	0	1	0	1	1	1
0	0	1	1	1	1	1	1
0	1	0	0	0	1	1	1
0	1	1	0	0	1	1	1
1	0	0	1	0	0	1	0
1	0	1	1	1	1	1	1
1	1	0	0	0	0	0	0
1	1	1	0	0	0	1	0

- If x, y are rational then $x+y$ is rational.
  $\underbrace{x, y \text{ are rational}}_{a}$ then $\underbrace{x+y \text{ is rational}}_{b}$
  - Converse $b \to a$
  - Inverse $\sim a \to \sim b$
  - Contradiction $\sim b \to \sim a$

If $x+y$ is rational then x, y is rational. Converse
If x, y is not rational then $x+y$ is also not rational Inverse
If $x+y$ is not rational then x, y is not rational. contrapositive

DNF →

① $p \vee (\sim p \to (q \wedge (q \to \sim r)))$

$p \vee (\sim\sim p \vee (q \wedge (q \to \sim r)))$

$p \vee (p \vee (q \wedge (q \to \sim r)))$

$p \vee (p \vee (q \wedge (\sim q \vee \sim r)))$

$p \vee (p \vee (q \wedge \sim q) \vee (q \wedge \sim r))$

$p \vee (p \vee (q \wedge \sim r))$

$p \vee ((p \vee q) \wedge (p \vee \sim r))$

$(p \vee p \vee q) \wedge (p \vee p \vee \sim r)$

$(p \vee q) \wedge (p \vee \sim r)$

$(p \wedge p \vee \sim r) \vee (q \wedge p \vee \sim r)$

$(p \vee \sim r) \vee (q \wedge p \vee \sim r)$

$p \vee (\sim p \to (q \wedge (q \to \sim r)))$

$p \vee (p \vee (q \wedge (q \to \sim r)))$

$p \vee (p \vee (q \wedge (\sim q \vee \sim r)))$

$p \vee (p \vee (q \wedge \sim q) \vee (q \wedge \sim r))$

$p \vee (p \vee (q \wedge \sim r))$

$p \vee [(p \vee q) \wedge (p \vee \sim r)]$

$(p \vee p) \vee (p \vee q) \wedge (p \vee \sim r)$

$p \vee (p \vee q) \wedge (p \vee \sim r)$

$(p \wedge p) \vee (p \wedge \sim r) \vee (p \vee q)$

$p \vee (p \wedge \sim r) \vee (p \vee q)$

$p \vee [(p \vee q) \wedge (p \vee \sim r)]$

$(p \vee p \vee q) \wedge (p \vee p \vee \sim r)$

$p \vee (p \vee q) \wedge p \vee (p \vee \sim r)$

$(p \vee p) \vee (p \vee q) \wedge (p \vee p) \vee (p \vee \sim r)$

$(p \vee (p \vee q)) \wedge (p \vee (p \vee \sim r))$

$((p \vee p) \vee q) \wedge (p \vee p) \vee \sim r)$

$(p \vee q) \wedge (p \vee \sim r)$

$(p \wedge p) \vee (p \wedge \sim r) \vee (q \wedge p) \vee (q \wedge \sim r)$

$p \vee (p \wedge \sim r) \vee (q \wedge p) \vee (q \wedge \sim r)$

$\sim((p \leftrightarrow q) \vee r) \equiv \sim(r \to q) \vee p$

$\sim(((p \to q) \wedge (q \to p)) \vee r)$

$\sim((\sim p \vee q) \wedge (\sim q \vee p) \vee r)$

$\sim(\sim p \wedge (\sim q \vee p) \vee q \wedge (\sim q \vee p) \vee r$

$p \vee (\sim p \to (q \wedge (q \to \sim r)))$

$p \vee (\sim p \to (q \wedge (\sim q \vee \sim r)))$

$\to (q \wedge \sim q) \vee (q \wedge \sim r)$

$p \vee (\sim p \to (q \wedge \sim r))$

$p \vee (p \vee q \wedge \sim r)$

$p \to ((p \to q) \wedge \sim (\sim q \vee \sim p))$

$p \to [(\sim p \vee q) \wedge (q \wedge p)]$

$p \to [(\sim p \vee q \wedge q) \wedge (\sim p \vee q \wedge p)]$

$p \to [\sim p \vee q \wedge q]$

$p \to (\sim p \vee q)$ $\qquad (\sim p \wedge q \wedge p) \vee (q \wedge p)$

$\sim p \vee \sim p \vee q$

$\sim p \vee q$

$(\sim p \to r) \wedge (q \leftrightarrow p)$

$(\sim\sim p \vee r) \wedge (q \leftrightarrow p)$

$(p \vee r) \wedge (q \to p) \wedge (p \to q)$

$(p \vee r) \wedge (\sim q \vee p) \wedge (\sim p \vee q)$

$(p \to q) \wedge (\sim p \wedge q)$

$(\sim p \vee q) \wedge (\sim p \wedge q)$

$(\sim p \wedge \sim p \wedge q) \vee (q \wedge \sim p \wedge q)$

$(\sim p \wedge q) \vee (q \wedge \sim p)$

$\sim (p \to (q \wedge r))$

$\sim (\sim p \vee (q \wedge r))$

$\sim ((\sim p \vee q) \wedge (\sim p \vee r))$

$(p \wedge \sim q \vee p \wedge \sim r)$

$p \wedge (\sim q \vee \sim r)$

$(p \to (q \wedge r)) \wedge (\sim p \to (\sim q \wedge \sim r))$

$(\sim p \vee (q \wedge r)) \wedge (p \vee (\sim q \wedge \sim r))$

$(\sim p \vee q) \wedge (\sim p \vee r) \wedge (p \vee \sim q) \wedge (p \vee \sim r)$

$(q \vee (p \wedge r)) \wedge \sim ((p \vee r) \wedge q)$

$(q \vee p) \wedge (q \vee r) \wedge ((\sim p \vee r) \vee \sim q)$

$(q \vee p) \wedge (q \vee r) \wedge (\sim p \vee \sim q) \wedge (\sim q \vee r)$

Idempotence law

$p \vee p \to p$

$p \wedge p \to p$

Commutative

$p \vee q \to p \wedge q$

$p \wedge q \to p \vee q$

Associative

$p \wedge (q \wedge R) \to (p \wedge q) \wedge R$

$p \vee (q \vee R) \to (p \vee q) \vee R$

Demorgan $\to$

$\sim (p \vee q) \to \sim p \wedge \sim q$

$\sim (p \wedge q) \to \sim p \vee \sim q$

compliment

$\sim p \vee p \to T$

$\sim p \wedge p \to T$

Introduction to set Theory →

set — collection of similar type of data set.

Roaster / Tabular Form — $\{1, 2, 3\}$

set builder form — $\{x : x \text{ is natural number} < 4\}$

$\{x : x \in N,\ 2 < x < \omega\}$

(: such that; $\in$ Belong to; N Natural number)

- N – set of natural number
- Z – set of integers
- Z+ – set of positive integer
- Q – set of rational number
- R – set of Real number
- W – set of whole number

Cardinality of set → Number of elements present in a set.

Operation's on set →

- ∪ Union (or)
- ∩ Intersection (and)
- − Difference of sets
- ^ symmetric Difference (eliminating common values)

$$A \cap B = n(A) + n(B) - n(A \cup B)$$

Equal set – if same element present.

Equivalent set – Same number of elements.

$$\underline{A} \subset \underline{B}$$

(A: subset; B: super set)

Power set →

$A = \{1, 2, 3\}$

$2^3 \Rightarrow 8$ subset's

$\{\}, \{\phi\}$

Proper sub set →

$A = \{1, 2, 3\}$

$B = \{1\}, \{2\}, \{3\}, \{1, 2\}, \{2, 3\}, \{3, 1\}, \{1, 2, 3\}$

all are proper except → $\{1, 2, 3\}$

Is not a proper set

Relation →

sets $A = \{1, 2, 3\}$

$B = \{4, 5\}$

$A \times B = \{ (1,4), (1,5)$
$(2,4), (2,5)$
$(3,4), (3,5) \}$

$B \times A = \{ (4,1), (4,2), (4,3)$
$(5,1), (5,2), (5,3) \}$

$(1,4) \neq (4,1)$

in relation.

1) Relation → If two sets are considered, the relation between them will be established if there exists a connection between the elements of two or more non-empty sets.

$\underline{A \times A}$

$A = \{1, 2, 3\}$

$A \times A = \{ (1,1), (1,2), (1,3)$
$(2,1), (2,2), (2,3)$
$(3,1), (3,2), (3,3) \}$

Types of relation →

1) Reflexive Relation →

$\forall a \quad aRa \in R \qquad (a,a) \in R \ \forall a \in$ "set"

2) Irr-Reflexive Relation →

$\forall a \quad aRa \notin R$

3) Non-Reflexive →

Some $aRa \in R$

4) Symmetric Relation →

If $(a,b) \in R$ then $(b,a) \in R$

5) Asymetric Relation $\longrightarrow$

if $(a,b) \in R$ then $(b,a) \notin R$

6) Anti Symmetric Relation $\longrightarrow$

if $(a,b) \in R$ then $(b,a) \notin R$ if $a \neq b$

if $(a,b) \in R$ and if $a=b$ then $(b,a) \in R$

7) Transitive Relation $\longrightarrow$

if $(a,b) \in R$ and $(b,c) \in R$ then $(a,c) \in R$

Q $A = \{1,2,3,4,5,6,7\}$

$aRb \in R$ $\{$if $(a-b)$ is divisible by $3\} = R$

Prove this is equivalance relation

① Reflexive ② Symmetric ③ Transitive

① Reflexive $\forall a$ $aRa \in R$

$a - a = 0$ $a = 1 \in R$

0 is divisible by 3. $1-1=0$ is divisible by 3

$\therefore$ $aRa \in R$

② Symmetric if $(a,b) \in R$ then $(b,a) \in R$

$(a-b)$ is divisible by 3 $1-2=-1$ is divisible by 3

$(b-a)$ is also divisible by 3 $2-1=1$ is divisible by 3

③ Transitive if $(a,b) \in R$ and $(b,c) \in R$ then (a,c) also belong to R.

$a-b$ is divisible by 3 $\in R$ $2-3 = -1$ divisible by 3

$b-c$ is divisible by 3 $\in R$ $3-7 = -4$ divisible by 3

$a-c$ is divisible by 3 $\in R$ $2-7 = -5$ divisible by 3

Q R and S are equivalence relation, then $R \cup S$?

	R	S	$R \cup S$
① Reflexive	✓	✓	✓
② Symmetric	✓	✓	✓
③ Transitive	✓	✓	✓

$R = (1,2), (2,4), (1,4)$
$S = (2,6), (6,3), (2,3)$ } both are Transitive

if $(1,2)$ and $(2,6) \in R$
then $(1,6)$ should also but $\notin R$
therefore can't say.

$R \cup S$

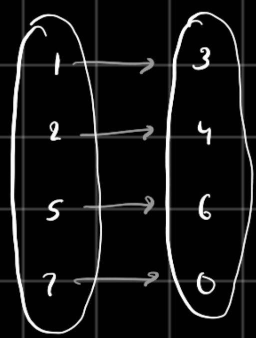
∵ Therefore we can't say in term's of Union, In Intersection its equivalent.

Inverse Relation →

$R = \{(1,3), (2,4), (5,6), (7,8)\}$

$\Rightarrow R^{-1} = \{(3,1), (4,2), (6,5), (8,7)\}$

Function →



Domain Range

Function is the relationship from elements of one set X to elements of another set Y.

① One-one / injective →

For every element in domain has an unique image in the range.

② many-one →

Elements inside the domain can have same image in the range.

③ onto →

Every element in range has an image in domain.

(4) into $\longrightarrow$

Every element in range does not have an pre image in domain.

Q $f(x)\ R \to R$ $f(x) = \frac{1}{x}$ and $x \neq 0$

(1) Bijective $\longrightarrow$ (one-one and onto)

one-one $\rightarrow$

- Let x_1 and x_2 are different inputs

$f(x_1) = f(x_2)$

$\frac{1}{x_1} \neq \frac{1}{x_2}$

one-one

$f(x_1) = f(x_2)$

$1/x_1 = 1/x_2$

onto $\rightarrow$

$1/x = y$

$x = (1/y) \longrightarrow$ real

$f(x) = 2x$

$g(x) = x^2$

$h(x) = x+3$

$f \circ g(x) = 2x^2$

$f \circ g \circ h(x) = 2(x+3)^2$

CAINC

Group $\longrightarrow$

$G = \{1, 2, 3, 4, 5\}$ '+'

$\langle G, \oplus \rangle$

(1) Closure $(a+b) \in G$ then $a \in G$ and $b \in G$

(2) Associative $(a+b)+c = a+(b+c) \in G$

(3) Identity $a \in G$ then $a+e = a$

(4) Inverse $a * a^{-1} = e$

Closure

Asso

Identity $a R e = a$

Inverse $a R a^{-1} = e$

Commutative $ab \quad ba$

Refl $aRa \in R \ \forall a \in R$

$ab \quad ba$

$ab \quad bc \quad ac$

$G = \{i, 1, -1, -i\}$ $i^2 = -1$

$\langle G, \times \rangle$

① Closure

	1	-1	i	-i
1	1	-1	i	-i
-1	-1	1	-i	i
i	i	-i	-1	1
-i	-i	i	1	-1

close

② Associative

$(a \times b) \times c = a \times (b \times c)$

always associative

③ Identity

$a \times e = a \Rightarrow 1$ is the identity.

④ Inverse

	1	-1	i	-i
1	1	-1	i	-i
-1	-1	1	-i	i
i	i	-i	-1	1
-i	-i	i	1	-1

every case contains an identity element therefore it has inverse

Q $G = \{1, \omega, \omega^2\}$

$\langle G, \times \rangle$ $\omega^3 = 1$

	ω	ω^2	1
ω	ω^2	1	ω
ω^2	1	ω	ω^2
1	ω	ω^2	1

① closure

② Associative

multiplication is always

③ Identity

④ Inverse

→ $\langle G, \oplus \rangle$
set of +ve Integers

$a \oplus b = ab/2$

① Close

$a \oplus b = ab/2 \notin G$

$\forall\ a \in G$ and $b \in G$

∵ multiplication of integer is always integer but division of integer is not integer always.

→ $\langle G, \oplus \rangle$ set of Real numbers

$a \oplus b = ab/2$

① Closure

$a \oplus b = ab/2 \in G$

where $a \in G$ and $b \in G$

∵ as G is set of Real number.

② Associative

$(a \oplus b) \oplus c = a \oplus (b \oplus c)$

$\frac{\left(\frac{ab}{2}\right)c}{2} = \frac{a\left(\frac{bc}{2}\right)}{2}$

$\frac{abc}{8} = \frac{abc}{8}$

$\frac{\frac{ab}{2} \times}{2/1}$ $\frac{ab}{4}\ \frac{c}{2}$

③ Identity →

$a \oplus e = a$

$\frac{ae}{2} = a$

$\underline{\underline{e = 2}}$

④ Inverse

$a \oplus a^{-1} = e$

$\frac{a \times a^{-1}}{2} = e$

$\frac{a a^{-1}}{2} = 2$ $a^{-1} = \frac{4}{a}$

Q → $G = \{0, 1, 2, 3, 4, 5, 6\}$

$\langle G, +_6 \rangle$

(a+b) if ≥ 6 then divide it by '6' and return the remainder

	0	1	2	3	4	5	6
0	0	1	2	3	4	5	0
1	1	2	3	4	5	0	1
2	2	3	4	5	0	1	2
3	3	4	5	0	1	2	3
4	4	5	0	1	2	3	4
5	5	0	1	2	3	4	5
6	0	1	2	3	4	5	6

The column that is same as row is the identity element.

Thus $G = \{0, 1, 2, 3, 4, 5\}$

Cyclic group →

$G = \{0, 1, i, -i\}$

Every cyclic group has '2' generators

$i^0 = 1$ $\quad$ $(-i)^0 = 1$

$i^1 = i$ $\quad$ $(-i)^1 = -i$

$i^2 = -1$ $\quad$ $(-i)^2 = -1$

$i^3 = -i$ $\quad$ $(-i)^3 = i$

$i^4 = 1$

Are the generator

Gener · has a generation.

· by raising the power of an element we get all the elements in the group.

Group →

(1) Identity element is unique

(2) Inverse of element is unique

Min no. of generator = 2 "number" and its "inverse"

min number of element required to test group is $\{a, a^{-1}, e\}$

(1) → $(G, \times)$

e_1 $\quad$ e_2

$a \times e_1 = a$ $\quad$ $a \times e_2 = a$

$\cancel{a} \times e_1 = \cancel{a} \times e_2$ {left cancelation law}

$\therefore \; e_1 = e_2$

② → $(G, \times)$

a_1^{-1} $\quad$ a_2^{-1}

$a \times a_1^{-1} = e \qquad a \times a_2^{-1} = e$

$\cancel{a} \times a_1^{-1} = \cancel{a} \times a_2^{-1}$ $\quad$ {left cancelation}

$a_1^{-1} = a_2^{-1}$

Q → $\langle G, \odot \rangle$ ← set of Real numbers

$a \odot b = ab + \frac{b}{2}$

① closure

$a \in G, \; b \in G$ also $ab \in G$

$a \odot b = ab + \frac{b}{2}$

$a \Rightarrow R \qquad R \times R = R$

$b \Rightarrow R$

$\frac{b}{2} \Rightarrow R/2$ is also 'R'

② Associative

$a \odot (b \odot c) = (a \odot b) \odot c$

$a \odot \left(bc + \frac{c}{2}\right) \qquad \left(ab + \frac{b}{2}\right) \odot c$

$abc + \frac{ac}{2} + \frac{bc}{2} + \frac{c}{4} \neq abc + \frac{bc}{2} + \frac{c}{2}$

$G \quad a, b \in G$

$(a * b)^{-1} = a^{-1} * b^{-1}$

$I = (a*b) * (b^{-1} * a^{-1})$

$= a * (b * b^{-1}) * a^{-1}$ $\qquad a * a^{-1} = e$

$= a * e * a^{-1}$

$= a * a^{-1}$

$= I / e$

If G is a group

$H_1 \qquad H_2$ is a sub group.

Then "$H_1 \cup H_2$" will not be a group.

"$H_1 \cup H_2$" $\leq G$ if and only if

$H_1 \leq H_2$ or $H_2 \leq H_1$

$H_1 \cap H_2 \leq G$

$A = \{1, 2, 3, 4, 5\}$ '$\leq$'

$R = \{(1,3), (2,5) \ldots\}$

Equivalence Relation

Partialy ordered set (POSET)

- Reflexive
- Anti symmetric
- Transative

$A = \{1, 3, 6, 9, 12, 18\}$ '/' (Partial ordered set)

1) Reflexive

$aRa \Rightarrow a/a$ = divisible

2) Anti symmetric

$a/b \in R$, $b/a \notin R$ and $a \neq b$

$2/6 \in R$, $6/2 \notin R$ and

Q $\{a,b,c\}$ '$\subseteq$'

- Finds its power set

$\{\phi\}, \{a\}, \{b\}, \{c\}, \{a,b\}, \{b,c\}, \{a,c\}, \{a,b,c\}$

1) Reflexive

$aRa \quad a \subseteq a \in R$

2) Anti symmetric

$a \subseteq b \neq b \subseteq a$ and $a \neq b$

from Reflexive as $a \subseteq a$ and not $a \subseteq b$ and $b \subseteq a$

3) Transitive

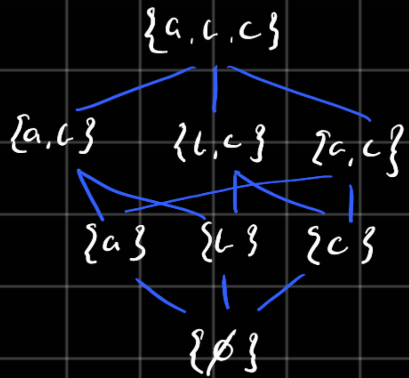
$a \subseteq b \in R \quad b \subseteq c \in R$ then $a \subseteq c$ also.

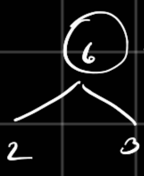
Partial ordered set

$A = \{a,b,c\}$

$R = \{ab : a,b \in P(A)$ and $a \subseteq b\}$

Hasse diagram

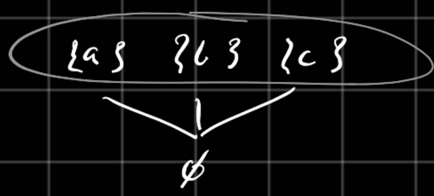


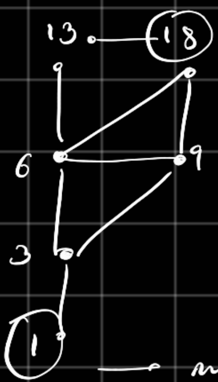


→ maximum

→ minimal elements $\{2,3\}$

maximal



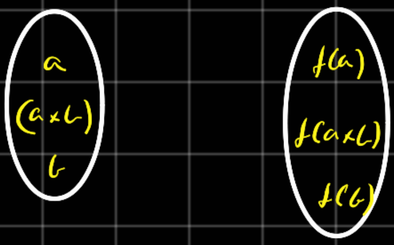


→ maximum

→ minimum

Homomorphism →

$(G, \times) \quad f \quad (G', \pm)$



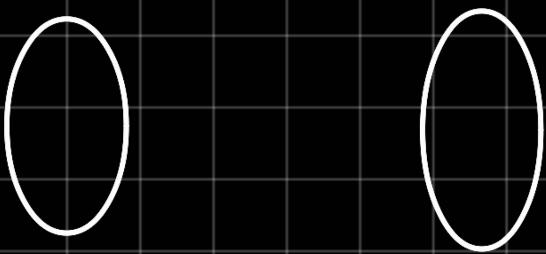
$$f(a \times b) = f(a) + f(b)$$

$\forall\, a, b \in G$

$$f(a \times b) = f(a) \pm f(b)$$

eg →

$(G, +) \qquad f(x) = 2^x \qquad (G', \times)$



$$f(a+b) = f(a) \times f(b)$$

$$f(a+b) = 2^{a+b}$$
$$= 2^a \cdot 2^b$$
$$f(a+b) = f(a) \times f(b)$$

$G, \times \quad f \quad G, \pm$

$$f(a \times b) = f(a) + f(b)$$

$\forall a, b \in G$

$$f(a \times b) = f(a) \pm f(b)$$

eg →

$f(x) = 2x$

$(G, +)$ 'Z' $\qquad (G', +)$ 'Z'

$$f(a+b) = f(a) + f(b)$$
$$= 2(a+b)$$
$$= 2a + 2b$$
$$= f(a) + f(b)$$

$f(a+b) = f(a) + f(b)$

$2(a+b) = 2a + 2b$

$2a + 2b = f(a) + f(b)$

$f(x) = 2^x$

Isomorphism →

1. one - one
2. Onto
3. Homomorphism

$f(x_1 + x_2) = f(x_1) + f(x_2)$

$f(x) = 2x$

$= 2(x_1 + x_2)$

$= 2x_1 + 2x_2$

$= f(x_1) + f(x_2)$

one - one

Let x_1 and x_2 be 2 different elements

then, $f(x_1) = f(x_2)$

$\cancel{2}x_1 = \cancel{2}x_2$

$x_1 = x_2$

∴ Therefore one - one

$f(x) = 2x$ if Range $\in$ 'Z'

$y = 2x$

$\frac{y}{2} = x$ $\quad y/2 \in R$

Determine weather this a group

$(Z, *)$

$a * b = a + b + 1$

① closure

② Associative

③ Identity

$a * e = a$

$a + e + 1 = a$

$e = -1$

$\cancel{a} + e + 1 = \cancel{a}$

$e = -1$

$a + a^{-1} + 1 = e$

$a + a^{-1} = -2$

$a^{-1} = -2 - a$

④ Inverse

$$a \times a^{-1} = e$$

$$a + a^{-1} + 1 = e$$

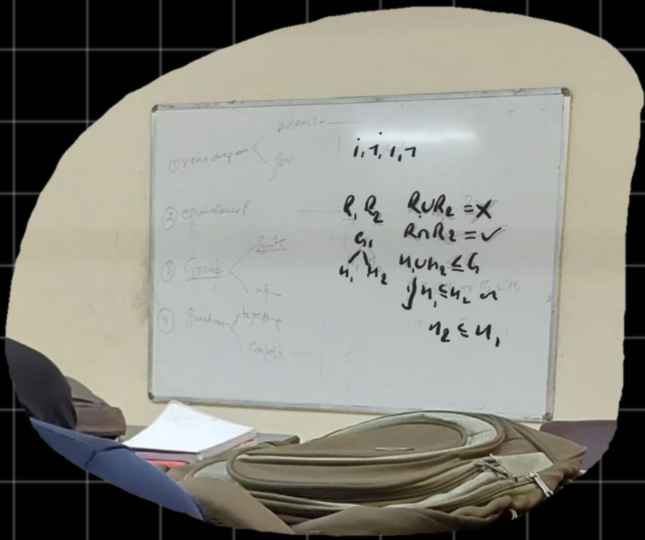
$$a^{-1} = -a - 2$$

Vendiagram — $A \cup (B \cap C)$ ...
— Lopez cricket.

Equivalance Relation

Group — Finite
— Infinite

Function — bijective
— Composite



$f: R \to R \quad f(x) = 2x + 3$

injective ⟶

Let x_1 and x_2 are different

$$f(x_1) = f(x_2)$$

$$2x_1 + 3 = 2x_2 + 3$$

$$2x_1 = 2x_2$$

$$\boxed{x_1 = x_2}$$ one – one injective

bijective ⟶

$$f(x) = 2x + 3$$

$$y = 2x + 3$$

$$2x = y - 3$$

$$x = \frac{y-3}{2}$$ bijective, onto

$y - 3 \in R$

$\frac{y-3}{2}$ also $\in R$

– $G\{1, 2, 3, 4\}$ '$\times_5$'

$\times_5$	1	2	3	4
1	1	2	3	4
2	2	4	1	3
3	3	1	4	2
4	4	3	2	1

identity — Identity element exists

inverse — as there exists an identity element

closure — as all the elements of the set are present in the matrix

Associative → $a(b \times c) = (a \times b) \times c$

Cyclic → Generators $2^3 \Rightarrow 2 \times_5 2 \times_5 2$

$3^n \Rightarrow$

→ $R = \{x, y : |x-y| < 1\}$

① Reflex

$a - a = 0$

but $|x-y| < 1$

therefore not reflexive

② symmetric

$|a-b| < 1$

$\Rightarrow a = 1$

$\Rightarrow b = 0.5$

$|1-0.5| = 0.5 < 1$

$|0.5-1| = 0.5 < 1$

Transitive

$|a-b| < 1$

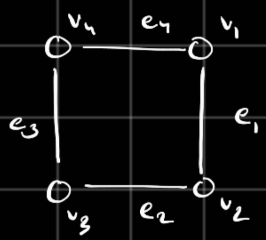
$(+)\ |b-c| < 1$

$|a-c| < 2$ → Therefore not transitive.

∴ Therefore only symmetric.

$(A \cup B \cup C) = A + B + C - (A \cup B \ (B \cup C \ A \cup C + A \cap B \cap C$

Graph →

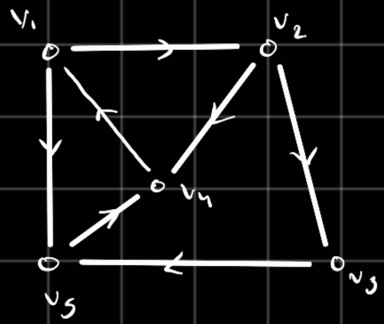


• Graph is a collection of edges and vertices.

Tree	Graph
• If any graph is connected by min number of edges.	•
number of edges = number of vertex − 1	• number of edges can be > vertexes.
• Can't be cyclic.	• Cyclic

Degree of node →

Total number of edges connected to a vertex.



– in degree	
v_1	1
v_2	1
v_3	1
v_4	2
v_5	2

– out degree	
v_1	2
v_2	1
v_3	1
v_4	1
v_5	1

Total degree of the graph →

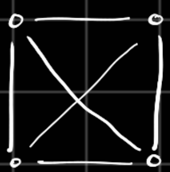
- Sum of total number of degree of vertexes.

Types of Graph →

1. Complete graph →

   Every vertex is connected to each other.

   every ✗ ✓

   may be.

Number of edges, in complete graph having n number of vertices.

$$\frac{n(n+1)}{2}$$

2. Regular Graph →

   Every node/vertex having same degree.

Q Draw a graph total degree '17'.

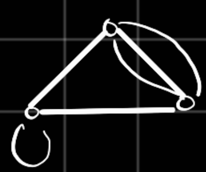
Total degree is always even.

– Self loop →

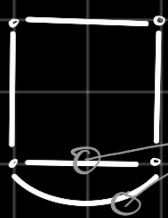
Total degree → 4

Pseudograph →

Multigraph with loops

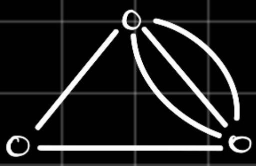


– Parallel edge →



Parallel edges

- **Multigraph** →



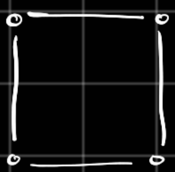
a graph which is permitted to have <u>multiple edges</u> also known as parallel edges.

3. **Planer Graph** →

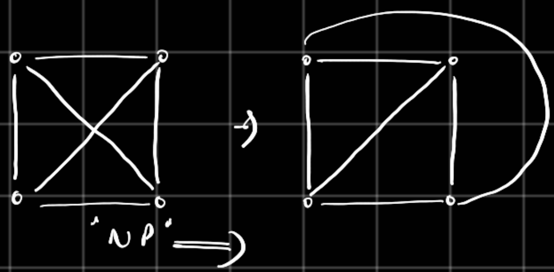
A grap which can be drawn on 2D-plane is called a planer.

Non planer →

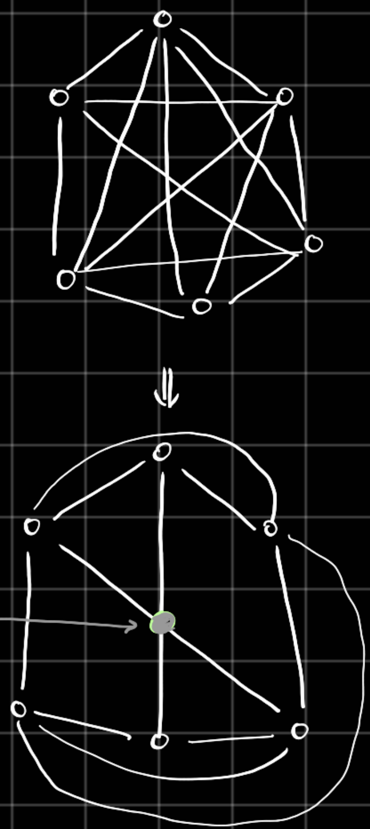
In a graph if edges are intersecting each other then its Non planer.



planer



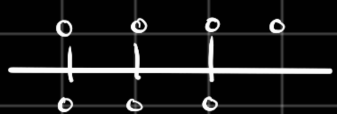
Planer

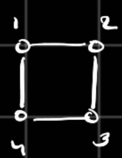


No possible way of drawing "NP"

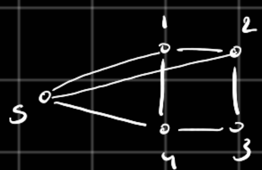
4. **Bipertite Graph** →

Divide the number of nodes into '2' graph such that their exists no edges between them.





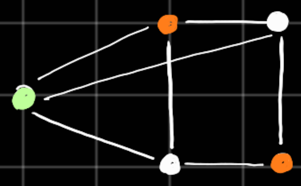
$\frac{1\quad 3}{2\quad 4}$ Bipertite.



$\frac{1\quad 3\quad \cancel{5}}{2\quad 4\quad \cancel{5}}$ } there for not Bipertite.

4. Chromatic No. of graph →

Minimum number of color required to color a graph such that 2 nodes have same color.



number of chromatic number of →

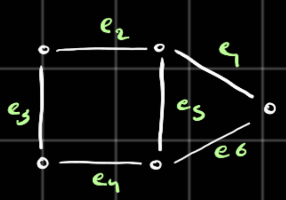
1) Complete graph → number of nodes
2) Regular graph →
3) Planar graph →

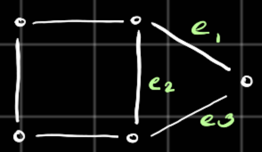
} can't say (need to calculate)

4) Biphertite → always be '2'.

- Walk →

Sequence of nodes and edges alternatively. Can be close or open.





- Trail →

Can't repeat any edge.

- Circuit →

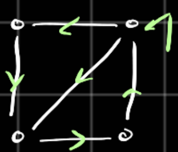
Can't repeat any edge.
Start and end point should be same.

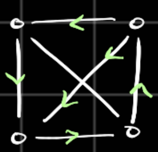
- Path →

No edge and node should be repeated.

Euler Graph →

Travel complete graph without repeating the edges.

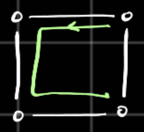


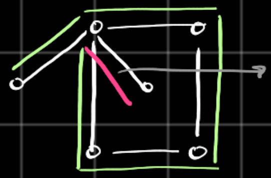


→ won't be able to travel all edges of the graph.

Hamiltonian Graph →

Travel all the nodes without repeated vertex.

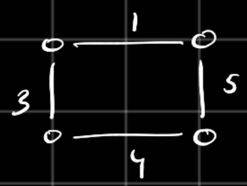




Repetation of
vertex for traveling of all nodes

Weighted Graph →

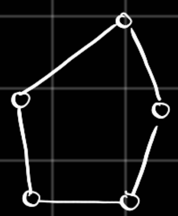
Edges with a graph.



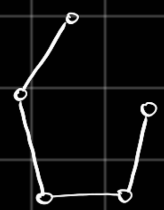
Spanning tree →

Subgraph of an undirected connected graph.

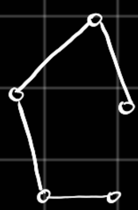
- It includes all vertices
- Least possible number of edges.



Graph

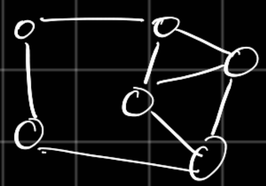


Spanning tree 1



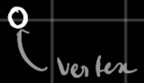
Spanning tree 2

Draw a graph having 6 vertices out of them 4v – 3 degree
2v – 2 degree



Trivial Graph →

A graph consisting only one vertex and no edges.

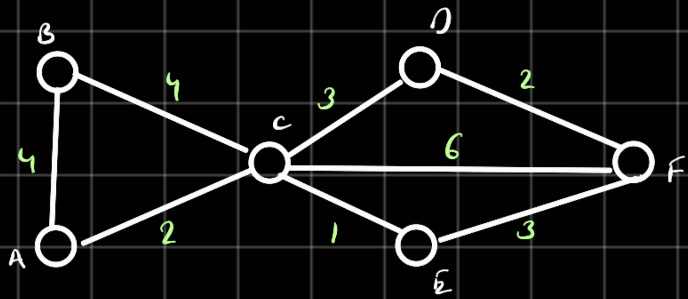


Null graph →

Graph consisting 'n' vertices and no edges.

Dijkstras Algorithm →

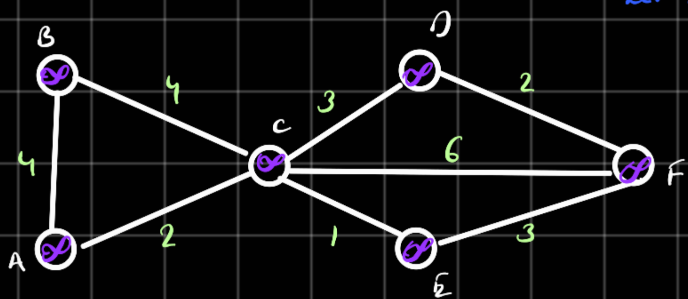
To find shortest path between any two vertices of a graph.



If $d(u) + c(u,v) < d(v)$ then

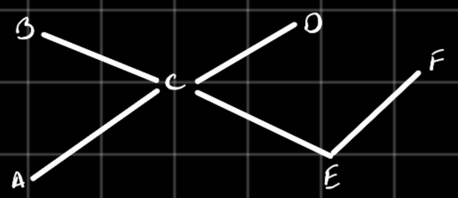
$d(v) = d(u) + c(u,v)$

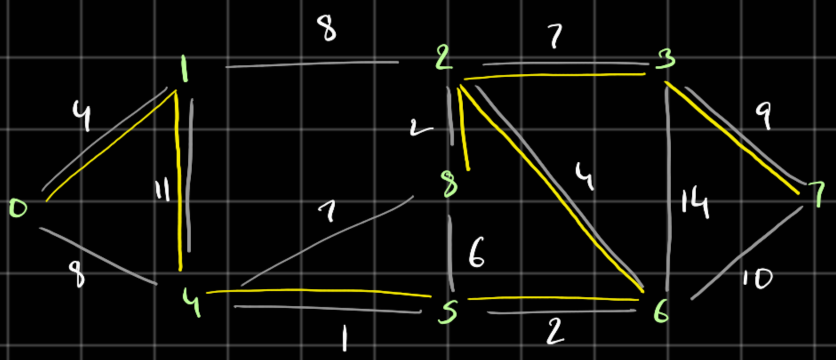
Let 'A' as starting point.



Source A	B	C	D	E	F
	∞	∞	∞	∞	∞
	4	2	∞	∞	∞
C	4	2	5	3	∞
E	4	2	5	3	6

B	4	2	5	1	6
D	4	2	5	1	6





0	1	2	3	4	5	6	7	8
	4	∞	∞	8	∞	∞	∞	∞
1	4	12	∞	8	∞	∞	∞	∞
4	4	12	∞	8	9	∞	∞	15
5	4	12	∞	8	9	11	∞	14
6	4	12	25	8	9	11	21	14
2	4	12	19	8	9	11	21	14
8	4	12	19	8	9	11	21	14
3	4	12	19	8	9	11	21	14

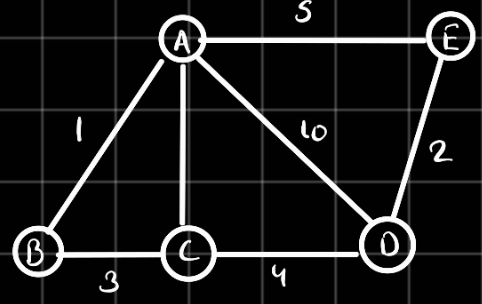
Kruskal Minimum Spanning Tree (MST) Algorithm →

Spanning tree →

Is the subgraph of an undirected connected graph.

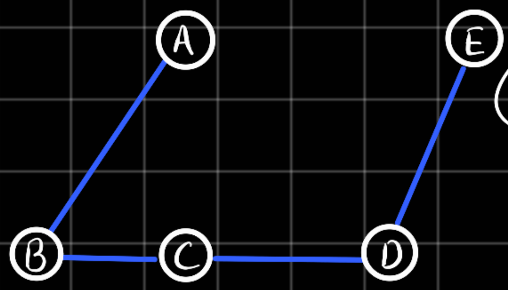
Minimum Spanning tree →

Spanning tree in which sum of the weights of the edge is minimum.



AB – 1 ✓
ED – 2 ✓
BC – 3 ✓
CD – 4 ✓
AE – 5 X
AC – 7 X
AD – 10 X

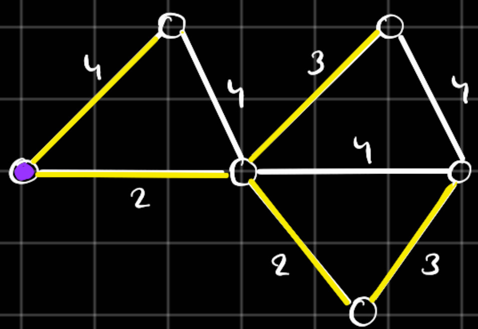
Arranging all edges in assending order



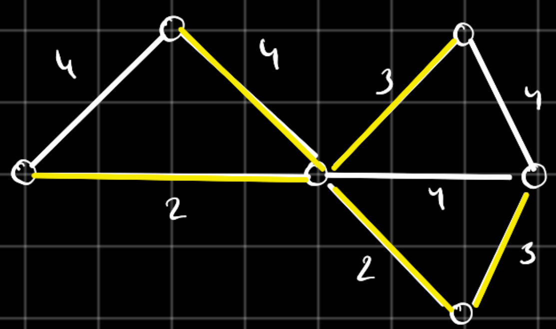
Minimum ST can't be closed

Prim's Algorithm →

- Choose a start vertex
- Choose next vertex with lowest weight amoungst all the edges.
- Keep on repeating these steps



Vs Krushical



SOP (Sum of Product)

- When function $f = 1$
- $A \rightarrow 1$
  $\bar{A} \rightarrow 0$
- eg:— 00, 10
  $\bar{A}\bar{B} + A\bar{B}$
- $\Sigma m(\,)$ Minterms → SOP → when $f = 1$
  So put '1' in Kmap

POS (Product of Sum)

- When function $f = 0$
- $A \rightarrow 0$
  $\bar{A} \rightarrow 1$
- eg:— 00, 01
  $(A+B)(A+\bar{B})$
- $\Pi m(\,)$ Maxterms → POS → when $f = 0$
  So put '0' in Kmap

Q — Simplify boolean function using K-map in SOP and POS forms.

$$f(A,B,C) = \Sigma m(0,2,3,4,5,6)$$

A \ BC	00	01	11	10
0	1 (0)	(1)	1 (3)	1 (2)
1	1 (4)	1 (5)	(7)	1 (6)

$$f = \bar{C} + \bar{A}B + A\bar{B}$$

K-map — SOP Σ, POS Π

Q $a\bar{b} + bc + \bar{a}\bar{c}$

$$a\bar{b}(c+\bar{c}) + (a+\bar{a})bc + \bar{a}\bar{c}(b+\bar{b})$$

$$a\bar{b}c + a\bar{b}\bar{c} + abc + \bar{a}bc + \bar{a}\bar{c}b + \bar{a}\bar{b}\bar{c}$$

$$101 + 100 + 111 + 011 + 001 + 000$$

$$\Sigma(5 \quad 4 \quad 7 \quad 3 \quad 1 \quad 0)$$