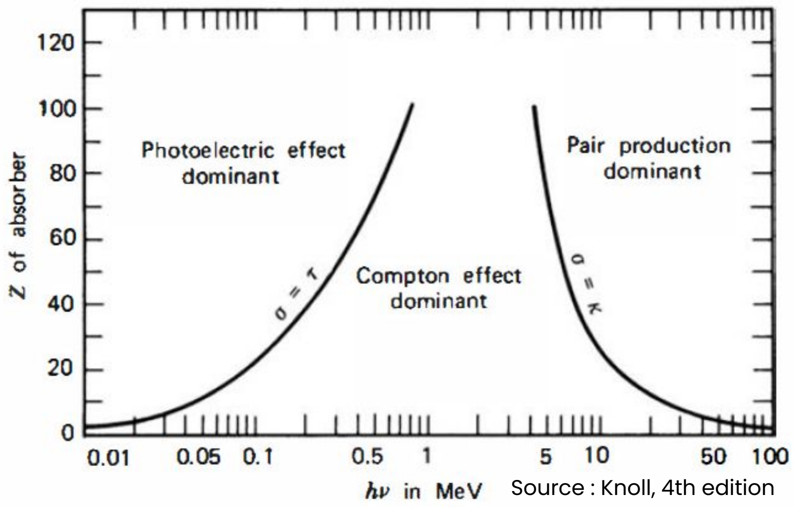
GRB Prompt Polarisation: What are we looking for?

By : Yashowardhan Rai,
Third Year EP

INDEX

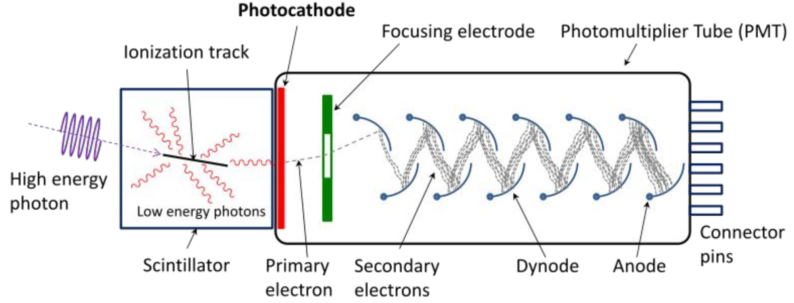
1a. Interaction of x-rays with matter



Source : Knoll, 4th edition

- Energy of incoming Photon on X axis
- Atomic number on the y axis
- Contours represent Equi-Cross section Lines

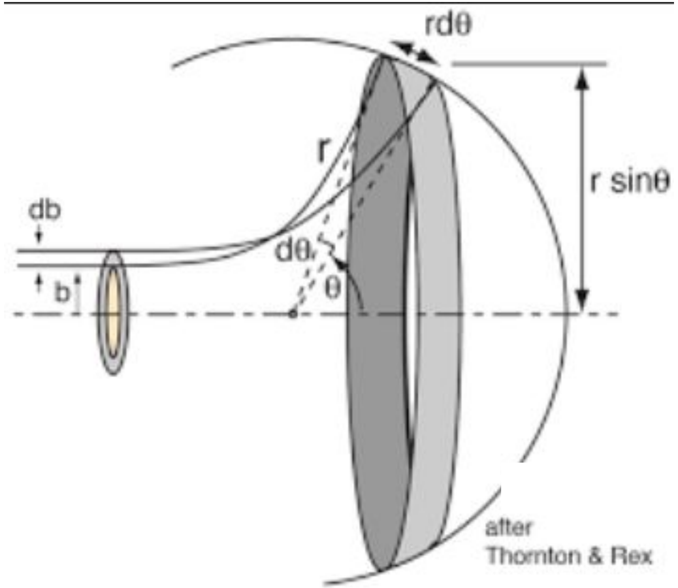
Atomic Numbers of
Cd ~ 48
Te ~ 52
Zinc ~30



Cadmium Zinc Telluride has effective atomic number of 50.

Source : wikimedia commons, scintillation counter

1b. Klein-Nishina Compton Cross-Section



$$\frac{d\sigma}{d\Omega} = \frac{r_o^2}{2}\frac{E'^2}{E^2}\left(\frac{E'}{E} + \frac{E}{E'} - 2\sin^2\theta\cos^2\phi\right).$$

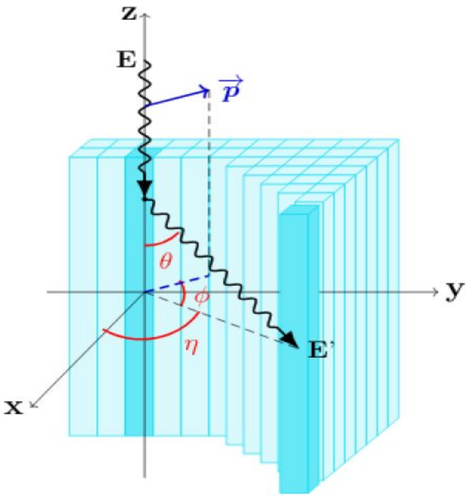
$$d\Omega = \sin\theta\, d\theta\, d\varphi,$$

At cos(phi)=+-1
D has its minima

At cos(phi)=0
D has its maxima

Image Source : hyperphysics mechanics

1b. Klein-Nishina Compton Cross-Section



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$$d\Omega = \sin\theta\, d\theta\, d\varphi,$$

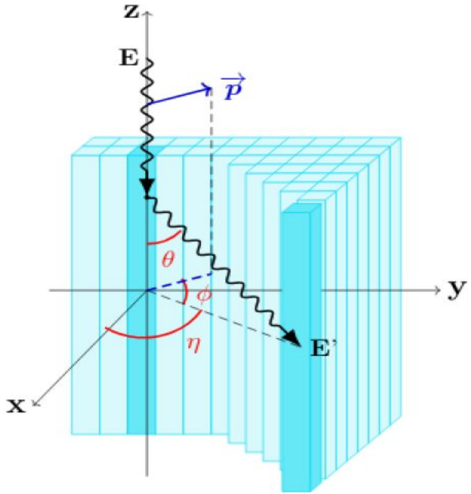
At cos(phi)=1
D has its minima

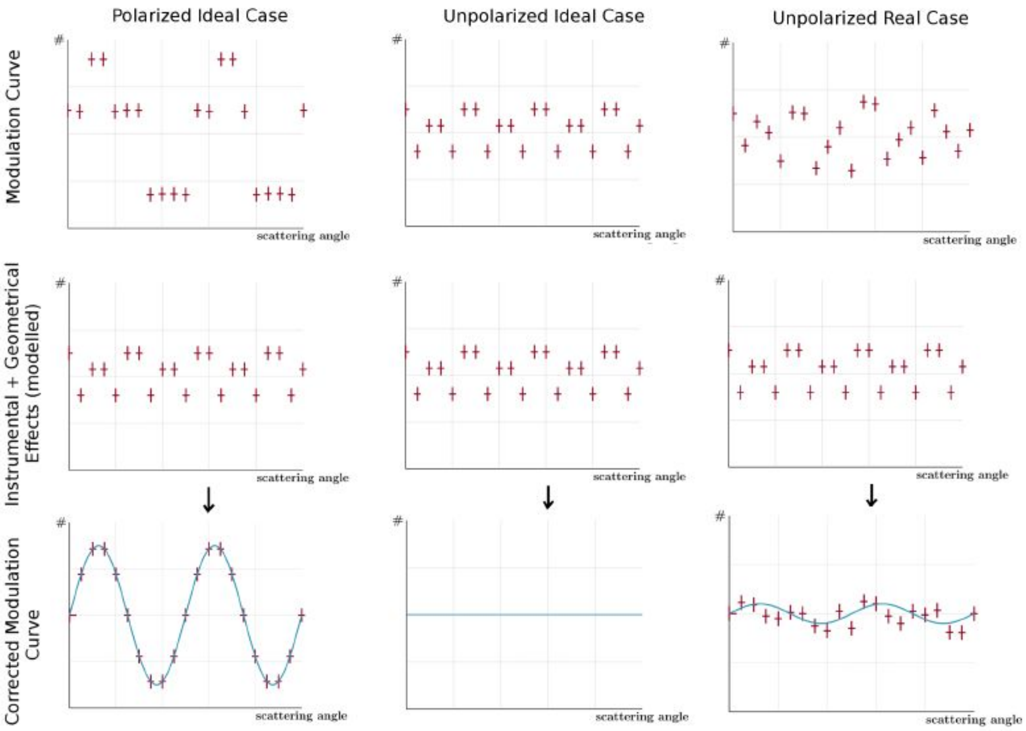
At cos(phi)=0
D has its maxima

Hence we expect, if the electric field vector is along X axis, maximum scattering will occur around +- Y axis, and vary in between sinusoidally.

Credits : Merlin Kole et al (2019) :https://arxiv.org/abs/2204.05844

1c. Compton Events and Modulation Curve



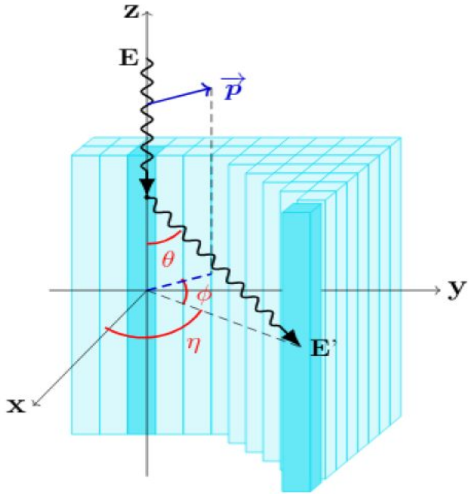


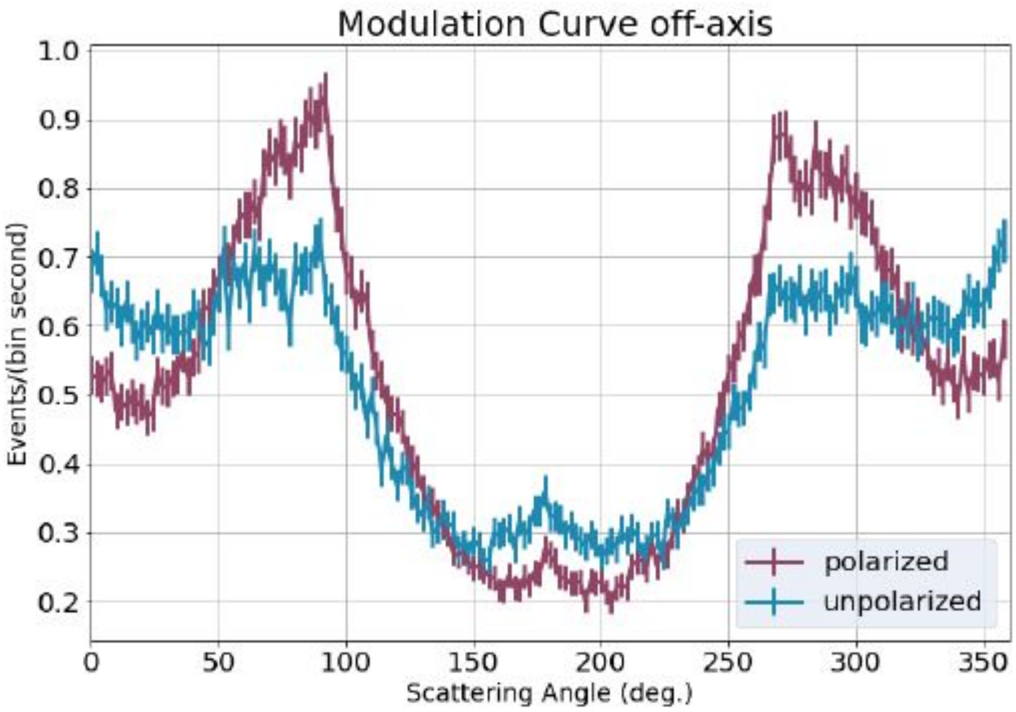
Astrosat allows for double counts in only neighbouring pixels, giving rise to 8 angular bins only.

Polar removes the neighbouring-only requirement, but geometrical effects come into play.
These have to be modelled and removed.

Credits : Merlin Kole et al (2019) :https://arxiv.org/abs/2204.05844

1c. Compton Events and Modulation Curve





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These have to be modelled and removed.

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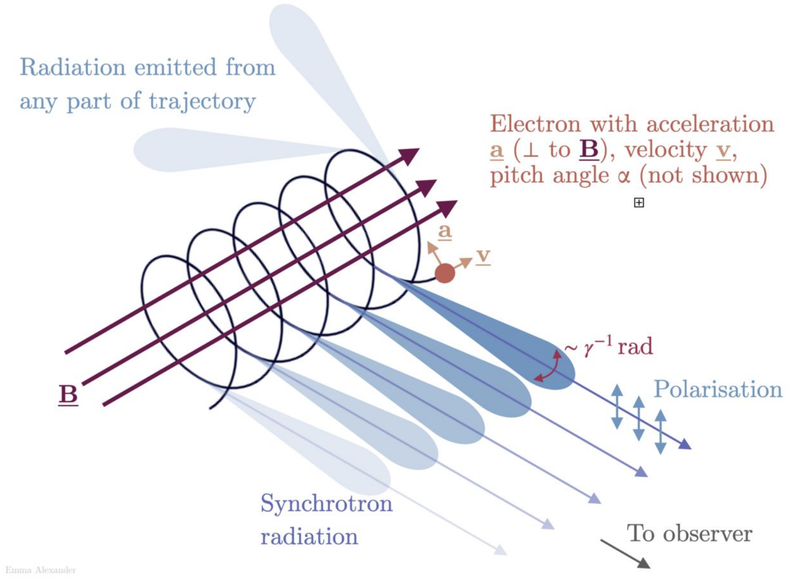
2a. Synchrotron Emission



However,

polarisation at micro-scale
!=
macro-scale observable polarisation

The net effect of the ~10^50 polarisations summed could still be 0.
For example the magnetic field could be completely random, giving no preferred direction.

However macro-scale geometries would give rise to overall net polarisation
if observed in the right way

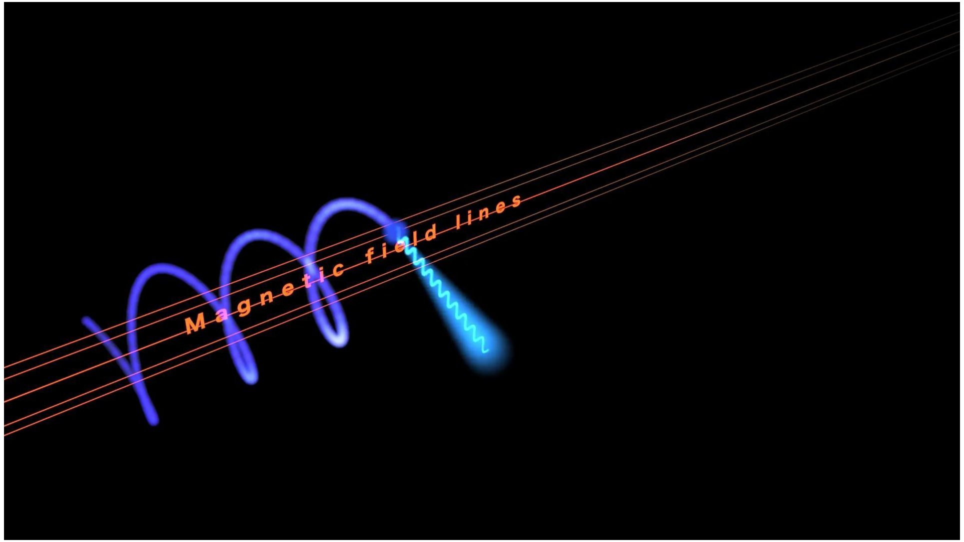
Source : wikimedia commons : synchrotron radiation

2a. Synchrotron Emission

2 popular models that can potentially explain the polarisation we observe are using synchrotron are

1) Synchrotron emission + **Random** B field
2) Synchrotron emission + **Ordered** B field
   a) **Parallel** B
   b) **Perpendicular** B
   c) **Toroidal** B (all Wrt axis of Jet)

Some more models proposed include **inverse compton drag and photospheric emission** .
They have different emission mechanisms from simple synchrotron.

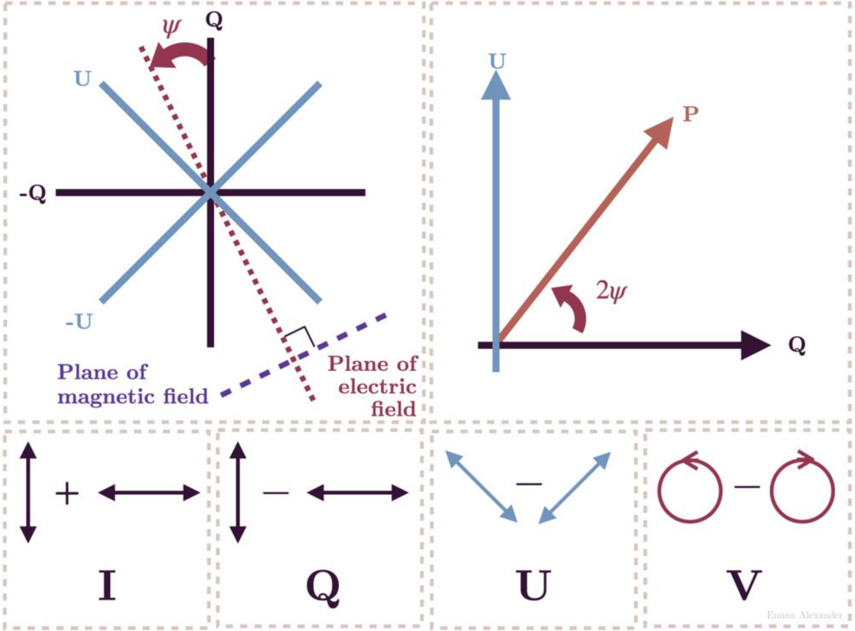


Credits :NASA Scientific Visualisation Studio (SVS)

2b. Stokes parameters

To properly calculate and characterise polarised emissions

We can take the help of stokes' parameters.



$$I = S_0$$

$$p = \frac{\sqrt{S_1^2 + S_2^2 + S_3^2}}{S_0}$$

Using Stokes' Parameters we can describe the entire vector space of partially and fully polarised emissions.

Source : Wikimedia Commons : Stokes Parameters

2b. Stokes parameters & Synchrotron

Assuming a Thin spherical shell, moving at relativistic speeds, with a spectral shape f(v). The spectral fluence is given by:

$$I_\nu = \frac{1+z}{d_L^2}\int d\phi \int d(\cos\theta) r_0^2 \frac{A_0 f(\nu')}{\gamma^2(1-\beta\cos\theta)^2}, \qquad (4)$$

The local Stokes Parameters upon being integrated over give the following integral :

$$\begin{Bmatrix} Q_\nu \\ U_\nu \end{Bmatrix} = \frac{1+z}{d_L^2}\int d\phi \int d(\cos\theta) r_0^2 \frac{A_0 f(\nu')}{\gamma^2(1-\beta\cos\theta)^2}\Pi_0 \begin{Bmatrix} \cos(2\chi) \\ \sin(2\chi) \end{Bmatrix}. \qquad (5)$$

The Final Polarisation degree/ Polarisation fraction is given by :

$$\Pi = \frac{\sqrt{Q^2+U^2}}{I}. \qquad (6)$$

Kenji Toma et al 2009: https://arxiv.org/abs/0812.2483

2c. Micro to Macro (for SO Model)

Synchrotron emission has the following emission spectrum (typical values known by spectrometry)

$$\tilde{f}(x) = \begin{cases} x^{-\alpha} e^{-x}, & \text{for } x \leqslant \beta - \alpha, \\ x^{-\beta}(\beta - \alpha)^{\beta-\alpha} e^{\alpha-\beta}, & \text{for } x \geqslant \beta - \alpha. \end{cases} \quad (7)$$

The Local Polarisation degree is given by

$$\Pi_0 = \Pi_0^{\mathrm{syn}} \equiv \begin{cases} (\alpha + 1)/(\alpha + \frac{5}{3}), & \text{for } x \leqslant \beta - \alpha, \\ (\beta + 1)/(\beta + \frac{5}{3}), & \text{for } x \geqslant \beta - \alpha. \end{cases} \quad (8)$$

Voila! with some variable changes the eqn 6 becomes in [v1,v2]:

$$\Pi = \left| \int_{\nu_1}^{\nu_2} d\nu \int_0^{(1+q)^2 y_j} \frac{dy}{(1+y)^2} \right.$$
$$\left. \times \int_{-\Delta\phi(y)}^{\Delta\phi(y)} d\phi\, \tilde{f}(x)(\sin\theta'_B)^{\alpha+1} \Pi_0^{\mathrm{syn}}(x) \cos(2\chi) \right|$$
$$\times \left[\int_{\nu_1}^{\nu_2} d\nu \int_0^{(1+q)^2 y_j} \frac{dy}{(1+y)^2} \int_{-\Delta\phi(y)}^{\Delta\phi(y)} d\phi\, \tilde{f}(x)(\sin\theta'_B)^{\alpha+1} \right]^{-1}, \quad (11)$$

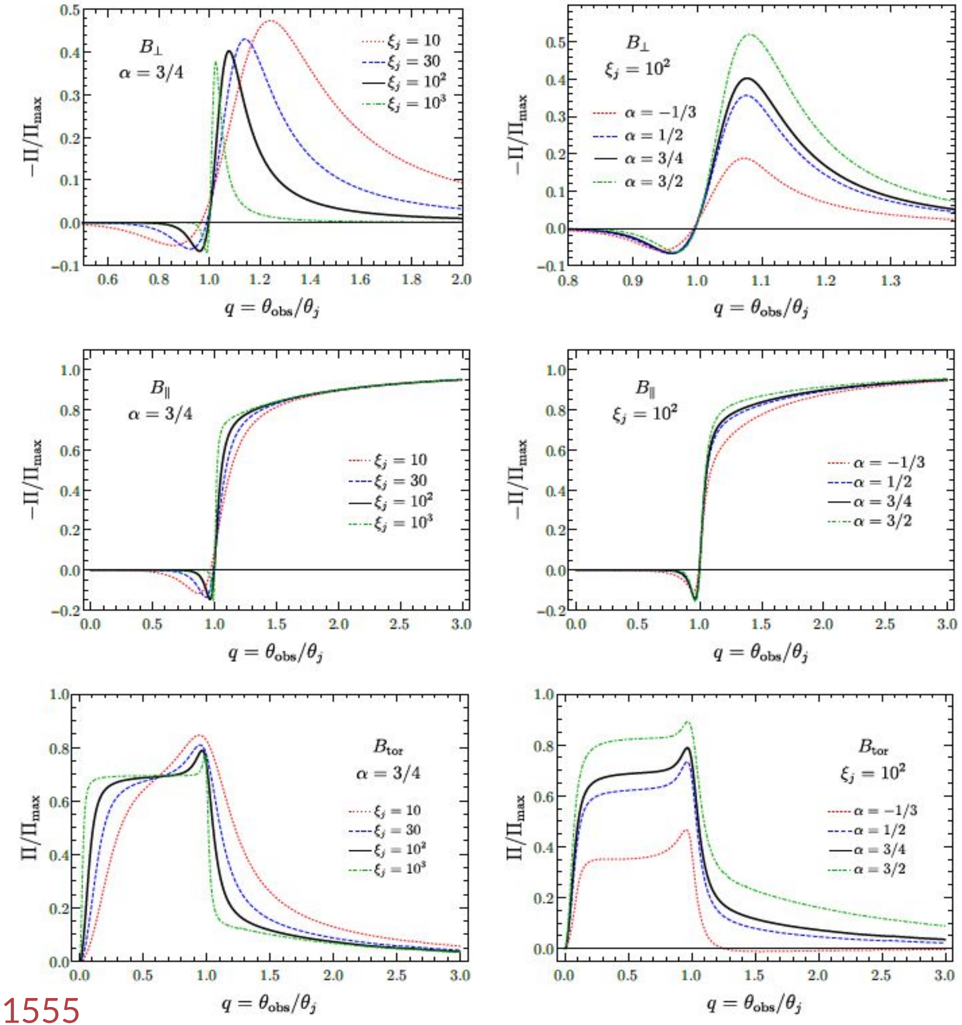
Kenji Toma et al 2009: https://arxiv.org/abs/0812.2483

3a. Top Hat jets - analytical predictions

Similar to SO, we can derive other expressions for other models.

Studying the plots gives us what to expect if we were to view the jet.

Special features appear in the graph When **q= Θ viewing / Θjet** Crosses
{1-1/Γ, 1 , 1+1/Γ}



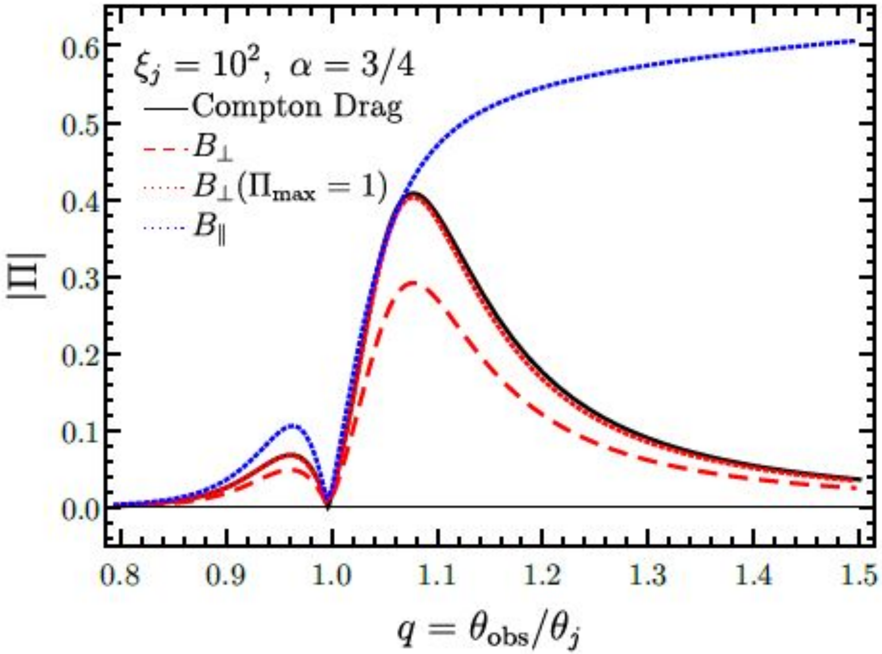
Ramandeep Gill 2019 :https://arxiv.org/abs/1811.11555

3a. Top Hat jets - analytical predictions

Similar to SO, we can derive other expressions for other models.

The Compton Drag Model ends up giving us very similar predictions to B perpendicular and B parallel as well for the correct parameter values.

Special features appear in the graph
When **q= Θ viewing / Θjet**
Crosses
{1-1/Γ, 1 , 1+1/Γ}

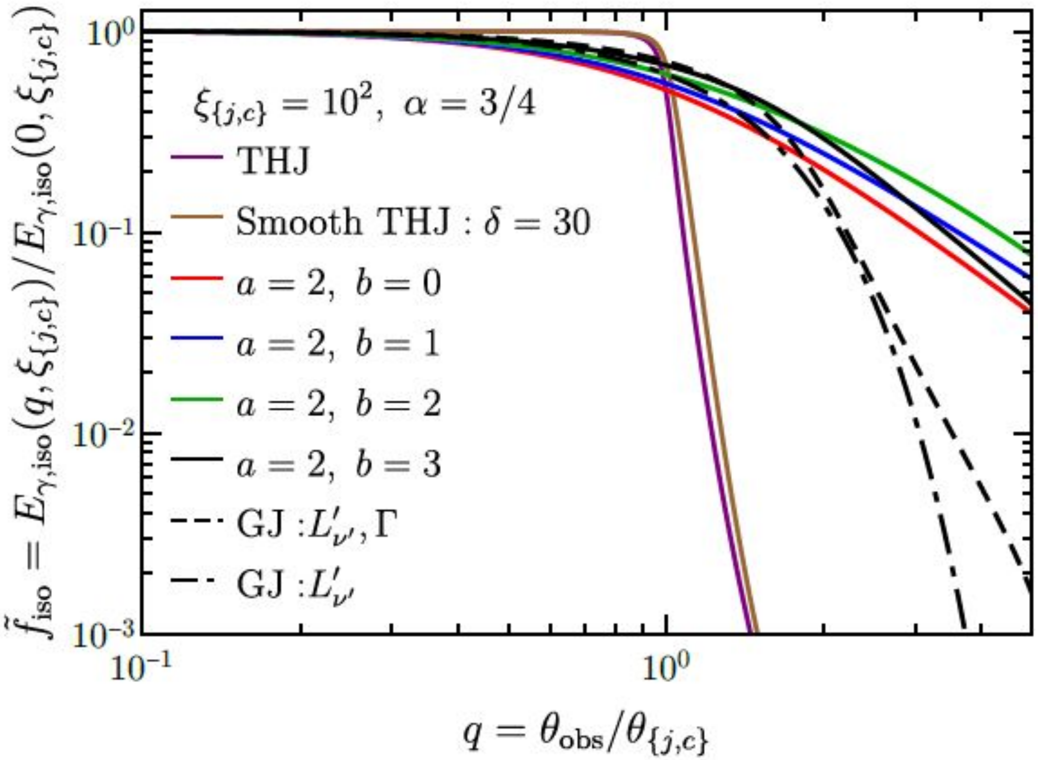


Ramandeep Gill 2019 :https://arxiv.org/abs/1811.11555

3a. Top Hat jets - analytical predictions

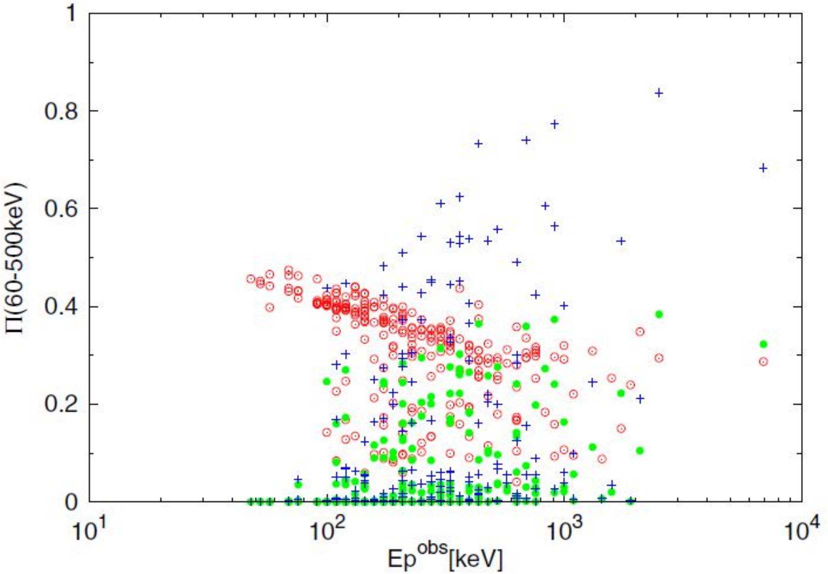
One of the many complications in actually observing this PF is the natural fall off in intensity with viewing angle.

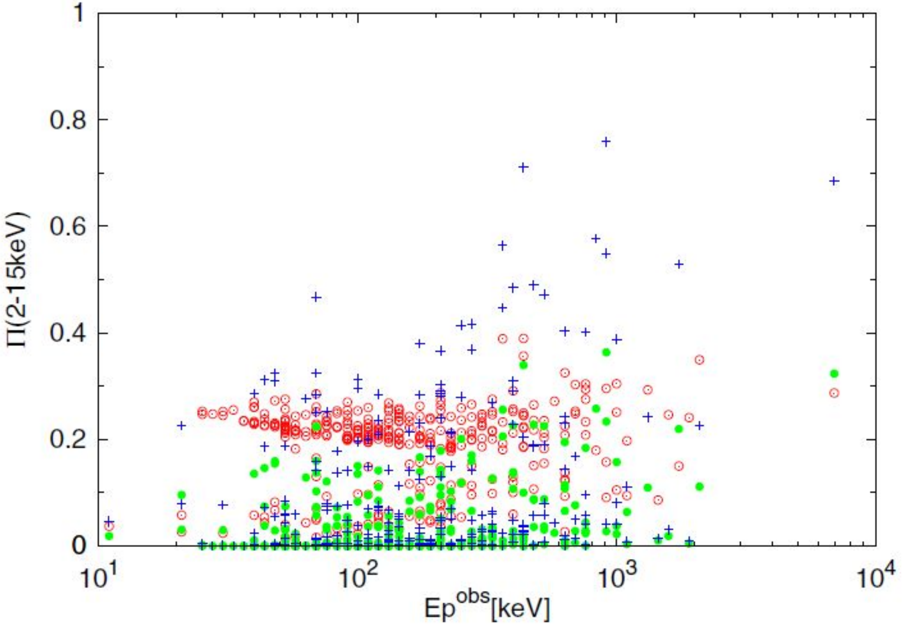
This inherently makes higher q observations less likely as they are less likely to be picked up by our instruments in all energy bands.



Ramandeep Gill 2019 :https://arxiv.org/abs/1811.11555

3a. Ensemble Predictions for top hat jet





Synchrotron Ordered, "Inherent" Synchrotron Random, Compton Drag "Geometrical Models"

Kenji Toma et al 2009: https://arxiv.org/abs/0812.2483

3b. Structured Jets

Fastest Decaying Wings ($\Delta \sim 0$, $\delta \sim 100$)

Middle Decaying Wings ($\Delta \sim 0.5$, $\delta \sim 10$)

Slowest Decaying Wings ($\Delta \sim 1$, $\delta \sim 2$)

Only Emissivity Varies

(i) *Exponential wings* - the emission falls off exponentially outside of the uniform core, such that

$$\frac{L'_{\nu'}}{L'_{\nu',0}} = \begin{cases} 1 & \xi \leqslant \xi_j \,, \\ \exp[(\sqrt{\xi_j} - \sqrt{\xi})/\Delta] & \xi > \xi_j \,, \end{cases} \tag{36}$$

where $L'_{\nu',0}$ is the uniform spectral luminosity.

(ii) *Power-law wings* - the emission declines as a power law outside of the uniform core, such that

$$\frac{L'_{\nu'}}{L'_{\nu',0}} = \begin{cases} 1 & \xi \leqslant \xi_j \,, \\ \left(\frac{\xi}{\xi_j}\right)^{-\delta/2} & \xi > \xi_j \,. \end{cases} \tag{37}$$

We can add in Variation of the Lorentz factor as well.

(i) *Gaussian Jet (GJ)*: Both the spectral luminosity and the kinetic energy of the emitting material per unit rest mass, $\Gamma - 1$, have a gaussian profile with a characteristic core angle θ_c:

$$\frac{L'_{\nu'}}{L'_{\nu',0}} = \frac{\Gamma(\theta) - 1}{\Gamma_c - 1} = \max\left[\exp\left(-\frac{\theta^2}{2\theta_c^2}\right) , \exp\left(-\frac{\theta_*^2}{2\theta_c^2}\right)\right] , \tag{38}$$

where Γ_c is the LF of the core and θ_* implies a floor, which corresponds to some finite $\beta_{\rm min}$, that is both physically motivated and numerically convenient, and is chosen to be sufficiently small so that it does not affect any of the results.

(ii) *Power-law Jet (PLJ)*: The spectral luminosity and the kinetic energy per unit rest mass of the emitting material decay as a power law outside of the core:

$$\frac{L'_{\nu'}}{L'_{\nu',0}} = \Theta^{-a} , \quad \frac{\Gamma(\theta) - 1}{\Gamma_c - 1} = \Theta^{-b} , \quad \Theta \equiv \sqrt{1 + \left(\frac{\theta}{\theta_c}\right)^2} \tag{39}$$

Ramandeep Gill 2019 :https://arxiv.org/abs/1811.11555

3b. Structured Jets - Jet Wings

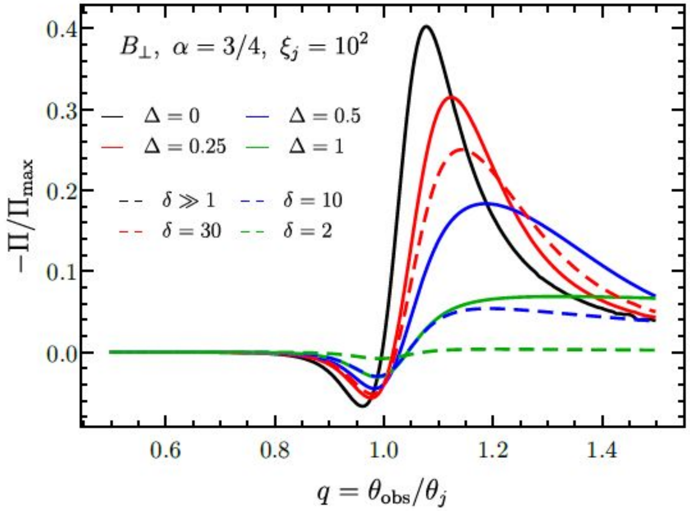
Adding Jet wings ends up adding complicated as well as simple features to look for in the PF vs Q plot.

Fastest Decaying Wings (Δ ~ 0 , δ ~100)

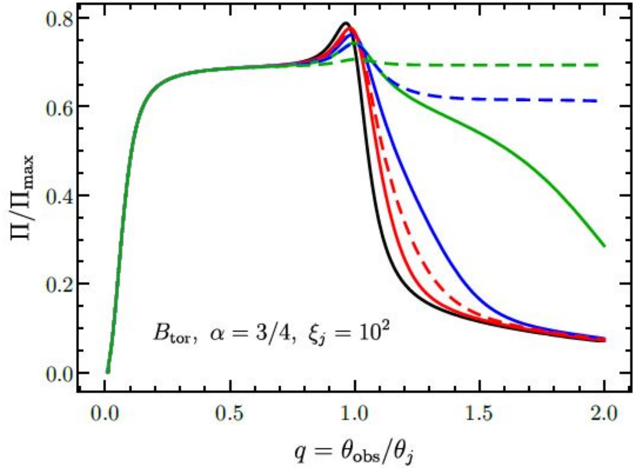
Middling Decaying Wings (Δ ~ 0.5 ,δ ~10)

Slowest Decaying Wings (Δ ~ 1, δ ~ 2)

Only Emissivity Varies.



Faster the decay, closer the behaviour is to a top hat jet. Fast drop off = fast symmetry breaking.



Ramandeep Gill 2019 :https://arxiv.org/abs/1811.11555

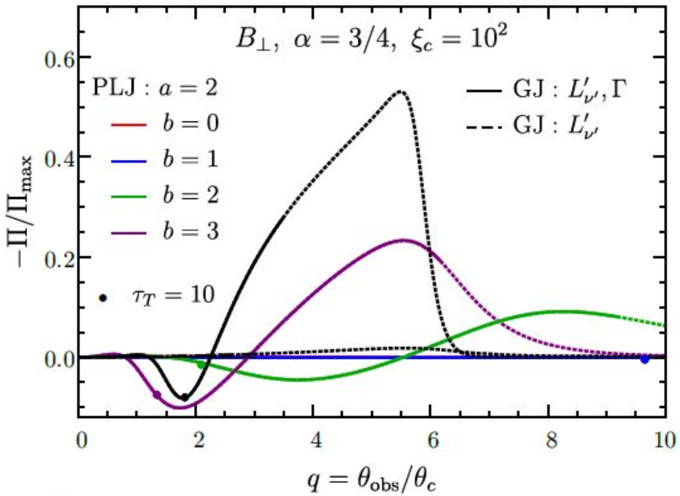
3b. Structured jet
- Lorentz factor

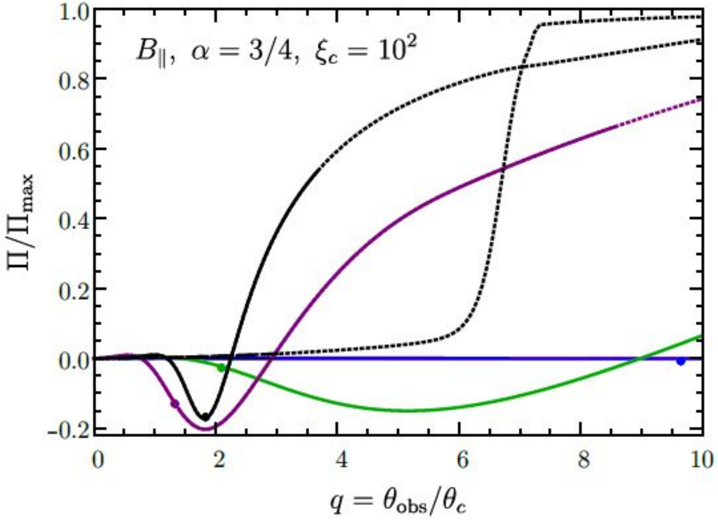
Adding Lorentz factor variation makes it so that we have access to larger values of q to observe the GRB from. But multiple further considerations need to be made.

(...........) dotted part of the curve represents when brightness falls below 1%

(**Solid Circle**) represents the part of the curve at which thomson cross section reaches 10)

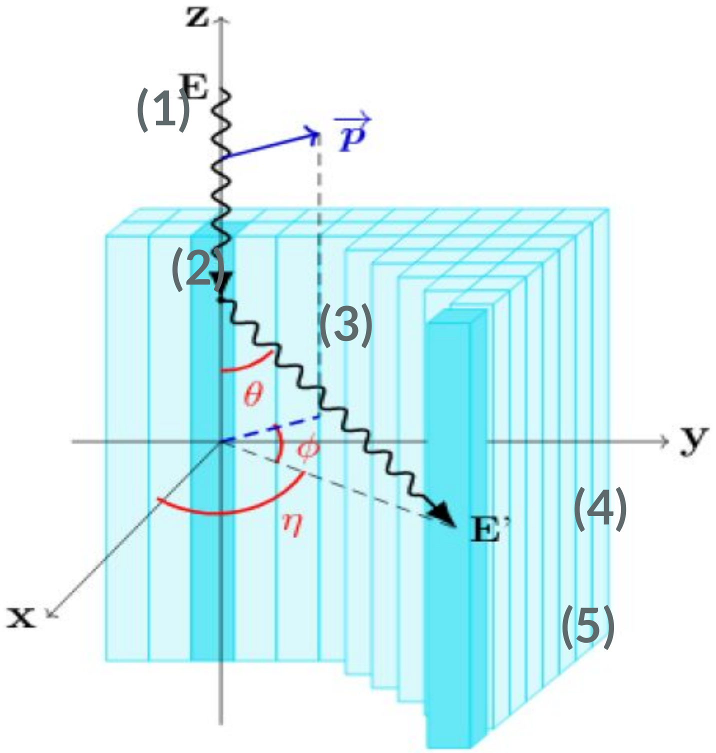
"For low values of Γ, the flow becomes optically thick to annihilation and results in the production of e- e+-pairs, which suppresses the emission of gamma ray photons."





Ramandeep Gill 2019 :https://arxiv.org/abs/1811.11555

4. My Future Work : MCMC
Monte-Carlo Modulation Curves!



1. Generate a Photon with a certain electric Polarization, Energy, Incidence Angle and Incidence Point (call that the origin)

2. Simulate the position where it will have its first compton interaction (pull from an exponential distribution)

3. Use Klein Nishina Cross section to find the new velocity vector.

4. Find the position of the second interaction, if it happens at all inside the detector.

5. Construct the histograms of azimuthal scattering angle, compare with the actual CZTI histograms

Thank you for listening!

Here are some key takeaways I'd want you to keep in mind :

1. Measuring Polarisation of High Energy photons lies at the intersection of Quantum Field Theory, Nuclear Physics, Electromagnetism, Material Science, Electronics and Algorithm Design!

2. We always will need more photons, thus having more satellites looking for GRBs is always a good thing :)

3. Spectral studies leave alot on the table, like jet geometry information. Involving polarisation measurements can crucially constrain GRB models.

4. Simplistic Top-Hat jet models tell us a lot; however we must be aware nature doesn't do discrete jumps (At this scale). Thus modelling with jet wings is important as well.

Acknowledgements and Image credits

1. "Statistical Properties of Gamma Ray Bursts Polarisation" - Kenji Toma et. al. (2009)
2. "Linear Polarisation in gamma-ray burst prompt emission" - Ramandeep Gill et. al. (2019)
3. "Gamma ray polarimetry of transient sources with POLAR" - Merlin Kole et. al. (2022)
4. "Radiation Detection and Measurement" - Glenn F Knoll, 4th edition.
5. NASA Scientific Visualisation Studio - Synchrotron
6. Wikimedia commons - multiple images
    a. https://en.wikipedia.org/wiki/Scintillation_counter
    b. https://en.wikipedia.org/wiki/Klein%E2%80%93Nishina_formula
    c. https://en.wikipedia.org/wiki/Stokes_parameters