EE774 - Lab 5

For all the problems below, please adhere to the following:

- Put each C++ source file in a separate directory, such as `prob1`, `prob2` etc.
- Add a `Makefile` in each directory to build the source code
- Try to accept the file input as a command line argument. Examples have been provided in class.
- Submit the assignment as a single `zip` or `tar.gz` file with all the subdirectories.

1. (10 points) Solve the questions below.

   a. Write a C++ program using Armadillo that generates a 4×4 matrix whose entries are real and Gaussian distributed. Then, using its SVD, find the best rank 1, rank 2 and rank 3 approximations, along with the corresponding norm errors, and print these onto the screen. That is, inspect the matrices $\sigma_1 u_1 v_1^T$, $\sigma_1 u_1 v_1^T + \sigma_2 u_2 v_2^T$ and $\sigma_1 u_1 v_1^T + \sigma_2 u_2 v_2^T + \sigma_3 u_3 v_3^T$, where $\sigma_1 \geq \sigma_2 \geq \sigma_3 \geq \sigma_4$ are the singular values.

   b. Check that each of these matrices added in computing the successive rank summations are rank 1 and orthogonal. In addition, evaluate $u_i v_i^T (u_j v_j^T)^T$ for $i \neq j$. What do you observe?

2. (20 points) In class, we used a Vandermonde matrix based approach to find the polynomial of best fit. We will now use the following approach to learn more about Vandermonde matrices. The included `data.txt` has $n + 1$ points (x_i, y_i) with x_i mutually distinct. Construct a Vandermonde matrix (square) to use it for interpolating a polynomial of degree n that interpolates through the given points. The following are the steps that you can follow:

   a. Read the columns of `data.txt` that has M entries. The first column has the x_i values and the second column has the corresponding y_i values.

   b. Construct the Vandermonde matrix using x_i, viz. $[X_{ij}] = x_i^{j-1}$. What is the rank of this matrix? Also find the condition number.

   c. We are now going to solve the equation using approximations of the matrix X. Let the SVD of X yield matrices U, Σ and V such that $X = U \Sigma V^T$, where these matrices have their usual meanings. Construct the matrix $\hat{X}$ from X by setting all but the r largest entries in Σ's diagonal as zero, where

r is the rank of X. What do you observe?
Note: use the solve function, since that solves linear equations even for the rank deficient cases.