1. Likelihood Func$^n$ : Let $\theta_1 = \mu$ $\theta_2 = \sigma^2$

$$L(\theta_1, \theta_2 \mid x_1, \dots, x_n) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(x_1-\mu)^2/2\sigma^2} \times \dots \times \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(x_n-\mu)^2/2\sigma^2}$$

Take log on both the sides

$$\ln\left[L(\theta_1, \theta_2 \mid x_1, \dots, x_n)\right] = \ln\left(\frac{1}{\sqrt{2\pi\sigma^2}} e^{-(x_1-\mu)^2/2\sigma^2} \times \dots \times \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(x_n-\mu)^2/2\sigma^2}\right)$$

$$= \ln\left(\frac{1}{\sqrt{2\pi\sigma^2}} e^{-(x_1-\mu)^2/2\sigma^2}\right) + \dots + \ln\left(\frac{1}{\sqrt{2\pi\sigma^2}} e^{-(x_n-\mu)^2/2\sigma^2}\right)$$

$$= -\frac{1}{2}\ln(2\pi) - \ln(\sigma) - \frac{(x_1-\mu)^2}{2\sigma^2} - \dots - \frac{1}{2}\ln(2\pi) - \ln(\sigma) - \frac{(x_n-\mu)^2}{2\sigma^2}$$

$$\ln\left[L(\theta_1, \theta_2 \mid x_1, \dots x_n)\right] = -\frac{n}{2}\ln(2\pi) - n\ln(\sigma) - \frac{(x_1-\mu)^2}{2\sigma^2} - \dots - \frac{(x_n-\mu)^2}{2\sigma^2}$$

$$\frac{\partial}{\partial \mu} \ln\left[L(\theta_1, \theta_2 \mid x_1 \dots x_n)\right] = 0 - 0 + \frac{(x_1-\mu)}{\sigma^2} + \dots + \frac{(x_n-\mu)}{\sigma^2}$$

$$= \frac{1}{\sigma^2}\left[(x_1 + \dots + x_n) - n\mu\right] \quad \text{—— ①}$$

$$\frac{\partial}{\partial \sigma} \ln\left[L(\theta_1, \theta_2 \mid x_1 \dots x_n)\right] = 0 - \frac{n}{\sigma} + \frac{1}{\sigma^3}\left[(x_1-\mu)^2 + \dots + (x_n-\mu)^2\right]$$

—— ②

$$\frac{\partial}{\partial \mu} \ln\left[L(\theta_1, \theta_2 \mid x_1 \dots x_n)\right] = 0$$

$$x_1 + x_2 + \dots + x_n = n\mu$$

$$\boxed{\theta_1(\text{or})\mu = \frac{x_1 + x_2 + \dots + x_n}{n}}$$

$$\frac{\partial}{\partial \sigma} \ln\left[L(\theta_1, \theta_2 \mid x_1, \dots x_n)\right] = 0$$

$$\sigma^2 n = (x_1-\mu)^2 + \dots + (x_n-\mu)^2$$

$$\boxed{\theta_2(\text{or})\,\sigma^2 = \frac{(x_1-\mu)^2 + \dots + (x_n-\mu)^2}{n}}$$

$$\boxed{\sigma = \sqrt{\frac{(x_1-\mu)^2 + \dots + (x_n-\mu)^2}{n}}}$$

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2. Likelihood Func$^n$ :

$$L(\theta | x_1 \dots x_n) = \prod_{i=1}^{n} \binom{m}{x_i} \theta^{x_i} (1-\theta)^{m-x_i}$$

$$\ln [L(\theta | x_1 \dots x_n)] = \sum_{i=1}^{n} \left( \log \binom{m}{x_i} + x_i \log \theta + (m - x_i) \log (1-\theta) \right)$$

$$\frac{\partial}{\partial \theta} \ln [L(\theta | x_1 \dots x_n)] = 0$$

$$\frac{\sum_{i=1}^{n} x_i}{\theta} - \frac{\sum_{i=1}^{n} (m - x_i)}{1-\theta} = 0$$

$$\frac{1-\theta}{\theta} = \frac{nm - \Sigma x_i}{\Sigma x_i}$$

$$\frac{1}{\theta} = \frac{nm}{\Sigma x_i}$$

$$\boxed{\theta = \frac{\Sigma x_i}{nm}}$$