fall2022_hw3

November 15, 2022

1 CS171-EE142 - Fall 2022 - Homework 3

2 Due: Tuesday, November 15, 2022 @ 11:59pm

2.0.1 Maximum points: 80 pts

2.1 Submit your solution to Gradescope:

1. Submit a single PDF to **HW3**
2. Submit your jupyter notebook to **HW3-code**

See the additional submission instructions at the end of this notebook

2.2 Enter your information below:

2.2.1 Your Name (submitter): Yash Aggarwal

2.2.2 Your student ID (submitter): 862333037

By submitting this notebook, I assert that the work below is my own work, completed for this course. Except where explicitly cited, none of the portions of this notebook are duplicated from anyone else's work or my own previous work.

2.3 Academic Integrity

Each assignment should be done individually. You may discuss general approaches with other students in the class, and ask questions to the TAs, but you must only submit work that is yours . If you receive help by any external sources (other than the TA and the instructor), you must properly credit those sources, and if the help is significant, the appropriate grade reduction will be applied. If you fail to do so, the instructor and the TAs are obligated to take the appropriate actions outlined at http://conduct.ucr.edu/policies/academicintegrity.html . Please read carefully the UCR academic integrity policies included in the link.

3 Overview

In this assignment you will implement a two-layer neural network. You will implement the loss functions, gradients, optimizers to train the network and test its performance on MNIST dataset.

For this assignment we will use the functionality of Pandas (https://pandas.pydata.org/), Matplotlib (https://matplotlib.org/), and Numpy (http://www.numpy.org/).

If you are asked to **implement** a particular functionality, you should **not** use an existing implementation from the libraries above (or some other library that you may find). When in doubt, please ask.

Before you start, make sure you have installed all those packages in your local Jupyter instance

3.1 Read *all* cells carefully and answer all parts (both text and missing code)

You will complete all the code marked `TODO` and answer descriptive/derivation questions

[1]:
```
import numpy as np
import matplotlib.pyplot as plt
import math
from sklearn.utils import shuffle

# make sure you import here everything else you may need
```

3.1.1 Load MNIST Dataset

For this assignment, we will use MNIST handwritten digits data set. The dataset consists 10 handwritten digits (0,1,...,9). It is a widely used dataset to demonstrate simple image classification problem.

MNIST dataset is publicly available from different sources. We will be using MNIST from Keras package. If you do not have Keras installed, you can find the installation guide here.

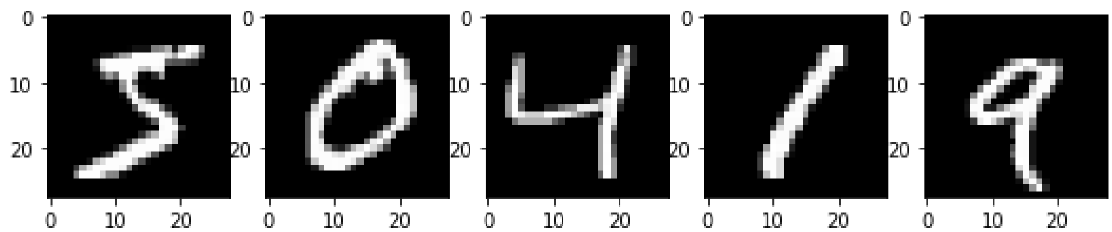
In short, you need to run `conda install -c anaconda keras` or `pip install keras`

The training data consists of 60000 images of size 28×28 pixels; the test data consists of 10000 images.

[2]:
```
from keras.datasets import mnist
(x_train, y_train), (x_test, y_test) = mnist.load_data()

print('Training data shape:',x_train.shape)
print('Test data shape:',x_test.shape)

n_img=5
plt.figure(figsize=(n_img*2,2))
plt.gray()
for i in range(n_img):
```

```
    plt.subplot(1,n_img,i+1)
    plt.imshow(x_train[i])
plt.show()
```

```
Training data shape: (60000, 28, 28)
Test data shape: (10000, 28, 28)
```



We will be vectorizing the training and test images. So, the size of each vector will be 784.

[3]:
```
x_train=x_train.reshape(x_train.shape[0],-1)
x_test=x_test.reshape(x_test.shape[0],-1)

print('Training data shape after reshaping:',x_train.shape)
print('Test data shape after reshaping::',x_test.shape)
```

```
Training data shape after reshaping: (60000, 784)
Test data shape after reshaping:: (10000, 784)
```

3.2 Question 1: Binary classification using neural network [45 pts]

We will start with classification of images for two different digits using a two-layer network with a cross entropy loss.

In the next question, we will extend the same architecture to multi-class classification.

Pick any two digits out of ten for our classification (say 5 and 8), which we will assign label "0" or "1".

Pick same number of images from each class for training and create arrays for input and output (say 1000).

```
# train_x -- N x 784 array of training input
# train_y -- N x 1 array of binary labels
```

If you use 1000 images from each class N = 2000. You can increase the number of training samples if you like. It is just a suggestion.

We also need to transpose the dimension of the data so that their size becomes $784 \times N$. It will be helpful to feed it to our model based on our notations.

[4]:

```
def extract_binary_classification_dataset(x, y, label1, label2, num_samples):
    """Make a subset dataset from MNIST, containing only 2 classes for binary␣
 ↪classification task
    Args:
        x (numpy.ndarray): data, can be x_train or x_test
        y (numpy.ndarray): labels of data, can be y_train or y_test
        label1 (int): the first class you pick, e.g. 5
        label2 (int): the second class you pick, e.g. 8
        num_samples (int): the number of images you select for each class, e.g.␣
 ↪1000
    Returns:
        x_ (numpy.ndarray): the data for 2 picked classes
        y_ (numpy.ndarray): the corresponding labels for 2 picked classes
    """
    # for class 1
    x1 = x[y == label1]
    x1 = x1[:num_samples]
    y1 = np.zeros(len(x1))

    # for class 2
    x2 = x[y == label2]
    x2 = x2[:num_samples]
    y2 = np.ones(len(x2))

    # combine 2 classes
    x_ = np.concatenate((x1,x2),axis=0)
    y_ = np.concatenate((y1,y2),axis=0)
    return x_, y_


# Pick your own digits
label1 = 5
label2 = 8
num_samples = 1000

# Train & test data
train_x, train_y = extract_binary_classification_dataset(x_train, y_train,␣
 ↪label1, label2, num_samples)
test_x, test_y = extract_binary_classification_dataset(x_test, y_test, label1,␣
 ↪label2, num_samples)

# reshape data
# Images are stored row wise, store it column wise
train_x = train_x.T
test_x = test_x.T
print("Training data shape:", train_x.shape)
print("Training labels shape:", train_y.shape)
```

```
print("Test data shape:", test_x.shape)
print("Test labels shape:", test_y.shape)
```

```
Training data shape: (784, 2000)
Training labels shape: (2000,)
Test data shape: (784, 1866)
Test labels shape: (1866,)
```

3.2.1 Network Architecture

We will be using a two layer neural network in our experiment. The input layer will have 784 nodes, the hidden layer will have 256 nodes and the output layer will have 1 node. Each node will have *sigmoid* activation function.

The equations for feedforward operation will be the following:

$$\mathbf{z}^{(1)} = W^{(1)}\mathbf{x} + \mathbf{b}^{(1)}\mathbf{y}^{(1)} = \varphi(\mathbf{z}^{(1)})\mathbf{z}^{(2)} = W^{(2)}\mathbf{y}^{(1)} + \mathbf{b}^{(2)}\mathbf{y}^{(2)} = \varphi(\mathbf{z}^{(2)})$$

where $\mathbf{x} \in \mathbb{R}^{784}$ is the input layer, $\mathbf{y}^{(1)} \in \mathbb{R}^{256}$ is the hidden layer, $\mathbf{y}^{(2)} \in \mathbb{R}$ is the output layer, $W^{(1)} \in \mathbb{R}^{256\times 784}$ is the first layer weights, $W^{(2)} \in \mathbb{R}^{1\times 256}$ is the second layer weights, $\mathbf{b}^{(1)} \in \mathbb{R}^{256}$ is the first layer bias, $\mathbf{b}^{(2)} \in \mathbb{R}$ is the second layer bias, $\varphi(\cdot)$ is the activation function.

3.2.2 Network initialization [5 pts]

We initialize the weights for $W^{(1)}$ and $W^{(2)}$ with random values drawn from normal distribution with zero mean and 0.01 standard deviation. We will initialize bias vectors $\mathbf{b}^{(1)}$ and $\mathbf{b^{(2)}}$ with zero values.

We can fix the seed for random initialization for reproducibility.

[5]:
```
def TwoLayerNetwork(layer_dims=[784,256,1]):

    # Fix the seed
    np.random.seed(3)

    mean = 0
    std = 0.01

    # TODO
    # Your code goes here
    params = {}
    params['w1'] = np.random.normal(mean, std,␣
 ↪size=(layer_dims[1],layer_dims[0]))
    params['b1'] = np.zeros(layer_dims[1])
    params['w2'] = np.random.normal(mean, std,␣
 ↪size=(layer_dims[2],layer_dims[1]))
    params['b2'] = np.zeros(layer_dims[2])
```

```
    return params
```

3.2.3 Sigmoid activation function

Now we will write the sigmoid activation function as

$$\varphi(z) = \frac{1}{1 + e^{-z}}$$

Note that derivative of **sigmoid** is $\varphi'(z) = \varphi(z)(1 - \varphi(z))$.

[6]:
```
def sigmoid(Z):
    # Input: Z -- numpy.ndarray
    # TODO
    Y = (1 / (1 + np.exp(-Z)))
    return Y
```

3.2.4 Cross entropy loss function [5 pts]

We will minimize the binary cross entropy loss function. You will use the true labels and predicted labels of a batch of N samples.

Binary crossentropy loss for i^{th} sample can be written as

$$Loss_i = -y_i \log y_i^{(2)} - (1 - y_i) \log(1 - y_i^{(2)})$$

where y_i is the true label. We can find the average loss for a batch of N samples as $Loss = \frac{1}{N} \sum_{i=1}^{N} Loss_i$.

Note that the gradient of the cross entropy loss w.r.t. the output is

$$\nabla_{y^{(2)}} Loss_i = -\frac{y_i}{y_i^{(2)}} + \frac{1 - y_i}{1 - y_i^{(2)}} = \frac{y_i^{(2)} - y_i}{y_i^{(2)}(1 - y_i^{(2)})}.$$

We can also show that

$$\delta^{(2)} = \nabla_{\mathbf{z}^{(2)}} Loss_i = \nabla_{y^{(2)}} Loss_i \odot \varphi'(\mathbf{z}) = y_i^{(2)} - y_i,$$

where $\odot$ denotes element-wise multiplication of the arrays.

[7]:
```
def CrossEntropyLoss(Y_true, Y2):
    # TODO
    # Write your code here

    loss = 0
    for y_true,y_pred in zip(Y_true, Y2.T):
```

```
        l = (y_true*np.log(y_pred)) + ((1-y_true)*np.log(1-y_pred))
        l = -l
        loss += l

    return loss[0] / len(Y2)
```

3.2.5 Forward propagation [5 pts]

Next, we will write the code for the forward pass for two layer network. Each layer consists of an affine function (fully-connected layer) followed by an activation function. You wil also return the intermediate results ($\mathbf{x}, \mathbf{z}^{(1)}, \mathbf{y}^{(1)}, \mathbf{z}^{(2)}$) in addition to final output ($\mathbf{y}^{(2)}$). You will need the intermediate outputs for the backpropagation step.

[8]:
```
def forward(X, params):

    # TODO
    # Write your codes here
    # X -- 784 x N array
    # params --
    # w1 -- 256 x 784 matrix
    # b1 -- 256 x 1 vector
    # W2 -- 1 x 256 matrix
    # b2 -- 1 x 1 scalar
    # Y2 -- 1 x N output

    intermediate = {}

    intermediate['X'] = X
    intermediate['z1'] = params['w1'].dot(intermediate['X'])+params['b1'].
↪reshape(params['b1'].shape[0], 1)
    intermediate['Y1'] = sigmoid(intermediate['z1'])
    intermediate['z2'] = params['w2'].dot(intermediate['Y1'])+params['b2'].
↪reshape(params['b2'].shape[0], 1)
    Y2 = sigmoid(intermediate['z2'])

    return Y2, intermediate
```

3.2.6 Backpropagation step [10 pts]

Now we will implement the backpropagation step for the two layer neural network.

You will need the gradient of the Loss w.r.t. $W^{(l)}, \mathbf{b}^{(l)}$ for $l = 1, 2$ for all the training samples.

We saw that we can write the gradient of Loss with respect to $W^{(l)}, \mathbf{b}^{(l)}$ for a single sample as

$$\nabla_{W^{(l)}} Loss_i = \delta^{(l)} \mathbf{y}^{(l-1)T},$$

$$\nabla_{\mathbf{b}^{(l)}} Loss_i = \delta^{(l)},$$

where

$$\delta^{(l)} = \nabla_{\mathbf{z}^{(l)}} Loss_i = \nabla_{\mathbf{y}^{(l)}} Loss_i \odot \varphi'(\mathbf{z}^{(l)}).$$

For the the last layer, we can compute $\delta^{(L)}$ by plugging the value of $\nabla_{\mathbf{y}^{(L)}} Loss$ as described above.

For the intermediate layers $l < L$, we can write

$$\delta^{(l)} = W^{(l+1)T} \delta^{(l+1)} \odot \varphi'(\mathbf{z}^{(l)}).$$

Once we have the gradients $\nabla_{W^{(l)}} Loss_i, \nabla_{\mathbf{b}^{(l)}} Loss_i$ for all i. We can compute their average to compute the gradient of the total loss function $\frac{1}{N}\sum_{i=1}^{N} Loss_i$ as

$$\nabla_{W^{(l)}} Loss = \frac{1}{N} \sum_i \nabla_{W^{(l)}} Loss_i,$$
$$\nabla_{\mathbf{b}^{(l)}} Loss = \frac{1}{N} \sum_i \nabla_{\mathbf{b}^{(l)}} Loss_i.$$

Please refer to the slides and lectures for more details.

[9]:
```python
def backward(Y_true, Y2, intermediate, params):

    # Inputs:
      # Y_true -- 1 x N true labels
      # Y2 -- 1 x N output of the last layer
      # intermediate -- X, Z1, Y1, Z2
      # params -- W1, b1, W2, b2

    # Outputs:
      # grads -- [grad_W1, grad_b1, grad_W2, grad_b2]

    # TODO
    # Write your codes here

    grads = {}

    Y_true_reshaped = train_y.reshape(1,train_y.shape[0])

    delta_2 = Y2 - Y_true_reshaped
    grads['grad_w2'] = delta_2.dot(intermediate['Y1'].T)
    grads['grad_b2'] = np.sum(delta_2,axis=1)

    delta_1 = params['w2'].T.dot(delta_2) * (sigmoid(intermediate['z1'])*(1 -␣
↪sigmoid(intermediate['z1'])))
    grads['grad_w1'] = delta_1.dot(intermediate['X'].T)
```

```
    grads['grad_b1'] = np.sum(delta_1,axis=1)

    return grads
```

3.2.7 Optimizer [5 pts]

We will use a standard gradient descent-based optimizer to minimize the loss function. You have already implemented gradient descent in HW2. You may have to adjust learning rate that provides you best training/validation performance. In this exercise, we are not using validation data; in practice, you should use it to tune your hyperparameters such as learning rate, network architecture etc.

You can use same learning rate for all weights in this assignment.

You should update $W^1, \mathbf{b}^1, W^2, \mathbf{b}^2$ as

$$W^1 \leftarrow W^1 - \alpha \nabla_{W^1} Loss$$
$$\mathbf{b}^1 \leftarrow \mathbf{b}^1 - \alpha \nabla_{\mathbf{b}^1} Loss$$
$$W^2 \leftarrow W^2 - \alpha \nabla_{W^2} Loss$$
$$\mathbf{b}^2 \leftarrow \mathbf{b}^2 - \alpha \nabla_{\mathbf{b}^2} Loss$$

α is the learning rate.

[10]:
```
def GD(params, grads, learning_rate):

    # updated params = old params - learning rate * gradient of Loss computed
 ↪at old params
    # TODO
    # Write your codes here

    params['w2'] = params['w2'] - learning_rate*grads['grad_w2']
    params['b2'] = params['b2'] - learning_rate*grads['grad_b2']
    params['w1'] = params['w1'] - learning_rate*grads['grad_w1']
    params['b1'] = params['b1'] - learning_rate*grads['grad_b1']


    return params
```

[11]:
```
def predict(x, params):
  Y2, _ = forward(x, params)
  Y2 = np.array([1 if y_i > 0.5 else 0 for y_i in Y2.T]).reshape(1,-1)
  return Y2
```

[12]:
```
def accuracy(y, y_pred):
  aa = y_pred.reshape(-1)
  bb = np.array([y == aa])
  acc = np.sum(bb) / bb.shape[1]
```

```python
    return acc
```

3.2.8 Train the Model [5 pts]

We will train the model using the functions we wrote above.

First, we specify the number of nodes in the layers, number of epochs and learning rate. Then we initialize the network.

[13]:
```python
layer_dims = [train_x.shape[0],256,1]
epochs = 100
lr = 0.00001

params = TwoLayerNetwork(layer_dims)
```

Then we train the network for the number of epochs specified above. In every epoch, we will do the following: 1. Calculate the forward pass to get estimated labels. 2. Use the estimated labels calculate loss. We will be recording loss for every epoch. 3. Use backpropagation to calculate gradients. 4. Use gradient descent to update the weights and biases.

You should store the loss value after every epoch in an array `loss_history` and print the loss value after every few epochs (say 20).

[14]:
```python
# TODO
# Write your codes here

loss_history = []
train_acc = []
test_acc = []
for epoch in range(epochs):

    print ('For Epoch : ', epoch)

    Y2, intermediate = forward(train_x, params)

    loss = CrossEntropyLoss(train_y, Y2)
    loss_history.append(loss)

    train_pred = predict(train_x, params)
    train_accuracy = accuracy(train_y, train_pred)
    train_acc.append(train_accuracy)

    test_pred = predict(test_x, params)
    test_accuracy = accuracy(test_y, test_pred)
    test_acc.append(test_accuracy)

    print ('loss', loss)
    print ('train_accuracy', train_accuracy)
```

```python
    print ('test_accuracy', test_accuracy)

    grads = backward(train_y, Y2, intermediate, params)
    params = GD(params, grads, lr)

    print('-------------')
```

```
For Epoch :  0
loss 1382.6803822154936
train_accuracy 0.5005
test_accuracy 0.5219721329046088
-------------
For Epoch :  1
loss 1344.6835766468082
train_accuracy 0.5615
test_accuracy 0.5659163987138264
-------------
For Epoch :  2
loss 1312.5122541437577
train_accuracy 0.719
test_accuracy 0.6854233654876741
-------------
For Epoch :  3
loss 1281.2094111728147
train_accuracy 0.813
test_accuracy 0.7867095391211146
-------------
For Epoch :  4
loss 1249.7418429811107
train_accuracy 0.872
test_accuracy 0.8397642015005359
-------------
For Epoch :  5
loss 1218.2988593657276
train_accuracy 0.8925
test_accuracy 0.87513397642015
-------------
For Epoch :  6
loss 1186.226182628
train_accuracy 0.9125
test_accuracy 0.897642015005359
-------------
For Epoch :  7
loss 1153.0783868022802
train_accuracy 0.9215
test_accuracy 0.912647374062165
-------------
```

```
For Epoch :  8
loss 1118.9972745434084
train_accuracy 0.9275
test_accuracy 0.9185423365487674
-------------
For Epoch :  9
loss 1084.7422031586223
train_accuracy 0.931
test_accuracy 0.9217577706323687
-------------
For Epoch :  10
loss 1049.964695444493
train_accuracy 0.9355
test_accuracy 0.9287245444801715
-------------
For Epoch :  11
loss 1015.0413282089033
train_accuracy 0.9395
test_accuracy 0.9297963558413719
-------------
For Epoch :  12
loss 980.2564032048854
train_accuracy 0.9425
test_accuracy 0.9330117899249732
-------------
For Epoch :  13
loss 945.7846655431864
train_accuracy 0.9445
test_accuracy 0.935155412647374
-------------
For Epoch :  14
loss 911.9151813785345
train_accuracy 0.945
test_accuracy 0.9367631296891747
-------------
For Epoch :  15
loss 878.7881808361788
train_accuracy 0.9455
test_accuracy 0.9367631296891747
-------------
For Epoch :  16
loss 846.4799514354353
train_accuracy 0.9455
test_accuracy 0.9372990353697749
-------------
For Epoch :  17
loss 815.1096598949241
train_accuracy 0.946
```

```
test_accuracy 0.9378349410503751
-------------
For Epoch :  18
loss 784.7641641162679
train_accuracy 0.947
test_accuracy 0.9389067524115756
-------------
For Epoch :  19
loss 755.7800848341215
train_accuracy 0.949
test_accuracy 0.9389067524115756
-------------
For Epoch :  20
loss 728.2806699924226
train_accuracy 0.95
test_accuracy 0.9410503751339764
-------------
For Epoch :  21
loss 700.7745553112132
train_accuracy 0.95
test_accuracy 0.9421221864951769
-------------
For Epoch :  22
loss 675.5968491512332
train_accuracy 0.9505
test_accuracy 0.939978563772776
-------------
For Epoch :  23
loss 651.7580419153363
train_accuracy 0.9505
test_accuracy 0.9437299035369775
-------------
For Epoch :  24
loss 629.4161590733192
train_accuracy 0.9525
test_accuracy 0.9405144694533762
-------------
For Epoch :  25
loss 607.9837002835761
train_accuracy 0.954
test_accuracy 0.9431939978563773
-------------
For Epoch :  26
loss 588.0771111520274
train_accuracy 0.954
test_accuracy 0.9405144694533762
-------------
For Epoch :  27
```

```
loss 568.9542933708474
train_accuracy 0.954
test_accuracy 0.9437299035369775
-------------
For Epoch :  28
loss 550.9514971628929
train_accuracy 0.954
test_accuracy 0.9421221864951769
-------------
For Epoch :  29
loss 534.1031271491797
train_accuracy 0.957
test_accuracy 0.9442658092175777
-------------
For Epoch :  30
loss 517.9299618387331
train_accuracy 0.9555
test_accuracy 0.9437299035369775
-------------
For Epoch :  31
loss 502.8528300972274
train_accuracy 0.9595
test_accuracy 0.9448017148981779
-------------
For Epoch :  32
loss 488.54445990220347
train_accuracy 0.957
test_accuracy 0.9448017148981779
-------------
For Epoch :  33
loss 475.0297021935305
train_accuracy 0.9605
test_accuracy 0.9453376205787781
-------------
For Epoch :  34
loss 462.1930003750459
train_accuracy 0.9575
test_accuracy 0.9453376205787781
-------------
For Epoch :  35
loss 450.09908950239543
train_accuracy 0.962
test_accuracy 0.9464094319399786
-------------
For Epoch :  36
loss 438.6311993035542
train_accuracy 0.9605
test_accuracy 0.9448017148981779
```

```
-------------
For Epoch :  37
loss 427.95115157440546
train_accuracy 0.9625
test_accuracy 0.947481243301179
-------------
For Epoch :  38
loss 417.53504663422353
train_accuracy 0.9605
test_accuracy 0.9448017148981779
-------------
For Epoch :  39
loss 407.81528601071926
train_accuracy 0.9635
test_accuracy 0.9469453376205788
-------------
For Epoch :  40
loss 398.14437162628786
train_accuracy 0.9625
test_accuracy 0.9458735262593784
-------------
For Epoch :  41
loss 389.1551206294519
train_accuracy 0.965
test_accuracy 0.947481243301179
-------------
For Epoch :  42
loss 380.2852605283457
train_accuracy 0.9635
test_accuracy 0.9464094319399786
-------------
For Epoch :  43
loss 372.0185010513821
train_accuracy 0.966
test_accuracy 0.947481243301179
-------------
For Epoch :  44
loss 363.8777274380071
train_accuracy 0.9645
test_accuracy 0.9480171489817792
-------------
For Epoch :  45
loss 356.370319027147
train_accuracy 0.969
test_accuracy 0.9469453376205788
-------------
For Epoch :  46
loss 349.07849445408755
```

```
train_accuracy 0.9655
test_accuracy 0.9480171489817792
-------------
For Epoch :  47
loss 342.04857086558974
train_accuracy 0.9705
test_accuracy 0.9469453376205788
-------------
For Epoch :  48
loss 335.3163594747206
train_accuracy 0.968
test_accuracy 0.9480171489817792
-------------
For Epoch :  49
loss 328.92587601927966
train_accuracy 0.971
test_accuracy 0.947481243301179
-------------
For Epoch :  50
loss 322.72953350925746
train_accuracy 0.968
test_accuracy 0.947481243301179
-------------
For Epoch :  51
loss 316.77669957919323
train_accuracy 0.9715
test_accuracy 0.9490889603429796
-------------
For Epoch :  52
loss 310.9018892917722
train_accuracy 0.9695
test_accuracy 0.9480171489817792
-------------
For Epoch :  53
loss 305.53306522956825
train_accuracy 0.9715
test_accuracy 0.9496248660235799
-------------
For Epoch :  54
loss 300.11038717171647
train_accuracy 0.9715
test_accuracy 0.9490889603429796
-------------
For Epoch :  55
loss 295.1209981975711
train_accuracy 0.9725
test_accuracy 0.9506966773847803
-------------
```

```
For Epoch :  56
loss 289.971813327418
train_accuracy 0.9725
test_accuracy 0.9490889603429796
-------------
For Epoch :  57
loss 285.5585209905177
train_accuracy 0.9735
test_accuracy 0.9506966773847803
-------------
For Epoch :  58
loss 280.860778194801
train_accuracy 0.9735
test_accuracy 0.9496248660235799
-------------
For Epoch :  59
loss 276.7921480922436
train_accuracy 0.975
test_accuracy 0.9512325830653805
-------------
For Epoch :  60
loss 272.38521213530464
train_accuracy 0.974
test_accuracy 0.9496248660235799
-------------
For Epoch :  61
loss 268.724036334955
train_accuracy 0.975
test_accuracy 0.9512325830653805
-------------
For Epoch :  62
loss 264.609903551408
train_accuracy 0.974
test_accuracy 0.9490889603429796
-------------
For Epoch :  63
loss 261.0684204095201
train_accuracy 0.9755
test_accuracy 0.9506966773847803
-------------
For Epoch :  64
loss 257.04486153668853
train_accuracy 0.9755
test_accuracy 0.9490889603429796
-------------
For Epoch :  65
loss 253.70201526598004
train_accuracy 0.976
```

```
test_accuracy 0.9506966773847803
-------------
For Epoch :  66
loss 249.83451093039628
train_accuracy 0.976
test_accuracy 0.9496248660235799
-------------
For Epoch :  67
loss 246.4130977127861
train_accuracy 0.977
test_accuracy 0.9506966773847803
-------------
For Epoch :  68
loss 242.64651414104762
train_accuracy 0.9765
test_accuracy 0.9496248660235799
-------------
For Epoch :  69
loss 239.4074944738534
train_accuracy 0.978
test_accuracy 0.9506966773847803
-------------
For Epoch :  70
loss 236.0271812055234
train_accuracy 0.977
test_accuracy 0.9496248660235799
-------------
For Epoch :  71
loss 232.89682599493784
train_accuracy 0.9785
test_accuracy 0.9512325830653805
-------------
For Epoch :  72
loss 229.79909876179318
train_accuracy 0.9775
test_accuracy 0.9496248660235799
-------------
For Epoch :  73
loss 227.1067108766107
train_accuracy 0.9785
test_accuracy 0.9512325830653805
-------------
For Epoch :  74
loss 224.23680445752015
train_accuracy 0.978
test_accuracy 0.9496248660235799
-------------
For Epoch :  75
```

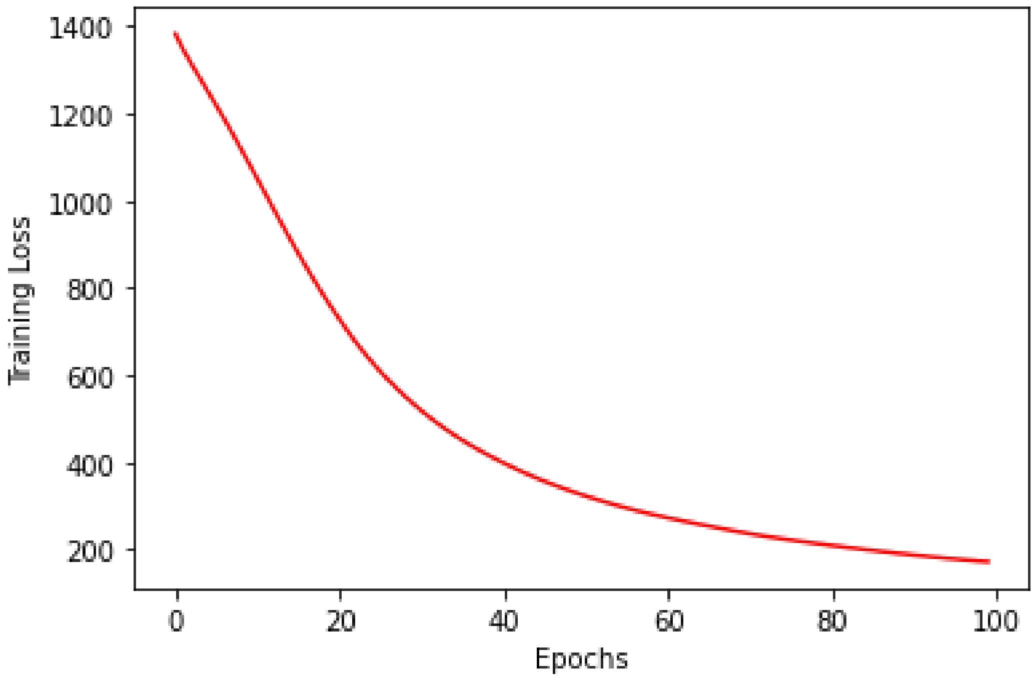
```
loss 221.61447361462677
train_accuracy 0.9785
test_accuracy 0.9512325830653805
-------------
For Epoch :  76
loss 218.79707518410154
train_accuracy 0.978
test_accuracy 0.9496248660235799
-------------
For Epoch :  77
loss 216.46582885639745
train_accuracy 0.979
test_accuracy 0.9506966773847803
-------------
For Epoch :  78
loss 213.7656832981032
train_accuracy 0.9785
test_accuracy 0.9496248660235799
-------------
For Epoch :  79
loss 211.51896219284143
train_accuracy 0.9795
test_accuracy 0.9506966773847803
-------------
For Epoch :  80
loss 208.85735739359754
train_accuracy 0.979
test_accuracy 0.9496248660235799
-------------
For Epoch :  81
loss 206.79491827445753
train_accuracy 0.981
test_accuracy 0.9506966773847803
-------------
For Epoch :  82
loss 204.14006675541643
train_accuracy 0.979
test_accuracy 0.9496248660235799
-------------
For Epoch :  83
loss 202.2052593845316
train_accuracy 0.9815
test_accuracy 0.9506966773847803
-------------
For Epoch :  84
loss 199.5561368809616
train_accuracy 0.979
test_accuracy 0.9490889603429796
```

```
-------------
For Epoch :  85
loss 197.71899577138498
train_accuracy 0.982
test_accuracy 0.9506966773847803
-------------
For Epoch :  86
loss 195.14735017403595
train_accuracy 0.979
test_accuracy 0.9490889603429796
-------------
For Epoch :  87
loss 193.41418262125447
train_accuracy 0.9825
test_accuracy 0.9506966773847803
-------------
For Epoch :  88
loss 190.9266269003494
train_accuracy 0.979
test_accuracy 0.9512325830653805
-------------
For Epoch :  89
loss 189.2729323683954
train_accuracy 0.9825
test_accuracy 0.9506966773847803
-------------
For Epoch :  90
loss 186.88529500257565
train_accuracy 0.9795
test_accuracy 0.9512325830653805
-------------
For Epoch :  91
loss 185.26135078765068
train_accuracy 0.9825
test_accuracy 0.9506966773847803
-------------
For Epoch :  92
loss 183.06718545245567
train_accuracy 0.981
test_accuracy 0.9512325830653805
-------------
For Epoch :  93
loss 181.5041770167333
train_accuracy 0.9825
test_accuracy 0.9501607717041801
-------------
For Epoch :  94
loss 179.50214930850638
```

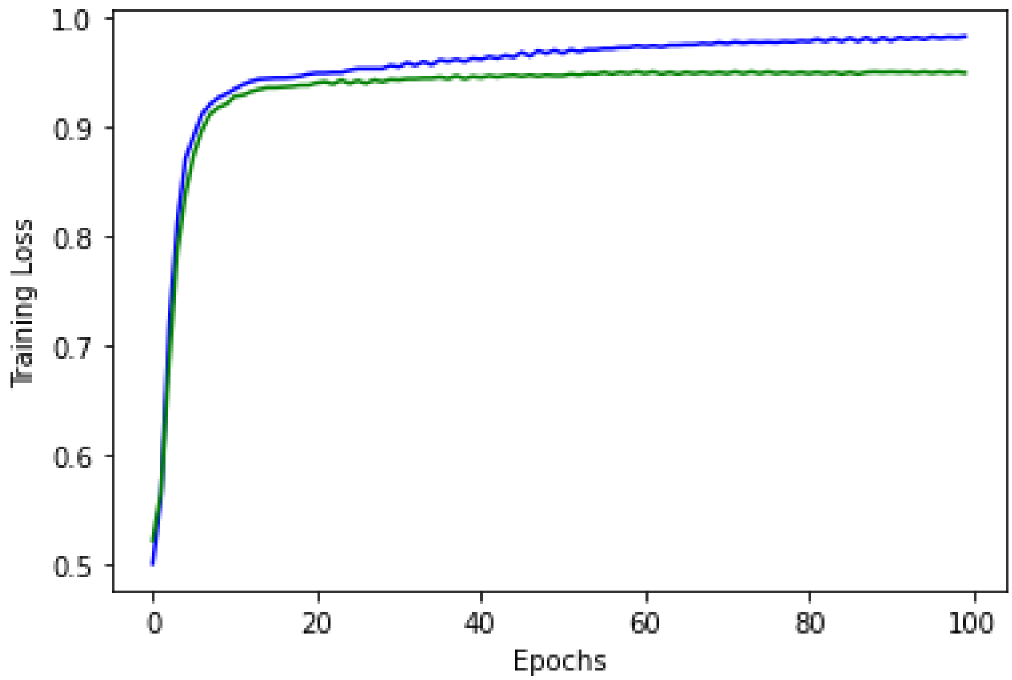
```
train_accuracy 0.981
test_accuracy 0.9512325830653805
-------------
For Epoch :  95
loss 178.0433922053425
train_accuracy 0.983
test_accuracy 0.9501607717041801
-------------
For Epoch :  96
loss 176.16560085132468
train_accuracy 0.982
test_accuracy 0.9512325830653805
-------------
For Epoch :  97
loss 174.81504470879568
train_accuracy 0.983
test_accuracy 0.9501607717041801
-------------
For Epoch :  98
loss 173.07481262019627
train_accuracy 0.9825
test_accuracy 0.9512325830653805
-------------
For Epoch :  99
loss 171.91108795963078
train_accuracy 0.9835
test_accuracy 0.9501607717041801
-------------
```

Now we will plot the recorded loss values vs epochs. We will observe the training loss decreasing with the epochs.

[15]:
```
plt.figure()
plt.plot(loss_history, color = 'red')
plt.xlabel("Epochs")
plt.ylabel("Training Loss")
plt.show()
```



[16]:
```
plt.figure()
# plt.plot(np.log(loss_history), color = 'red')
plt.plot(train_acc, color = 'blue')
plt.plot(test_acc, color = 'green')
plt.xlabel("Epochs")
plt.ylabel("Training Loss")
plt.show()
```



3.2.9 Evaluation on test data [5 pts]

Now we will be evaluating the accuracy we get from the trained model. We feed training data and test data to the forward model along with the trained parameters.

Note that, we need to covert the output probability of the forward pass to binary labels before evaluating accuracy. Since the model provides the posterior probability $p(y = 1|x)$ in range [0,1]. We can binarize them using 0.5 as a theshold (i.e. if $y_i^{(2)} \geq 0.5$, $y_i^{(2)} \leftarrow 1$ otherwise $y_i^{(2)} \leftarrow 0$).

[17]:
```
# TODO
y_pred = predict(train_x, params)
acc = accuracy(train_y, y_pred)
print("Training accuracy:", acc)

y_pred = predict(test_x, params)
acc = accuracy(test_y, y_pred)
print("Test accuracy:", acc)
```

```
Training accuracy: 0.9825
Test accuracy: 0.9512325830653805
```

3.2.10 Visualize some of the correct/miscalassified images [5 pts]

Now we will look at some images from training and test sets that were misclassified.

Training set. Pick 5 images from each class that are correcly and incorreclty classified. True/False Positive/Negatives

Test set. Pick 5 images from each class that are correcly and incorreclty classified. True/False Positive/Negatives

[18]:
```
# TODO
# Training set
print("Training set examples for true/false positive/negative")
Y_hat = predict(train_x, params)
Y_hat = Y_hat.reshape(-1)

positive = []
negative = []
false_positive = []
false_negative = []

idx = 0

for y_hat,y_true,x_i in zip(Y_hat, train_y, train_x.T):
  idx += 1
  if y_hat == y_true:
    if y_hat == 1:
      # print('positive')
      positive.append(x_i)
    else :
      # print('negative')
      negative.append(x_i)
  else :
    if y_hat == 0:
      # print('false_positive')
      false_positive.append(x_i)
    else:
      # print('false_negative')
      false_negative.append(x_i)


positive = np.array(positive)
negative = np.array(negative)
false_positive = np.array(false_positive)
false_negative = np.array(false_negative)

print ('positive', positive.shape)
print ('negative', negative.shape)
print ('false_positive', false_positive.shape)
print ('false_negative', false_negative.shape)
```

```
Training set examples for true/false positive/negative
```

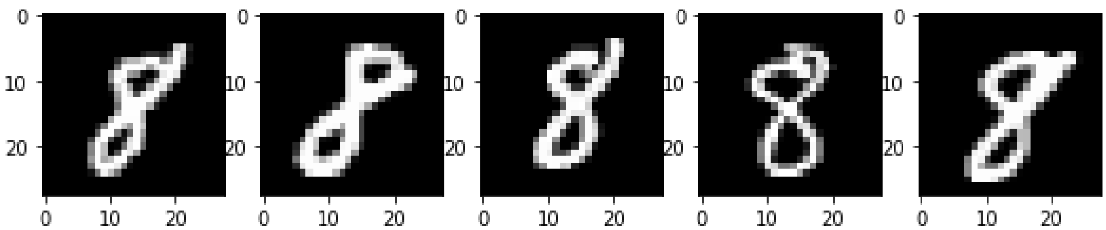
```
positive (977, 784)
negative (988, 784)
false_positive (23, 784)
false_negative (12, 784)
```

[19]:

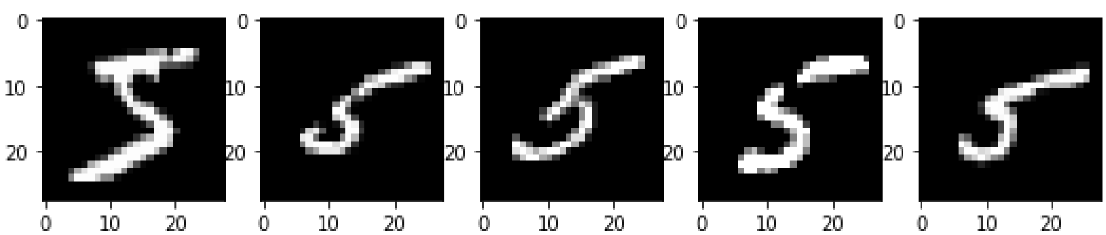
```python
print (' --------- Positives -----------')
print (' --------- Label: 8, Predicted: 8 -----------')
print ()

n_img=5
plt.figure(figsize=(n_img*2,2))
plt.gray()
for i in range(n_img):
    plt.subplot(1,n_img,i+1)
    plt.imshow(positive[i].reshape(28,28))
plt.show()

print ()
print (' --------- Negatives -----------')
print (' --------- Label: 5, Predicted: 5 -----------')
print ()

n_img=5
plt.figure(figsize=(n_img*2,2))
plt.gray()
for i in range(n_img):
    plt.subplot(1,n_img,i+1)
    plt.imshow(negative[i].reshape(28,28))
plt.show()

print ()
print (' --------- False Positives -----------')
print (' --------- Label: 8, Predicted: 5 -----------')
print ()

n_img=5
plt.figure(figsize=(n_img*2,2))
plt.gray()
for i in range(n_img):
    plt.subplot(1,n_img,i+1)
    plt.imshow(false_positive[i].reshape(28,28))
plt.show()

print ()
print (' --------- False Negatives -----------')
print (' --------- Label: 5, Predicted: 8 -----------')
print ()
```

```
n_img=5
plt.figure(figsize=(n_img*2,2))
plt.gray()
for i in range(n_img):
    plt.subplot(1,n_img,i+1)
    plt.imshow(false_negative[i].reshape(28,28))
plt.show()
```

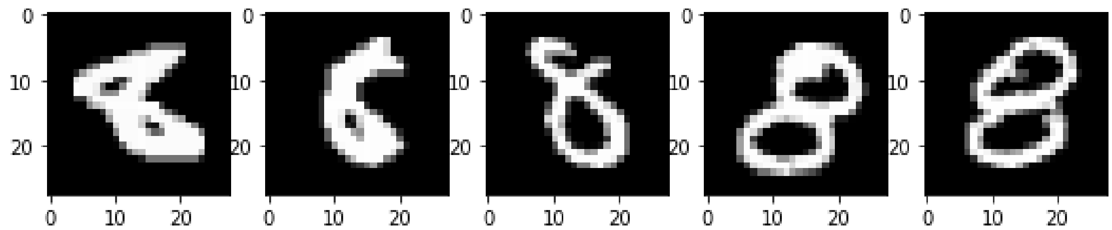
```
--------- Positives -----------
--------- Label: 8, Predicted: 8 -----------
```



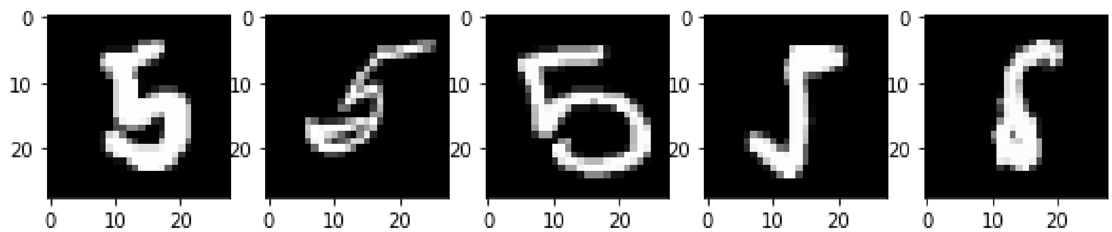
```
--------- Negatives -----------
--------- Label: 5, Predicted: 5 -----------
```



```
--------- False Positives -----------
--------- Label: 8, Predicted: 5 -----------
```



```
--------- False Negatives -----------
--------- Label: 5, Predicted: 8 -----------
```



[20]:
```
# Test set
print("Test set examples for true/false positive/negative")
Y_hat = predict(test_x, params)
Y_hat = Y_hat.reshape(-1)

positive = []
negative = []
false_positive = []
false_negative = []

idx = 0

for y_hat,y_true,x_i in zip(Y_hat, test_y, test_x.T):
  idx += 1
  if y_hat == y_true:
    if y_hat == 1:
      # print('positive')
      positive.append(x_i)
    else :
      # print('negative')
      negative.append(x_i)
  else :
```

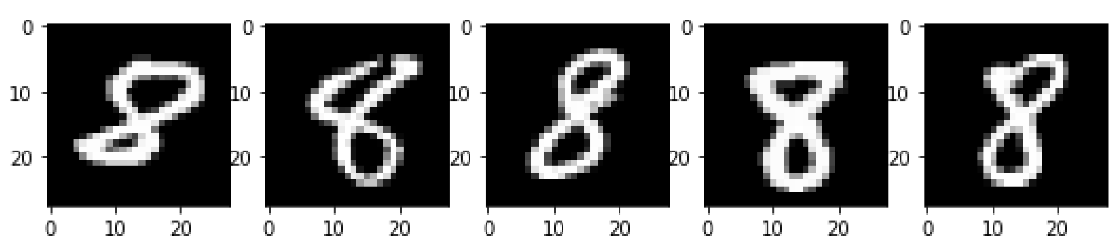
```
    if y_hat == 0:
      # print('false_positive')
      false_positive.append(x_i)
    else:
      # print('false_negative')
      false_negative.append(x_i)


positive = np.array(positive)
negative = np.array(negative)
false_positive = np.array(false_positive)
false_negative = np.array(false_negative)

print ('positive', positive.shape)
print ('negative', negative.shape)
print ('false_positive', false_positive.shape)
print ('false_negative', false_negative.shape)
```

```
Test set examples for true/false positive/negative
positive (927, 784)
negative (848, 784)
false_positive (47, 784)
false_negative (44, 784)
```

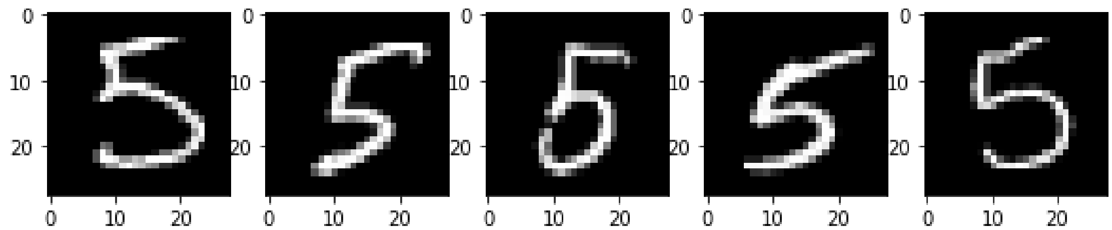
[21]:
```
print (' --------- Positives -----------')
print (' --------- Label: 8, Predicted: 8 -----------')
print ()

n_img=5
plt.figure(figsize=(n_img*2,2))
plt.gray()
for i in range(n_img):
    plt.subplot(1,n_img,i+1)
    plt.imshow(positive[i].reshape(28,28))
plt.show()

print ()
print (' --------- Negatives -----------')
print (' --------- Label: 5, Predicted: 5 -----------')
print ()

n_img=5
plt.figure(figsize=(n_img*2,2))
plt.gray()
for i in range(n_img):
    plt.subplot(1,n_img,i+1)
    plt.imshow(negative[i].reshape(28,28))
```

```
plt.show()

print ()
print (' --------- False Positives -----------')
print (' --------- Label: 8, Predicted: 5 -----------')
print ()

n_img=5
plt.figure(figsize=(n_img*2,2))
plt.gray()
for i in range(n_img):
    plt.subplot(1,n_img,i+1)
    plt.imshow(false_positive[i].reshape(28,28))
plt.show()

print ()
print (' --------- False Negatives -----------')
print (' --------- Label: 5, Predicted: 8 -----------')
print ()

n_img=5
plt.figure(figsize=(n_img*2,2))
plt.gray()
for i in range(n_img):
    plt.subplot(1,n_img,i+1)
    plt.imshow(false_negative[i].reshape(28,28))
plt.show()
```

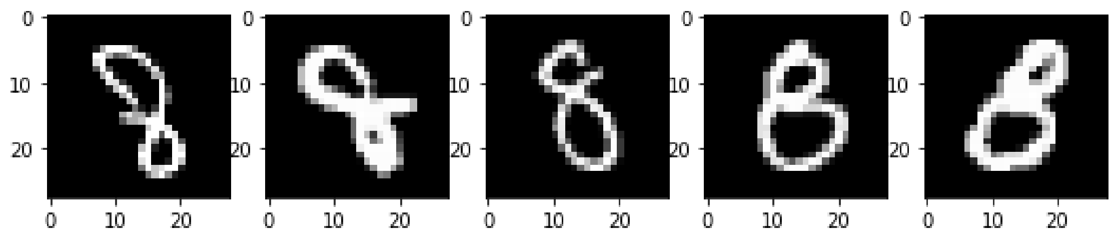
```
--------- Positives -----------
--------- Label: 8, Predicted: 8 -----------
```



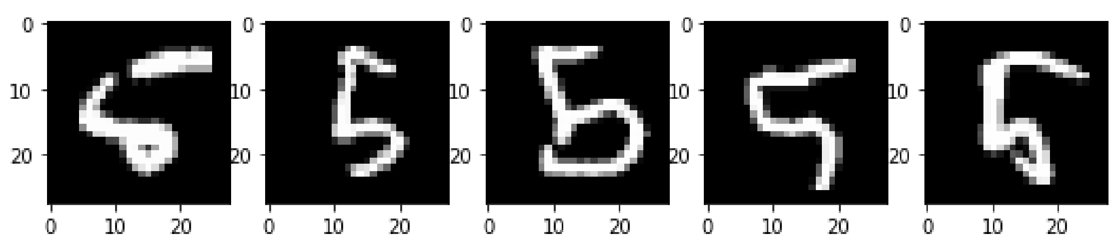

```
--------- Negatives -----------
--------- Label: 5, Predicted: 5 -----------
```



```
--------- False Positives -----------
--------- Label: 8, Predicted: 5 -----------
```



```
--------- False Negatives -----------
--------- Label: 5, Predicted: 8 -----------
```



3.3 Question 2. Multiclass classification [35 pts]

Now we will build a classifier to separate all the digits. For this purpose, we will only change the last layer and the loss.

Instead of using a single output, we will provide 10 outputs; and instead of using a binary cross entropy loss, we will use mutli-class cross entropy loss.

In multinomal logistic regression (aka softmax regression), we define the posterior probability of label $y \in \{0, \ldots, K-1\}$ as

$$p(y = c|\mathbf{x}) = \frac{\exp(\mathbf{w}_c^T \mathbf{x})}{\sum_{k=1}^{K} \exp(\mathbf{w}_k^T \mathbf{x})} = \mathbf{p}_c.$$

In other words, last layer of the network provides a probability vector $\mathbf{p} \in \mathbb{R}^K$, such that each $0 \leq \mathbf{p}_c \leq 1$ and $\sum_c \mathbf{p}_c = 1$.

3.3.1 Softmax function [5 pts]

Let us first define the softmax function, which is a multinomal extension of the sigmoid function that maps a vector of length K to a probability vector.

We can define `softmax` function on a vector $\mathbf{z} \in \mathbb{R}^K$ as $\mathbf{p} = \text{softmax}(\mathbf{z})$:

$$\mathbf{p}_c(\mathbf{z}) = \frac{\exp(\mathbf{z}_c)}{\sum_{k=1}^{K} \exp(\mathbf{z}_k)}$$

[22]:
```
def softmax_2(Z):
    # Z -- K x N numpy.ndarray, K is the number of classes, N is the number of␣
 ↪samples
    # TODO
    # your code goes here...
    exp = np.exp(Z)
    exp_sum = exp.sum(0, keepdims=True)
    probs = exp/exp_sum

    return probs
```

We have to note that the numerical range of floating point numbers in numpy is limited. For `float64` the upper bound is 10^{308}. For exponential, its not difficult to overshoot that limit, in which case python returns `nan`.

To make our softmax function numerically stable, we simply normalize the values in the vector, by multiplying the numerator and denominator with a constant `C` as

$$\begin{aligned}
\mathbf{p}_c &= \frac{\exp(\mathbf{z}_c)}{\sum_{k=1}^{K} \exp(\mathbf{z}_k)} \\
&= \frac{C\exp(\mathbf{z}_c)}{C\sum_{k=1}^{K} \exp(\mathbf{z}_k)} \\
&= \frac{\exp(\mathbf{z}_c + \log C)}{C\sum_{k=1}^{K} \exp(\mathbf{z}_k + \log C)}.
\end{aligned}$$

We can choose an arbitrary value for `log(C)` term, but generally `log(C) = -max(z)` is chosen

[23]:
```
def stable_softmax_2(Z):
    # Z -- K x N numpy.ndarray, K is the number of classes, N is the number of␣
 ↪samples
    # TODO (this is optional)
    # your code goes here
    emax = -np.amax(Z)
    regularizer = math.e**emax
    exp = np.exp(Z + np.log(regularizer))
    exp_sum = exp.sum(0, keepdims=True)
    probs = exp/exp_sum
    return probs
```

3.3.2 Derivative of the softmax function

We can show that the derivative of the **softmax** function with respect to any input can be written as

$$\frac{\partial \mathbf{p}_i}{\partial \mathbf{z}_j} = \begin{cases} \mathbf{p}_i(1-\mathbf{p}_j) & i = j \\ \mathbf{p}_i(-\mathbf{p}_j) & i \neq j. \end{cases}$$

More info here

3.3.3 Multiclass cross entropy loss function [5 pts]

We will minimize the cross entropy loss. You will use the true labels and predicted labels of a batch of N samples.

The multi-class cross entropy loss for i^{th} sample can be written as

$$Loss_i = -\sum_c \mathbf{1}(y_i = c) \log \mathbf{p}_c$$

where y_i is the true label and

$$\mathbf{1}(y_i = c) = \begin{cases} 1 & y_i = c \\ 0 & \text{otherwise} \end{cases}$$

is an indicator function.

We can find the average loss for a batch of N samples as $Loss = \frac{1}{N}\sum_{i=1}^{N} Loss_i$.

[24]:
```
def one_hot(X):
  # X -- N x 1 array

  X_int = train_y.astype(int)
  X_one_hot = np.eye(np.max(X_int)+1)[X_int]
  return X_one_hot
```

[25]:
```
def MultiClassCrossEntropyLoss_2(Y_true, probs):

    # TODO
    # Write your code here

    # probs -- K x N array
    # Y_true -- 1 x N array
    # loss --  sum Loss_i over N samples
    Y_true = Y_true.astype(int)
    logprobs = -np.log(probs.T[range(Y_true.shape[0]),Y_true])
    l = np.sum(logprobs)
    loss = (1.0/Y_true.shape[0]) * l

    return loss
```

3.3.4 Derivative of the cross entropy loss

Let us assume that $\mathbf{p} = \text{softmax}(\mathbf{z})$.

Note that the derivative of the loss w.r.t. $\mathbf{p}_j$ can be written as

$$\frac{\partial Loss_i}{\partial \mathbf{p}_j} = \begin{cases} -1/\mathbf{p}_j & j = y_i \\ 0 & j \neq y_i \end{cases}.$$

Note that we can use *total derivative* to compute the derivative of the loss for ith sample w.r.t. jth entry in $\mathbf{z}$ as

$$\frac{\partial Loss_i}{\partial \mathbf{z}_j} = \sum_c \frac{\partial Loss_i}{\partial \mathbf{p}_c} \frac{\partial \mathbf{p}_c}{\partial \mathbf{z}_j}.$$

From our discussion above, we know that the $\frac{\partial Loss_i}{\partial \mathbf{p}_c} = 0$ if $c \neq y_i$.

$$\begin{aligned} \frac{\partial Loss_i}{\partial \mathbf{z}_j} &= -\frac{1}{\mathbf{p}_c} \frac{\partial \mathbf{p}_c}{\partial \mathbf{z}_j} \\ &= \begin{cases} \mathbf{p}_j - 1 & j = y_i \\ \mathbf{p}_j & j \neq y_i. \end{cases} \end{aligned}$$

Therefore,

$$\delta^{(2)} = \nabla_{\mathbf{z}^{(2)}} Loss_i = \mathbf{p} - \mathbf{1}_{y_i}.$$

where $\mathbf{1}_{y_i}$ is a **one-hot vector** that has length K and is zero everywhere except 1 at index same as y_i.

3.3.5 Training data

Let us pick training data for multi-class classification.

Pick same number of images from each class for training and create arrays for input and output.

```
# train_x -- N x 784 array of training input
# train_y -- N x 1 array of labels
```

If you use 1000 images from each class N = 10000. You can increase the number of training samples if you like. You may also use unequal number of images in each class.

We also need to transpose the dimension of the data so that their size becomes $784 \times N$. It will be helpful to feed it to our model based on our notations.

[25]:

```

```

[26]:

```python
# Pick training samples
num_samples = 1000

# Training data
x = np.zeros((0,784))
y = np.zeros((0))
for label in range(10):
  x1 = x_train[y_train == label]
  x1 = x1[:num_samples]
  y1 = y_train[y_train == label]
  y1 = y1[:num_samples]

  x = np.concatenate((x,x1),axis=0)
  y = np.concatenate((y,y1),axis=0)

train_x = x
train_y = y
print("Training data shape:", train_x.shape)


# Test data
test_x = x_test
test_y = y_test
print("Test data shape:", test_x.shape)

# reshape data
train_x = train_x.T
test_x = test_x.T
print("Training data shape:", train_x.shape)
print("Training label shape:", train_y.shape)
print ()
print("Test data shape:", test_x.shape)
```

```
print("Test label shape:", test_y.shape)
```

```
Training data shape: (10000, 784)
Test data shape: (10000, 784)
Training data shape: (784, 10000)
Training label shape: (10000,)

Test data shape: (784, 10000)
Test label shape: (10000,)
```

3.3.6 Network Architecture

We will be using a two layer neural network in our experiment. The input layer has 784 nodes, the hidden layer will have 256 nodes and the output layer will have 10 nodes. First layer will have **sigmoid** activation and second layer will have **softmax** activation.

The equations for feedforward operation will be as follows.

$$\mathbf{z}^{(1)} = W^{(1)}\mathbf{x} + \mathbf{b}^{(1)}\mathbf{y}^{(1)} = \text{sigmoid}(\mathbf{z}^{(1)})\mathbf{z}^{(2)} = W^{(2)}\mathbf{y}^{(1)} + \mathbf{b}^{(2)}\mathbf{p} = \mathbf{y}^{(2)} = \text{softmax}(\mathbf{z}^{(2)})$$

where $\mathbf{x} \in \mathbb{R}^{784}$ is the input layer, $\mathbf{y}^{(1)} \in \mathbb{R}^{256}$ is the hidden layer, $\mathbf{y}^{(2)} \in \mathbb{R}$ is the output layer, $W^{(1)} \in \mathbb{R}^{256\times 784}$ is the first layer weights, $W^{(2)} \in \mathbb{R}^{10\times 256}$ is the second layer weights, $\mathbf{b}^{(1)} \in \mathbb{R}^{256}$ is the first layer bias, $\mathbf{b}^{(2)} \in \mathbb{R}^{10}$ is the second layer bias vector.

3.3.7 Network initialization [5 pts]

We initialize the weights for $W^{(1)}$ and $W^{(2)}$ with random values drawn from normal distribution with zero mean and 0.01 standard deviation. We will initialize bias vectors $\mathbf{b}^{(1)}$ and $\mathbf{b}^{(2)}$ with zero values.

We can fix the seed for random initialization for reproducibility.

[27]:
```
def sigmoid_2(Z):
    # Input: Z -- numpy.ndarray
    # TODO
    Y = (1 / (1 + np.exp(-Z)))
    return Y
```

[28]:
```
def TwoLayerNetwork_2(layer_dims=[784,256,10]):
    # TODO
    # Your code goes here

    # Fix the seed
    np.random.seed(3)

    #Initialize the weights
```

```
    mean = 0
    std = 0.01

    params = {}
    params['w1'] = np.random.normal(mean, std,␣
 ↪size=(layer_dims[1],layer_dims[0]))
    params['b1'] = np.zeros((layer_dims[1],1))
    params['w2'] = np.random.normal(mean, std,␣
 ↪size=(layer_dims[2],layer_dims[1]))
    params['b2'] = np.zeros((layer_dims[2],1))

    return params
```

3.3.8 Forward propagation

Next, we will write the code for the forward pass for two layer network. Each layer consists of an affine function (fully-connected layer) followed by an activation function. You wil also return the intermediate results $(\mathbf{x}, \mathbf{z}^{(1)}, \mathbf{y}^{(1)}, \mathbf{z}^{(2)})$ in addition to final output $(\mathbf{y}^{(2)})$. You will need the intermediate outputs for the backpropagation step.

[29]:
```
def forward_2(X, params):

    # TODO
    # Write your codes here

    # X -- 784 x N array
    # params --
      # W1 -- 256 x 784 matrix
      # b1 -- 256 x 1 vector
      # W2 -- 10 x 256 matrix
      # b2 -- 10 x 1 scalar
    # probs -- 10 x N output

    intermediate = {}
    intermediate['X'] = X
    intermediate['z1'] = params['w1'].dot(intermediate['X'])+params['b1']
    intermediate['Y1'] = sigmoid_2(intermediate['z1'])
    intermediate['z2'] = params['w2'].dot(intermediate['Y1'])+params['b2']
    Y2 = stable_softmax_2(intermediate['z2'])

    return Y2, intermediate
```

3.3.9 Backpropagation step [10 pts]

Now we will implement the backpropagation step for the two layer neural network using softmax layer and loss function.

You will need the gradient of the Loss w.r.t. $W^{(l)}, \mathbf{b}^{(l)}$ for $l = 1, 2$ for all the training samples.

We saw that we can write the gradient of Loss with respect to $W^{(l)}, \mathbf{b}^{(l)}$ for a single sample as

$$\nabla_{W^{(l)}} Loss_i = \delta^{(l)} \mathbf{y}^{(l-1)T},$$

$$\nabla_{\mathbf{b}^{(l)}} Loss_i = \delta^{(l)},$$

where

$$\delta^{(l)} = \nabla_{\mathbf{z}^{(l)}} Loss = \nabla_{\mathbf{y}^{(l)}} Loss \odot \varphi'(\mathbf{z}^{(l)}).$$

We saw above that for an ith sample, $\delta^{(2)} = \nabla_{\mathbf{z}^{(2)}} Loss_i = \mathbf{p} - \mathbf{1}_{y_i}$, where $\mathbf{1}_{y_i}$ is a **one-hot vector** that has length K and is zero everywhere except 1 at index same as y_i and $\mathbf{p}$ is the outpu probability vector for the ith sample.

Once we have the gradients $\nabla_{W^{(l)}} Loss_i, \nabla_{\mathbf{b}^{(l)}} Loss_i$ for all i. We can compute their average to compute the gradient of the total loss function as

$$\nabla_{W^{(l)}} Loss = \frac{1}{N} \sum_i \nabla_{W^{(l)}} Loss_i,$$
$$\nabla_{\mathbf{b}^{(l)}} Loss = \frac{1}{N} \sum_i \nabla_{\mathbf{b}^{(l)}} Loss_i.$$

Please refer to the slides and lectures for more details.

[30]:
```python
def backward_2(Y_true, probs, intermediate, params):

    # Inputs:
      # Y_true -- true labels
      # probs -- 10 x N output of the last layer
      # intermediate -- X, Z1, Y1, Z2
      # params -- W1, b1, W2, b2

    # Outputs:
      # grads -- [grad_W1, grad_b1, grad_W2, grad_b2]

    # TODO
    # Write your codes here

    grads = {}

    train_y_one_hot = one_hot(Y_true)
```

```
    train_y_one_hot_transposed = train_y_one_hot.T

    delta_2 = probs - train_y_one_hot_transposed
    grads['w2'] = delta_2.dot(intermediate['Y1'].T)
    grads['b2'] = np.sum(delta_2,axis=1,keepdims=True)

    delta_1 = params['w2'].T.dot(delta_2) * (sigmoid_2(intermediate['z1'])*(1 -␣
 ↪sigmoid_2(intermediate['z1'])))
    grads['w1'] = delta_1.dot(intermediate['X'].T)
    grads['b1'] = np.sum(delta_1,axis=1,keepdims=True)

    return grads
```

[31]:

```
def GD_2(params, grads, learning_rate):

    # updated params = old params - learning rate * gradient of Loss computed␣
 ↪at old params
    # TODO
    # Write your codes here

    params['w2'] = params['w2'] - learning_rate*grads['w2']
    params['b2'] = params['b2'] - learning_rate*grads['b2']
    params['w1'] = params['w1'] - learning_rate*grads['w1']
    params['b1'] = params['b1'] - learning_rate*grads['b1']


    return params
```

[32]:

```
def predict_2(train_x,params):
  y_pred,_ = forward_2(train_x,params)
  return np.argmax(y_pred,axis=0)

def accuracy_2(Y_true, y_pred):
  aa = np.sum(y_pred == Y_true)
  return aa / len(Y_true)
```

3.3.10 Train the model [5 pts]

We will use the forward and backward functions defined above with the same optimizer defined in the previous question to train our multi-class classificaiton model.

We will specify the number of nodes in the layers, number of epochs and learning rate and initialize the network

[33]:

```
layer_dims = [train_x.shape[0],256,10]
epochs = 100
lr = 0.00001
```

```
params = TwoLayerNetwork_2(layer_dims)
```

Then we train the network for the number of epochs specified above. In every epoch, we will do the following: 1. Calculate the forward pass to get estimated labels. 2. Use the estimated labels calculate loss. We will be recording loss for every epoch. 3. Use backpropagation to calculate gradients. 4. Use gradient descent to update the weights and biases.

You should store the loss value after every epoch in an array `loss_history` and print the loss value after every few epochs (say 20).

[34]:
```
# TODO
# Write your codes here

loss_history = []
train_acc = []
test_acc = []
for i in range(epochs):

  print ('epoch ==> ', i)

  probs, intermediate = forward_2(train_x, params)
  loss = MultiClassCrossEntropyLoss_2(train_y, probs)
  loss_history.append(loss)

  train_pred = predict_2(train_x, params)
  test_pred = predict_2(test_x, params)

  train_accuracy = accuracy_2(train_y, train_pred)
  test_accuracy = accuracy_2(test_y,test_pred)

  print (' loss ', loss)
  print (' train_accuracy ', train_accuracy)
  print (' test_accuracy ', test_accuracy)

  test_acc.append(test_accuracy)
  train_acc.append(train_accuracy)

  grads = backward_2(train_y, probs, intermediate, params)
  params = GD_2(params, grads, lr)
```

```
epoch ==>  0
 loss  2.3156506076429806
 train_accuracy  0.0954
 test_accuracy  0.0957
epoch ==>  1
 loss  2.173161610307765
 train_accuracy  0.5641
```

```
 test_accuracy  0.5646
epoch ==>  2
 loss  2.0563830122829168
 train_accuracy  0.697
 test_accuracy  0.6924
epoch ==>  3
 loss  1.9373658497805297
 train_accuracy  0.7371
 test_accuracy  0.7303
epoch ==>  4
 loss  1.8171994054672316
 train_accuracy  0.7483
 test_accuracy  0.7467
epoch ==>  5
 loss  1.7021259261242867
 train_accuracy  0.7569
 test_accuracy  0.7545
epoch ==>  6
 loss  1.5964476895658268
 train_accuracy  0.7631
 test_accuracy  0.7619
epoch ==>  7
 loss  1.5095160233574614
 train_accuracy  0.7637
 test_accuracy  0.7633
epoch ==>  8
 loss  1.432854846455317
 train_accuracy  0.7435
 test_accuracy  0.743
epoch ==>  9
 loss  1.3969053218555136
 train_accuracy  0.7393
 test_accuracy  0.7381
epoch ==>  10
 loss  1.3865557757074205
 train_accuracy  0.6427
 test_accuracy  0.6435
epoch ==>  11
 loss  1.413788637602713
 train_accuracy  0.6706
 test_accuracy  0.6657
epoch ==>  12
 loss  1.2600190798995918
 train_accuracy  0.7212
 test_accuracy  0.7176
epoch ==>  13
 loss  1.1391045674392837
 train_accuracy  0.7708
```

```
 test_accuracy  0.7754
epoch ==>  14
 loss  1.022420874739453
 train_accuracy  0.8328
 test_accuracy  0.8305
epoch ==>  15
 loss  0.9649164117193981
 train_accuracy  0.8189
 test_accuracy  0.8229
epoch ==>  16
 loss  0.912099084149136
 train_accuracy  0.8523
 test_accuracy  0.8442
epoch ==>  17
 loss  0.8849288899167499
 train_accuracy  0.8285
 test_accuracy  0.8326
epoch ==>  18
 loss  0.862095212428969
 train_accuracy  0.8491
 test_accuracy  0.8355
epoch ==>  19
 loss  0.8700436674181552
 train_accuracy  0.809
 test_accuracy  0.8116
epoch ==>  20
 loss  0.8677989858385019
 train_accuracy  0.8222
 test_accuracy  0.8065
epoch ==>  21
 loss  0.8458458311711123
 train_accuracy  0.801
 test_accuracy  0.8036
epoch ==>  22
 loss  0.786888397071406
 train_accuracy  0.8434
 test_accuracy  0.8259
epoch ==>  23
 loss  0.7124839267602593
 train_accuracy  0.8451
 test_accuracy  0.8502
epoch ==>  24
 loss  0.665525613032068
 train_accuracy  0.8773
 test_accuracy  0.864
epoch ==>  25
 loss  0.637398822653663
 train_accuracy  0.8694
```

```
 test_accuracy  0.8641
epoch ==>  26
 loss  0.621795156027863
 train_accuracy  0.8856
 test_accuracy  0.8697
epoch ==>  27
 loss  0.617331999016253
 train_accuracy  0.8627
 test_accuracy  0.8564
epoch ==>  28
 loss  0.6280031461909735
 train_accuracy  0.8666
 test_accuracy  0.8504
epoch ==>  29
 loss  0.6578550041405236
 train_accuracy  0.8253
 test_accuracy  0.8194
epoch ==>  30
 loss  0.6906992991506035
 train_accuracy  0.8159
 test_accuracy  0.805
epoch ==>  31
 loss  0.7029244601459791
 train_accuracy  0.7863
 test_accuracy  0.7796
epoch ==>  32
 loss  0.6691350753278987
 train_accuracy  0.8204
 test_accuracy  0.8105
epoch ==>  33
 loss  0.6046847666824171
 train_accuracy  0.8351
 test_accuracy  0.8234
epoch ==>  34
 loss  0.5544965837852426
 train_accuracy  0.8691
 test_accuracy  0.8607
epoch ==>  35
 loss  0.5173935070874454
 train_accuracy  0.8809
 test_accuracy  0.8705
epoch ==>  36
 loss  0.4966461766126244
 train_accuracy  0.8899
 test_accuracy  0.8752
epoch ==>  37
 loss  0.4874442605160869
 train_accuracy  0.8931
```

```
 test_accuracy  0.8833
epoch ==>  38
 loss  0.47291222612839506
 train_accuracy  0.8943
 test_accuracy  0.8798
epoch ==>  39
 loss  0.46428951266012414
 train_accuracy  0.9019
 test_accuracy  0.8851
epoch ==>  40
 loss  0.4495991850660177
 train_accuracy  0.9005
 test_accuracy  0.8861
epoch ==>  41
 loss  0.4405671405173176
 train_accuracy  0.9107
 test_accuracy  0.889
epoch ==>  42
 loss  0.4295938773175547
 train_accuracy  0.9052
 test_accuracy  0.891
epoch ==>  43
 loss  0.41799888137287966
 train_accuracy  0.9163
 test_accuracy  0.8927
epoch ==>  44
 loss  0.40707128343854365
 train_accuracy  0.9109
 test_accuracy  0.8981
epoch ==>  45
 loss  0.39850548358664184
 train_accuracy  0.9184
 test_accuracy  0.8944
epoch ==>  46
 loss  0.3939196173154426
 train_accuracy  0.9115
 test_accuracy  0.8969
epoch ==>  47
 loss  0.3919043214990302
 train_accuracy  0.9147
 test_accuracy  0.8919
epoch ==>  48
 loss  0.39523370595552293
 train_accuracy  0.9081
 test_accuracy  0.8948
epoch ==>  49
 loss  0.39505623719488436
 train_accuracy  0.9082
```

```
 test_accuracy  0.8857
epoch ==>  50
 loss  0.412016812486422
 train_accuracy  0.8985
 test_accuracy  0.8838
epoch ==>  51
 loss  0.4113422970603993
 train_accuracy  0.8972
 test_accuracy  0.8742
epoch ==>  52
 loss  0.4250038876130226
 train_accuracy  0.8877
 test_accuracy  0.8746
epoch ==>  53
 loss  0.41068623896375667
 train_accuracy  0.8926
 test_accuracy  0.8714
epoch ==>  54
 loss  0.41232606510124753
 train_accuracy  0.8919
 test_accuracy  0.8782
epoch ==>  55
 loss  0.3862734436034813
 train_accuracy  0.9025
 test_accuracy  0.8831
epoch ==>  56
 loss  0.3701302245521263
 train_accuracy  0.9098
 test_accuracy  0.8941
epoch ==>  57
 loss  0.3580933879023865
 train_accuracy  0.9113
 test_accuracy  0.8954
epoch ==>  58
 loss  0.34874978727572215
 train_accuracy  0.9192
 test_accuracy  0.8996
epoch ==>  59
 loss  0.3448099245583374
 train_accuracy  0.9147
 test_accuracy  0.8983
epoch ==>  60
 loss  0.3357251756111439
 train_accuracy  0.9248
 test_accuracy  0.9016
epoch ==>  61
 loss  0.3320036446407125
 train_accuracy  0.9218
```

```
 test_accuracy  0.9021
epoch ==>  62
 loss  0.32584183462007543
 train_accuracy  0.9276
 test_accuracy  0.903
epoch ==>  63
 loss  0.3216413668764804
 train_accuracy  0.9245
 test_accuracy  0.9028
epoch ==>  64
 loss  0.31972193375798663
 train_accuracy  0.9298
 test_accuracy  0.9018
epoch ==>  65
 loss  0.32390571254272876
 train_accuracy  0.9217
 test_accuracy  0.9015
epoch ==>  66
 loss  0.3336694970588721
 train_accuracy  0.92
 test_accuracy  0.8941
epoch ==>  67
 loss  0.3490731044082739
 train_accuracy  0.9091
 test_accuracy  0.8909
epoch ==>  68
 loss  0.3702191701189866
 train_accuracy  0.9028
 test_accuracy  0.8803
epoch ==>  69
 loss  0.3717300543355166
 train_accuracy  0.8983
 test_accuracy  0.8818
epoch ==>  70
 loss  0.37543297465606745
 train_accuracy  0.9014
 test_accuracy  0.8781
epoch ==>  71
 loss  0.3560217449468707
 train_accuracy  0.9052
 test_accuracy  0.8866
epoch ==>  72
 loss  0.33558247223273574
 train_accuracy  0.917
 test_accuracy  0.8912
epoch ==>  73
 loss  0.31218416877665367
 train_accuracy  0.9206
```

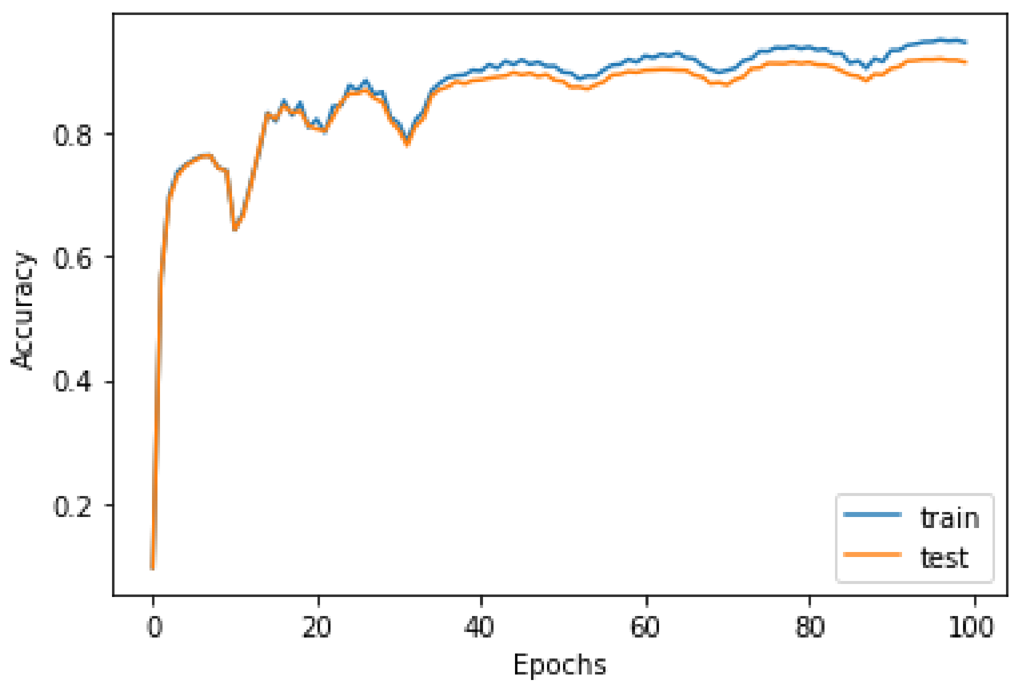
```
 test_accuracy  0.9045
epoch ==>  74
 loss  0.2914009352166016
 train_accuracy  0.9325
 test_accuracy  0.9056
epoch ==>  75
 loss  0.2824673631885681
 train_accuracy  0.932
 test_accuracy  0.9129
epoch ==>  76
 loss  0.2726404963427077
 train_accuracy  0.9383
 test_accuracy  0.9127
epoch ==>  77
 loss  0.2692442464975002
 train_accuracy  0.9374
 test_accuracy  0.9124
epoch ==>  78
 loss  0.2650540354567803
 train_accuracy  0.9404
 test_accuracy  0.9147
epoch ==>  79
 loss  0.2651195834010936
 train_accuracy  0.9366
 test_accuracy  0.9126
epoch ==>  80
 loss  0.2638569401021088
 train_accuracy  0.9393
 test_accuracy  0.9143
epoch ==>  81
 loss  0.26826886099944564
 train_accuracy  0.9344
 test_accuracy  0.9105
epoch ==>  82
 loss  0.27215346296203236
 train_accuracy  0.9359
 test_accuracy  0.9101
epoch ==>  83
 loss  0.28436335201425067
 train_accuracy  0.928
 test_accuracy  0.9067
epoch ==>  84
 loss  0.2973733110277197
 train_accuracy  0.9287
 test_accuracy  0.9004
epoch ==>  85
 loss  0.323059467765083
 train_accuracy  0.913
```

```
 test_accuracy  0.8943
epoch ==>  86
 loss  0.32309000947040517
 train_accuracy  0.9169
 test_accuracy  0.8919
epoch ==>  87
 loss  0.34854848909745834
 train_accuracy  0.905
 test_accuracy  0.8846
epoch ==>  88
 loss  0.3149280806460704
 train_accuracy  0.9207
 test_accuracy  0.8955
epoch ==>  89
 loss  0.3139014254842209
 train_accuracy  0.9154
 test_accuracy  0.8947
epoch ==>  90
 loss  0.27934786094806946
 train_accuracy  0.9338
 test_accuracy  0.9046
epoch ==>  91
 loss  0.26416564342249227
 train_accuracy  0.9341
 test_accuracy  0.9075
epoch ==>  92
 loss  0.24690310049430575
 train_accuracy  0.9421
 test_accuracy  0.9165
epoch ==>  93
 loss  0.23739219403010986
 train_accuracy  0.9442
 test_accuracy  0.9173
epoch ==>  94
 loss  0.22972178059180398
 train_accuracy  0.9474
 test_accuracy  0.9189
epoch ==>  95
 loss  0.22615980298870939
 train_accuracy  0.9478
 test_accuracy  0.9188
epoch ==>  96
 loss  0.22395702971021506
 train_accuracy  0.9509
 test_accuracy  0.92
epoch ==>  97
 loss  0.2242875781099915
 train_accuracy  0.9488
```

```
 test_accuracy  0.9179
epoch ==>  98
 loss  0.22357547759081786
 train_accuracy  0.9501
 test_accuracy  0.9178
epoch ==>  99
 loss  0.22643837929628483
 train_accuracy  0.9469
 test_accuracy  0.9148
```

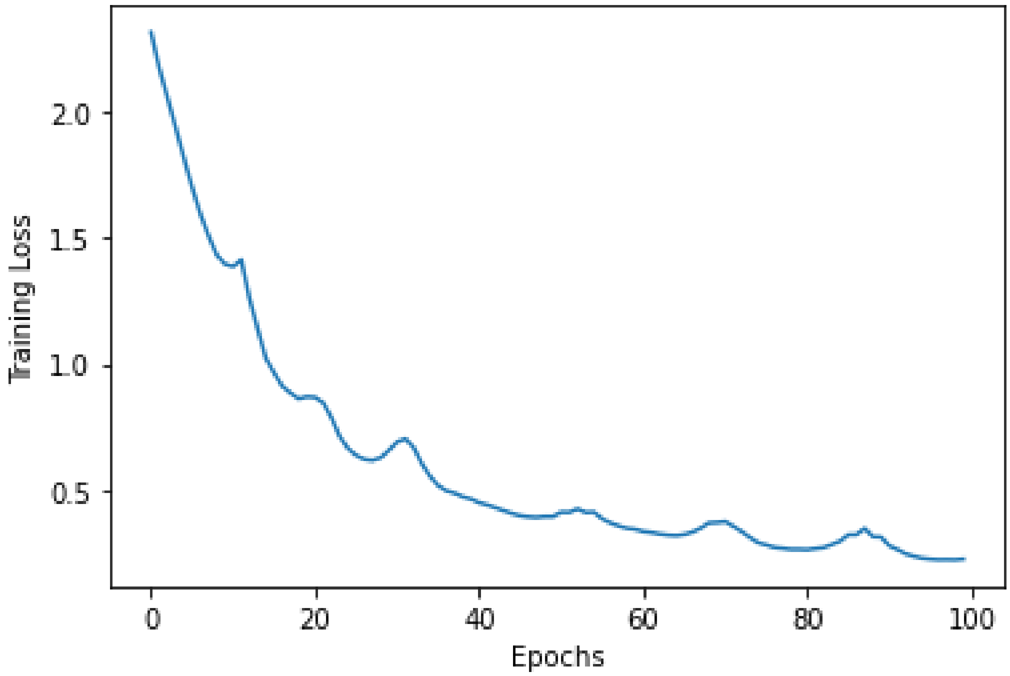
Now we will plot the recorded loss values vs epochs. We will observe the training loss decreasing with the epochs.

[35]:
```
plt.figure()
plt.plot(train_acc,label='train')
plt.plot(test_acc,label='test')
plt.xlabel("Epochs")
plt.ylabel("Accuracy")
plt.legend()
plt.show()
```



[36]:
```
plt.figure()
plt.plot(loss_history)
plt.xlabel("Epochs")
plt.ylabel("Training Loss")
```

```
plt.show()
```



3.3.11 Evaluation on test data [5 pts]

Now we will be evaluating the accuracy we get from the trained model. We feed training data and test data to the forward model along with the trained parameters.

Note that, we need to convert the (probability) output of the forward pass into labels before evaluating accuracy. We can assign label based on the maximum probability.

We assign estimated labels

$$\hat{y}_i = \arg\max_c \mathbf{p}_c$$

for every probility vector.

[37]:
```
y_train_pred = predict_2(train_x, params)
train_accuracy = accuracy_2(train_y, y_train_pred)


print("Training accuracy:",train_accuracy)

y_test_pred = predict_2(test_x, params)
test_accuracy = accuracy_2(test_y, y_test_pred)

print("Test accuracy:",test_accuracy)
```

```
Training accuracy: 0.9483
Test accuracy: 0.9166
```

3.3.12 Visualize some of the correct/miscalassified images [optional]

Now we will look at some images from training and test sets that were misclassified.

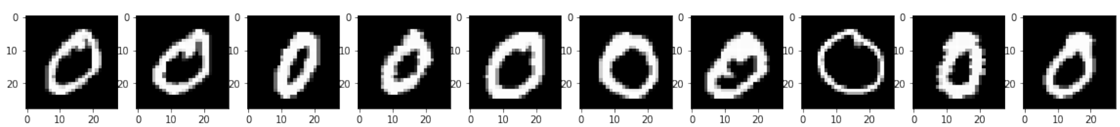
Training set. Pick example from each class that are correcly and incorreclty classified. True/False Positive/Negatives

Test set. Pick examples from each class that are correcly and incorreclty classified. True/False Positive/Negatives

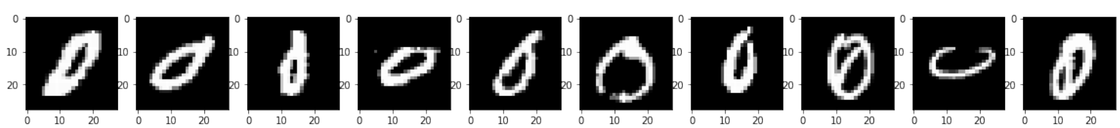
[38]:
```python
# TODO
# Training set
print("Training set examples for true/false positive/negative")
Y_hat = predict_2(train_x, params)
Y_hat = Y_hat.reshape(-1)

positive = []
negative = []
false_positive = []
false_negative = []

idx = 0

for y_hat,y_true,x_i in zip(Y_hat, train_y, train_x.T):
  idx += 1
  if y_hat == y_true:
      positive.append(x_i)
  else :
    negative.append(x_i)


positive = np.array(positive)
negative = np.array(negative)

print ('positive', positive.shape)
print ('negative', negative.shape)


print (' --------- Positives -----------')
print ()

n_img=10
plt.figure(figsize=(n_img*2,2))
plt.gray()
for i in range(n_img):
```

```
    plt.subplot(1,n_img,i+1)
    plt.imshow(positive[i].reshape(28,28))
plt.show()

print ()
print (' --------- Negatives -----------')
print ()


plt.figure(figsize=(n_img*2,2))
plt.gray()
for i in range(n_img):
    plt.subplot(1,n_img,i+1)
    plt.imshow(negative[i].reshape(28,28))
plt.show()
```

```
Training set examples for true/false positive/negative
positive (9483, 784)
negative (517, 784)
 --------- Positives -----------
```



```
 --------- Negatives -----------
```

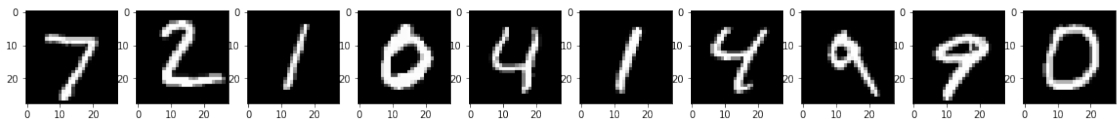


[39]:

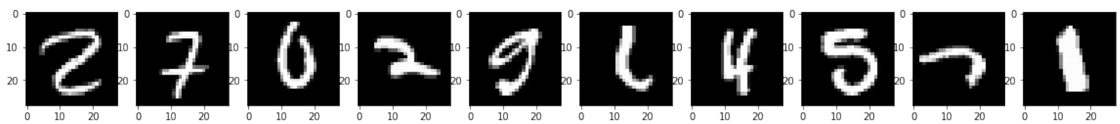
```
# TODO
# Training set
print("Training set examples for true/false positive/negative")
Y_hat = predict_2(test_x, params)
Y_hat = Y_hat.reshape(-1)

positive = []
negative = []
```

```
false_positive = []
false_negative = []

idx = 0

for y_hat,y_true,x_i in zip(Y_hat, test_y, test_x.T):
  idx += 1
  if y_hat == y_true:
      positive.append(x_i)
  else :
    negative.append(x_i)


positive = np.array(positive)
negative = np.array(negative)

print ('positive', positive.shape)
print ('negative', negative.shape)


print (' --------- Positives -----------')
print ()

n_img=10
plt.figure(figsize=(n_img*2,2))
plt.gray()
for i in range(n_img):
    plt.subplot(1,n_img,i+1)
    plt.imshow(positive[i].reshape(28,28))
plt.show()

print ()
print (' --------- Negatives -----------')
print ()


plt.figure(figsize=(n_img*2,2))
plt.gray()
for i in range(n_img):
    plt.subplot(1,n_img,i+1)
    plt.imshow(negative[i+300].reshape(28,28))
plt.show()
```

```
Training set examples for true/false positive/negative
positive (9166, 784)
negative (834, 784)
 --------- Positives -----------
```



```
--------- Negatives -----------
```



3.3.13 Note about implementation

This is a note on two problems I have seen in the past and how they can be easily fixed.

1. Summation along different axes ?
2. Summation of gradients over samples ?

1. Summation to create probability vectors in the Softmax function

Suppose X is a d x N array, in our case, it is 784 x 10000.

```
Z2 = W2 Y1 + b2  will be 10 x 10000 array
```

`softmax(Z2)` will be a `10 x 10000` array in which we want to apply a softmax function on every column of `Z2` by first computing exponential and then normalizing the column to sum to 1, which is needed for it to be a probability vector.

We can do that as

```
probs = np.exp(Z2)

# now you want to sum up each column and divide the column by the sum so that each column is a

probs /= np.sum(probs,axis=0,keepdims=True) # this makes sum of each column to 1
```

The **WRONG** thing to do is

```
probs /= np.sum(probs)
# This is WRONG. np.sum() computes sum of the entire array.
```

2. Computing gradient for the entire loss function

(this involves summation of N rank-one matrices in our notation.)

Suppose you have computed delta1, delta2 properly

Let's assume you computed

```
# delta2 is a 10 x 10000 array
# Y1 is a 256 x 10000 array
# N is 10000
# grad_W2 should be a 10 x 256 array
```

We can expand the formula for the gradient of the overall loss.

$$\nabla_{W^{(2)}} Loss = \frac{1}{N} \sum_i \nabla_{W^{(2)}} Loss_i,$$

where

$$\nabla_{W^{(2)}} Loss_i = \delta^{(2)} y^{(1)T}$$

is the gradient of the loss for ith training sample, where $\delta^{(2)}$ is a column of length 10 and $y^{(1)T}$ is a row of length 256, corresponding to ith training sample. Matrix product of column and row gives a a rank-1 matrix of size 10 x 256.

To compute the gradient of loss over all the training samples, we need to average the rank-1 matrices for all N training samples.

We can write the code for that as

```
# Sum gradient of loss for each sample
for i in range(N):
    grad_W2 += (1/N)*delta2[:,i,None].dot(Y1[:,i,None].T)

# OR we can compute grad_W2 without for loop as
grad_W2 = 1/N*np.dot(delta2,Y1.T)
```

To see why this is true, you can convince yourself that matrix product of an `M x N` matrix with an `N x K` matrix can be written as a summation of N `M x K` rank-one matricess.

Suppose

$$A = [\mathbf{a}_1 \ \cdots \ \mathbf{a}_N] \text{ and } B \ = \begin{bmatrix} \mathbf{b}_1^T \\ \vdots \\ \mathbf{b}_N^T \end{bmatrix},$$

where $\mathbf{a}_i, \mathbf{b}_i$ are columns of length M, K, respectively.

We can write AB as

$$AB = \sum_{i=1}^{N} \mathbf{a}_i \mathbf{b}_i^T.$$

3.4 Submission instructions

1. Download this Colab to ipynb, and convert it to PDF. Follow similar steps as here but convert to PDF.

- Download your .ipynb file. You can do it using only Google Colab. `File -> Download -> Download .ipynb`
- Reupload it so Colab can see it. Click on the `Files` icon on the far left to expand the side bar. You can directly drag the downloaded .ipynb file to the area. Or click `Upload to session storage` icon and then select & upload your .ipynb file.
- Conversion using %%shell. `!sudo apt-get update !sudo apt-get install texlive-xetex texlive-fonts-recommended texlive-generic-recommended !jupyter nbconvert --log-level CRITICAL --to pdf name_of_hw.ipynb`
- Your PDF file is ready. Click 3 dots and `Download`.

2. Upload the PDF to Gradescope, select the correct pdf pages for each question. **Important!**

3. Upload the ipynb file to Gradescope

Notice: In case of errors in conversion, please check your LaTeX and debug. In Markdown, when you write in LaTeX math mode, do not leave any leading and trailing whitespaces inside the dollar signs ($). For example, write `(dollarSign)\mathbf(dollarSign)(dollarSign)` instead of `(dollarSign)(space)\mathbf{w}(dollarSign)`. Otherwise, nbconvert will throw an error and the generated pdf will be incomplete. This is a bug of nbconvert.

[40]:
```
!sudo apt-get update
!sudo apt-get install texlive-xetex texlive-fonts-recommended␣
↪texlive-generic-recommended
```

```
Hit:1 http://archive.ubuntu.com/ubuntu bionic InRelease
Get:2 http://archive.ubuntu.com/ubuntu bionic-updates InRelease [88.7 kB]
Get:3 http://security.ubuntu.com/ubuntu bionic-security InRelease [88.7 kB]
Get:4 https://cloud.r-project.org/bin/linux/ubuntu bionic-cran40/ InRelease
[3,626 B]
Get:5 http://archive.ubuntu.com/ubuntu bionic-backports InRelease [83.3 kB]
Get:6 http://ppa.launchpad.net/c2d4u.team/c2d4u4.0+/ubuntu bionic InRelease
[15.9 kB]
Ign:7 https://developer.download.nvidia.com/compute/machine-
learning/repos/ubuntu1804/x86_64  InRelease
Hit:8 https://developer.download.nvidia.com/compute/cuda/repos/ubuntu1804/x86_64
InRelease
Hit:9 https://developer.download.nvidia.com/compute/machine-
learning/repos/ubuntu1804/x86_64  Release
Get:10 https://cloud.r-project.org/bin/linux/ubuntu bionic-cran40/ Packages [101
kB]
```

```
Hit:11 http://ppa.launchpad.net/cran/libgit2/ubuntu bionic InRelease
Hit:12 http://ppa.launchpad.net/deadsnakes/ppa/ubuntu bionic InRelease
Get:13 http://archive.ubuntu.com/ubuntu bionic-updates/main amd64 Packages
[3,472 kB]
Hit:14 http://ppa.launchpad.net/graphics-drivers/ppa/ubuntu bionic InRelease
Get:16 http://ppa.launchpad.net/c2d4u.team/c2d4u4.0+/ubuntu bionic/main Sources
[2,218 kB]
Get:17 http://ppa.launchpad.net/c2d4u.team/c2d4u4.0+/ubuntu bionic/main amd64
Packages [1,135 kB]
Fetched 7,207 kB in 2s (2,960 kB/s)
Reading package lists… Done
Reading package lists… Done
Building dependency tree
Reading state information… Done
The following package was automatically installed and is no longer required:
  libnvidia-common-460
Use 'sudo apt autoremove' to remove it.
The following additional packages will be installed:
  fonts-droid-fallback fonts-lato fonts-lmodern fonts-noto-mono fonts-texgyre
  javascript-common libcupsfilters1 libcupsimage2 libgs9 libgs9-common
  libijs-0.35 libjbig2dec0 libjs-jquery libkpathsea6 libpotrace0 libptexenc1
  libruby2.5 libsynctex1 libtexlua52 libtexluajit2 libzzip-0-13 lmodern
  poppler-data preview-latex-style rake ruby ruby-did-you-mean ruby-minitest
  ruby-net-telnet ruby-power-assert ruby-test-unit ruby2.5
  rubygems-integration t1utils tex-common tex-gyre texlive-base
  texlive-binaries texlive-latex-base texlive-latex-extra
  texlive-latex-recommended texlive-pictures texlive-plain-generic tipa
Suggested packages:
  fonts-noto apache2 | lighttpd | httpd poppler-utils ghostscript
  fonts-japanese-mincho | fonts-ipafont-mincho fonts-japanese-gothic
  | fonts-ipafont-gothic fonts-arphic-ukai fonts-arphic-uming fonts-nanum ri
  ruby-dev bundler debhelper gv | postscript-viewer perl-tk xpdf-reader
  | pdf-viewer texlive-fonts-recommended-doc texlive-latex-base-doc
  python-pygments icc-profiles libfile-which-perl
  libspreadsheet-parseexcel-perl texlive-latex-extra-doc
  texlive-latex-recommended-doc texlive-pstricks dot2tex prerex ruby-tcltk
  | libtcltk-ruby texlive-pictures-doc vprerex
The following NEW packages will be installed:
  fonts-droid-fallback fonts-lato fonts-lmodern fonts-noto-mono fonts-texgyre
  javascript-common libcupsfilters1 libcupsimage2 libgs9 libgs9-common
  libijs-0.35 libjbig2dec0 libjs-jquery libkpathsea6 libpotrace0 libptexenc1
  libruby2.5 libsynctex1 libtexlua52 libtexluajit2 libzzip-0-13 lmodern
  poppler-data preview-latex-style rake ruby ruby-did-you-mean ruby-minitest
  ruby-net-telnet ruby-power-assert ruby-test-unit ruby2.5
  rubygems-integration t1utils tex-common tex-gyre texlive-base
  texlive-binaries texlive-fonts-recommended texlive-generic-recommended
  texlive-latex-base texlive-latex-extra texlive-latex-recommended
  texlive-pictures texlive-plain-generic texlive-xetex tipa
```

```
0 upgraded, 47 newly installed, 0 to remove and 17 not upgraded.
Need to get 146 MB of archives.
After this operation, 460 MB of additional disk space will be used.
Get:1 http://archive.ubuntu.com/ubuntu bionic/main amd64 fonts-droid-fallback
all 1:6.0.1r16-1.1 [1,805 kB]
Get:2 http://archive.ubuntu.com/ubuntu bionic/main amd64 fonts-lato all 2.0-2
[2,698 kB]
Get:3 http://archive.ubuntu.com/ubuntu bionic/main amd64 poppler-data all
0.4.8-2 [1,479 kB]
Get:4 http://archive.ubuntu.com/ubuntu bionic/main amd64 tex-common all 6.09
[33.0 kB]
Get:5 http://archive.ubuntu.com/ubuntu bionic/main amd64 fonts-lmodern all
2.004.5-3 [4,551 kB]
Get:6 http://archive.ubuntu.com/ubuntu bionic/main amd64 fonts-noto-mono all
20171026-2 [75.5 kB]
Get:7 http://archive.ubuntu.com/ubuntu bionic/universe amd64 fonts-texgyre all
20160520-1 [8,761 kB]
Get:8 http://archive.ubuntu.com/ubuntu bionic/main amd64 javascript-common all
11 [6,066 B]
Get:9 http://archive.ubuntu.com/ubuntu bionic-updates/main amd64 libcupsfilters1
amd64 1.20.2-0ubuntu3.1 [108 kB]
Get:10 http://archive.ubuntu.com/ubuntu bionic-updates/main amd64 libcupsimage2
amd64 2.2.7-1ubuntu2.9 [18.6 kB]
Get:11 http://archive.ubuntu.com/ubuntu bionic/main amd64 libijs-0.35 amd64
0.35-13 [15.5 kB]
Get:12 http://archive.ubuntu.com/ubuntu bionic/main amd64 libjbig2dec0 amd64
0.13-6 [55.9 kB]
Get:13 http://archive.ubuntu.com/ubuntu bionic-updates/main amd64 libgs9-common
all 9.26~dfsg+0-0ubuntu0.18.04.17 [5,092 kB]
Get:14 http://archive.ubuntu.com/ubuntu bionic-updates/main amd64 libgs9 amd64
9.26~dfsg+0-0ubuntu0.18.04.17 [2,267 kB]
Get:15 http://archive.ubuntu.com/ubuntu bionic/main amd64 libjs-jquery all
3.2.1-1 [152 kB]
Get:16 http://archive.ubuntu.com/ubuntu bionic-updates/main amd64 libkpathsea6
amd64 2017.20170613.44572-8ubuntu0.1 [54.9 kB]
Get:17 http://archive.ubuntu.com/ubuntu bionic/main amd64 libpotrace0 amd64
1.14-2 [17.4 kB]
Get:18 http://archive.ubuntu.com/ubuntu bionic-updates/main amd64 libptexenc1
amd64 2017.20170613.44572-8ubuntu0.1 [34.5 kB]
Get:19 http://archive.ubuntu.com/ubuntu bionic/main amd64 rubygems-integration
all 1.11 [4,994 B]
Get:20 http://archive.ubuntu.com/ubuntu bionic-updates/main amd64 ruby2.5 amd64
2.5.1-1ubuntu1.12 [48.6 kB]
Get:21 http://archive.ubuntu.com/ubuntu bionic/main amd64 ruby amd64 1:2.5.1
[5,712 B]
Get:22 http://archive.ubuntu.com/ubuntu bionic-updates/main amd64 rake all
12.3.1-1ubuntu0.1 [44.9 kB]
Get:23 http://archive.ubuntu.com/ubuntu bionic/main amd64 ruby-did-you-mean all
```

```
1.2.0-2 [9,700 B]
Get:24 http://archive.ubuntu.com/ubuntu bionic/main amd64 ruby-minitest all
5.10.3-1 [38.6 kB]
Get:25 http://archive.ubuntu.com/ubuntu bionic/main amd64 ruby-net-telnet all
0.1.1-2 [12.6 kB]
Get:26 http://archive.ubuntu.com/ubuntu bionic/main amd64 ruby-power-assert all
0.3.0-1 [7,952 B]
Get:27 http://archive.ubuntu.com/ubuntu bionic/main amd64 ruby-test-unit all
3.2.5-1 [61.1 kB]
Get:28 http://archive.ubuntu.com/ubuntu bionic-updates/main amd64 libruby2.5
amd64 2.5.1-1ubuntu1.12 [3,073 kB]
Get:29 http://archive.ubuntu.com/ubuntu bionic-updates/main amd64 libsynctex1
amd64 2017.20170613.44572-8ubuntu0.1 [41.4 kB]
Get:30 http://archive.ubuntu.com/ubuntu bionic-updates/main amd64 libtexlua52
amd64 2017.20170613.44572-8ubuntu0.1 [91.2 kB]
Get:31 http://archive.ubuntu.com/ubuntu bionic-updates/main amd64 libtexluajit2
amd64 2017.20170613.44572-8ubuntu0.1 [230 kB]
Get:32 http://archive.ubuntu.com/ubuntu bionic-updates/main amd64 libzzip-0-13
amd64 0.13.62-3.1ubuntu0.18.04.1 [26.0 kB]
Get:33 http://archive.ubuntu.com/ubuntu bionic/main amd64 lmodern all 2.004.5-3
[9,631 kB]
Get:34 http://archive.ubuntu.com/ubuntu bionic/main amd64 preview-latex-style
all 11.91-1ubuntu1 [185 kB]
Get:35 http://archive.ubuntu.com/ubuntu bionic/main amd64 t1utils amd64 1.41-2
[56.0 kB]
Get:36 http://archive.ubuntu.com/ubuntu bionic/universe amd64 tex-gyre all
20160520-1 [4,998 kB]
Get:37 http://archive.ubuntu.com/ubuntu bionic-updates/main amd64 texlive-
binaries amd64 2017.20170613.44572-8ubuntu0.1 [8,179 kB]
Get:38 http://archive.ubuntu.com/ubuntu bionic/main amd64 texlive-base all
2017.20180305-1 [18.7 MB]
Get:39 http://archive.ubuntu.com/ubuntu bionic/universe amd64 texlive-fonts-
recommended all 2017.20180305-1 [5,262 kB]
Get:40 http://archive.ubuntu.com/ubuntu bionic/universe amd64 texlive-plain-
generic all 2017.20180305-2 [23.6 MB]
Get:41 http://archive.ubuntu.com/ubuntu bionic/universe amd64 texlive-generic-
recommended all 2017.20180305-1 [15.9 kB]
Get:42 http://archive.ubuntu.com/ubuntu bionic/main amd64 texlive-latex-base all
2017.20180305-1 [951 kB]
Get:43 http://archive.ubuntu.com/ubuntu bionic/main amd64 texlive-latex-
recommended all 2017.20180305-1 [14.9 MB]
Get:44 http://archive.ubuntu.com/ubuntu bionic/universe amd64 texlive-pictures
all 2017.20180305-1 [4,026 kB]
Get:45 http://archive.ubuntu.com/ubuntu bionic/universe amd64 texlive-latex-
extra all 2017.20180305-2 [10.6 MB]
Get:46 http://archive.ubuntu.com/ubuntu bionic/universe amd64 tipa all 2:1.3-20
[2,978 kB]
Get:47 http://archive.ubuntu.com/ubuntu bionic/universe amd64 texlive-xetex all
```

```
2017.20180305-1 [10.7 MB]
Fetched 146 MB in 2s (65.1 MB/s)
debconf: unable to initialize frontend: Dialog
debconf: (No usable dialog-like program is installed, so the dialog based
frontend cannot be used. at /usr/share/perl5/Debconf/FrontEnd/Dialog.pm line 76,
<> line 47.)
debconf: falling back to frontend: Readline
debconf: unable to initialize frontend: Readline
debconf: (This frontend requires a controlling tty.)
debconf: falling back to frontend: Teletype
dpkg-preconfigure: unable to re-open stdin:
Selecting previously unselected package fonts-droid-fallback.
(Reading database … 123942 files and directories currently installed.)
Preparing to unpack …/00-fonts-droid-fallback_1%3a6.0.1r16-1.1_all.deb …
Unpacking fonts-droid-fallback (1:6.0.1r16-1.1) …
Selecting previously unselected package fonts-lato.
Preparing to unpack …/01-fonts-lato_2.0-2_all.deb …
Unpacking fonts-lato (2.0-2) …
Selecting previously unselected package poppler-data.
Preparing to unpack …/02-poppler-data_0.4.8-2_all.deb …
Unpacking poppler-data (0.4.8-2) …
Selecting previously unselected package tex-common.
Preparing to unpack …/03-tex-common_6.09_all.deb …
Unpacking tex-common (6.09) …
Selecting previously unselected package fonts-lmodern.
Preparing to unpack …/04-fonts-lmodern_2.004.5-3_all.deb …
Unpacking fonts-lmodern (2.004.5-3) …
Selecting previously unselected package fonts-noto-mono.
Preparing to unpack …/05-fonts-noto-mono_20171026-2_all.deb …
Unpacking fonts-noto-mono (20171026-2) …
Selecting previously unselected package fonts-texgyre.
Preparing to unpack …/06-fonts-texgyre_20160520-1_all.deb …
Unpacking fonts-texgyre (20160520-1) …
Selecting previously unselected package javascript-common.
Preparing to unpack …/07-javascript-common_11_all.deb …
Unpacking javascript-common (11) …
Selecting previously unselected package libcupsfilters1:amd64.
Preparing to unpack …/08-libcupsfilters1_1.20.2-0ubuntu3.1_amd64.deb …
Unpacking libcupsfilters1:amd64 (1.20.2-0ubuntu3.1) …
Selecting previously unselected package libcupsimage2:amd64.
Preparing to unpack …/09-libcupsimage2_2.2.7-1ubuntu2.9_amd64.deb …
Unpacking libcupsimage2:amd64 (2.2.7-1ubuntu2.9) …
Selecting previously unselected package libijs-0.35:amd64.
Preparing to unpack …/10-libijs-0.35_0.35-13_amd64.deb …
Unpacking libijs-0.35:amd64 (0.35-13) …
Selecting previously unselected package libjbig2dec0:amd64.
Preparing to unpack …/11-libjbig2dec0_0.13-6_amd64.deb …
Unpacking libjbig2dec0:amd64 (0.13-6) …
```

```
Selecting previously unselected package libgs9-common.
Preparing to unpack …/12-libgs9-common_9.26~dfsg+0-0ubuntu0.18.04.17_all.deb
…
Unpacking libgs9-common (9.26~dfsg+0-0ubuntu0.18.04.17) …
Selecting previously unselected package libgs9:amd64.
Preparing to unpack …/13-libgs9_9.26~dfsg+0-0ubuntu0.18.04.17_amd64.deb …
Unpacking libgs9:amd64 (9.26~dfsg+0-0ubuntu0.18.04.17) …
Selecting previously unselected package libjs-jquery.
Preparing to unpack …/14-libjs-jquery_3.2.1-1_all.deb …
Unpacking libjs-jquery (3.2.1-1) …
Selecting previously unselected package libkpathsea6:amd64.
Preparing to unpack …/15-libkpathsea6_2017.20170613.44572-8ubuntu0.1_amd64.deb
…
Unpacking libkpathsea6:amd64 (2017.20170613.44572-8ubuntu0.1) …
Selecting previously unselected package libpotrace0.
Preparing to unpack …/16-libpotrace0_1.14-2_amd64.deb …
Unpacking libpotrace0 (1.14-2) …
Selecting previously unselected package libptexenc1:amd64.
Preparing to unpack …/17-libptexenc1_2017.20170613.44572-8ubuntu0.1_amd64.deb
…
Unpacking libptexenc1:amd64 (2017.20170613.44572-8ubuntu0.1) …
Selecting previously unselected package rubygems-integration.
Preparing to unpack …/18-rubygems-integration_1.11_all.deb …
Unpacking rubygems-integration (1.11) …
Selecting previously unselected package ruby2.5.
Preparing to unpack …/19-ruby2.5_2.5.1-1ubuntu1.12_amd64.deb …
Unpacking ruby2.5 (2.5.1-1ubuntu1.12) …
Selecting previously unselected package ruby.
Preparing to unpack …/20-ruby_1%3a2.5.1_amd64.deb …
Unpacking ruby (1:2.5.1) …
Selecting previously unselected package rake.
Preparing to unpack …/21-rake_12.3.1-1ubuntu0.1_all.deb …
Unpacking rake (12.3.1-1ubuntu0.1) …
Selecting previously unselected package ruby-did-you-mean.
Preparing to unpack …/22-ruby-did-you-mean_1.2.0-2_all.deb …
Unpacking ruby-did-you-mean (1.2.0-2) …
Selecting previously unselected package ruby-minitest.
Preparing to unpack …/23-ruby-minitest_5.10.3-1_all.deb …
Unpacking ruby-minitest (5.10.3-1) …
Selecting previously unselected package ruby-net-telnet.
Preparing to unpack …/24-ruby-net-telnet_0.1.1-2_all.deb …
Unpacking ruby-net-telnet (0.1.1-2) …
Selecting previously unselected package ruby-power-assert.
Preparing to unpack …/25-ruby-power-assert_0.3.0-1_all.deb …
Unpacking ruby-power-assert (0.3.0-1) …
Selecting previously unselected package ruby-test-unit.
Preparing to unpack …/26-ruby-test-unit_3.2.5-1_all.deb …
Unpacking ruby-test-unit (3.2.5-1) …
```

```
Selecting previously unselected package libruby2.5:amd64.
Preparing to unpack …/27-libruby2.5_2.5.1-1ubuntu1.12_amd64.deb …
Unpacking libruby2.5:amd64 (2.5.1-1ubuntu1.12) …
Selecting previously unselected package libsynctex1:amd64.
Preparing to unpack …/28-libsynctex1_2017.20170613.44572-8ubuntu0.1_amd64.deb
…
Unpacking libsynctex1:amd64 (2017.20170613.44572-8ubuntu0.1) …
Selecting previously unselected package libtexlua52:amd64.
Preparing to unpack …/29-libtexlua52_2017.20170613.44572-8ubuntu0.1_amd64.deb
…
Unpacking libtexlua52:amd64 (2017.20170613.44572-8ubuntu0.1) …
Selecting previously unselected package libtexluajit2:amd64.
Preparing to unpack
…/30-libtexluajit2_2017.20170613.44572-8ubuntu0.1_amd64.deb …
Unpacking libtexluajit2:amd64 (2017.20170613.44572-8ubuntu0.1) …
Selecting previously unselected package libzzip-0-13:amd64.
Preparing to unpack …/31-libzzip-0-13_0.13.62-3.1ubuntu0.18.04.1_amd64.deb …
Unpacking libzzip-0-13:amd64 (0.13.62-3.1ubuntu0.18.04.1) …
Selecting previously unselected package lmodern.
Preparing to unpack …/32-lmodern_2.004.5-3_all.deb …
Unpacking lmodern (2.004.5-3) …
Selecting previously unselected package preview-latex-style.
Preparing to unpack …/33-preview-latex-style_11.91-1ubuntu1_all.deb …
Unpacking preview-latex-style (11.91-1ubuntu1) …
Selecting previously unselected package t1utils.
Preparing to unpack …/34-t1utils_1.41-2_amd64.deb …
Unpacking t1utils (1.41-2) …
Selecting previously unselected package tex-gyre.
Preparing to unpack …/35-tex-gyre_20160520-1_all.deb …
Unpacking tex-gyre (20160520-1) …
Selecting previously unselected package texlive-binaries.
Preparing to unpack …/36-texlive-
binaries_2017.20170613.44572-8ubuntu0.1_amd64.deb …
Unpacking texlive-binaries (2017.20170613.44572-8ubuntu0.1) …
Selecting previously unselected package texlive-base.
Preparing to unpack …/37-texlive-base_2017.20180305-1_all.deb …
Unpacking texlive-base (2017.20180305-1) …
Selecting previously unselected package texlive-fonts-recommended.
Preparing to unpack …/38-texlive-fonts-recommended_2017.20180305-1_all.deb …
Unpacking texlive-fonts-recommended (2017.20180305-1) …
Selecting previously unselected package texlive-plain-generic.
Preparing to unpack …/39-texlive-plain-generic_2017.20180305-2_all.deb …
Unpacking texlive-plain-generic (2017.20180305-2) …
Selecting previously unselected package texlive-generic-recommended.
Preparing to unpack …/40-texlive-generic-recommended_2017.20180305-1_all.deb
…
Unpacking texlive-generic-recommended (2017.20180305-1) …
Selecting previously unselected package texlive-latex-base.
```

```
Preparing to unpack …/41-texlive-latex-base_2017.20180305-1_all.deb …
Unpacking texlive-latex-base (2017.20180305-1) …
Selecting previously unselected package texlive-latex-recommended.
Preparing to unpack …/42-texlive-latex-recommended_2017.20180305-1_all.deb …
Unpacking texlive-latex-recommended (2017.20180305-1) …
Selecting previously unselected package texlive-pictures.
Preparing to unpack …/43-texlive-pictures_2017.20180305-1_all.deb …
Unpacking texlive-pictures (2017.20180305-1) …
Selecting previously unselected package texlive-latex-extra.
Preparing to unpack …/44-texlive-latex-extra_2017.20180305-2_all.deb …
Unpacking texlive-latex-extra (2017.20180305-2) …
Selecting previously unselected package tipa.
Preparing to unpack …/45-tipa_2%3a1.3-20_all.deb …
Unpacking tipa (2:1.3-20) …
Selecting previously unselected package texlive-xetex.
Preparing to unpack …/46-texlive-xetex_2017.20180305-1_all.deb …
Unpacking texlive-xetex (2017.20180305-1) …
Setting up libgs9-common (9.26~dfsg+0-0ubuntu0.18.04.17) …
Setting up libkpathsea6:amd64 (2017.20170613.44572-8ubuntu0.1) …
Setting up libjs-jquery (3.2.1-1) …
Setting up libtexlua52:amd64 (2017.20170613.44572-8ubuntu0.1) …
Setting up fonts-droid-fallback (1:6.0.1r16-1.1) …
Setting up libsynctex1:amd64 (2017.20170613.44572-8ubuntu0.1) …
Setting up libptexenc1:amd64 (2017.20170613.44572-8ubuntu0.1) …
Setting up tex-common (6.09) …
debconf: unable to initialize frontend: Dialog
debconf: (No usable dialog-like program is installed, so the dialog based
frontend cannot be used. at /usr/share/perl5/Debconf/FrontEnd/Dialog.pm line
76.)
debconf: falling back to frontend: Readline
update-language: texlive-base not installed and configured, doing nothing!
Setting up poppler-data (0.4.8-2) …
Setting up tex-gyre (20160520-1) …
Setting up preview-latex-style (11.91-1ubuntu1) …
Setting up fonts-texgyre (20160520-1) …
Setting up fonts-noto-mono (20171026-2) …
Setting up fonts-lato (2.0-2) …
Setting up libcupsfilters1:amd64 (1.20.2-0ubuntu3.1) …
Setting up libcupsimage2:amd64 (2.2.7-1ubuntu2.9) …
Setting up libjbig2dec0:amd64 (0.13-6) …
Setting up ruby-did-you-mean (1.2.0-2) …
Setting up t1utils (1.41-2) …
Setting up ruby-net-telnet (0.1.1-2) …
Setting up libijs-0.35:amd64 (0.35-13) …
Setting up rubygems-integration (1.11) …
Setting up libpotrace0 (1.14-2) …
Setting up javascript-common (11) …
Setting up ruby-minitest (5.10.3-1) …
```

```
Setting up libzzip-0-13:amd64 (0.13.62-3.1ubuntu0.18.04.1) …
Setting up libgs9:amd64 (9.26~dfsg+0-0ubuntu0.18.04.17) …
Setting up libtexluajit2:amd64 (2017.20170613.44572-8ubuntu0.1) …
Setting up fonts-lmodern (2.004.5-3) …
Setting up ruby-power-assert (0.3.0-1) …
Setting up texlive-binaries (2017.20170613.44572-8ubuntu0.1) …
update-alternatives: using /usr/bin/xdvi-xaw to provide /usr/bin/xdvi.bin
(xdvi.bin) in auto mode
update-alternatives: using /usr/bin/bibtex.original to provide /usr/bin/bibtex
(bibtex) in auto mode
Setting up texlive-base (2017.20180305-1) …
mktexlsr: Updating /var/lib/texmf/ls-R-TEXLIVEDIST…
mktexlsr: Updating /var/lib/texmf/ls-R-TEXMFMAIN…
mktexlsr: Updating /var/lib/texmf/ls-R…
mktexlsr: Done.
tl-paper: setting paper size for dvips to a4:
/var/lib/texmf/dvips/config/config-paper.ps
tl-paper: setting paper size for dvipdfmx to a4:
/var/lib/texmf/dvipdfmx/dvipdfmx-paper.cfg
tl-paper: setting paper size for xdvi to a4: /var/lib/texmf/xdvi/XDvi-paper
tl-paper: setting paper size for pdftex to a4:
/var/lib/texmf/tex/generic/config/pdftexconfig.tex
debconf: unable to initialize frontend: Dialog
debconf: (No usable dialog-like program is installed, so the dialog based
frontend cannot be used. at /usr/share/perl5/Debconf/FrontEnd/Dialog.pm line
76.)
debconf: falling back to frontend: Readline
Setting up texlive-fonts-recommended (2017.20180305-1) …
Setting up texlive-plain-generic (2017.20180305-2) …
Setting up texlive-generic-recommended (2017.20180305-1) …
Setting up texlive-latex-base (2017.20180305-1) …
Setting up lmodern (2.004.5-3) …
Setting up texlive-latex-recommended (2017.20180305-1) …
Setting up texlive-pictures (2017.20180305-1) …
Setting up tipa (2:1.3-20) …
Regenerating '/var/lib/texmf/fmtutil.cnf-DEBIAN'… done.
Regenerating '/var/lib/texmf/fmtutil.cnf-TEXLIVEDIST'… done.
update-fmtutil has updated the following file(s):
        /var/lib/texmf/fmtutil.cnf-DEBIAN
        /var/lib/texmf/fmtutil.cnf-TEXLIVEDIST
If you want to activate the changes in the above file(s),
you should run fmtutil-sys or fmtutil.
Setting up texlive-latex-extra (2017.20180305-2) …
Setting up texlive-xetex (2017.20180305-1) …
Setting up ruby2.5 (2.5.1-1ubuntu1.12) …
Setting up ruby (1:2.5.1) …
Setting up ruby-test-unit (3.2.5-1) …
Setting up rake (12.3.1-1ubuntu0.1) …
```

```
Setting up libruby2.5:amd64 (2.5.1-1ubuntu1.12) …
Processing triggers for mime-support (3.60ubuntu1) …
Processing triggers for libc-bin (2.27-3ubuntu1.6) …
Processing triggers for man-db (2.8.3-2ubuntu0.1) …
Processing triggers for fontconfig (2.12.6-0ubuntu2) …
Processing triggers for tex-common (6.09) …
debconf: unable to initialize frontend: Dialog
debconf: (No usable dialog-like program is installed, so the dialog based
frontend cannot be used. at /usr/share/perl5/Debconf/FrontEnd/Dialog.pm line
76.)
debconf: falling back to frontend: Readline
Running updmap-sys. This may take some time… done.
Running mktexlsr /var/lib/texmf … done.
Building format(s) --all.
        This may take some time… done.
```

[41]:
```
!jupyter nbconvert --log-level CRITICAL --to pdf Copy_of_fall2022_hw3.ipynb #
↪make sure the ipynb name is correct
```

```
This application is used to convert notebook files (*.ipynb)
        to various other formats.

        WARNING: THE COMMANDLINE INTERFACE MAY CHANGE IN FUTURE RELEASES.

Options
=======
The options below are convenience aliases to configurable class-options,
as listed in the "Equivalent to" description-line of the aliases.
To see all configurable class-options for some <cmd>, use:
    <cmd> --help-all

--debug
    set log level to logging.DEBUG (maximize logging output)
    Equivalent to: [--Application.log_level=10]
--show-config
    Show the application's configuration (human-readable format)
    Equivalent to: [--Application.show_config=True]
--show-config-json
    Show the application's configuration (json format)
    Equivalent to: [--Application.show_config_json=True]
--generate-config
    generate default config file
    Equivalent to: [--JupyterApp.generate_config=True]
-y
    Answer yes to any questions instead of prompting.
    Equivalent to: [--JupyterApp.answer_yes=True]
--execute
    Execute the notebook prior to export.
```

```
    Equivalent to: [--ExecutePreprocessor.enabled=True]
--allow-errors
    Continue notebook execution even if one of the cells throws an error and
include the error message in the cell output (the default behaviour is to abort
conversion). This flag is only relevant if '--execute' was specified, too.
    Equivalent to: [--ExecutePreprocessor.allow_errors=True]
--stdin
    read a single notebook file from stdin. Write the resulting notebook with
default basename 'notebook.*'
    Equivalent to: [--NbConvertApp.from_stdin=True]
--stdout
    Write notebook output to stdout instead of files.
    Equivalent to: [--NbConvertApp.writer_class=StdoutWriter]
--inplace
    Run nbconvert in place, overwriting the existing notebook (only
            relevant when converting to notebook format)
    Equivalent to: [--NbConvertApp.use_output_suffix=False
--NbConvertApp.export_format=notebook --FilesWriter.build_directory=]
--clear-output
    Clear output of current file and save in place,
            overwriting the existing notebook.
    Equivalent to: [--NbConvertApp.use_output_suffix=False
--NbConvertApp.export_format=notebook --FilesWriter.build_directory=
--ClearOutputPreprocessor.enabled=True]
--no-prompt
    Exclude input and output prompts from converted document.
    Equivalent to: [--TemplateExporter.exclude_input_prompt=True
--TemplateExporter.exclude_output_prompt=True]
--no-input
    Exclude input cells and output prompts from converted document.
            This mode is ideal for generating code-free reports.
    Equivalent to: [--TemplateExporter.exclude_output_prompt=True
--TemplateExporter.exclude_input=True]
--log-level=<Enum>
    Set the log level by value or name.
    Choices: any of [0, 10, 20, 30, 40, 50, 'DEBUG', 'INFO', 'WARN', 'ERROR',
'CRITICAL']
    Default: 30
    Equivalent to: [--Application.log_level]
--config=<Unicode>
    Full path of a config file.
    Default: ''
    Equivalent to: [--JupyterApp.config_file]
--to=<Unicode>
    The export format to be used, either one of the built-in formats
            ['asciidoc', 'custom', 'html', 'latex', 'markdown', 'notebook',
'pdf', 'python', 'rst', 'script', 'slides']
            or a dotted object name that represents the import path for an
```

```
            `Exporter` class
    Default: 'html'
    Equivalent to: [--NbConvertApp.export_format]
--template=<Unicode>
    Name of the template file to use
    Default: ''
    Equivalent to: [--TemplateExporter.template_file]
--writer=<DottedObjectName>
    Writer class used to write the
                                        results of the conversion
    Default: 'FilesWriter'
    Equivalent to: [--NbConvertApp.writer_class]
--post=<DottedOrNone>
    PostProcessor class used to write the
                                        results of the conversion
    Default: ''
    Equivalent to: [--NbConvertApp.postprocessor_class]
--output=<Unicode>
    overwrite base name use for output files.
                can only be used when converting one notebook at a time.
    Default: ''
    Equivalent to: [--NbConvertApp.output_base]
--output-dir=<Unicode>
    Directory to write output(s) to. Defaults
                                  to output to the directory of each notebook.
To recover
                                  previous default behaviour (outputting to the
current
                                  working directory) use . as the flag value.
    Default: ''
    Equivalent to: [--FilesWriter.build_directory]
--reveal-prefix=<Unicode>
    The URL prefix for reveal.js (version 3.x).
            This defaults to the reveal CDN, but can be any url pointing to a
copy
            of reveal.js.
            For speaker notes to work, this must be a relative path to a local
            copy of reveal.js: e.g., "reveal.js".
            If a relative path is given, it must be a subdirectory of the
            current directory (from which the server is run).
            See the usage documentation
            (https://nbconvert.readthedocs.io/en/latest/usage.html#reveal-js-
html-slideshow)
            for more details.
    Default: ''
    Equivalent to: [--SlidesExporter.reveal_url_prefix]
--nbformat=<Enum>
    The nbformat version to write.
```

```
            Use this to downgrade notebooks.
    Choices: any of [1, 2, 3, 4]
    Default: 4
    Equivalent to: [--NotebookExporter.nbformat_version]

Examples
--------

    The simplest way to use nbconvert is

            > jupyter nbconvert mynotebook.ipynb

            which will convert mynotebook.ipynb to the default format (probably
HTML).

            You can specify the export format with `--to`.
            Options include ['asciidoc', 'custom', 'html', 'latex', 'markdown',
'notebook', 'pdf', 'python', 'rst', 'script', 'slides'].

            > jupyter nbconvert --to latex mynotebook.ipynb

            Both HTML and LaTeX support multiple output templates. LaTeX
includes
            'base', 'article' and 'report'.  HTML includes 'basic' and 'full'.
You
            can specify the flavor of the format used.

            > jupyter nbconvert --to html --template basic mynotebook.ipynb

            You can also pipe the output to stdout, rather than a file

            > jupyter nbconvert mynotebook.ipynb --stdout

            PDF is generated via latex

            > jupyter nbconvert mynotebook.ipynb --to pdf

            You can get (and serve) a Reveal.js-powered slideshow

            > jupyter nbconvert myslides.ipynb --to slides --post serve

            Multiple notebooks can be given at the command line in a couple of
            different ways:

            > jupyter nbconvert notebook*.ipynb
            > jupyter nbconvert notebook1.ipynb notebook2.ipynb

            or you can specify the notebooks list in a config file, containing::
```

```
            c.NbConvertApp.notebooks = ["my_notebook.ipynb"]

        > jupyter nbconvert --config mycfg.py

To see all available configurables, use `--help-all`.
```

[41]:

```

```