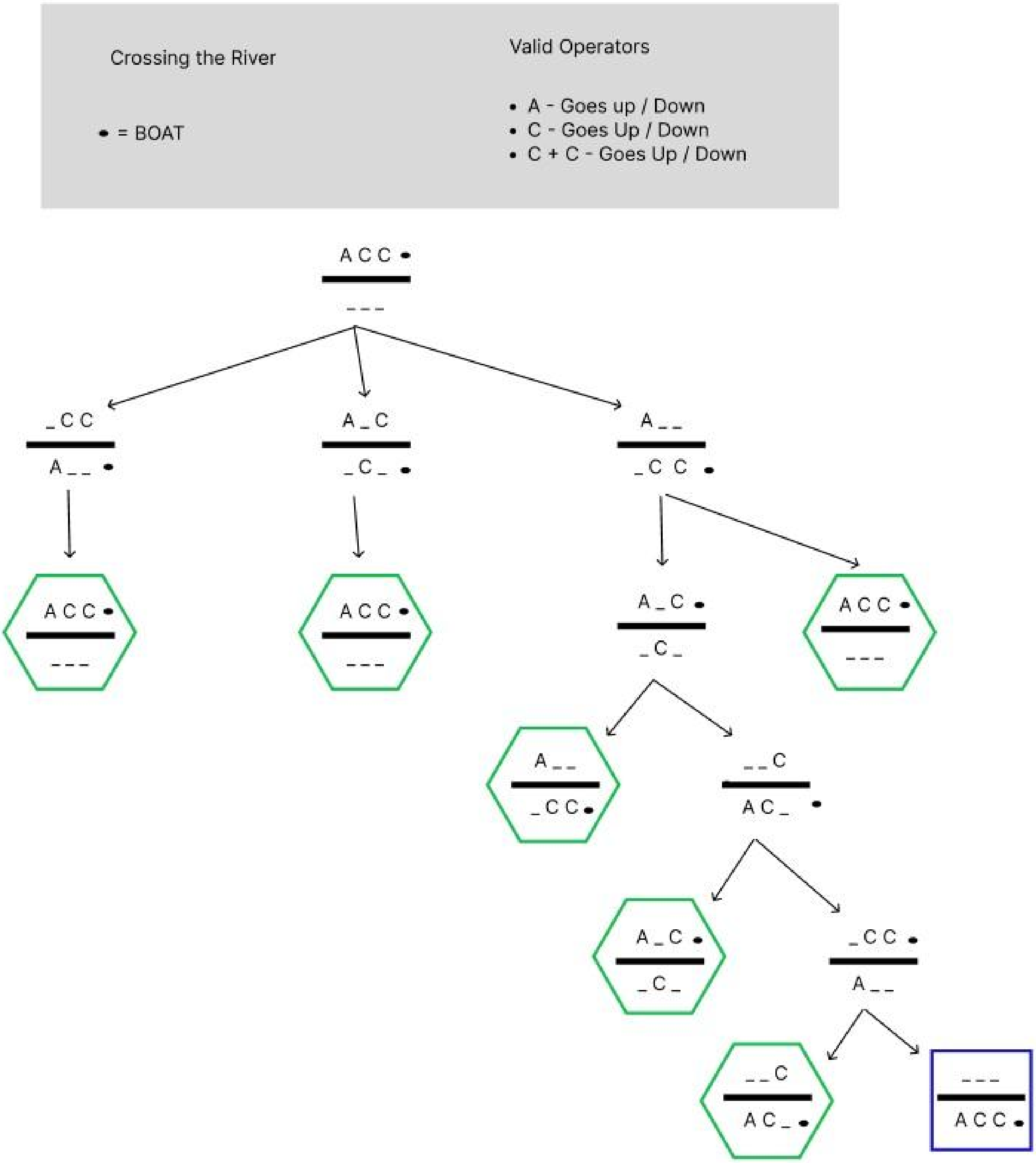
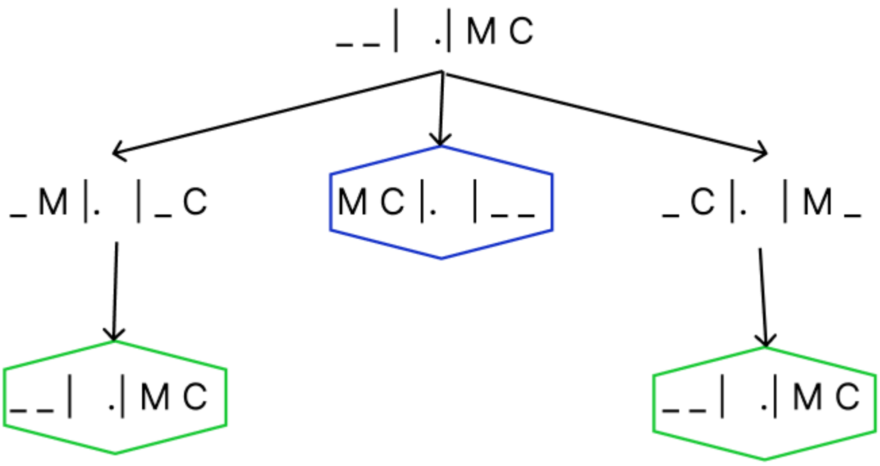
Cross the River

Green Hexagon represents **Duplicate State**
Blue Square represents **Goal State**
Red Circle represents **Invalid State**



Missionaries and Cannibals

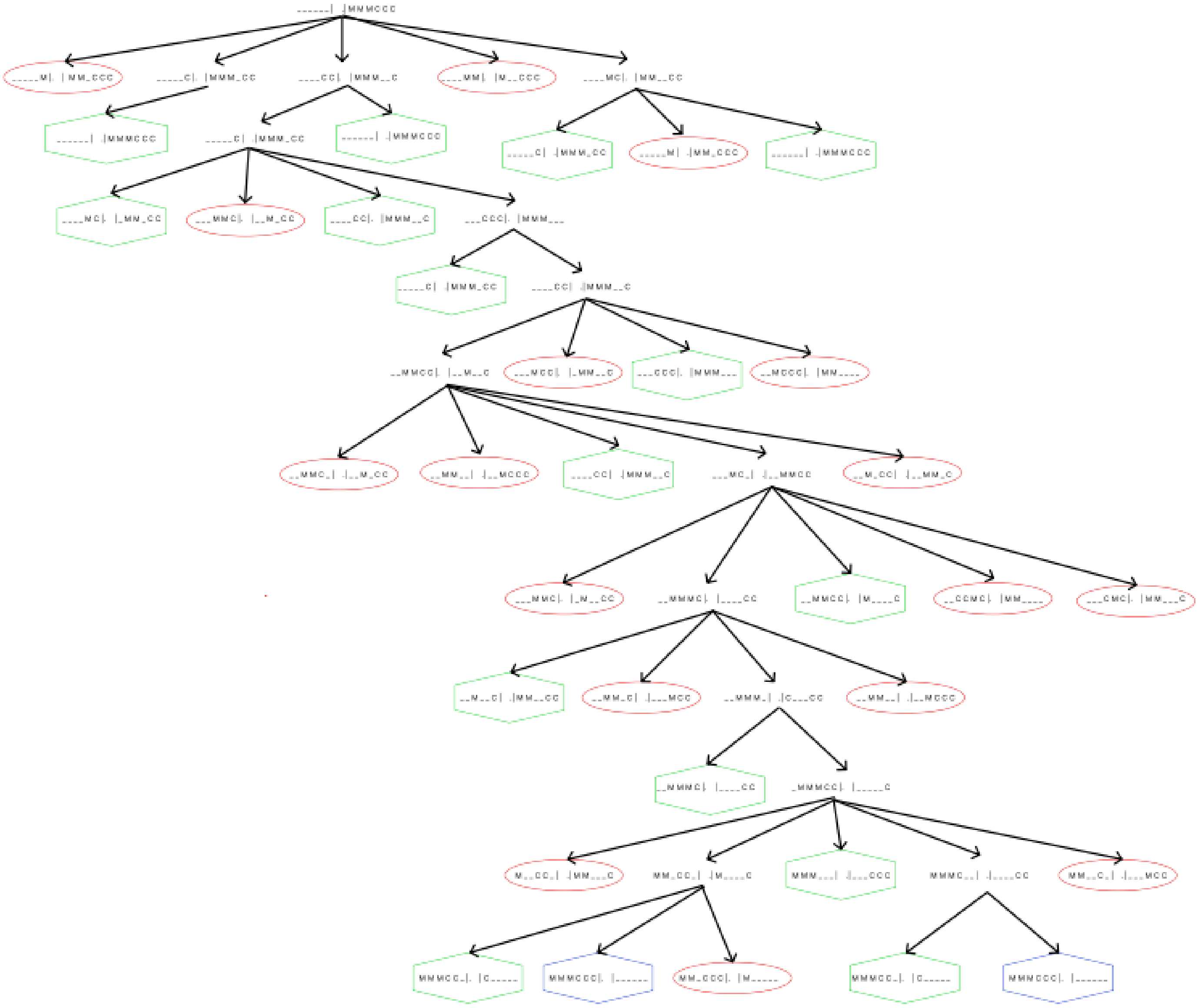
As it is not specified how many missionaries and cannibals to be taken and 3,3 combination is standard but not fixed, we can solve the 1,1 Missionaries and Cannibals problem for extra credit using this search space



In case this trick did not work and we need to provide the solution for 3,3 Missionaries and cannibals, here is the search space for 3,3 missionaries and cannibals.

Green Hexagon represents **Duplicate State**
Blue Square represents **Goal State**
Red Circle represents **Invalid State**

Link to high resolution image :
https://drive.google.com/file/d/1SjwP_JZUm0mUtSlQwHqX5bUNWfy0kgpe/view?usp=share_link



Exhaustive Search

A)

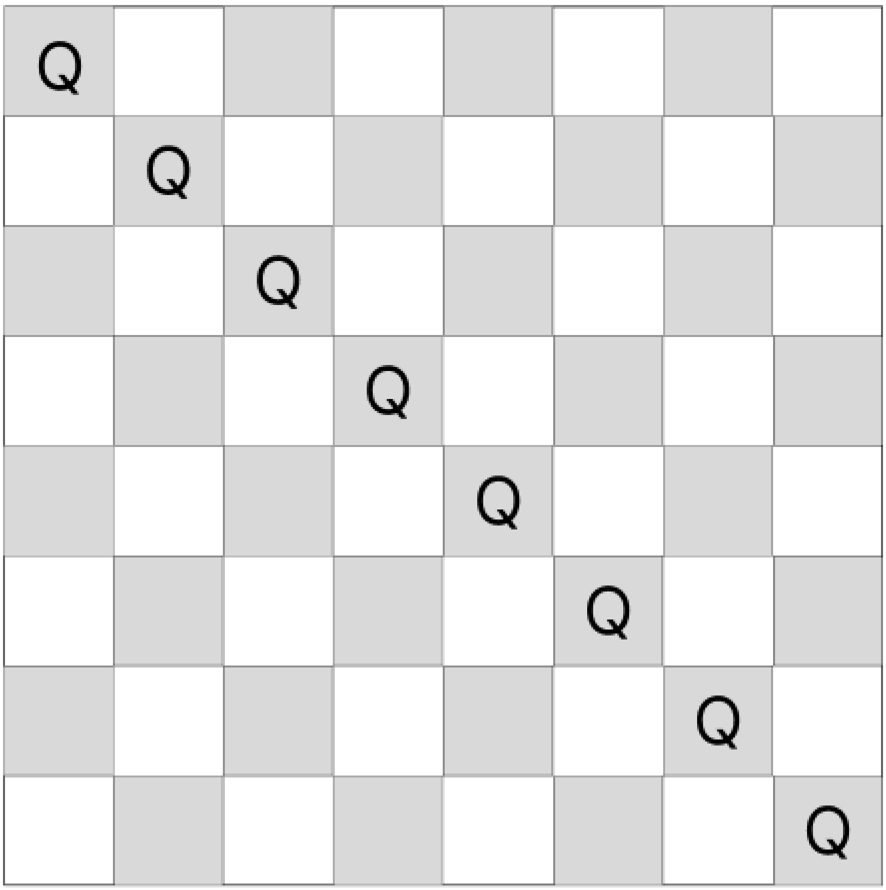
Assumptions -

1) To limit the search space, we will not place a queen if we already have a queen in that row or column
2) We can place a queen in the diagonal of other queen just for the search space

Arrangements -
Total Rows = 8

Number of ways to put queen in first row	= 64
Number of ways to put queen in second row	= 49
Number of ways to put queen in third row	= 36
Number of ways to put queen in fourth row	= 25
Number of ways to put queen in fifth row	= 16
Number of ways to put queen in sixth row	= 9
Number of ways to put queen in seventh row	= 4
Number of ways to put queen in eighth row	= 1

Total Arrangements possible = 64*49*36*25*16*9*4*1 = 1625702400
Arrangements per second = 1000
Total Time = 1625702400 / 1000 = ~452 hrs or 19 days



In Case we do not make the assumption that Queens can not be placed in same row or column if other queen exists there, then

Total Arrangements = 64C8 = 4,426,165,368 arrangements
Time = Approximately 51.2 days

Assumptions -

3) To limit the search space, we will not place a queen if we already have a queen in that row or column
4) We can not place a queen in the diagonal of other queen just for the search space

Arrangements -
Total Rows = 8

Number of ways to put queen in first row	= 64
Number of ways to put queen in second row	= 42
Number of ways to put queen in third row	= 25
Number of ways to put queen in fourth row	= 15
Number of ways to put queen in fifth row	= 9
Number of ways to put queen in sixth row	= 5
Number of ways to put queen in seventh row	= 2
Number of ways to put queen in eighth row	= 1

Total Arrangements possible = 64*42*25*15*9*5*2*1 = 90720000
Arrangements per second = 1000
Total Time = 90720000 / 1000 = ~25.2 hrs

B)
We can not comment on how fast or slowly will we get the solution, It depends on the arrangement of the board and the algorithm, if we have a favorable arrangement and algorithm is A* then we may find a solution in seconds. If we have an empty board or the worst possible arrangement and we use depth first search, we may have to wait the max possible time.

Assuming that we only use optimal arrangements and can not place queens in the row, column and diagonal of another queen while making arrangements, then we have a total of 90720000 (64*42*25*15*9*5*2*1) arrangements. Also assuming that solutions are randomly distributed and we can compute around 1000 arrangements per second, we can assume that a solution can be computed in **1/12 (90720000 / 1000) seconds = ~2.1 hrs (divide by 12 as there are 12 randomly distributed solutions and we need to find only 1)**

C)
In case we can use exhaustive search to construct and search through the entire search space without the time and space limitations, then proving that the solution does not exist would not be possible as we have access to the entire search space. Even if it takes 100 billion years, the search space is finite and thus exhaustive search can provide an answer.

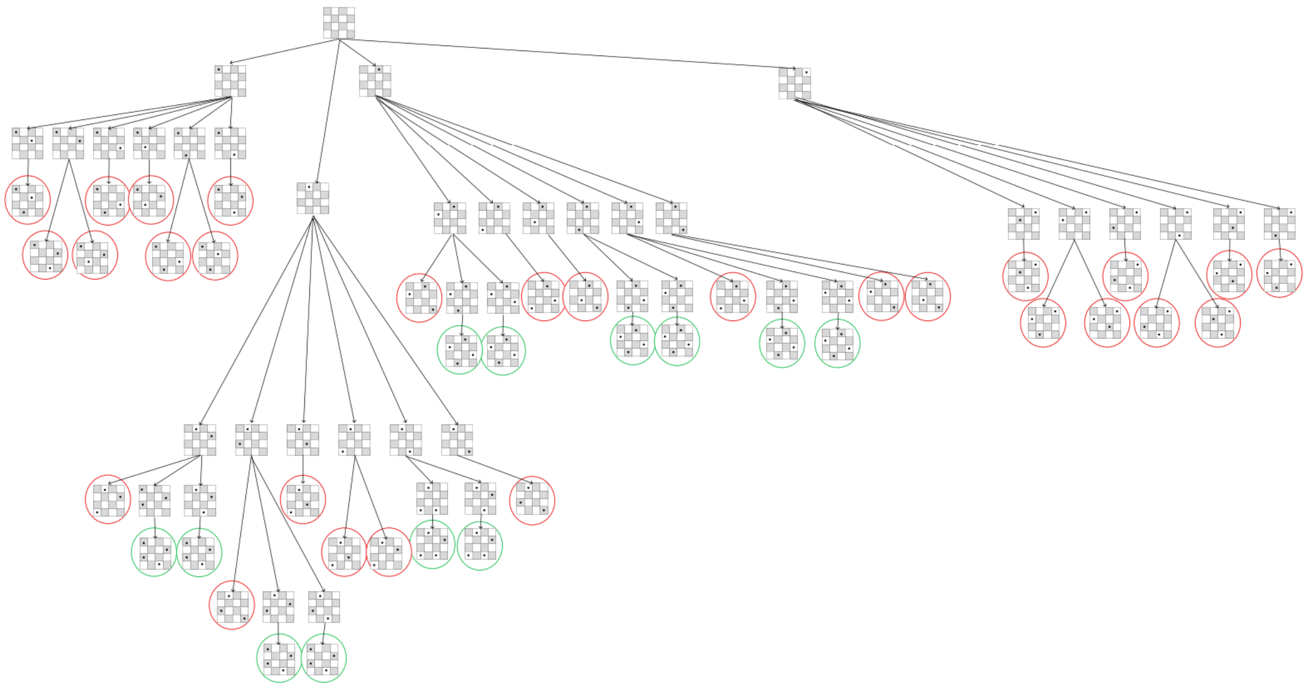
However, in case we do not have a finite search space and the problem that we are trying to solve is open ended problems that do not have a solution found till date. These are the types of problems for which we might not be able to show searching the entire search space will not have any solution. Any problem that belongs in NP-hard category or problems like collatz conjecture(see below) that do not have a solution till date.

Note:
Collatz conjecture, for a any integer n, perform the following operations, n = n/2 if n % 2 == 0 else n = 3n + 1
The task is to find an integer that will not be 1 eventually after applying the operations.
PS - There is no solution in the first 20 million numbers as tried by me.
PPS - There is no solution in the first 2^68 ≈ 2.95×10^20 numbers according to google.

D)

Green Circle represents **Goal State**
Red Circle represents **Invalid State**

link to high resolution image -
https://drive.google.com/file/d/1XYVysz76kTRfUz3snKN6nvqf3s2SHP36/view?usp=share_link



Cubes Puzzle

1)

The brute force search space is (4C1 * 24) x (3C1 * 24) x (2C1 * 24) x (1C1 * 24) = 7962624 arrangements large. This is assuming a standard dice with 6 different colors and each dice has 24 unique way to be placed (4 arrangements per color * 6 colors).
The way to get this search space is there are 4 dice to pick for the bottom most position and for each dice we have 6 possible combinations. Similarly for second position we have 3 dice and so on.

In case we chose from the given dice with repetitive colors, we get
Arrangements for dice 1 = 16 [Total Unique arrangements]
Arrangements for dice 2 = 24 [Total Unique arrangements]
Arrangements for dice 3 = 12 [Total Unique arrangements]
Arrangements for dice 4 = 24 [Total Unique arrangements]

Total Arrangements = 16*24*12*24* 4P4 = 2654208

The different values of different arrangements is due to the placement of common colors. In some places the same colors have no effect while in others, the same colors generate the same arrangements.
For the 4 dice given, we can have 4 permutations and thus we can multiply all the arrangements with total permutations to get the final search space.

2)

In cube 1, we have 3 red faces, we can hide 2 of the faces by placing them at the top and at the bottom and only use 1 red face. However, if we were to add another red face to the cube one, then we would have 2 red faces shown on 2 sides all the time. Given then we have 4 colors, there is a high chance that now no such possibility exist that satisfies the problem statement