Q1) 1. In this game, the player's actions have a positive effect on each other's payoffs, leading to mutual benefit. The positive coefficient α in the best-response function implies that if one player increases their action, it will increase the marginal benefit of other player's actions as well. This creates a situation where players have a tendency to coordinate their actions and select similar strategies.

Therefore, based on the given best-response function, the game is of strategic complements.

2. To find the unique equilibrium of the given game, we need to solve Nash Equilibrium, where no player has an incentive to deviate from their chosen strategy, given the strategies of the other players.

$$a = \begin{pmatrix} a_1 \\ a_n \end{pmatrix} \qquad b = \begin{pmatrix} b_1 \\ b_m \end{pmatrix} \qquad W$$

$$a = b + \alpha W a$$

Solving for a,

$$a - \alpha W a = b$$

$$[I - \alpha W] a = b$$

$$[I - \alpha W]^{-1}$$

In summary, the unique equilibrium of the game is given by $a^* = (a_1^*, a_2, a_3, \dots a_n)$ where $a_i = (\alpha / x_i)$

3) The network topology affects the threshold value of α that determines whether the equilibrium exists or not. If α is greater than or equal to the centrality of the least central ~~control~~ node in the network, then an equilibrium exists. But, if ~~there~~ α is less than the centrality of the least central node, then there is no equilibrium.

Network topology plays a crucial role in determining the unique equilibrium of the game and whether it exists or not. In this game, it is a irreducible matrix and a fully connected graph, and there are no isolated nodes. Thus, Nodes with high centrality are more influential in determinig the equilibrium, and the threshold value of α depends on the centrality of the least central node in the network.

4) If $\alpha < 0$, then the best response function (BR_i) for each agent i becomes -

$$BR_i(a_{-i}) = \alpha \sum_j w_{ij} a_j + b_i < b_i$$

Since $\alpha < 0$, the term $\alpha \sum_j w_{ij} a_j$ is negative, which means that each agent's best response is strictly decreasing in the actions of the other agents. Therefore, the game is no longer a strategic complement game, but instead becomes a strategic substitute game.

5) One measure to quantify the inefficiency in the Nash Equilibrium of the game ~~relative~~ is the Price of Anarchy (POA). PoA measures the ratio of the worst case social welfare in the Nash Equilibrium to the maximum ~~to~~ social welfare achievable.

$S \rightarrow$ set of all action profiles

$W(a) \rightarrow$ Social welfare function

socially optimal outcome -

$a^* = \text{argmax}\_ a \in S \; W(a)$

$a_N \rightarrow$ Nash Equilibrium

$PoA = \max\_ a \in S \; (W(a) / W(a_N))$

If $PoA = 1$, $a_N = a^*$

$PoA > 1$, a_N is socialy inefficient and the gap between a_N and a^* is larger

Thus, PoA can be a measure to quantify inefficiency.

Q2) 1) From the given figure, there can be two different ways to travel from town A to town B. Driver can either choose path ACB or ADB - two strategies.

Let's say 500 cars choose to travel from ACB and 500 from ADB - this will be Nash equilibrium which is yielded by equal balance.

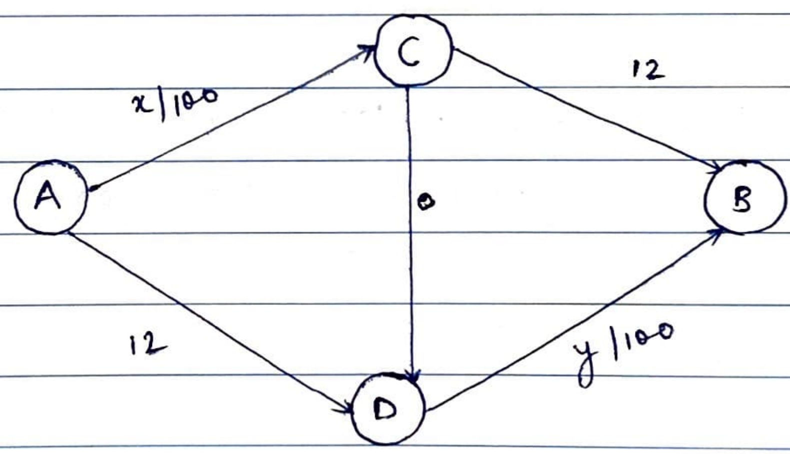
So, travel time for driver on ACB and ADB will be 500/100 + 12 = 17.

If x and y are not equal to 500, then, the two routes will have unequal travel times, and any driver can switch the routes any time. Thus, any values where x and y are not equal to 500; cannot be Nash equilibrium.

∴ Nash equilibrium value of $x = 500$

Nash equilibrium value of $y = 500$

2) The network after construction of new road from C to D -



Nash Equilibrium for new network -

If some driver chooses path ADB, the cost of travel is $12 + 1000/100 = 22$, this is because the driver travels using the path DB. The travel time can be reduced if the driver chooses the path ACDB, because the cost for this path is $(x + 1000 + 1000/100) < 22$.

Now, if a driver reaches point C, he won't choose route CB because cost of CDB is $1000/100 = 10 < 12$.

Thus, the driver will choose ACB and will reach B using path DB.

Hence, we can say that every driver will choose path ACDB and the Nash equilibrium for x and y is 1000.

The total cost of travel after new road -

For route AC, $1000/100 = 10$

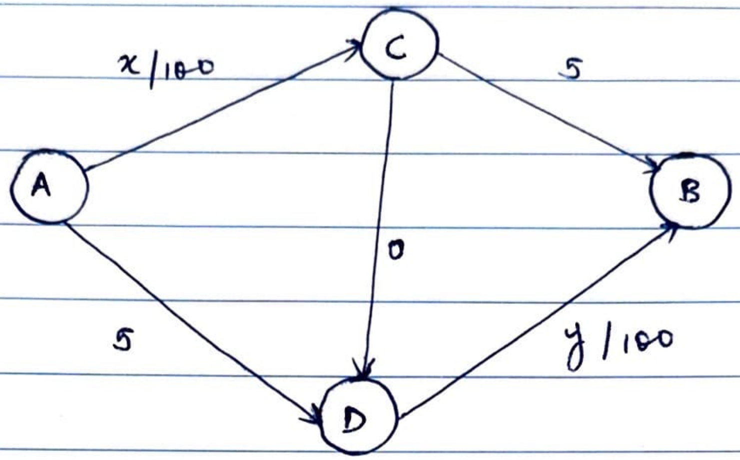
For path C, the cost will be 0

For path DB cost will be $1000/100 = 10$.

Thus, total travel cost $= 1000/100 + 0 + 1000/100 = 20$.

And total cost of network $= 20 \times 1000 = 20{,}000$.

3) Network -



Nash Equilibrium for this network -

when 500 cars choose ADB and other 500 choose ACB -

Travel time for both paths = 500 / 100 + 5 = 10. This is Nash equilibrium because if any driver switches the path the travel time wille be >10, which is not a good value.

Total cost of travel = 10 X 1000 = 10,000

Because no one chooses the road CD, the total cost of travel if the government closes the road from C to D will remain the same.