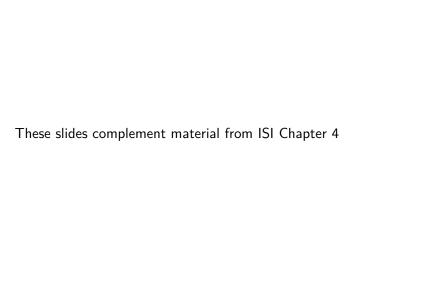
# Known Discrete Random Variables STAT-S520

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## Binomial distribution and PMF

Let  $X \sim Binomial(n, p)$ 

► The PMF of X is

$$f(x) = \binom{n}{x} p^{x} (1-p)^{n-x}$$

for x = 1, 2, ..., n, and f(x) = 0 for other values of x.

## PMF of binomial disribution using R

In R we use the function dbinom(). For the birthday party example  $X \sim Binomial(50, 0.8)$ . The probability that exactly 30 friends attend the party:

$$f(30) = P(X = 30) = {50 \choose 30} 0.8^{30} (1 - 0.8)^{50 - 30}.$$

can be calculated in R as follows:

$$dbinom(x = 30, size = 50, prob = 0.8)$$

## [1] 0.0006117722

The argument x in the R function is the same x that appears in our f(x) math notation. The arguments size and prob correspond to the n and p parameters in the binomial distribution, respectively.

# PMF of binomial disribution using R

## [1] 0.0006117722

As long as you enter the arguments in the proper order, you do not need to include the argument names, simply the values. Observe how both lines below give the same result:

```
dbinom(x = 30, size = 50, prob = 0.8)
## [1] 0.0006117722
dbinom(30, 50, 0.8)
```

### The CDF of the binomial distribution

The CDF is given by

$$F(y) = \sum_{x: x \in L(y)} f(x)$$

where L(y) is the set of values in X(S) that are less than or equal to y. If  $y \in X(S)$  this is simply

$$F(y) = \sum_{x=0}^{y} f(x)$$

# CDF of binomial disribution using R

For the birthday party example  $X \sim Binomial(50, 0.8)$ . If we want to find the probability that 30 friends or less attend the party, then

$$F(30) = P(X \le 30) = F(30) = \sum_{x=0}^{30} f(x) = f(0) + f(1) + \dots + f(30)$$

In R we use the function pbinom():

$$pbinom(q = 30, size = 50, prob = 0.8)$$

```
## [1] 0.0009324365
```

# Relating PMF and CDF in R

For the b-day party example  $X \sim Binomial(50, 0.8)$ , there are three ways to find  $F(7) = P(X \le 7) = f(0) + f(1) + \cdots + f(7)$ :

```
pbinom(7, 50, 0.8)
```

```
## [1] 1.918335e-23
```

```
dbinom(0,50, 0.8)+dbinom(1,50, 0.8)+dbinom(2,50, 0.8)+
dbinom(3,50, 0.8)+dbinom(4,50, 0.8)+dbinom(5,50,0.8)+
dbinom(6,50, 0.8)+dbinom(7,50, 0.8)
```

```
## [1] 1.918335e-23
```

```
sum(dbinom(0:7, 50, 0.8))
```

```
## [1] 1.918335e-23
```

#### Exercise 1

For the b-day party example  $X \sim Binomial(50, 0.8)$ , find the probability that:

- You do not celebrate alone.
- At most 35 friend attend.
- At least 24 friend attend.
- ▶ No more than 15 but less than 40 friends attend.

Let's work in R

# Expected value, variance, and standard deviation for the binomial distribution

If 
$$X \sim Binomial(n, p)$$
 then  $EX = np$ ,  $VarX = np(1 - p)$ , and  $SD = \sqrt{np(1 - p)}$ .

In our b-day party example with  $X \sim Binomial(50,0.8)$  we expect for  $50 \cdot 0.8 = 40$  friends to attend give or take  $\sqrt{50 \cdot 0.8 \cdot 0.2} = \sqrt{8} \approx 2.83$  friends.

## The geometric distribution

▶ X is the number of trials before a success.  $X(S) = \{0, 1, ...\}$  then

$$X \sim geometric(p)$$

- $f(x) = p(1-p)^x$
- ► EX = (1 p)/p
- ►  $VarX = (1 p)/p^2$
- R: dgeom (PMF) and pgeom (CDF)

## The Poisson distribution

- ▶ X is a Poisson distribution with  $X(S) = \{0, 1, ...\}$  and we write  $X \sim Poisson(\lambda)$
- $f(x) = P(X = x) = \frac{\lambda^x e^{-\lambda}}{\sqrt{1}}$
- $\triangleright$   $EX = \lambda$
- $ightharpoonup Var X = \lambda$
- R: dpois (PMF) and ppois (CDF)