Random Variables 1

STAT-S520

Arturo Valdivia

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These slides complement ISI Section 3.5

Random variable

- A random variable (RV) is a function that assigns a real number to each outcome of an experiment.
  - We use uppercase letters, sometimes with sub-indices, to denote random variables; e.g., X, Y, X_1, Z_3 etc.
  - If X is a random variable we can write $X : S \to \mathbb{R}$

Example 1

- Experiment: Toss a fair coin twice and observe the top faces
- The sample space can be given by $S = \{HH, HT, TH, TT\}$
- Let X be a random variable that assigns to each outcome its total number of heads. We then have:
  - $X(HH) = 2$,
  - $X(HT) = 1$,
  - $X(TH) = 1$, and
  - $X(TT) = 0$

The range of a random variable

- The range of a random variable is the set of all the numbers assigned to each possible outcome
  - If X is a random variable with corresponding sample space S, we write $X(S)$ to denote the range of X
  - In our previous example $S = \{HH, HT, TH, TT\}$ and $X(S) = \{0, 1, 2\}$

Events and random variables

- We can define events based on random variables
- Let S be the sample space and X a random variable
- For any real number $y \in \mathbb{R}$ we can define

$$\{s \in S : X(s) \leq y\}$$

- The expression above is an event (set of outcomes) which depends on y
  - A random variable requires that for any number y the event defined above has a well defined probability
  - This is always true if the sample space if finite

Exercise 1

Using the example above, recall that if we toss a fair coin twice, $S = \{HH, HT, TH, TT\}$, and X assigns the number of heads to each outcome. Let's find the probability for event

$$\{s \in S : X(s) \leq y\}$$

for different values of y

- If $y = 10$ then $P\left(\{s \in S : X(s) \leq 10\}\right) =$
- If $y = -3$ then $P\left(\{s \in S : X(s) \leq -3\}\right) =$
- If $y = \pi \cdot (1/2)^2$ then $P\left(\{s \in S : X(s) \leq \pi \cdot (1/2)^2\}\right) =$

Cumulative distribution function (CDF)

The exercise above illustrates a very useful function. Let X be a random variable. The cumulative distribution function (CDF) of X,

$$F : \mathbb{R} \to [0, 1]$$

is defined as

$$F(y) = P(\{s \in S : X(s) \leq y\})$$

for any $y \in \mathbb{R}$

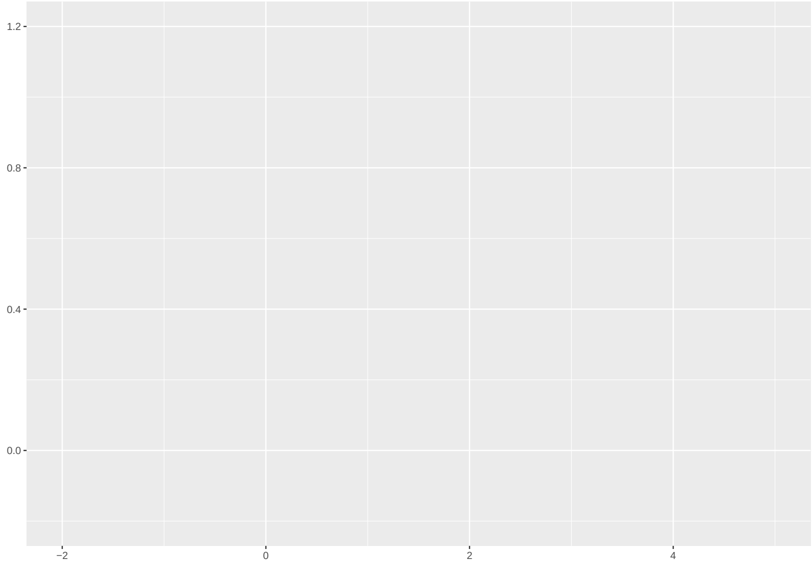
Example 1 continued

We toss a fair coin twice, $S = \{HH, HT, TH, TT\}$, and X assigns the number of heads to each outcome. Then

- $F(10) =$
- $F(-3) =$
- $F(\pi \cdot (1/2)^2) =$

Example 1 continued: The function F for the coin example

Example 1 continued: The graph of F for the coin example



Exercise 1

- Experiment: Draw 2 tickets with replacement from the urn

$$[1, 1, 1, 1, 1, 3, 3, 3, 7, 7]$$

and let Y be the random variable that assigns the sum of both tickets

a. What is the sample space, S?
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b. What is the range of Y, $Y(S)$?
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c. If F_Y is the CDF of Y, what is $F(3\pi)$?
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