Continuous Random Variables 1

STAT-S520

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02-02-23

These slides complement ISI Chapter 5

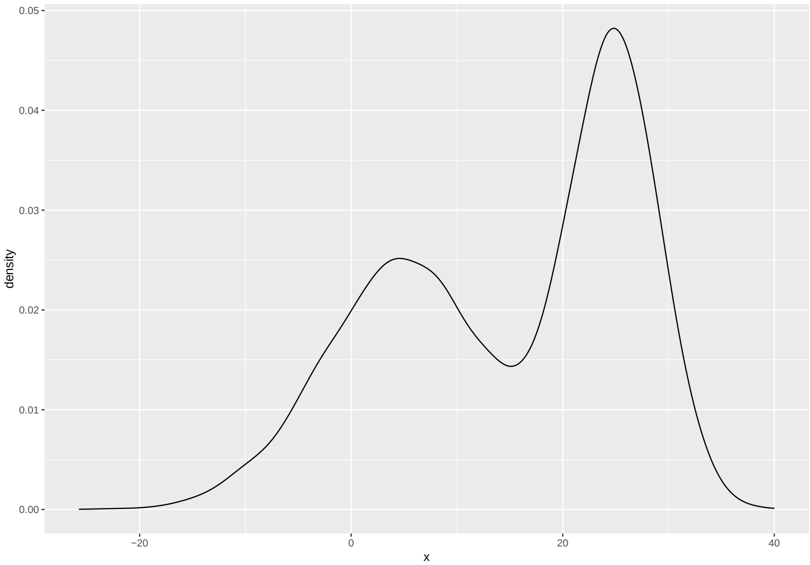
Probability density function (PDF)

A probability density function (PDF) is a function $f : \mathbb{R} \to \mathbb{R}$ such that

1. $f(x) \geq 0$ for every $x \in \mathbb{R}$
2. $Area_{(-\infty,\infty)}(f) = \int_{-\infty}^{\infty} f(x)dx = 1$

Note: The function f here is different than the PMF (also f) that we used for discrete random variables.

PDF Example



Continuous random variable

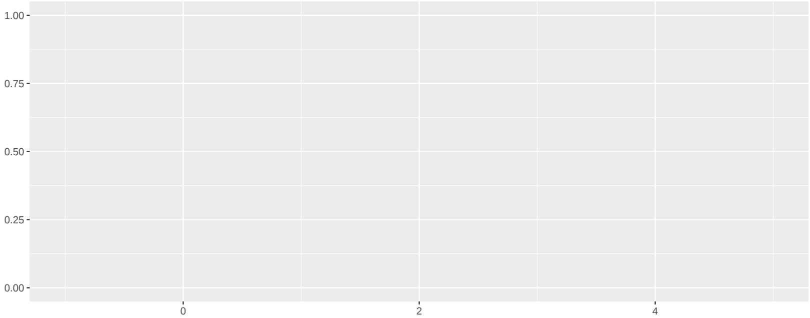
A random variable X is continuous if there exists a probability density function f such that

$$P(X \in [a, b]) = \int_a^b f(x)dx$$

Example 1

Let X be a random variable with PDF given by

$$f(x) = \begin{cases} 0 & x \in (-\infty, 0) \\ 1/4 & x \in [0, 4) \\ 0 & x \in [4, \infty) \end{cases}$$



What is $P(X \in (1, 2))$?

Continuous random variable and CDF

The CDF of a continuous random variable X is defined as before:

$$F(y) = P(X \leq y)$$

Based on the definition of a continuous random variable observe that:

$$F(y) = P(X \leq y) = P(X \in (-\infty, y]) = \int_{-\infty}^{y} f(x)dx$$

CDF and PDF

By applying the Fundamental Theorem of Calculus, the PDF of a continuous random variable is the derivative of its CDF:

$$\frac{d}{dy}F(y) = \frac{d}{dy}\int_{-\infty}^{y} f(x)dx = f(y)$$

Expected value and variance

If X is a continuous random variable the expected value of X is

$$\mu = EX = \int_{-\infty}^{\infty} xf(x)dx$$

and the variance is

$$\sigma^2 = VarX = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)dx$$

Properties of the expected value and variance

The properties are the same as those for dicrete random variables. Let X, Y be random variables and a, b scalars (constant values).

- $E(a + X) = a + EX$
- $E(bX) = bEX$
- $E(X + Y) = EX + EY$

If in addition, X and Y are independent:

- $Var(a + X) = VarX$
- $Var(bX) = b^2 \cdot VarX$
- $Var(X + Y) = VarX + VarY$

Exercise 1: ISI Section 5.6. Exercise 5

Exercise 2: ISI Section 5.6. Exercise 4