# Quantiles

## STAT-S520

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- These slides complement material from ISI Chapter 6

# Quantile

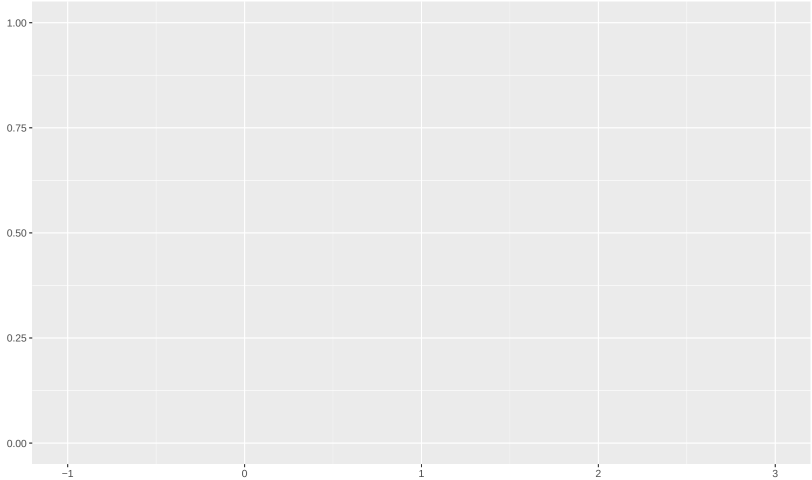
Let X be a random variable and $\alpha \in (0, 1)$. Let $q = q(X; \alpha)$ a function such that:

$$P(X < q) \leq \alpha \qquad \text{and} \qquad P(X > q) \leq 1 - \alpha$$

then $q$ is called the $\alpha$-quantile of $X$.

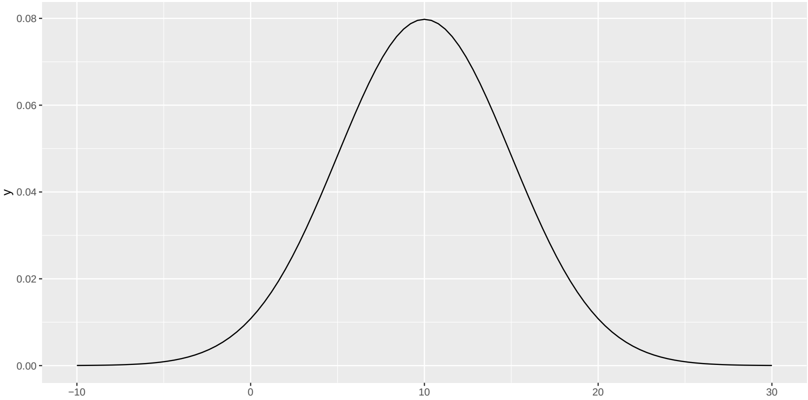
# Example 1:

Let $X \sim Uniform(0, 2)$, let's find the 0.6-quantile of $X$.



# Example 2:

Let $Y \sim Normal(10, 25)$, let's find the 0.6-quantile of $Y$. In R we use `qnorm()`.




```r
qnorm(p = 0.6, mean = 10, sd = sqrt(25))
```

# Example 3:

Let $X$ be discrete with PMF

$$f(x) = \begin{cases} 0.4 & x = 1 \\ 0.4 & x = 2 \\ 0.2 & x = 3 \\ 0 & \text{otherwise} \end{cases}$$

What are the 0.6, 0.7, and 0.8-quantiles of $X$?

# Note

- If the random variable is continuous, there is a single $q$ for each $\alpha$ (one-to-one correspondence).
- If the random variable is discrete, the one-to-one breaks down for certain regions, for both $q$ and $\alpha$.

# Commonly used terminology

- Quartiles: $q_1(X)$, $q_2(X)$, and $q_3(X)$
  - They divide the range of $X$ in four equal parts.
  - E.g., the first quartile is the 0.25-quantile
  - The median is the second quartile, $q_2(X)$
- Percentiles: They divide the range of $X$ in 100 equal parts
  - There are 99 percentiles: the 1st, 2nd, . . . , 99th.
  - E.g., the 57th percentile is the 0.57-quantile
- The interquartilerange (IQR) of $X$ is

$$iqr(X) = q_3(X) - q_1(X)$$

# Example 4:

Let $Y \sim Normal(10, 25)$, let's find the 83th percentile and the IQR of $Y$

# Symmetry

- Let $X$ be a continuous random variable with PDF f. If there exists a value $\theta \in \mathbb{R}$ such that

$$f(\theta + x) = f(\theta - x)$$

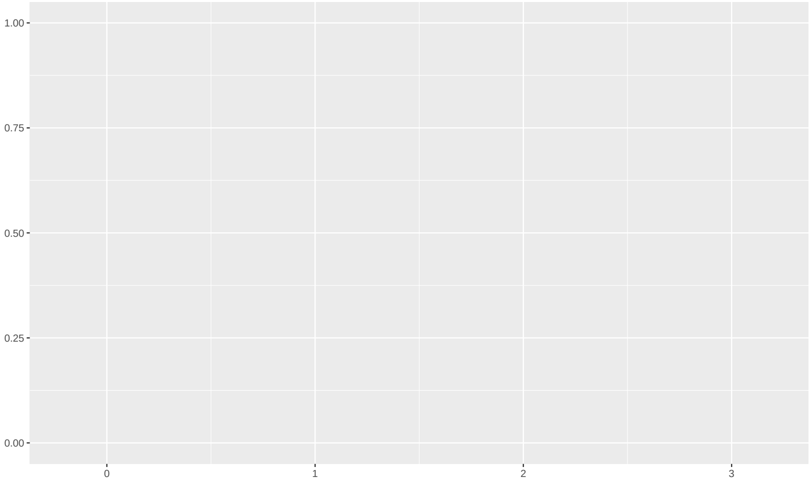
  for every $x \in \mathbb{R}$, then $X$ is a symmetric random variable and $\theta$ is its center of symmetry
- If $Y$ is not symmetric, there is not a single way to measure centrality

# Theorem 6.1

Let $X$ be a random variable with population median $q_2$ and population mean $\mu = EX$. Then

1. The value of $c$ that minimizes $E|X - c|$ is $c = q_2$
2. The value of $c$ that minimizes $E(X - c)^2$ is $c = EX$

# ISI 6.4 Exercise 2



# ISI 6.4 Exercise 2 (cont.)

# ISI 6.4 Exercise 7