Two sample problems

$X_1, X_2, \ldots, X_{n_1} \sim \mathbb{P}_1 \qquad EX_i = \mu_1 \quad Var\, X_i = \sigma_1^2 \qquad i = 1, \ldots, n_1$

Independent

$Y_1, Y_2, \ldots, Y_{n_2} \sim \mathbb{P}_2 \qquad EY_j = \mu_2 \quad Var\, Y_j = \sigma_2^2 \qquad j = 1, \ldots, n_2$

$$\bar{X}_{n_1} \sim N\left(\mu_1, \frac{\sigma_1^2}{n_1}\right)$$

$$\bar{Y}_{n_2} \sim N\left(\mu_2, \frac{\sigma_2^2}{n_2}\right)$$

$H_0: \mu_1 = \mu_2 \qquad \overbrace{\mu_1 - \mu_2}^{\Delta} = 0$ Delta

$H_1: \mu_1 \neq \mu_2 \qquad \overbrace{\mu_1 - \mu_2}^{\Delta} \neq 0$

$$\boxed{\bar{X}_{n_1} - \bar{Y}_{n_2} \sim N\left(\overbrace{\mu_1 - \mu_2}^{\Delta}, \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}\right)}$$

Our first task is to recognize whether the problem is a 1-sample or a 2-sample problem

Problem Set B Each of the following scenarios can be modelled as a 1- or 2-sample location problem. For 1-sample problems, let X_i denote the random variables of interest and let $\mu = EX_i$. For 2-sample problems, let X_i and Y_j denote the random variables of interest; let $\mu_1 = EX_i$, $\mu_2 = EY_j$, and $\Delta = \mu_1 - \mu_2$. For each scenario, you should answer/do the following:

(a) What is the experimental unit?

(b) From how many populations were the experimental units drawn? Identify the population(s). How many units were drawn from each population? Is this a 1- or a 2-sample problem?

(c) How many measurements were taken on each experimental unit? Identify them.

(d) Define the parameter(s) of interest for this problem. For 1-sample problems, this should be μ; for 2-sample problems

(b) From how many populations were the experimental units drawn? Identify the population(s). How many units were drawn from each population? Is this a 1- or a 2-sample problem?

(c) How many measurements were taken on each experimental unit? Identify them.

(d) Define the parameter(s) of interest for this problem. For 1-sample problems, this should be μ; for 2-sample problems, this should be Δ.

(e) State appropriate null and alternative hypotheses.

Example

5. A political scientist theorizes that women tend to be more opposed to military intervention than do men. To investigate this theory, he devises an instrument on which a subject responds to several recent U.S. military interventions on a 5-point Likert scale (1="strongly support,"...,5="strongly oppose"). A subject's score on this instrument is the sum of his/her individual responses. The scientist randomly selects 50 married couples in which neither spouse has a registered party affiliation and administers the instrument to each of the 100 individuals so selected. How might he use his results to determine if his theory is correct? (Respond to (a)–(e) above.)

- The experimental unit is a married couple
- This is a 1-sample problem

Two ~~m~~ measurements were taken per experimental unit.

$$X_i = W_i - H_i \qquad i = 1, \ldots, 50$$

$$\mu = E\bar{X}_{50}$$

$$H_0: \mu \le 0$$

$$H_1: \mu > 0$$