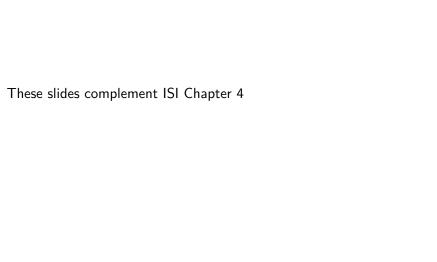
# Discrete Random Variables 1 STAT-S520

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01-26-23



#### Discrete random variable

- Let X be a random variable. X is discrete if X(S) is countable
  - ► A set is countable if it is finite or denumerable
  - $\blacktriangleright$  A set is denumerable if there is a one-to-one correspondence with  $\mathbb N$

- ► Let  $X(S) = \{0, 1, 2\}$ , then X is . . .
- ▶ Let  $Y(S) = \mathbb{N}$  (naturals), then Y is . . .
- ▶ Let  $X_1(S) = \mathbb{R}$  (reals), then  $X_1$  is . . .
- ▶ Let  $X_2(S) = \mathbb{Z}$  (integers), then  $X_2$  is . . .
- ▶ Let  $X_3(S) = \mathbb{Q}$  (rationals), then  $X_3$  is . . .
- ▶ Let Z(S) = [0, 1], then Z is . . .

# Probability Mass Function

Let X be a discrete random variable. The probability mass function (PMF) of X is a function f, with  $f: \mathbb{R} \to [0,1]$ , such that, for any  $y \in \mathbb{R}$ :

$$f(y) = \begin{cases} P(X = y) & \text{if } y \in X(S) \\ 0 & \text{otherwise} \end{cases}$$

We toss a fair coin twice and X is the number of heads. What is

- ightharpoonup f(1) =
- ightharpoonup f(2) =
- $ightharpoonup f(\pi) =$
- ightharpoonup f(10) =

## CDF and PMF

If X is a discrete random variable with range X(S) and for any given  $y \in \mathbb{R}$  the relationship between the PMF and CDF is given by:

$$F(y) = \sum_{x: x \in L(y)} f(x)$$

where L(y) is the set of numbers in the range of X that are less than or equal to y, or in math notation:  $L(y) = X(S) \cap (\infty, y]$ ,

We toss a fair coin twice and X is the number of heads. Let's find  $F(\sqrt(2))$  using the PMF

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#### Bernoulli trials

- A Bernoulli trial is a random variable where  $X(S) = \{0, 1\}$ .
  - It is customary to call X = 1 a success and X = 0 a failure.
- ► There is a family of Bernoulli trials with parameter p where p = P(X = 1) and we write

$$X \sim \text{Bernoulli}(p)$$

# PMF of a Bernoulli trial

▶ If  $X \sim \text{Bernoulli}(p)$ , the PMF of X is given by

$$f(x) = \begin{cases} p & \text{if } x = 1\\ 1 - p & x = 0\\ 0 & \text{otherwise} \end{cases}$$

We can simplify this notation by writing

$$f(x) = p^{x}(1-p)^{1-x}$$

for  $x \in \{0,1\}$  and f(x) = 0 otherwise.

## Binomial distribution

- Let  $X_1, X_2, ..., X_n$  independent Bernoulli trials with parameter p
- ▶ We construct a random variable as follows:

$$Y = \sum_{i=1}^{n} X_i = X_1 + \dots + X_n$$

We then say that Y follows a binomial distribution with parameters n and p and write

$$Y \sim binomial(n, p)$$

- ▶ You invite 50 friends to your birthday party. We assume that
  - Friends don't influence each other
  - Each friend has the same chance to attend
- ▶ There is a 0.8 chance that any given friend attends the party.

# PMF of a Binomial distribution

If  $Y \sim binomial(n, p)$ , the PMF of Y is given by

$$f(x) = P(Y = x) = \binom{n}{x} p^{x} (1 - p)^{n-x}$$

for  $x \in Y(S)$  and f(x) = 0 otherwise.

# Expected value

▶ Let X be a discrete random variable. The expected value of X is given by

$$EX = \sum_{x \in X(S)} x \cdot f(x)$$

► The expected value can be understood as the long-run average of values that X assigns when the experiment is performed many times

## Variance

► Let *X* be a discrete random variable. The variance of *X* is given by

$$VarX = E\left((X - \mu)^2\right) = \sum_{x \in X(S)} (x - \mu)^2 \cdot f(x)$$

The standard deviation of X is the square root of the variance

# Properties of the expected value and variance

Let X, Y be random variables and a, b scalars (constant values).

- $\triangleright$  E(a+X)=a+EX
- $\triangleright$  E(bX) = bEX
- $\triangleright$  E(X + Y) = EX + EY

If in addition, X and Y are independent:

- ightharpoonup Var(a+X) = VarX
- $ightharpoonup Var(bX) = b^2 \cdot VarX$
- ightharpoonup Var(X+Y)=VarX+VarY

Let X, Y be independent random variables with EX=1, EY=2, VarX=4, VarY=9. Find the expected value and variance of 2X+1 and X-Y