Inference Part 2

STAT-S520

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- These slides complement material from ISI Chapter 9

Hypothesis Testing

- A claim about μ is made (null hypothesis) about the mean of the population
- A random sample from this population is obtained and used to obtain the sample mean $\bar{x}_n$
- Assuming the claim about μ is correct, we determine how likely is to observe a value as extreme as or more extreme than $\bar{x}_n$
  - If, getting $\bar{x}_n$ is very unlikely, we reject the claim (reject the null hypothesis)

Hypotheses

- The null hypothesis, H_0, is what we initially assume to be true. It's a statement about μ.
  - For procedural reasons, we **always** include the equal sign in the statement under H_0
- The alternative hypothesis, H_1, is what we would conclude (about μ) if we were to reject H_0.

ISI Example 9.3

Test Statistic

The statistic is the method we use to assess the claim made under the null hypothesis. Many statistics exist, the one we encounter often looks like this:

$$statistic = \frac{estimator - parameter\ under\ H_0}{standard\ error}$$

Hypothesis about μ when σ is known

When the hypothesis is made about μ, and σ is known, the test statistic is given by

$$Z = \frac{\bar{X}_n - \mu_0}{\sigma/\sqrt{n}}$$

and due to the CLT, $Z \sim N(0, 1)$.

Hypothesis about μ when σ is unknown

When σ is unknown (as in most real-life problems), the test statistic is given by

$$T = \frac{\bar{X}_n - \mu_0}{S_n/\sqrt{n}}$$

where $S_n = \sqrt{\frac{\sum(X_i - \bar{X}_n)^2}{n-1}}$ is the sample standard deviation, another estimator. Given the added uncertainty of S_n, T is no longer normal, but T follows a T-distribution with $n-1$ degrees of freedom and we write $T \sim T_{n-1}$

Observed test statistic

Once you collect a (random) sample of n observations, the sample observed is $\overrightarrow{x} = (x_1, \ldots, x_n)$. If σ is known, the observed test statistic is

$$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$$

and if σ is unknown, the observed test statistic is

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

where $s = \sqrt{\frac{\sum(x_i - \bar{x})^2}{n-1}}$ is the sample standard deviation.

Significance Probability (p-value)

- The p-value is the probability of observing a test statistic that is as extreme or more extreme than the one observed in our data, when we assume that H_0 is true.
- The p-value depends on the hypotheses statements made

Conclusion and Interpretation of results

- We want to reject H_0 if the observed estimate is highly unlikely to have been obtained by chance
  - The smaller the p-value, the more evidence to reject H_0.
- While how small p-value needs to be is somewhat arbitrary, it should be guided by how important is not to make a mistake by rejecting H_0 when it is actually true (Type I Error) or by failing to reject H_0 when it is actually false (Type II Error).

Reasonable ranges for the p-value

To guide you in the decision process, some reasonable values (although still arbitrary) can be:

- If p-value > 0.1, do not reject H_0.
- If p-value < 0.001, reject H_0.
- If $0.001 \leq$ p-value ≤ 0.1, decide based on your own perception of this uncertainty (different people may make different decisions).
- Alternatively, come up with a significance level, α, such that if p-value $\leq \alpha$, we reject H_0, and if p-value $> \alpha$, we fail to reject H_0.
  - The choice of α should be set before collecting and/or observing the random sample.

ISI Example 9.3 (continued)