Sammanfattning av SI2360 Analytisk mekanik och klassisk fältteori

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Sammanfattning

Innehåll

1 Coordinates **1**

1 Coordinates

Coordinates A general set of coordinates on $\mathbb{R}^n$ is n numbers $x^a, a = 1, \ldots, n$ that uniquely define a point in the space.

Cartesian coordinates In cartesian coordinates we introduce an orthonormal basis $\mathbf{e}_i$. Vi kan då skriva $\mathbf{x} = x^i \mathbf{e}_i$.

Basis vectors There are two different choices of coordinate bases.

The first is the tangent basis of vectors

$$\mathbf{E}_a = \partial_{x_a} \mathbf{x} = \partial_a \mathbf{x}.$$

The second is the dual basis

$$\mathbf{E}^a = \vec{\boldsymbol{\nabla}} x^a.$$

Vector coordinates Any vector can now be written as

$$\mathbf{v} = v^a \mathbf{E}_a = v_a \mathbf{E}^a.$$

The v^a are called contravariant components and the v_a are called covariant components.

We can now compute the scalar product

$$\mathbf{E}_a \cdot \mathbf{E}^b = \partial_a \mathbf{x} \cdot \vec{\boldsymbol{\nabla}} x^a = \delta_a^b.$$

Coordinate transformations Suppose that a vector can be written as

$$\mathbf{v} = v^a \mathbf{E}_a = v^{a'} \mathbf{E}'_{a'}.$$

How do we transform between these? A single component is given by

$$v^{a'} = \mathbf{E}'_{a'} \cdot v^a \mathbf{E}_a = v^a (\vec{\boldsymbol{\nabla}} x'^{a'} \cdot \partial_a \mathbf{x}) = v^a \partial_a x'^{a'}.$$

Tangents to curves The tangent to a curve is given by

$$\dot{\gamma} = \frac{\mathrm{d}\mathbf{x}}{\mathrm{d}t} = \partial_a \mathbf{x} \frac{\mathrm{d}x^a}{\mathrm{d}t} = \dot{x}^a \mathbf{E}_a.$$

Gradients The gradient of a curve is given by

$$\vec{\boldsymbol{\nabla}} f = \partial_a f \vec{\boldsymbol{\nabla}} x^a = \mathbf{E}^a \partial_a f.$$

Rates of change along a curve The rate of change of a quantity along a path is given by

$$\frac{\mathrm{d}f}{\mathrm{d}t} = \partial_a f \frac{\mathrm{d}x^a}{\mathrm{d}t} = \vec{\boldsymbol{\nabla}} f \cdot \dot{\gamma}.$$

2 Tensors

Definition A tensor of rank N is a linear map from N vectors to a scalar.

Components of a tensor The components of a tensor are defined by

$$T(\mathbf{E}^{a_1}, \ldots, \mathbf{E}^{a_N}) = T^{a_1, \ldots, a_N}.$$