Summary of SI2510 Statistical Mechanics

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Abstract

This is a summary of SI2510 Statistical Mechanics.

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1 Basic Concepts

Phase Transitions Landau introduced the concept that phase transitions are defined by spontaneous symmetry breaking.

Order Parameters An order parameter describes spontaneous symmetry breaking. It is zero in one phase and non-zero in another.

Density Matrices The probability distribution is of the form

$$p_n = \frac{1}{\sum_m P_m} P_n = \frac{1}{Z} P_n,$$

where the summation is performed over some set of states. We now introduce the density matrix

$$\rho = \frac{1}{Z} \sum_n P_n \left|n\right\rangle\!\left\langle n\right|,$$

yielding

$$\langle O \rangle = \sum_n p_n O_{nn} = \sum_n \frac{1}{Z} P_n \langle n|O|n\rangle = \frac{1}{Z} \sum_n \sum_m P_n \langle n|m\rangle \langle m|O|n\rangle = \frac{1}{Z} \sum_n \sum_m \langle n|\rho|m\rangle \langle m|O|n\rangle = \mathrm{tr}(O\rho).$$

As a side note, ρ takes the form

$$\rho = \frac{1}{Z} e^{-\beta K},$$

where K is the Hamiltonian in the canonical ensemble and $H - \mu N$ in the grand canonical ensemble. In these cases, the partition function Z may be computed according to

$$Z = \sum_m P_m = \sum_m e^{-\beta K_m} = \sum_m \langle m|e^{-\beta K}|m\rangle = \mathrm{tr}\Big(e^{-\beta K}\Big).$$

The Ising Model The Ising model is a simple model of magnets. In this model, a magnet is a collection of interacting spins on a lattice under the influence of an external field. Its generalized coordinates are σ_i, which may take the values ± 1, signifying a particular spin pointing up or down. The Hamiltonian is

$$\mathcal{H} = -J \sum_i \sum_{j=\mathrm{nn}(i)} \sigma_i \sigma_j - h \sum_i \sigma_i.$$

The inner summation is carried out over the nearest neighbours of site i in the Ising model, but is generally a sum over the whole lattice. The order parameter defining its phase transition is $m = \langle \sigma_i \rangle$.

This model will be used to demonstrate core concepts in the course.