Summary of SI2380 Advanced Quantum Mechanics

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Abstract

This is a summary of SI2380 Advanced Quantum Mechanics.

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1 Basic Concepts

Observables An observable is a Hermitian operator whose orthonormal eigenvectors form a basis.

The Postulates of Quantum Mechanics The postulates of quantum mechanics are:

- At any fixed time the state of a physical system is specified by a ket in Hilbert space.
- Every measurable physical quantity corresponds to an operator on Hilbert space. This is a Hermitian observable. The possible outcomes of a measurement are the eigenvalues of A.
- The probability of measuring the value a of operator A in a normalized state $|\Psi\rangle$ is $P(a) = \langle\Psi|P_a|\Psi\rangle$, where P_a is the projector onto the subspace corresponding to the eigenvalue a given by $P_a = |a\rangle\langle a|$.
- If a measurement of an observable A gives an outcome a, the state of the system immediately after the measurement is the projection of the state onto the subspace with eigenvalue a.

Consequences of the Probability Picture The form of writing the projection operator implies $P(a) = |\langle a|\Psi\rangle|^2$, or $P(a)\,\mathrm{d}a = |\langle a|\Psi\rangle|^2\,\mathrm{d}a$ in the continuous case. In order for the probability interpretation to be consistent, i.e. for the sum of all probabilities to amount to 1, it must hold that $\langle\Psi|\Psi\rangle = 1$.

Expectation Values Expectation values are given by

$$\langle A\rangle = \sum aP(a) = \sum a\,\langle\Psi|P_a|\Psi\rangle = \langle\Psi|\sum a\,|a\rangle\,\langle a|\Psi\rangle = \langle\Psi|A|\Psi\rangle\,.$$

Physical States Modifying a state by a phase factor $e^{i\alpha}$ does not change any expectation values.

Mixed States

Density Matrix The density matrix is defined as

$$\rho = |\Psi\rangle\langle\Psi|\,.$$

It has some cool properties. For instance:

$$\begin{aligned}
&\mathrm{tr}\{\rho\} = \sum_n \langle n|\rho|n\rangle = \left\langle\psi\middle|\sum_n |n\rangle\langle n|\middle|\psi\right\rangle = \langle\Psi|\Psi\rangle = 1,\\
&\rho^\dagger = \rho,\\
&\langle A\rangle = \sum_{n,m}\langle\Psi|n\rangle\,\langle n|A|m\rangle\,\langle m|\Psi\rangle = \sum_{n,m}\langle m|\Psi\rangle\,\langle\Psi|n\rangle\,\langle n|A|m\rangle = \sum_{n,m}\langle m|\rho|n\rangle\,\langle n|A|m\rangle = \mathrm{tr}(\rho A),\\
&\rho^2 = \rho.
\end{aligned}$$

Note that the latter is only true for pure states. Mixed states have a density matrix of the form

$$\rho = \sum_j P_j\,|\Psi_j\rangle\langle\Psi_j|\,.$$