Notes for the Master Thesis

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Sammanfattning

This is a collection of notes pertaining to concepts I needed to learn for my master's thesis.

Innehåll

1 Topology

Topological Spaces Let X be a set and $T = \{U_i | i \in I\}$ be a collection of subsets of X (I is some set of indices). The pair (X, T) (sometimes we only explicitly write X) is defined as a topological space if

- $\emptyset,\ X \in T$.
- If J is any subcollection of I, the family $\{U_j | j \in J\}$ satisfies

$$\bigcup_{j \in J} U_j \in T.$$

- If J is any finite subcollection of I, the family $\{U_j | j \in J\}$ satisfies

$$\bigcap_{j \in J} U_j \in T.$$

If the two satisfy the definition, we say that T gives a topology to X. The U_i are called its open sets.

Two cases of little interest are $T = \{\emptyset,\ X\}$ and T being the collection of all subsets of X. The two are called the trivial and discrete topologies respectively.

Metrics A metric is a map $d : X \times X \to \mathbb{R}$ that satisfies

- $d(x, y) = d(y, x)$.
- $d(x, x) \geq 0$, with equality applying if and only if $x = y$.
- $d(x, y) + d(y, z) \geq d(x,)z$.

Metric Space Suppose X is endowed with a metric. The collection of open disks

$$U_\varepsilon = \{x \in X | d(x, x_0) < \varepsilon\}$$

then gives a topology to X called the metric topology. The pair forms a metric space.

Continuous Maps A map between two topological spaces X and Y is continuous if its inverse maps an open set in Y to an open set in X.

Neighborhoods N is a neighborhood of x if it is a subset of X and x belongs to at least one open set contained within N.

Hausdorff Spaces A topological space is a Hausdorff space if, for any two points x, y, there exists neighborhoods U_x, U_y of the two points that do not intersect. This is an important type of topological space, as examples in physics are practically always within this category.

Homeomorphisms A homeomorphism (not to be confused with a homomorphism, however hard it may be) is a continuous map between two topological spaces with a continuous inverse. Two topological spaces are homeomorphic if there exists a homeomorphism between them.

Donuts and Coffee Mugs Homeomorphicity defines an equivalence relation between topological spaces. This means that we can define topological spaces into categories based on homeomorphicity.

We are now in a position to introduce the poor man's notion of topology, which considers two bodies as equivalent if one can be deformed into the other without touching two parts of the surface or tearing a part of the body. These continuous deformations correspond to homeomorphisms, but we will try to keep the discussions more to the abstract.