# Summary of SH2372 General Relativity

Yashar Honarmandi
yasharh@kth.se

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**Abstract**

This is a summary of SH2372 General Relativity.

The course opens with a discussion of differential geometry. As I have extensive notes on the subject in my summary of SI2360, I only keep the bare minimum in this summary and refer to those notes for details.

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# 1 Differential Geometry

For details on much of this, notably the early parts on Euclidean space, please consult my summary of SI2360 Analytical Mechanics and Classical Field Theory.

**Euclidean and Affine Spaces** A Euclidean space is a set of points such that there to each point can be assigned a position vector. To such spaces we may assign a set of $n$ coordinates $\chi^a$ which together uniquely describe each point in the space locally.

**Tangent and Dual Bases** The tangent and dual bases are defined by

$$\mathbf{E}_a = \partial_{\chi^a}\mathbf{r} = \partial_a\mathbf{r},\ \mathbf{E}^a = \vec{\boldsymbol{\nabla}}\chi^a.$$

Using such bases, we may write

$$\mathbf{v} = v^a\mathbf{E}_a = v_a\mathbf{E}^a.$$

The components of these vectors are called contravariant and covariant components respectively.

**Christoffel Symbols** When computing the derivative of a vector quantity, one must account both for the change in the quantity itself and the change in the basis vectors. We define the Christoffel symbols according to

$$\partial_b\mathbf{E}_a = \Gamma^c{}_{ba}\mathbf{E}_c.$$

These can be computed according to

$$\mathbf{E}^c \cdot \partial_b\mathbf{E}_a = \mathbf{E}^c \cdot \Gamma^d{}_{ba}\mathbf{E}_d = \delta^c_d\Gamma^d{}_{ba} = \Gamma^c{}_{ba}.$$

Note that

$$\partial_a\mathbf{E}_b = \partial_a\partial_b\mathbf{r} = \partial_b\partial_a\mathbf{r} = \partial_b\mathbf{E}_a,$$

which implies

$$\Gamma^c{}_{ba} = \Gamma^c{}_{ab}.$$

Similarly, we might want to consider $\partial_b\mathbf{E}^a$, which might introduce new symbols. We find, however, that

$$\partial_a\mathbf{E}^b \cdot \mathbf{E}_c = \mathbf{E}^b \cdot \partial_a\mathbf{E}_c + \mathbf{E}_c \cdot \partial_a\mathbf{E}^b = 0,$$

which implies

$$\partial_a\mathbf{E}^b = -\Gamma^b{}_{ac}\mathbf{E}^c.$$

**Covariant Derivatives** Covariant derivatives are defined by

$$\vec{\boldsymbol{\nabla}}_a v^b = \partial_a v^b + \Gamma^b{}_{ac}v^c,$$

and thus satisfy

$$\partial_a\mathbf{v} = \mathbf{E}_b\vec{\boldsymbol{\nabla}}_a v^b.$$

**Tensors** To define tensors, we first define tensors of the kind $(0, n)$ as maps from $n$ vectors to scalars. Using this, we define tensors of the kind $(n, m)$ as linear maps from $(0, n)$ tensors to $(0, m)$ tensors.