Summary of SI2540 Complex Systems

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Abstract

This is a summary of SI2540 Complex Systems.

Contents

1 Basic Concepts

What is a Complex System? A complex system is a dynamical system characterized by at least one of the following:

- Nonlinearity.
- High sensitivity to initial conditions - the butterfly effect.
- The existence of bifurcations.
- Emergent phenomena - the formation of patterns in the solution.
- Feedback.
- Dissipation.

The Interesting Aspects of Complex Systems The interesting aspects of complex systems are

- long-term behaviour.
- dependence on initial conditions.
- parameter dependence.

Autonomous Systems An autonomous system is described by

$$\frac{\mathrm{d}\mathbf{x}}{\mathrm{d}t} = \mathbf{f}(\mathbf{x})$$

where $\mathbf{x}$ is the state vector of the system.

This is of course not a restriction to first-order systems, as all systems may be written in this form. It is neither a restriction to f being independent of t, as one in this case can simply extend $\mathbf{x}$ to contain t.

Deterministic Systems A deterministic system is a system without random noise. Such systems are entirely specified by $\mathbf{f}$ and an initial condition.

Conservative and Dissipative Systems Conservative systems satisfy $\vec{\boldsymbol{\nabla}} \cdot \mathbf{f} = 0$. Dissipative systems satisfy $\vec{\boldsymbol{\nabla}} \cdot \mathbf{f} < 0$.

Orbits An orbit is a solution to an autonomous system corresponding to some particular initial value. The set of all orbits is the set of flow lines of $\mathbf{f}$. Because the position in phase space fully determines the future solution, flow lines never cross.

Fixed Points A fixed point is a point that satisfies $\mathbf{f}(\mathbf{x}) = \mathbf{0}$. Close to such points, non-zero first derivatives produce exponential behaviour locally and first derivatives equal to zero produce evolution slower than exponential.

Bifurcations A bifurcation is a qualitative in the structure of $\mathbf{f}$ as some parameter is varied.

Uniqueness of Solutions The weakest condition for the existence and uniqueness of a solution to

$$\frac{\mathrm{d}\mathbf{x}}{\mathrm{d}t} = \mathbf{f}(\mathbf{x}),\ \mathbf{x}(t_0) = \mathbf{x}_0$$

in a finite time interval around t_0, which we will assume to hold, is the Lipschitz condition

$$|\mathbf{f}(\mathbf{x}) - \mathbf{f}(\mathbf{y})| \leq \kappa |\mathbf{x} - \mathbf{y}|$$

for some finite κ. This entails that $\mathbf{f}$ should be continuous and have piecewise continuous derivatives. If this condition holds, the solution is continuous in $\mathbf{x}_0$.