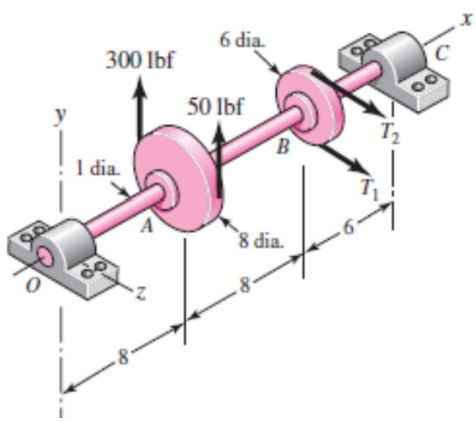
DESIGN OF MACHINE ELEMENTS ASSIGNMENT 1:

Problem statement 37:

The below figure shows a shaft mounted in bearings at A and D and having pulleys at B and C. The forces shown acting on the pulley surfaces represent the belt tensions. The shaft is to be made of AISI 1035 CD steel. Using a conservative failure theory with a design factor of 1.5, determine the minimum shaft diameter to avoid yielding.



Methodology used:

- Understanding the question and what we have to find in that (minimum shaft diameter to avoid yielding)
- Finding the tension T1 and T2 by assumptions given in the question.
- Finding the reaction forces in the xy and xz plane.
- Finding the shear force and bending moment diagram in xy and xz plane and getting the moment at point A and B.
- Finding the torque at point A and B.
- Analyzing the critical point and finding the stresses.
- Finding the components of stresses and shear stress
- Finding the Principal stresses
- By given safety factor and yield strength, the minimum radius or diameter of the shaft can be calculated using maximum shear stress theory (Tresca).

Methodology used in Ansys:

- First, converted the units in lbf, in format using '/units,bin'.
- After this, went to preference and chose structural, then took the element type as beam.
- Section is added (radius: 0.472441 inches is taken as standard size).
- Added young modulus and poisson ratio in the material properties. By looking at the structure, modeling of structure was the next procedure.
- Then meshing is done on the beam. The next procedure is to apply loads, so applying Fy: 350 lbf at point A, Fz: 450.98 lbf at point B, constraints added in O and C (in figure) which is at point O (constrained: UY, UZ, ROTX) and point C (constrained: UY, UZ, ROTX)
- Moment given to point A and B is 1000 lbf.in (MX). The solution is solved and the shear force, bending moment diagram and stresses are seen.

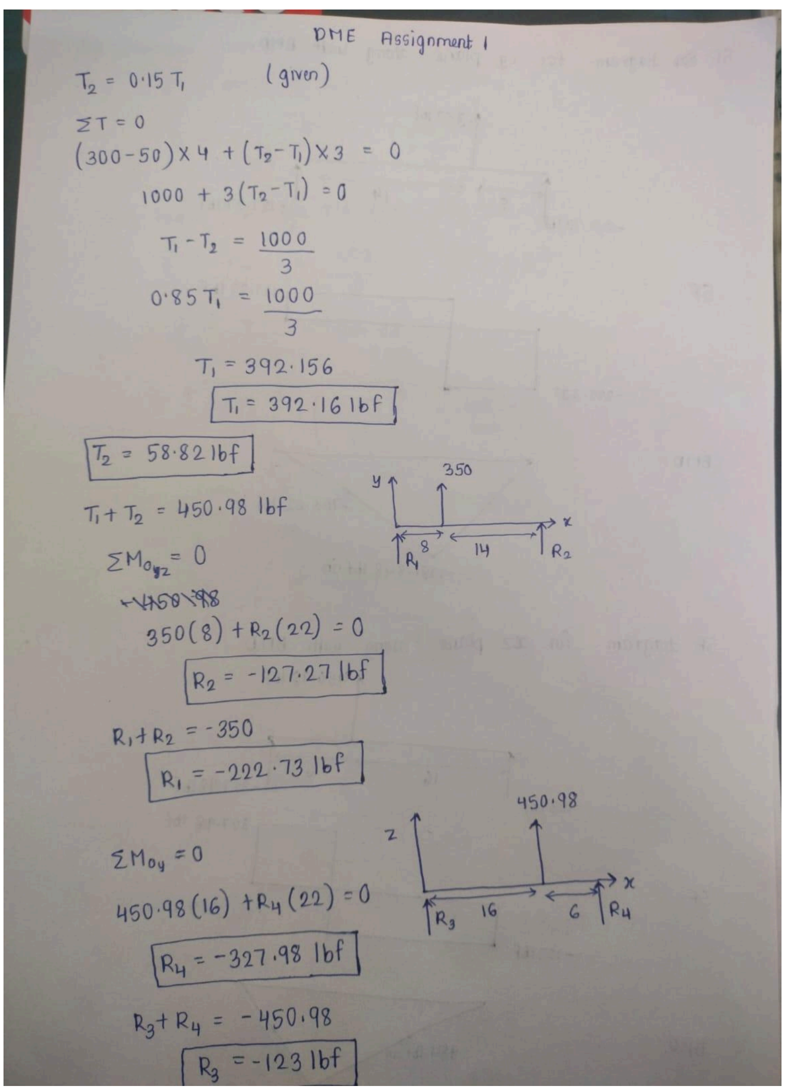
Assumptions made:

- The shaft is subjected to combined bending and torsion due to the belt tensions.
- The shaft is made from AISI 1035 cold-drawn steel (CD), whose material properties are:
  - [ ] Yield strength: 60200 psi
  - [ ] Young Modulus: 28400 psi
  - [ ] Poisson Ratio: 0.3
- A design factor of 1.5 is applied.
- Belt tension on the loose side of B is 15 percent of the tension on the tight side

Calculations:

Comparison between the Ansys results and the hand calculation part:

The reaction forces shown in the Ansys matches with the hand calculation part with negligible error. As shown in the figure:



DME Assignment 1

$T_2 = 0.15\,T_1$ (given)

$\Sigma T = 0$

$(300-50)\times 4 + (T_2 - T_1)\times 3 = 0$

$1000 + 3(T_2 - T_1) = 0$

$T_1 - T_2 = \frac{1000}{3}$

$0.85\,T_1 = \frac{1000}{3}$

$T_1 = 392.156$

$\boxed{T_1 = 392.16\ \text{lbf}}$

$\boxed{T_2 = 58.82\ \text{lbf}}$

$T_1 + T_2 = 450.98\ \text{lbf}$

$\Sigma M_{o_{yz}} = 0$

$350(8) + R_2(22) = 0$

$\boxed{R_2 = -127.27\ \text{lbf}}$

$R_1 + R_2 = -350$

$\boxed{R_1 = -222.73\ \text{lbf}}$

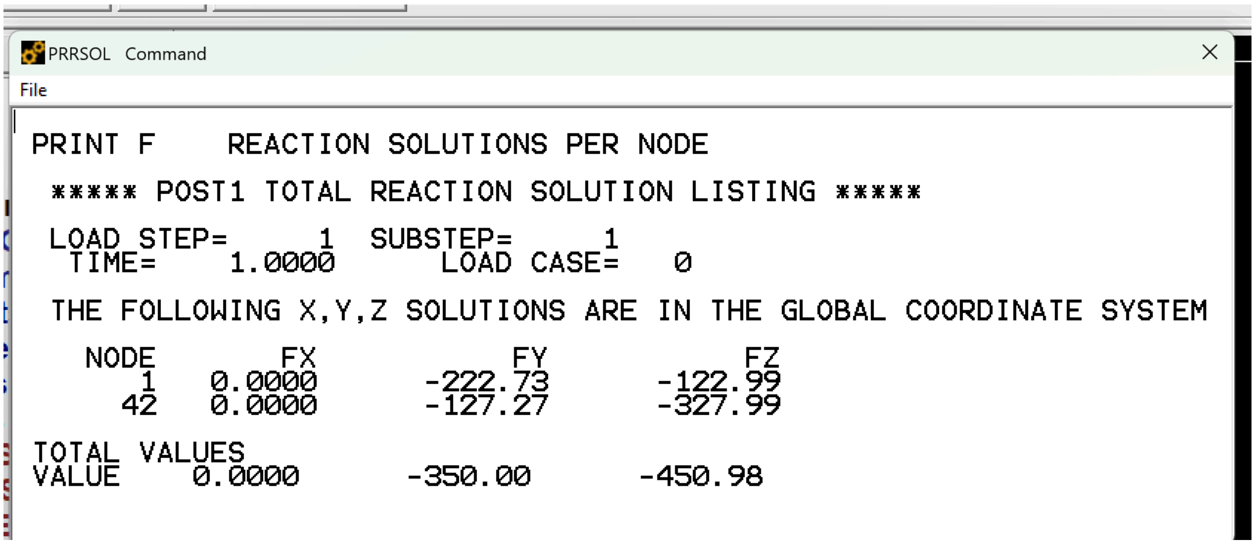
$\Sigma M_{oy} = 0$

$450.98(16) + R_4(22) = 0$

$\boxed{R_4 = -327.98\ \text{lbf}}$

$R_3 + R_4 = -450.98$

$\boxed{R_3 = -123\ \text{lbf}}$



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PRRSOL  Command
File

PRINT F    REACTION SOLUTIONS PER NODE

 ***** POST1 TOTAL REACTION SOLUTION LISTING *****

 LOAD STEP=      1  SUBSTEP=     1
  TIME=    1.0000      LOAD CASE=   0

 THE FOLLOWING X,Y,Z SOLUTIONS ARE IN THE GLOBAL COORDINATE SYSTEM

    NODE       FX          FY          FZ
       1   0.0000     -222.73     -122.99
      42   0.0000     -127.27     -327.99

 TOTAL VALUES
 VALUE    0.0000     -350.00     -450.98
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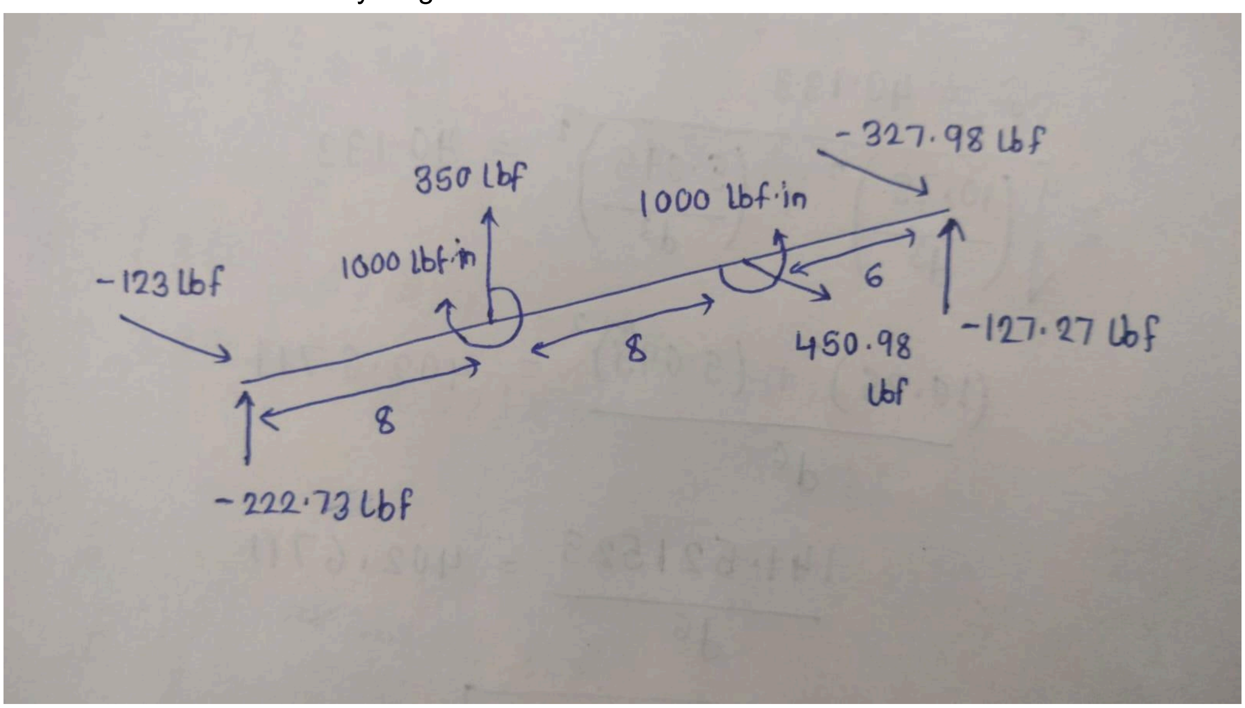
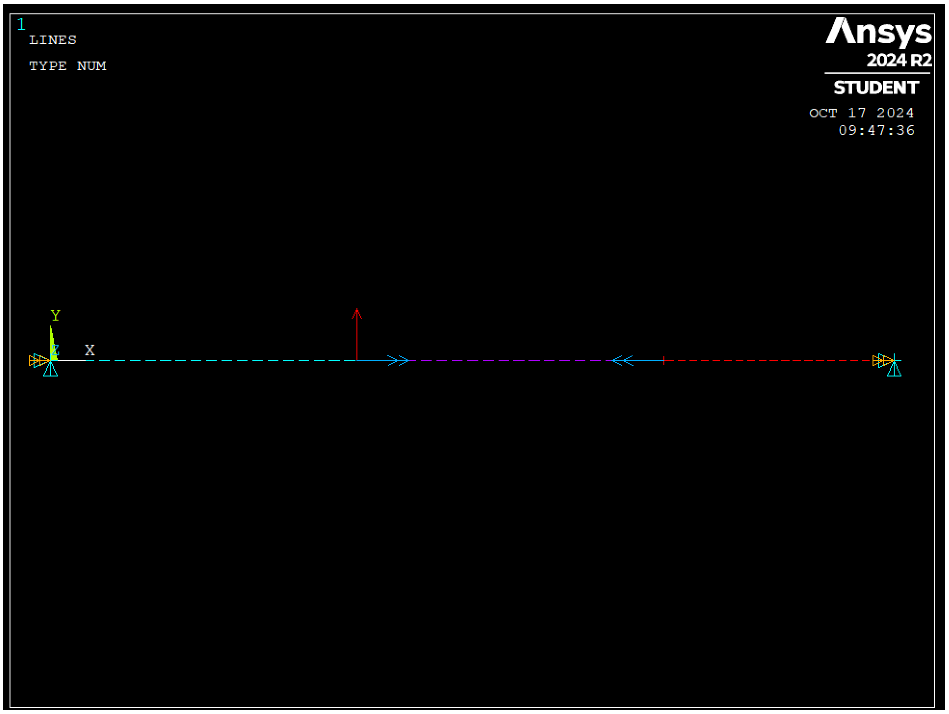
General postproc - List results - Reaction Solu

The reaction forces were found by taking the xy and xz plane separately and then calculated by showing equilibrium.

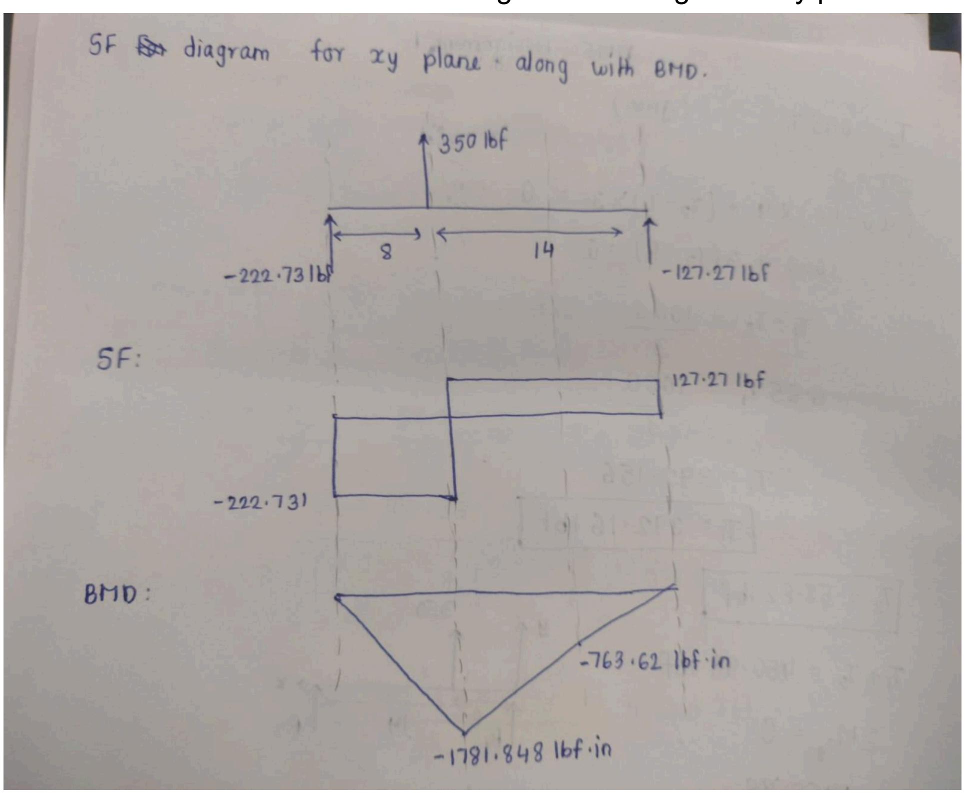
- The overall free body diagram:

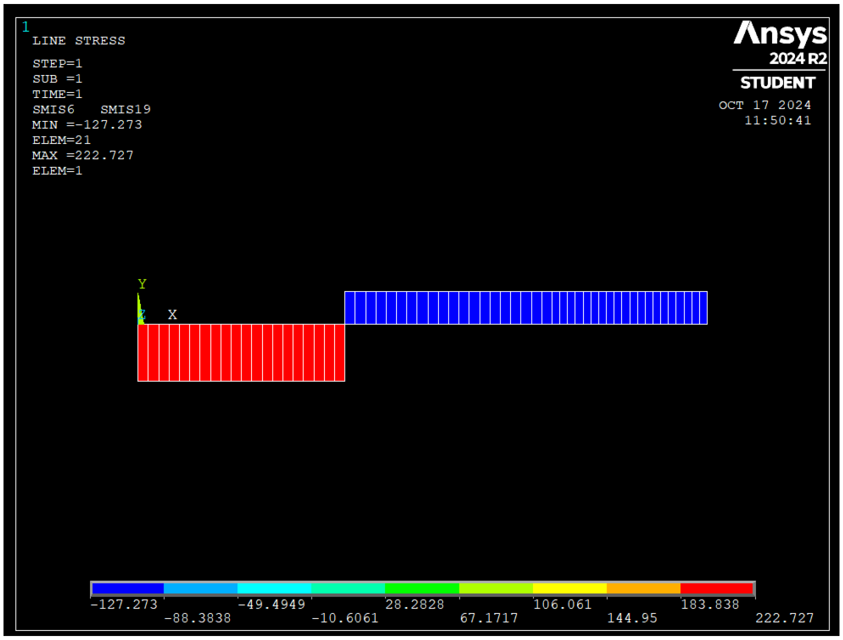




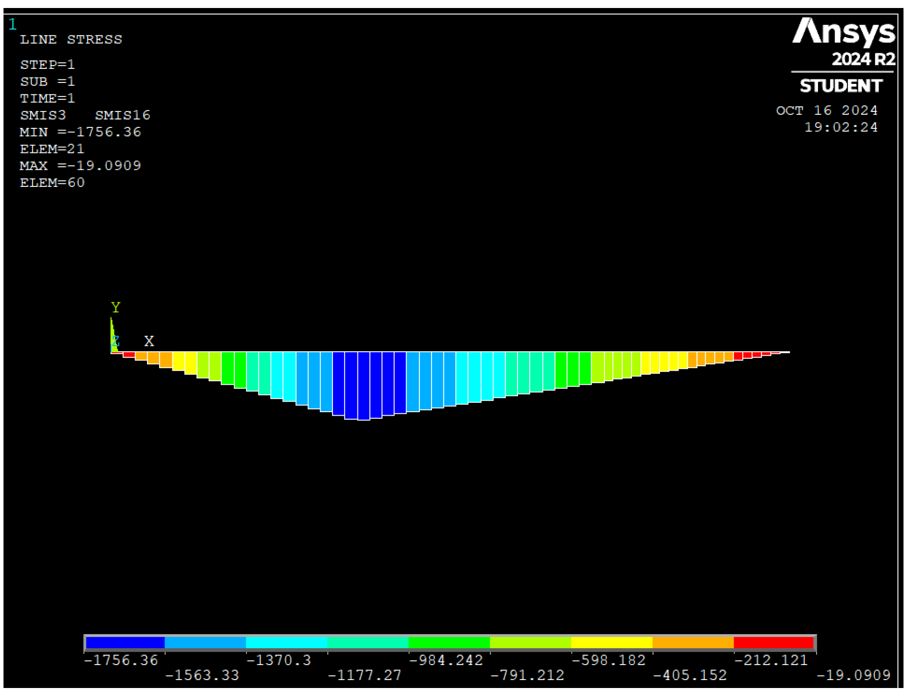
The loads are added as seen in the free body diagram, the Fy force is added and Fz force is added and moments are at in Mx, along with that constrained are added at the ends.

- For the shear force and bending moment diagram in xy plane:





Element table - Define Table - SMIS6, SMIS19,

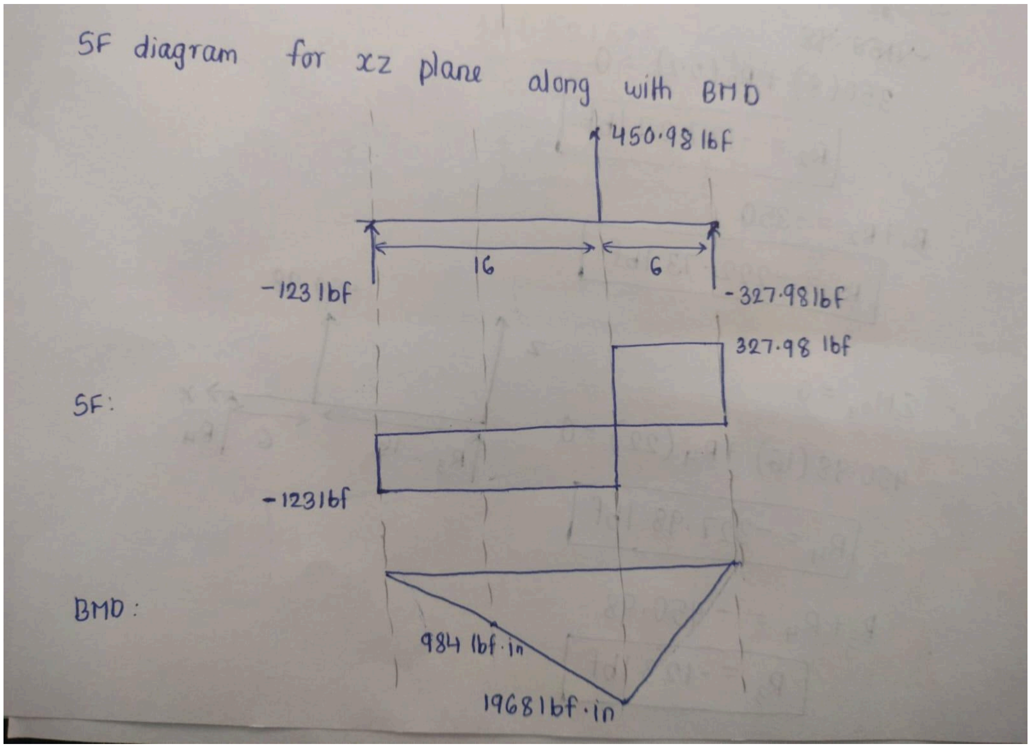


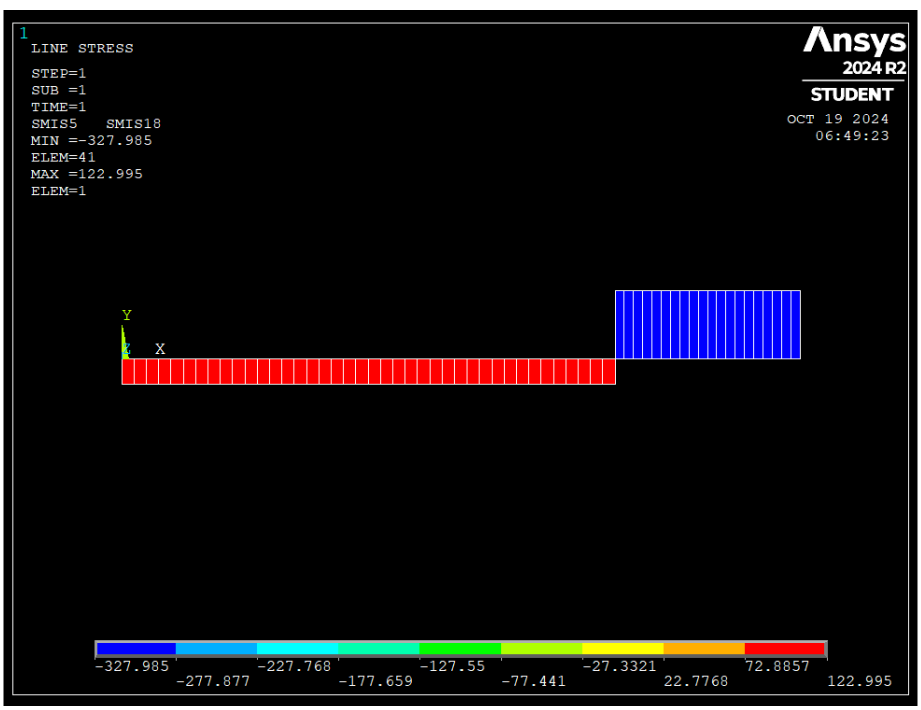
Element table - Define Table - SMIS3, SMIS16

The shear force diagram was drawn using the reaction forces. The bending moment diagram was drawn by multiplying the force with distance between the points.

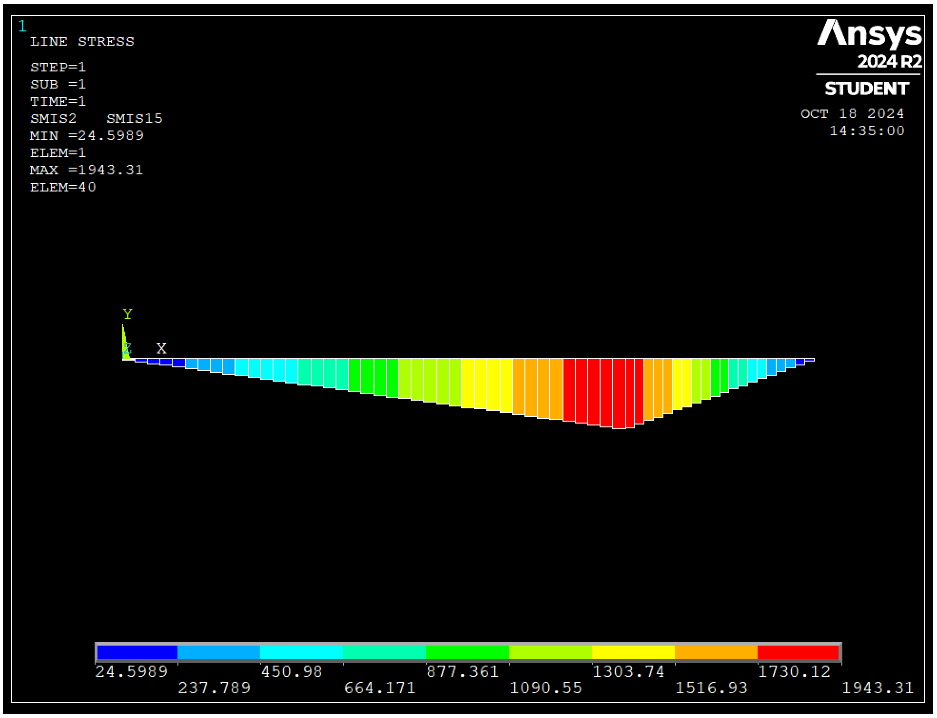
Error in the Ansys and the hand calculation part:
Shear force error : 0.00179%
Bending moment error at point A : 1.43%

For the shear force and bending moment diagram in xz plane:





Element table - Define Table - SMIS5, SMIS18



Element table - Define Table - SMIS2, SMIS15

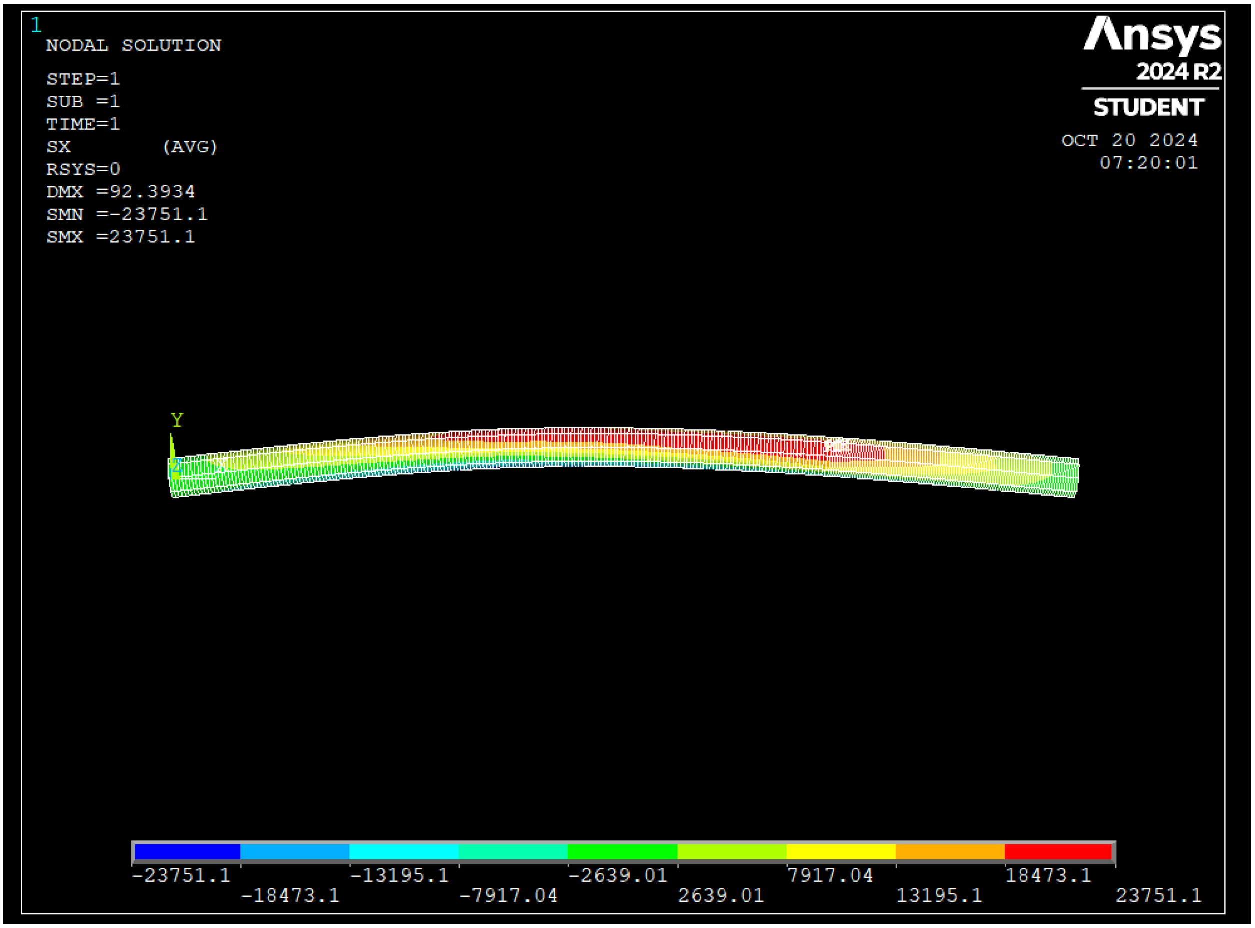
Error in the Ansys and the hand calculation part:
Shear force error : 0.004%
Bending moment error at point B : 1.25%

- For the stresses,

The Ansys and the hand calculation part are shown:
For the X component of stresses:

$$\sigma = \frac{Mc}{I} = \frac{2110.845 \times \frac{d}{2}}{\frac{\pi}{64} d^4} = \frac{32(2110.845)}{\pi d^3} = \frac{21511.79}{d^3}$$

$$\sigma = \frac{21511.79}{(0.944882)^3} = 25500.21384 \text{ psi}$$



General Postproc - Result viewer

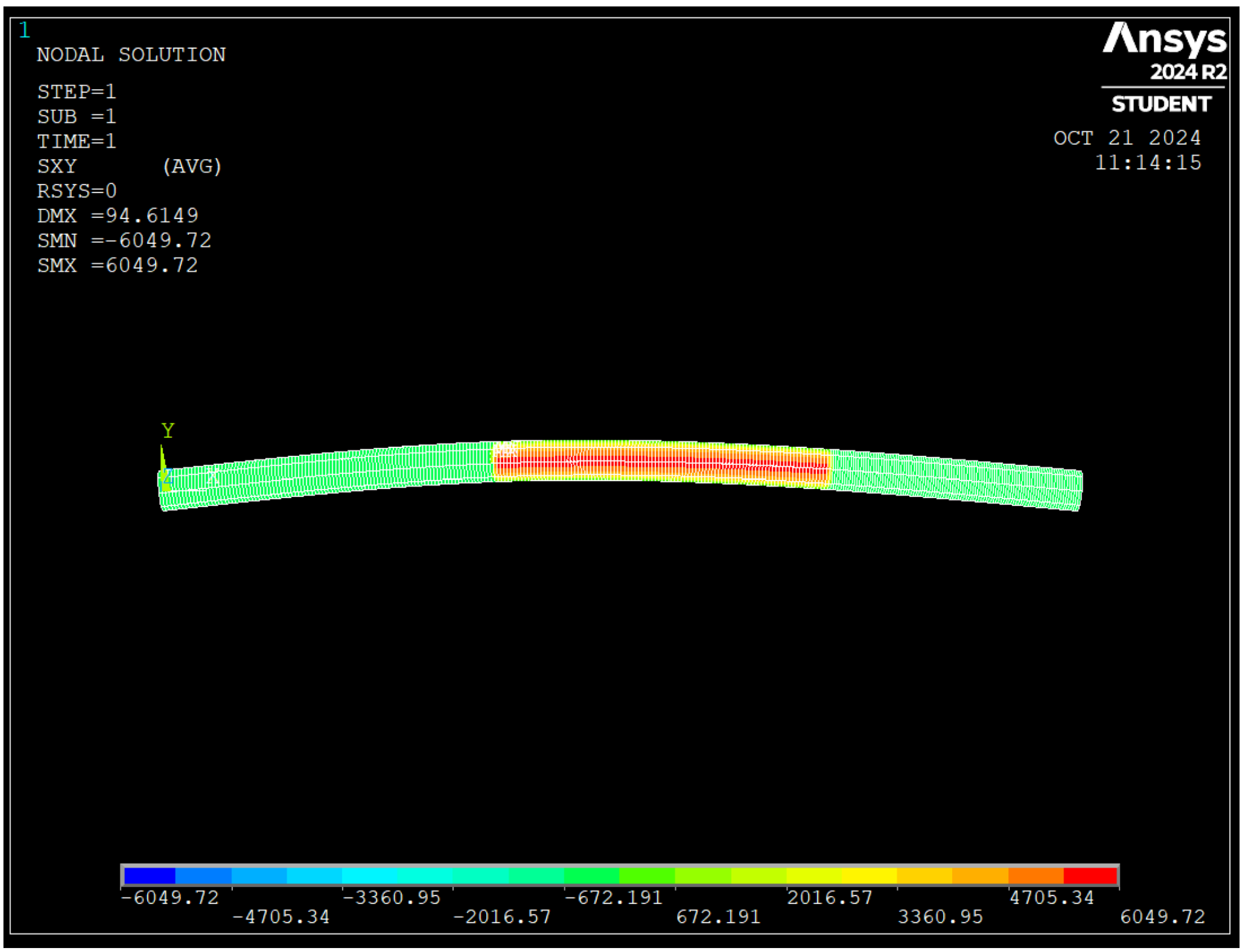
Error in the Ansys and the hand calculation part: 6.8%

- Reason for this error:
  In ANSYS, the choice of finite elements and mesh density can affect results. Finer meshes generally provide more accurate results, but coarse meshes can lead to approximations and deviations from analytical results.
- Finite element methods use interpolation functions to estimate stresses between nodes, which can introduce small inaccuracies compared to analytical solutions that assume ideal conditions.
- If boundary conditions in ANSYS are not applied exactly as assumed in hand calculations, results may differ. Small deviations in how constraints or loads are applied can cause variations in the stress distribution.

For the shear stress:

$$\tau = \frac{Tr}{J} = \frac{1000 \times d/2}{\frac{\pi}{32} d^4} = \frac{16000}{\pi d^3} = \frac{5095.54}{d^3}$$

$$\tau = \frac{5095.54}{(0.944882)^3} = 6040.2857 \text{ psi}$$



General Postproc - Result viewer
Error in the Ansys and the hand calculation part: 0.156%

For the 1st Principal stress:

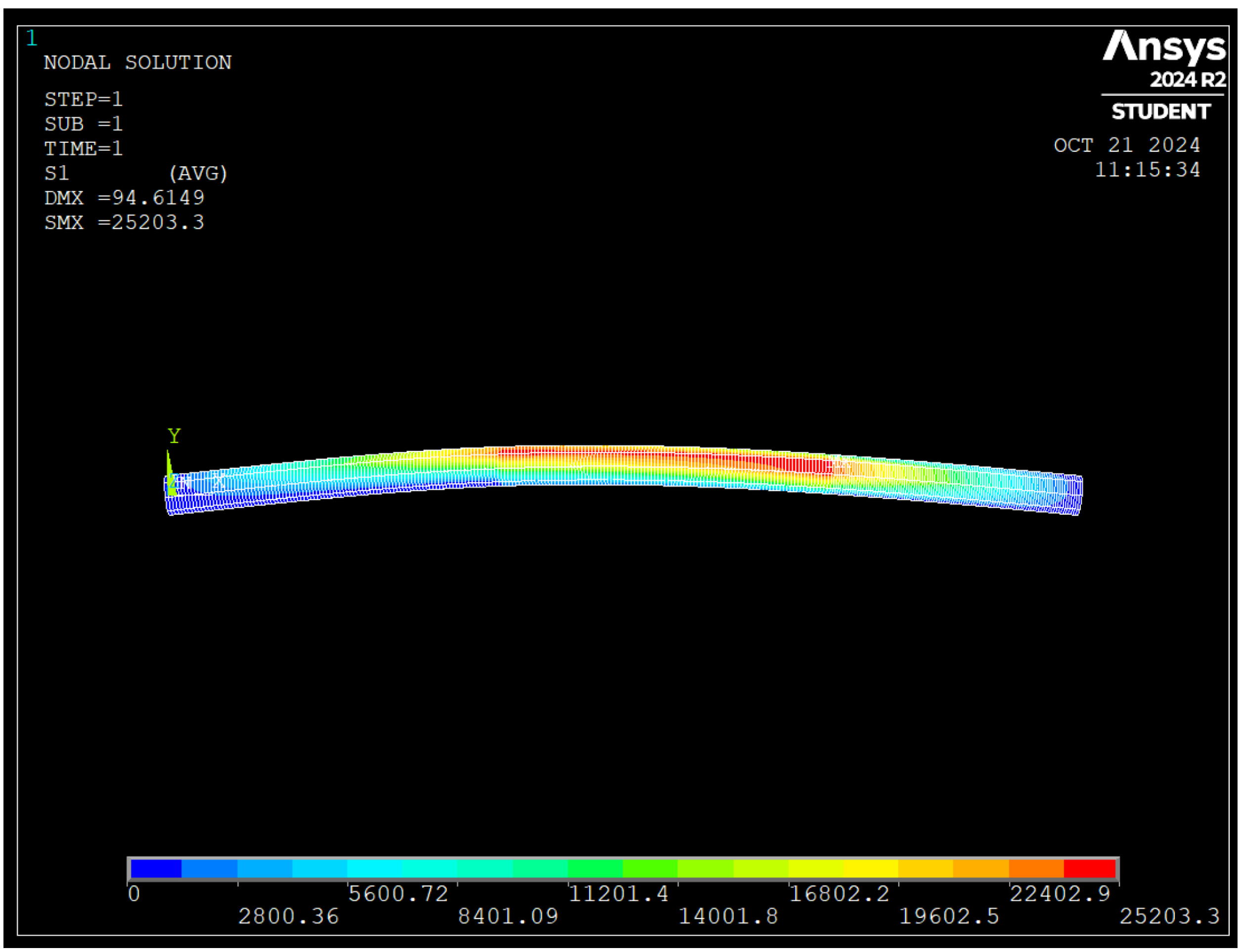
$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{25500.21384}{2} + \sqrt{\left(\frac{25500.21384}{2}\right)^2 + (6040.287)^2}$$

$$= 12750.106 + \sqrt{199050293.5}$$

$$= 12750.106 + 14108.51847$$

$$= 26858.62447 \text{ psi}$$



Error in the Ansys and the hand calculation part: 6.1 %

Reason for this error:

- In ANSYS, the choice of finite elements and mesh density can affect results. Finer meshes generally provide more accurate results, but coarse meshes can lead to approximations and deviations from analytical results.
- ANSYS uses numerical methods to approximate solutions, which may introduce discretization errors, especially if the problem domain is not well-refined.
- Finite element methods use interpolation functions to estimate stresses between nodes, which can introduce small inaccuracies compared to analytical solutions that assume ideal conditions.
- Hand calculations often assume idealized, uniform load distributions, whereas ANSYS allows for more complex, real-world load distributions, which can lead to discrepancies.

With the bending stress and shear force , principal stress was found.

Final Calculation:

At point B,

$$\sigma = \frac{Mc}{I} = \frac{2110.845 \times d/2}{\frac{\pi}{64} d^4} = \frac{32(2110.845)}{\pi d^3} = \frac{21511.79}{d^3} \text{ psi} = \frac{21.511}{d^3} \text{ kpsi}$$

$$\tau = \frac{Tr}{J} = \frac{1000 \times d/2}{\frac{\pi}{32} d^4} = \frac{16(1000)}{\pi d^3} = \frac{5095.54}{d^3} \text{ psi} = \frac{5.095}{d^3} \text{ kpsi}$$

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_1 = \frac{21.511}{2d^3} + \sqrt{\left(\frac{21.511}{2d^3}\right)^2 + \left(\frac{5.095}{d^3}\right)^2}$$

$$\sigma_2 = \frac{21.511}{2d^3} - \sqrt{\left(\frac{21.511}{2d^3}\right)^2 + \left(\frac{5.095}{d^3}\right)^2}$$

By maximum shear stress

$$\tilde{\sigma} = \max\left(|\sigma_1 - \sigma_2|, |\sigma_1|, |\sigma_2|\right)$$

$$\tilde{\sigma} = 2\sqrt{\left(\frac{21.511}{2d^3}\right)^2 + \left(\frac{5.095}{d^3}\right)^2}$$

Safety factor (for 1035 cold steel : 60.2 kpsi)

$$\bar{x} = 1.5$$

$$\bar{x} = \frac{\sigma_{yt}}{\tilde{\sigma}} = \frac{60.2}{\tilde{\sigma}} \Rightarrow 1.5 = \frac{60.2}{\tilde{\sigma}}$$

$$\tilde{\sigma} = 40.133$$

$$2\sqrt{\left(\frac{10.75}{d^3}\right)^2 + \left(\frac{5.095}{d^3}\right)^2} = 40.133$$

$$\frac{(10.75)^2 + (5.095)^2}{d^6} = 402.6711$$

$$\frac{141.521525}{d^6} = 402.6711$$

$$\boxed{d = 0.84 \text{ in}}$$

$r = 0.42$ in

Taking standard sizes,

12 mm → $r = 0.472441$ in

so, $d = 0.944882$ in

The minimum shaft diameter to avoid yielding is 0.84 in.
Standard Size taken for solving the question is 0.944882 in (diameter Ansys).

Conclusion:
The safety factor (design factor) was given through which by using the maximum shear stress formula, the minimum diameter of the shaft was found.
The reaction forces, the shear force and bending moment diagram and the bending stress and the shear stress were found and compared with the hand calculated solutions.
The errors were notified and the reason behind this error was understood.
The diameter was taken using the standard sizes and then they were compared with the bending and the shear stresses.
By solving it with Ansys and analytically it gave a knowledge and visualization of the question with better understanding of the solution.

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