



New Zealand Mathematical Olympiad Committee

2010 Squad Assignment Four

Geometry

Due: Wednesday, 14th April 2010

1. A cyclic quadrilateral $ABCD$ is given. Show that the line connecting the orthocentre of the triangle ABC with the orthocentre of the triangle ABD is parallel to the line CD .
2. Let ABC be an isosceles triangle with $|AC| = |BC|$. Its incircle touches AB and BC at D and E respectively. A line (different from AE) passes through A and intersects the incircle at F and G . The lines EF and EG intersect the line AB at K and L , respectively. Prove that $|DK| = |DL|$.
3. Two circles k_a and k_b are given, whose centres lie on the legs of length a and b respectively of a right triangle. Both circles are tangent to the hypotenuse of the triangle and pass through the vertex opposite the hypotenuse.

Let the circles have radii ρ_a and ρ_b respectively. Find the greatest real number p such that the inequality

$$\frac{1}{\rho_a} + \frac{1}{\rho_b} \geq p \left(\frac{1}{a} + \frac{1}{b} \right)$$

holds for all right triangles.

4. A line ℓ is given, and on it four points P, Q, R, S (in this order from left to right). Construct, with a straight edge and compasses, all squares $ABCD$ satisfying all of the following conditions:
 - (a) P lies on the line through A and D ;
 - (b) Q lies on the line through B and C ;
 - (c) R lies on the line through A and B ; and
 - (d) S lies on the line through C and D .

Note: Be sure to show that your construction works. You may describe it in terms of known constructions: for example, if you need a perpendicular bisector you may simply say “construct the perpendicular bisector to EF ”, without describing in detail how to do this.

5. Two distinct points O and T are given in the plane ω . Find the locus of vertices of all triangles lying in ω whose centroid is T and whose circumcentre is O .

6. The incircle k of a scalene triangle ABC has centre S and is tangent to the sides BC , CA and AB at the points P , Q and R respectively. The lines QR and BC intersect in the point M . A circle passing through B and C is tangent to k at the point N . The circumcircle of triangle MNP meets the line AP at a point L which is different to P . Prove that the points S , L , M are collinear.

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