Principles of Brain Computation KU

708.086 18S

Homework Sheet 3

Problems marked with * are optional.

Dynamical synapses [8+3*P]

In this task, we investigate the effects of short-term synaptic plasticity on the population rate of a number of LIF neurons using NEST.

The model we will use consists of 500 input neurons which spike randomly. This input population is connected to a population of LIF neurons via dynamical synapses (Fig. 1). By measuring the population rate of the latter population, we will examine the effects of short-term plasticity.

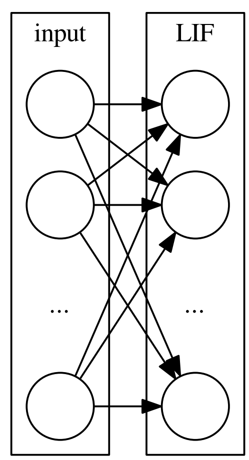

Figure 1: Network architecture.

Model details

The input neurons generate spikes according to Poisson point processes with a constant rate r_{input} (use the model `poisson_generator`). Each LIF neuron receives input from 100 randomly chosen input neurons (i.e. fixed indegree of 100). The synapses should have a weight of $100/r_{\mathrm{input}}$ and undergo short-term plasticity (set $u = 0$ at simulation begin, the other parameters given below for each individual experiment). The synaptic delays should be uniformly distributed between 1 and 10 ms. Note that you cannot connect a Poisson generator to neurons using plastic synapses in NEST, thus, you need to use `parrot_neurons` in between (which simply copy spikes from input to output).

The LIF neurons should have δ-shaped PSCs and have parameters $u_{\mathrm{rest}} = -65\,\mathrm{mV}$, $R_{\mathrm{m}} = 0.01\,\mathrm{G\Omega}$, $C_{\mathrm{m}} = 2000\,\mathrm{pF}$, $\vartheta = -50\,\mathrm{mV}$, $u_{\mathrm{reset}} = -80\,\mathrm{mV}$. No absolute refractory period should be used.

Dynamical synapses

We use the NEST model `tsodyks2_synapse` for dynamical synapses, which is based on a phenomenological model of synaptic behavior. The state of the synapse is updated every time a spike is transmitted. The weight of each connection at the time of the n-th presynaptic spike is given by

$$A_n = A\, u_n\, R_n\ , \tag{1}$$

where A is the absolute synaptic efficacy (i.e. the maximum weight), u_n describes the current utilization of synaptic resources, and R_n describes the current availability of synaptic resources. These are updated from some starting values for a delay of Δt since the last spike using

$$u_{n+1} = U + u_n(1 - U)\exp\left(-\frac{\Delta t}{F}\right)\ , \tag{2}$$

where U governs the increase of synaptic efficacy per spike and F is the time constant for facilitation, and

$$R_{n+1} = 1 + (R_n - R_n\, u_n - 1)\exp\left(-\frac{\Delta t}{D}\right)\ , \tag{3}$$

where D is the time constant for depression.

Population rate

The population rate of a population of N neurons in some time period $[t - \Delta t, t]$ is given by

$$A(t) = \left\langle \frac{1}{\Delta t} \cdot \frac{N_{\mathrm{spikes}}(t - \Delta t, t)}{N} \right\rangle\ , \tag{4}$$

where $N_{\mathrm{spikes}}(t_0, t_1)$ is the number of spikes which occured between t_0 and t_1. The brackets $\langle \cdot \rangle$ denote averaging over multiple trials.

Each experiment (details are given below) should last 2 seconds. For every experiment, you should compute the population rate over the entire simulation time using a bin size (i.e. Δt) of 20 ms and a bin overlap of 50%. Since the simulations we conduct are somewhat noisy, you should average this population rate over at least 10 independent trials (i.e. repetitions from the same initial condition).

You should use a NEST time step of 0.1 ms (the default value).

Code

The provided template script contains two functions which you should complete:

1. `populate_rate(spike_times, t_start, t_end, bin_width, bin_spacing)` to compute the population rate for all times between `t_start` and `t_end` using an array of spike times of a population of neurons and the given parameters.

2. `experiment(trials, rate_in, U, D, F)` to perform a single experiment with the given parameters (number of trials to average over, rate of the input neurons, U, D, and F parameters for the dynamical synapses).

Task 3a [2P]

Plot the population rate (averaged over at least 10 trials) for an experiment using the parameters for the facilitating synapse type F1 from [Gupta et al., 2000] ($U = 0.16$, $D = 0.045\,\text{s}$, $F = 0.376\,\text{s}$) using an input rate of $r_{\text{input}} = 20\,\text{Hz}$. Explain the the time course of the population rate.

Set $F = 0.1\,\text{s}$ and plot the population rate again. Compare the resulting curves. Why are the final stable values different?

Task 3b [2P]

Perform the same analysis for the depressing synapse type F2 from [Gupta et al., 2000] ($U = 0.25$, $D = 0.706\,\text{s}$, $F = 0.021\,\text{s}$). Compare the resulting behavior of the population rate using input rates of $r_{\text{input}} = 10\,\text{Hz}$ and $r_{\text{input}} = 20\,\text{Hz}$. Explain the results.

Task 3c [2P]

Use the facilitating parameters from 3a ($U = 0.16$, $D = 0.045\,\text{s}$, $F = 0.376\,\text{s}$) and compare the results for input rates of $r_{\text{input}} = 20, \text{Hz}$ and $r_{\text{input}} = 40\,\text{Hz}$. Explain the results. Why does the population rate decrease when r_{input} is increased?

Task 3d [2P]

Check your results by deriving the synaptic weight for long constant rate input. Using a fixed interspike interval of $\Delta t = 1/r_{\text{input}}$, derive the fixed points for A_n (remember that the absolute synaptic efficacy A was set to $100/r_{\text{input}}$). Give the resulting equations for u_∞ and R_∞ in the general case. Then, calculate A_∞ for all the cases considered above in 3a-c. It is not straightforward to compute the actual firing rates. However, the relative ordering of the weights should be equal to the ordering of the firing rates.

Task 3e [3*P]

Due to the simple neuron model which was used, it is possible to compute the actual firing rates for the fixed points derived above. Assuming all input weights are given by A_∞, each neuron receives a mean input current of

$$I_\infty = A_\infty \, C_{\text{pre}} \, r_{\text{pre}} \, \alpha \ , \tag{5}$$

where C_{pre} is the number of neurons from which this neuron receives input, and r_{pre} is the rate with which these neurons fire (in Hz). (Note that A_∞ is a function of r_{pre}. The scaling factor α (its exact form is due to the implementation of the `iaf_psc_delta` model in NEST) is given by

$$\alpha = \frac{1}{R_{\text{m}} \left(1 - \exp\left(-\Delta t / \tau_{\text{m}}\right)\right)} \cdot \frac{\Delta t}{1000} \ , \tag{6}$$

where R_{m} is the membrane resistance, τ_{m} is the membrane time constant, and $\Delta t = 0.1\,\text{ms}$ is the simulation time step. (Dividing Δt by 1000 changes the unit to s, which cancels with the unit of r_{pre}. Once the input current is known, the firing rate of the population neurons can be computed with the gain function of LIF neurons

$$f(I) = \left(\tau_{\text{m}} \log \left(\frac{R\,I + u_{\text{rest}} - u_{\text{reset}}}{R\,I + u_{\text{rest}} - \vartheta} \right) \right)^{-1} \ . \tag{7}$$

Using the parameters for the facilitating synapse type F1 from [Gupta et al., 2000] ($U = 0.16$, $D = 0.045\,\text{s}$, $F = 0.376\,\text{s}$), plot the theoretical firing frequency of the LIF neurons for constant input frequencies from 1 to 100 Hz. Draw vertical lines at the frequencies used in 3c and compare the theoretical values to the simulated ones. Explain the shape of the function from input frequency to LIF firing frequency.

Note: take caution when using time values / time constants since we typically use ms as unit, but $f(I)$ should be in Hz.

References

[Gupta et al., 2000] Gupta, A., Wang, Y., and Markram, H. (2000). Organizing principles for a diversity of gabaergic interneurons and synapses in the neocortex. *Science*, 287(5451):273–278.

Submit the code until 8:00 AM of the day of submission to `mueller@igi.tugraz.at` and `lydia.lindner@student.tugraz.at`. Use `PoBC HW3,` ⟨`name team member 1`⟩ ⟨`name team member 2`⟩ as email subject. Only one email per team is necessary. Submit regular Python code files (`*.py`). You need to hand in a printed version of your report at the submission session. Each team member needs to write their own report. Use the cover sheet provided on the course website.