Principles of Brain Computation KU
708.086 18S

Homework Sheet 2

Problems marked with * are optional.

Nonlinear integrate-and-fire models [7P]

In this task, we investigate a nonlinear integrate-and-fire (IAF) model, which we will fit to data recorded from a neuron. Nonlinear IAF models are generally described by

$$\frac{dV}{dt} = F(V) + \frac{1}{C_{\mathrm{m}}} I_{\mathrm{input}}(t) \tag{1}$$

where

- V is the membrane potential,
- $F(V)$ is a nonlinear function of the membrane potential,
- C_{m} is the membrane capacitance, and
- I_{input} is some input current.

Typically, $F(V)$ consists of a nonlinear term as well as a linear leak term $-g_{\mathrm{L}}(V - V_{\mathrm{rest}})$ with leak conductance g_{L}.

Spikes are generated as for linear IAF models: if the membrane potential reaches the firing threshold, a spike is generated and the membrane potential is reset to the reset potential V_{reset}:

$$\text{if } V \geq \vartheta_{\mathrm{reset}} : \; V \leftarrow V_{\mathrm{reset}} \; . \tag{2}$$

If there is an absolute refractory period $\Delta_{\mathrm{abs}} > 0$, the membrane potential is clamped to V_{reset} for the duration of Δ_{abs}, after which it may evolve freely.

You are given a 60 s recording of a neuron during which a fluctuating current (with known amplitude) is injected into the neuron and the resulting membrane potential is measured. The goal of this homework is to analyze the data to find good parameters for a model to fit to this data.

To estimate F, we assume the currents

$$\frac{dV}{dt} = -\frac{1}{C_{\mathrm{m}}} I_{\mathrm{m}}(V,t) + \frac{1}{C_{\mathrm{m}}} I_{\mathrm{input}}(t) + I_{\mathrm{noise}}(t) \tag{3}$$

are present (this follows from a basic electrophysiological model). Here, $I_{\mathrm{m}}(V,t)$ is the time- and voltage-dependent current across the cell membrane. $I_{\mathrm{noise}}(t)$ is additional noise which we cannot model. Comparing (3) to (1), we see that we can estimate $F(V)$ from the current $I_{\mathrm{m}}(V,t)$.

The pickle file you receive with this homework contains four vectors: a time vector, vectors for injected current and voltage at those times, and all times at which the neuron spikes.

Task 2a [2P]

First, we want to estimate the nonlinear function $F(V)$.

- Since we want to fit a model with a single differential equation, the APs and the reset should not be part of the model (instead, we will use a simple reset mechanism as sketched above). To remove the APs from the data, we can simply drop the 5 ms following each spike from our data. Remove these segments from the voltage and current vectors.

- Compute the time derivatives of the membrane potential $\frac{dV}{dt}$ using the first-order finite difference:

$$\left.\frac{dV}{dt}\right|_{t=t_0} = V(t_0+\Delta t) - V(t_0) \tag{4}$$

- For each time t_0, compute the noisy estimate of $\hat{F}(V,t_0)$ using

$$\cancel{\hat{F}(V,t_0) = -\left.\frac{dV}{dt}\right|_{t=t_0} + \frac{1}{C_{\mathrm{m}}} I_{\mathrm{input}}(t_0)\ .}$$

$$\hat{F}(V,t_0) = \left.\frac{dV}{dt}\right|_{t=t_0} - \frac{1}{C_{\mathrm{m}}} I_{\mathrm{input}}(t_0)\ . \tag{5}$$

  Provide a scatter plot for the values of $\hat{F}(V,t)$ as a function of V.

  Notes:

  - After computing $\frac{dV}{dt}$, we still have one large decrease of the membrane potential for large values of V left in the data. Exclude these values by dropping all large negative values of $\frac{dV}{dt}$ (e.g. all values where $\frac{dV}{dt} < -30$) and the corresponding values of I_{input}.
  - After recording from the neuron, the membrane capacitance was measured to be 920 pF.

- To get a real estimate of $F(V)$, we need to get rid of the time-dependency. Assuming zero-mean noise, a good estimate is

$$\hat{F}(V) = \text{Mean}\left[\hat{F}(V,t)\right] . \tag{6}$$

  Create bins of 1 mV width for $V \in [-80, -25]$ mV. For all values of $\hat{F}(V,t)$ falling into such a bin, compute the mean value. Plot the resulting curve of $\hat{F}(V)$ on top of the scatter plot.

Task 2b [2P]

Analyze the stability of the model (1) using the estimated curve $\hat{F}(V)$ from Task 2a.

- Which equilibrium points are present in the model? Briefly discuss their stability.
- Which features of neuron models do these equilibrium points correspond to?

Task 2c [3P]

Fit an exponential integrate-and-fire model to the data using the results obtained above. It is given by

$$C_\mathrm{m}\frac{dV}{dt} = -g_\mathrm{L}\left(V - E_\mathrm{L}\right) + g_\mathrm{L}\Delta_\mathrm{T}\exp\left(\frac{V - \vartheta_\mathrm{rh}}{\Delta_\mathrm{T}}\right) + I \tag{7}$$

with a reset mechanism

$$\text{if } V \geq \vartheta_\mathrm{reset} : \; V \leftarrow V_\mathrm{reset} \tag{8}$$

and an absolute refractory period of Δ_abs.

- Estimate (roughly) E_L and ϑ_rh from your plot of $\hat{F}(V)$. During the recording, the AP slope parameter was measured to be $\Delta_\mathrm{T} = 5$ mV. Use $\Delta_\mathrm{abs} = 2$ ms and $V_\mathrm{reset} = -80$ mV.
- Estimate g_L from the linear part of $\hat{F}(V)$.
- Using this parameters and the given input current, model the neuron for 500 ms.

  Notes:

  - Use a NEST `aeif_cond_exp` model. Be careful as NEST may use different names for parameters.
  - You need to set the `a` and `b` parameters of the neuron model to zero (they control adaptation, which we do not use here).

  - The voltage at the beginning of the simulation should be equal to the first measured voltage value.
  - Use the full data given for the input currents.

- Plot the first 500 ms of the recorded membrane potential along with your modeling results. How well does your model fit the data? Briefly discuss how the deviations from the recorded data may arise.

  Notes:

  - The exact value of g_{L} has a large influence on the result. You should possibly tweak it a little.
  - Your model will not be perfect.

Submit the code until 8:00 AM of the day of submission to `mueller@igi.tugraz.at` and `lydia.lindner@student.tugraz.at`. Use `PoBC HW2,` ⟨`name team member 1`⟩ ⟨`name team member 2`⟩ as email subject. Only one email per team is necessary. Submit regular Python code files (`*.py`). You need to hand in a printed version of your report at the submission session. Each team member needs to write their own report. Use the cover sheet provided on the course website.