# D.C. MATCHING ERRORS IN THE WILSON CURRENT SOURCE

*Indexing term: Constant-current sources*

A theoretical formula, accounting for parameter mismatch errors, and experimental measurements establish the necessity for using an additional transistor in the widely used Wilson current source if a direct-current transfer ratio centred on unity is to be achieved.

*Introduction:* The conventional Wilson current mirror[1] finds wide application in the design of bipolar monolithic linear integrated circuits. A basic parameter of the Wilson configuration is the direct-current transfer ratio $\lambda$, which, in precision design work, is often required to have a value of unity with as small a tolerance as possible. The application of the recently discussed early-intercept[2] voltage parameter facilitates the development of a practically useful formula, which is presented here, for estimating the nominal value of $\lambda$ and the tolerance limits on it, arising from mismatches in the operating conditions and the respective d.c. parameters of the transistors used.

*Theory:* Fig. 1 shows a standard Wilson current source, comprising closely matched monolithic bipolar transistors $T_1$, $T_2$, $T_3$, driven by an input current $I_i$ and producing an output current $I_o$. $I_i$ is assumed to be small enough to ensure low-level injection conditions in the transistors, but large enough to guarantee operation well above leakage-current levels. $\beta(\alpha)$ with the appropriate number subscript refers to the common-emitter (common-base) direct-current gain of the correspondingly numbered transistor.

The current ratio $P$ is given by

$$P = \{I + (\delta I)^{1} + (\delta I)^{11}\}/I \quad (1)$$

In eqn. 1, the collector current differential $(\delta I)^1$, arising from differences in the respective parameters of $T_1$, $T_2$ when they are operated at the same collector–base voltage, is conveniently expressed in terms of a base–emitter offset voltage of magnitude $v_{os}$.

By virtue of the logarithmic relationship between collector current and base–emitter voltage, and for the usual case $v_{os} \ll V_T$ (the thermal voltage $\simeq 25$ mV at 290 K), it is easily shown that

$$(\delta I)^1 \simeq \pm I(v_{os}/V_T) \quad (2)$$

Although $v_{os}$ is frequently attributed to emitter-area differences alone, such factors as basewidth differentials and unequal extrinsic base resistances do have a contributing effect, however secondary.

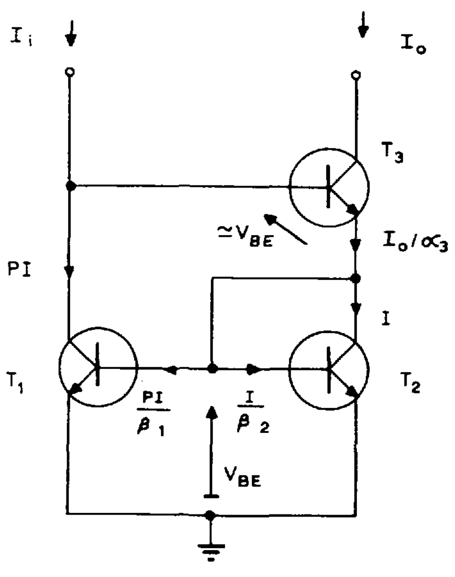


**Fig. 1** *Standard Wilson current source*

$(\delta I)^{11}$ is the collector-current differential that results from the collector–base voltage difference ($\simeq V_{BE}$) between $T_1$ and $T_2$. It can be expressed in terms of the l.f. output resistance $r_o$ of $T_1$. Thus

$$(\delta I)^{11} = (V_{BE}/r_o) = I(V_{BE}/Ir_o) \quad (3)$$

Now the product $Ir_o$ is independent of $I$, and, since $V_{BE} < 1$ V, it will be sensibly equal in magnitude to the early-intercept voltage[2] $(V_I)_o$ of T at zero collector–base voltage. Using eqns. 2 and 3, eqn. 1 becomes

$$P = 1 \pm (v_{os}/V_T) + \{V_{BE}/|(V_I)_o|\} \quad (4)$$

From Fig. 1, by current summation,

$$(I_o/\alpha_3) = I[1 + 1/\beta_2 + P/\beta_1] \quad (5)$$

or

$$(I_o/\alpha_3) \simeq I[1 + 2/\bar{\beta}] \quad (5a)$$

Eqn. 5*a* follows from eqn. 5 because $P \simeq 1$ and, by definition, $2/\bar{\beta} = 1/\beta_1 + 1/\beta_2$. Since, normally, $\beta_1$ and $\beta_2$ do not differ by more than about 10%, $\bar{\beta}$ can be identified as either the geometric or arithmetic mean of $\beta_1$ and $\beta_2$.

Using Kirchoff's current law,

$$I_i + I_o = PI + (I_o/\alpha_3). \quad (6)$$

Since $\beta_3 \gg 1$, $\alpha_3 \simeq 1 - 1/\beta_3$, eqn. 6 can be rewritten as

$$\{1 - 1/\beta_3\}\{I_i + I_o\} = \{1 - 1/\beta_3\}PI + I_o \quad (7)$$

Routine algebraic manipulation of eqn. 7, the use of the binominal expansion and appropriate engineering approximations commensurate with the order of magnitude of the terms involved finally yield

$$\lambda = I_o/I_i = 1 - \{V_{BE}/|(V_I)_o|\} \pm 2(\Delta\beta/\bar{\beta}^2) \pm (V_{os}/V_T) \quad (8)$$

in which $\Delta\beta = |\beta_3 - \bar{\beta}|$.

The nominal value for $\lambda$ is less than unity by an amount $V_{BE}/|(V_I)|_o$, which might well approach the magnitude of the tolerance on $\lambda$ resulting from $\beta$ and $V_{BE}$ mismatches. Thus, if

$V_{BE} = 0{\cdot}7$ V, $|(V_I)_o| = 70$ V, $\beta = 100$, $(\Delta\beta/\beta) = 0{\cdot}05$,

$v_{os} = 0{\cdot}25$ mV, then $\lambda = 0{\cdot}99 \pm 0{\cdot}011$. The nominal value can be made to approach unity more closely by the simple addition of the diode-strapped matched transistor $T_4$ in Fig. 2. This ensures that $T_1$, $T_2$ operate at virtually the same $V_{CB}$, i.e. zero, and hence sensibly eliminates the $V_{BE}$ term from eqn. 8. This 4-transistor current source has also been used, independently, by Schlotzhauer and Viswanathian,[3] but the error in the nominal value of $\lambda$ if $T_4$ is omitted has not previously, as far as the authors are aware, been quantified.

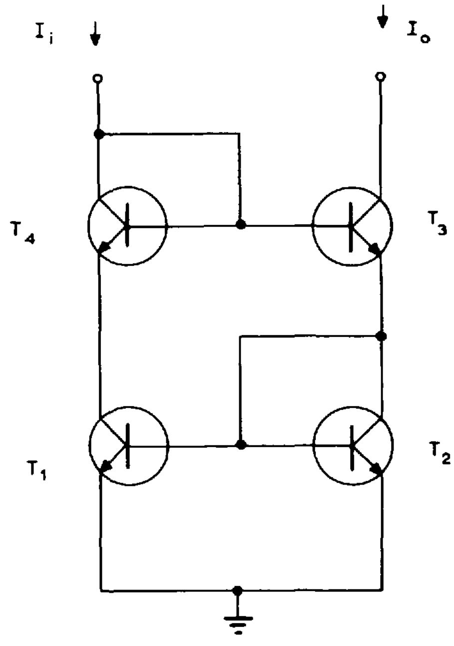


**Fig. 2** *Improved Wilson current source*

$T_1 + T_2$ = mono MAT-01 AH (Precision Monolithics)
$T_3 + T_4$ = mono MAT-01 AH

*Results:* To achieve a current transfer ratio $\lambda$ close to unity and to demonstrate the benefit of adding an extra transistor to the usual Wilson scheme, the circuit of Fig. 2 was constructed from two selected pairs of matched monolithic transistors of the type indicated. Test results that summarise the comparative performance of the circuits of Figs. 1 and 2 over the current range $10\,\mu A < I_i < 1$ mA are given in Table 1.

In columns (i) and (ii), $\lambda_1$, $\lambda_2$ refer, respectively, to the current transfer ratio of Figs. 1 and 2. $\lambda_1$ was actually obtained from measurements on the circuit of Fig. 2 with the base–emitter junction of $T_4$ short circuited. The values given for $\lambda_1$ and $\lambda_2$, to the accuracy stated, remain unchanged for an output voltage variation of 2 to 10 V. The performance of the improved Wilson source facilitated the design of a precision current-splitting circuit which has been described, briefly, elsewhere.[4]

*Discussion and conclusions:* Measurements made on the devices used gave $\bar{\beta} \simeq 370$ and $\Delta\beta = 50$ and the manufacturer's data sheet quotes $v_{os} = 0{\cdot}1$ mV (maximum) at $I_c = 10\,\mu$A. If offset voltage and $\beta$ mismatch were the sole source of error in the Wilson current source, it would be expected that $\lambda = 1 \pm 0{\cdot}005$.

Column (i) of Table 1 shows that $\lambda_1$ is clearly outside these tolerance limits, and column (ii) shows that $\lambda_2$ is within them. In this case, the major source of error in the departure of $\lambda$ from unity is clearly the collector–base differential of $T_1$, $T_2$ in the circuit of Fig. 1. Inspection of Table 1 shows that the relationship $\lambda_2 - \lambda_1 = V_{BE}/|(V_I)_o|$ is sensibly valid.

**Table 1**

| | (i) | (ii) | (iii) |
|---|---|---|---|
| $I_i$ | $\lambda_1$ | $\lambda_2$ | $V_{BE}/\lvert(V_I)_o\rvert$ |
| μA | | | |
| 10 | 0·984 | 0·998 | 0·011 |
| 100 | 0·986 | 0·998 | 0·012 |
| 1000 | 0·983 | 0·996 | 0·013 |

In conclusion, the magnitude of the error obtained with closely matched devices justifies the approximations made in the derivation and use of the formula for the current-mirror transfer ratio $\lambda$.

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*14th June 1976*

## References

1 WILSON, G. R.: 'A monolithic junction FET-*npn* operational amplifier', *IEEE J. Solid-State Circuits*, 1968, SC-3, pp. 341–348
2 HART, B. L., and BARKER, R. W. J.: 'Early-intercept voltage: a parameter of voltage-driven b.j.t.s', *Electron. Lett.*, 1976, 12, pp. 174–175
3 SCHLOTZHAUER, K., and VISWANATHIAN, T. R.: 'Improved integrated unity-gain current controlled sources', *Int. J. Electron.*, 1974, 37, pp. 27–31
4 BARKER, R. W. J., and HART, B. L.: 'A precision active current-splitting circuit technique'. Proceedings of 1st European solid state circuits conference (ESSCIRC), IEE Conf. Publ. 130, 1975, pp. 54–55

## DOUBLE-ERROR-CORRECTING BCH CODES FOR SOURCE ENCODING

*Indexing terms: Encoding, Error-correction codes*

It is known that error-correcting codes can be used to encode sources in the sense of data compression. We derive an expression for the average distortion per bit for double-error-correcting narrow-sense primitive BCH codes.

With reference to an equiprobable discrete memoryless source whose output is a sequence over GF(2), let us consider an $(n, k)$ binary group[1] code $V$ with minimum distance $d = 2t+1$. One of the ways in which $V$ can be used to compress the output of the source is as follows. Let $b$ represent an $n$ bit sequence coming out of the source. We can choose the codeword $v$ which has the shortest distance from $b$ and transmit, instead of $b$, the $k$ bit information sequence $c$ relevant to $v$. At the receiving end, $c$ is again converted into $v$.

The distance $\delta$ between $b$ and $v$ is the distortion. If $\bar{\delta}$ is the average of $\delta$ over the set of $2^n$ $n$ bit sequences, the average distortion $D$ per bit is given by $\bar{\delta}/n$. Using $V$ as one coset,[2] let us completely partition the set of $2^n$ $n$ bit sequences into cosets. If there are $\psi_\omega$ cosets with leaders of weight $\omega$, $D$ is given by

$$D = \frac{\Sigma\omega\psi_\omega}{2^{n-k}n} \qquad (1)$$

where the summation is over all relevant $\omega$. The compression ratio is given by $n/k$ and the rate distortion[2] $R(D)$ is given by $R(D) = 1 - D\log_2(1/D) - (1-D)\log_2(1/(1-D))$. It is known[2] that $D = 1/(n+1)$ for Hamming codes which are

$$(n = 2^s - 1,\ k = 2^s - 1 - s)$$

with $d = 3$ and that $D = 0{\cdot}1238$ for the Golay code which is (23, 12) with $d = 7$. The computation of $D$ in these cases is simple because these codes are perfect.

*Shortened-by-1-bit Hamming codes:* If $V$ is the

$$(n = 2^s - 2,\ k = 2^s - s - 2)$$

code obtained by shortening a Hamming code by 1 bit, it is known[3] that $V$ is a nearly perfect code. Using this fact it is easy to show, with reference to eqn. 1, that there would be one coset with the leader of weight zero, $n$ cosets with leaders of weight 1 and one coset with a leader of weight 2, so that eqn. 1 reduces to $D = 1/n$.

*Narrow-sense primitive BCH codes with $d = 5$:* With

$$n = 2^s - 1,$$

let us consider the $GF(2^s) = \{0, 1, a, a^2, \ldots, a^{n-1}\}$, where $a$ is a primitive element. Let $F_1(X)$ be the irreducible polynomial with $a$ as a root and $F_3(X)$ the irreducible polynomial with $a^3$ as a root. First, we establish that $n - k = 2s$.

It is known that $F_1(X)$ has degree $s$. If we partition the set $\{1, 2, \ldots, n-1\}$ into cyclotomic cosets, the coset with number 1 would be $1, 2, 2^2, \ldots 2^{s-1}$, implying that 3 does not belong to this coset. This means that $F_1(X) \neq F_3(X)$.

If $s$ is even, 3 divides $n$, so that $a^3$ has order $n/3$. If

$$(n/3) = 2^{s_1} - 1$$

for some $s_1 < s$, this would mean that

$$2^s - 1 = n = 2^{s_1+2} - 2^{s_1} - 2^2 + 1.$$

This is impossible, since $2^s - 1 = n = 1 + 2 + 2^2 + \ldots + 2^{s-1}$. Therefore $(n/3) \neq 2^{s_1} - 1$, implying that $F_3(X)$ has degree $s$.

If $s$ is odd, 3 does not divide $n$, so that $a^3$ has order $n$, implying that $F_3(X)$ has degree $s$.

Combining all these arguments, we have, in the context of the design of BCH codes, that a narrow-sense primitive BCH