An Adaptive Approach for Linearization of Temperature Sensor Characteristics

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Abstract

This paper discusses an adaptive approach, based on iterative construction of a piecewise linear approximant, for calibration of industrial sensors. Given the non-linear characteristic of the sensor, the approach consists of two steps: (i) the inverted characteristic is computed in pointwise form on the basis of iterative Newton-Raphson scheme; (ii) an iterative algorithm is employed in order to construct continuously improving set of piecewise linear approximations of the computationally inverted sensor characteristic. The main feature of the present approach is the irregular partitioning of the measurement range, induced from the error minimization procedure performed in (ii). On this basis, the lookup table necessary for calibration of the sensor is constructed in the "cheapest" possible way, allowing at one side optimal linear approximation in every particular measurement sub-range and an effective reduction of the necessary storage capacity at the other.

Key words: sensor's characteristic, signal processing, Newton-Raphson method, adaptive partitioning.

1. Introduction

Accurate temperature measurements are necessary in many cases of control and monitoring. In every industrial environment there are almost no situation where the precise measurement of the temperature to be not required. Whatever the application is, the designers always aim to construct an accurate measurement scheme at lowest possible price determined in financial and computational parameters. It is not surprising then the contemporary tendency to extend the possible application range of intelligent sensors and to introduce in the engineering practice the whole panoply of microtechnologies. The standard approach for conceptual and physical realization of intelligent sensors is based on the appropriate usage of low-cost microcontrollers. At comparatively low price one can also acquire onboard A/D and D/A converters, computational and communicational capabilities (CAN, SPI, I^{2}C, Ethernet) as well as a non-volatile memory for correction factors. A microcontroller can be used either to replace the existing analog functions such as linearization and filtering or to add enhanced functionality [8].

The present work is devoted to a problem concerning the optimal design using low-cost microcontrollers and complete analog front ends, which are oriented to measurement applications with microcontroller cores, for realization of intelligent temperature sensors. The constraints enforced from the architecture of the microcontrollers can be overcome using external software tools (programming environments, such as Mathlab, MathCAD etc.). In this way, such sometimes complicated tasks as interfacing, processing, correction and transmission of senor signals can be reduced to more simple tasks easily solvable with low-coast microcontrollers.

The central idea in this contribution is to reduce the computational cost for correction of the sensor's functionality on the basis of an optimal piecewise linear approximation, constructed for the inverted sensor characteristic. The meaning behind the notion of optimality should be understand in the following sense: on the basis of an iterative procedure, the measurement range is partitioned into a set of linear segments with irregular length so that the measurement error would be decreased under certain preliminary given threshold, dependent of the concrete measurement task. Such conceptual rationalization tends to avoid the main drawback of many approaches of sensors' calibration by shifting the necessary user-input form heuristically motivated number of segments to deterministically motivated threshold of the measurement error.

In a more rigor context, the computation of the final piecewise linear approximant is performed in two steps:

(i) Point wise reconstruction of inverted sensor characteristic,

(ii) Adaptive piecewise linearization of the inverse sensor characteristic.

In step (i) the sensor characteristics is computationally inverted using Newton-Raphson method for iterative evaluation of the reference points. On the basis of this reconstruction, in step (ii) an iterative procedure for adaptive partitioning of the measurement range is employed and the optimal linear approximant is evaluated. The procedure in step (ii) is gradientless and, hence, implemental for non-smooth sensor characteristics.

2. Calculation of the inverse characteristics

Consider the primary real variable x that possibly represents certain measurable quantity. The domain of x is called measurement range and will be denoted further with $M = [T_{min}, T_{max}] \in R$. The values T_{min} and T_{max} denote the lower and upper boundary of the measurement range. The measurement usually is performed in terms of other variable y that depends on x through certain physical effect. Examples in this context are the Seebeck's effect, piezoresistive effect and so on. The dependence between y and x is given in form of a regression polynomial constructed on the basis of some representative data set. This polynomial is called characteristics of the measurement device and will be denoted in the following way: $y = f(x)$. Most frequently in the engineering practice it is necessary to know the inverse characteristic $f^{-1}(y) = x$ because it allows one to obtain closed form expressions for the error in the measurable quantity x. However, the problem to find $f^{-1}(y)$ given f(x) is a non-trivial, since the regression map is a polynomial of order 10 and above and to invert it analytically is (almost) impossible. Much better approach is to invert the polynomial numerically using the Newton's method. From the available calibration standards it can be constructed the sequence $\{y_k\}$ at particular reference points in the interval $[f(T_{min}), f(T_{max})] \in R$. Then the following problem could be formulated:

Problem A. Considering the sequence $\{y_k\}_{k=1}^{n}$, and the closed form analytic characteristics $y = f(x)$, the sequence $\{x_k\}$, that constitutes the discrete representation of the inverse characteristics $f^{-1}(y)$, has to be found using certain iterative method.

Prior to describe the algorithm let us transform the function from the explicit $y = f(x)$ to an implicit form $F(x, y) = f(x) - y = 0$. For this function at every particular $y = y_k$ the following procedure should be performed:

1. Initialize the tolerance TOL, set the counter $j = 0$, and make an initial guess x^0 of the solution $\hat{x}_k$ at y_k.
2. Start an iterative process and compute from $x^{(j)}$ the next approximation $x^{(j+1)}$ of $\hat{x}_k$ according to the recurrence formula:
3.

$$x^{(j+1)} = x^{(j)} - \frac{F(x_j, y_k)}{F'(x_j, y_k)} = x^{(j)} - \frac{F(x_j, y_k)}{f'(x_j)} \qquad (1)$$

where $(.)' = d(.)/dx$.

4. Check for convergency according to

$$\left| \frac{F(x_j, y_k)}{F'(x_j, y_k)} \right| \le TOL \qquad (2)$$

If (2) is not satisfied, then go to step 2.

3. Adaptive approximation of the inverse characteristics

The inverse characteristic of the sensor was computed as a solution of the Problem A. However, in most of the cases such kind of solution is meaningless. The reason descents in its pointwise character that requires immense amount of storage space, thus, reducing significantly the operational capacity of the sensor defined in terms of performance and accuracy. To resolve such kind of problem another approach could be implemented. It is based on a piecewise linear approximation of the inverse characteristic. Essentially, the whole measurement region is subdivided (partitioned) into segments where in every segment the inverse characteristic is approximated with a straight line. The first order continuity requirements enforced on the boundary points lead to polygonal representation of the inverse characteristic, which is much more suitable for implementation in intelligent microcontrollers.

In the context of this linear approximation two important questions can be addressed:

(i) Should the measurement interval be partitioned in a regular manner; (ii) how much segments should be used in order to provide the piecewise linear approximation with the required level of accuracy. The answers of both questions, as the theory has shown, are mutually dependent. That is to say, the partition of the measurement range into a preliminary fixed number of sub intervals with the same length does not necessary provides "good" approximation. In addition, for some cases the approximation could be even worst (this depends on the regularity of the inverse characteristics). In order to provide a reasonable basis for non-regular subdivision of the measurement range, it should be firstly defined a reliable criterion, which would rationalize the quality of the approximation and after that, using this criterion, to identify in an iterative fashion the number and dimensions of the elements in the partitioning. The estimation of the quality of approximation is given through the so-called posteriori error. In our case this error is assumed to be the exact upper bound of the absolute approximation error. Accordingly, let the measurement range is divided into N subintervals. Let us denote the i[th] subinterval with $T_i = \left[T^i_{low} - T^i_{up}\right] \subset M$, where i = 1, 2, 3, ... , N. Solving the Problem A in every subinterval we could obtain the inverse characteristics in a pointwise form $\{y_k\}_{k=1}^m \rightarrow \{x_k\}_{k=1}^m$, where m determines the desired number of reference points $\{y_k\}_{k=1}^m$ per subinterval. Let also for the same sequence of reference points the linear approximation of the inverse function is calculated and given as $\{y_k\}_{k=1}^m \rightarrow \{\bar{x}_k\}_{k=1}^m$. The posteriori approximation error for the subinterval T_i is defined as

$$\varepsilon = \sup_{T_i}\{|x_k - \bar{x}_k|\} \tag{3}$$

Disposing on this posteriori error estimation, the partitioning refinement process has the following structure:

1. Construct an initial coarse partitioning P_0 representing sufficiently well the measurement range. Put an iteration indicator k = 0.
2. Build for the partitioning P_k the piecewise linear approximant and compute for every subinterval the posteriori error estimate ε.
3. If in every subinterval the posteriori error estimate is smaller than the preliminary specified threshold ε_{TOL}, then STOP. Otherwise, in all segments where $\varepsilon_{TOL} < \varepsilon$, refine the partitioning and construct the next subdivision P_{k+1}. Replace k by $k+1$ and return to step 2.

With such strategy the process of partitioning refinement goes subinterval by subinterval, providing smoothness of the computation and avoiding the instabilities due to round off errors.

4. Implementation of the algorithm

In this section we will discuss in more details the practical implementation of the algorithm for computation of inverse temperature characteristics for intelligent sensors based on low-cost microcontrollers and conventional thermocouple elements. Our intension is to illustrate with more details specific aspects of the adaptive partitioning process, approving its efficiency.

4.1. The transfer functions of a thermocouples and their linearization

The industrial applications require effective measurement of the temperature in broad ranges. Most frequently those measurements are performed using intelligent temperature sensors realized on the basis of two principal elements: thermocouples and resistive temperature sensors. The principle question that should be addressed then is how to build the look-up table for every one of those devices, according to its own specifics. Here we will consider only the thermocouples.

The thermo-couples have many advantages and by far are the most frequently used sensors for temperature measurements. Their broad measurement diapason varies from -270°C to 3000°C. They also have long-term stability and high reliability. The small size, compactness, the reduced response-time in order of milliseconds, and very low-cost add additional attractiveness to the characteristics of the thermocouples [1,6].

The characteristic of given thermocouple is usually tabulated, giving the output voltage in function of the temperature with respect to the reference point of 0 °C. In systems with computational capabilities like the microcontrollers, it is possible to use polynomial functions in order to interpolate between the discrete, tabulated values. The accuracy of interpolation depends on the data set used for calculation of the polynomial coefficients and the order of the interpolating polynomial. In general, the interpolating polynomial reads as,

$$E = \sum_{i=0}^{n} c_i (t_{90})^i , \tag{4}$$

where t_{90} is the temperature in °C, c_i are the polynomial coefficients; E is the output voltage (EMF) in mV and n is the order of the interpolating polynomial.

The tabulated values prescribe the reference correspondence between the temperature and the output voltage. However, in practice it is necessary to know the inverse correspondence (the inverse sensor characteristic), which describes the dependence of the temperature on the output voltage. In fact, the exact equations of the inverse transfer functions are very complex and their implementation in low cost microcontrollers is almost impossible, since the limitations imposed from the floating point arithmetic, ergo the exact inverse transfer functions, usually involve multiple high-order polynomial equations, most frequently up to 13-th order, with coefficients around 1.10^{-25}. For this reason, the direct implementation of the exact inverse sensor characteristics is not practically reasonable for thermocouples in microcontroller-based designs [7].

Figure 1 shows typical transfer functions for the thermocouple types in listed in Table 1. After taking care of cold junction compensation these transfer functions can be used to find the inverse sensor characteristics. The latter are significantly more nonlinear than the transfer functions, especially in the low temperature sub range.

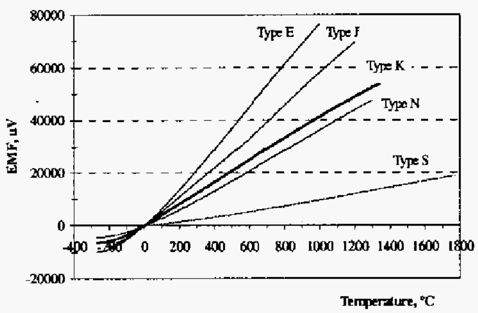

Fig. 1. Thermocouples transfer functions.

4.2. Error considerations

Using linear approximation in the range 0 – 1000°C results in maximal deviations around 20 °C. In order to obtain approximation errors in the diapason ±0,01°C it is necessary to use higher order polynomial approximation, which renders extremely difficult the calibration of the low-cost microcontrollers. As shown in Fig. 2, the piecewise continuous

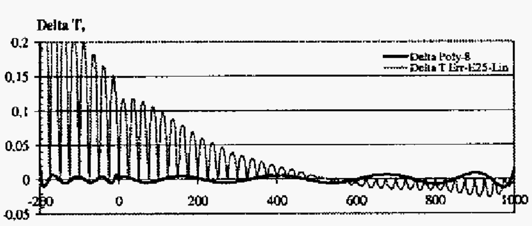

Fig. 2. Linearization error, for transfer function of E-Type thermocouple: a) after both, a polynomial approximation of 8-th /9-th order, b) after piecewise continuous linear approximation with regular partitions of 25 K.

linear approximation in regular intervals of 25°C, performed in the measurement range between 0°C and 1000 °C, reduces the approximation error below 0,15 °C. In the range [0 °C, 300 °C], the errors are below 0,05 °C and comparable with those obtained after 9^{th} order polynomial approximation.

Since the approximation error is a quantity influenced not only by the number of reference points but also by their replacement in the measurement interval, it is easy to guess that an adaptive procedure for partition of the measurement interval will be the most cheaper way to ensure accurate measurements. Under an 'adaptive procedure' we mean an algorithmmmically driven partitioning of the measurement range that to provide approximation error below preliminary prescribed limit.

Approximation errors under 0,15°C can be achieved with 25 linear segments (26 endpoint values) by the use of the adaptive procedure for the interval partitioning. This is a reduction of nearly 40% for the memory, needed for the look-up table. The complex high-order polynomial calculations are reduced to determining the segment number, on which the measured value falls; finding four values, which define the line segment, and computing the linearized value (four additions/ subtractions, one multiplication and one division).

5. Conclusions

The paper deals with an efficient algorithm for adaptive piecewise affine approximation of the inverse sensor characteristics. Although the discussion and the computational examples address the construction of adaptive approximants for temperature inverse characteristics, the algorithm could be easily extended to other process parameters, even when the approximation is based on the best fit-type linearization in the partitioning

The principal of the processing of sensor signals is to correct the defects of the primary sensor mechanism.

References

1. Pallás-Areny, R. and J.G. Webster, "Sensors and signal conditioning", John Wiley & Sons: New York, 2001.
2. Brignell, J.E. and N.M. White, Intelligent sensor systems, IOP, Bristol, 1994, ISBN 0-7503-0297-6.
3. Taner, A.H. and J.E. Brignell, "Aspects of intelligent sensor reconfiguration", Sensors and Actuators A, 1995.
4. Brignell, J.E., "Software techniques for sensor compensation", Sensors and Actuators A, Vol.25-27, 1991, 29-35.
5. Brignell, J. E., Digital Processing of Sensor Signals, pp. 3-107 – 3-114, 1989.
6. Fraden, J. , Handbook of modern sensors. Physics, designs and applications, AIP, Woodbury, NY, 1997.
7. Guide to thermocouple and resistance thermometry, Issue 6.0 by TC Ltd. Uxbridge England. http://www.tc.co.uk
8. Huddleston, C.: Digital signal controllers turn thermocouples into superstars. Sensors, January 2003.