Rashtrasant Tukadoji Maharaj Nagpur University

Department of Mathematics

M.Sc. (Mathematics) Semester I

Mid Semester Examination Winter-2022

Time: Two hour | **Algebra I** | **Maximum Marks: 40**

Note: Solve Q.1 (A), (B) **OR** (C), (D), Q.2 (A), (B) **OR** (C), (D)

Q. 1 (A) State and prove fundamental theorem of homomorphism. (10)

(B) Describe all the permutations on the set $\{1, 2, 3\}$. Find their respective order. Also, find all the even permutations and all the odd permutation. (10)

OR

(C) Let H be a subgroup of a group G. Prove that H is a normal subgroup of G if and only if $h \in H$ and $x \in G \Longrightarrow xhx^{-1} \in H$. (10)

(D) Describe the group of symmetries of a square (Dihedral group D_4). (10)
Find the number of elements of each possible oreder in dihedral group D_4, and dihedral group D_6.

Q. 2 (A) Prove that A_5 is a simple group. (10)

(B) Prove that every normal series of a group G is subnormal but converse may not be true. (10)

OR

(C) Prove that $|A_n| = \frac{n!}{2}$. (10)

(D) Prove that any subgroup H of a solvable group G is solvable. (10)

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DEPARTMENT OF MATHEMATICS
M.Sc. (Mathematics) Semester I
MID-SEMESTER EXAMINATION WINTER-2022
Ordinary Differential Equations

[Time: 02Hrs] [Max.Marks: 40]

1 (A) Let ϕ be any solution of an equation $L(y) = y'' + a_1y' + a_2y = 0$ on an interval I containing a point x_0. Then prove that for all x in I

$$\|\phi(x_0)\|e^{-k|x-x_0|} \leq \|\phi(x)\| \leq \|\phi(x_0)\|e^{k|x-x_0|}$$

where $\phi(x)$ is a solution of $L(y) = 0$. [10]

(B) Find two linearly independent solutions of the equation

$$(3x-1)^2y'' + (9x-3)y' - 9y = 0, \text{for } x > 1/3$$ [10]

OR

(C) If ϕ_1 is a solution of $L(y) = y'' + a_1(x)y' + a_2(x)y = 0$ on an interval I and $\phi_1(x) \neq 0$ on I, then prove that a second solution ϕ_2 is given by

$$\phi_2(x) = \phi_1(x)\int_{x_0}^{x}\frac{1}{[\phi_1(s)]^2}\exp\left[-\int_{x_0}^{s}a_1(t)dt\right]ds$$ [10]

(D) If $\phi_1(x) = x$ and $\phi_2(x) = x^3$ are solutions of

$$x^3y''' - 3xy' + 3y = 0, x > 0$$

then find a third independent solution of the equation. [10]

2 (A) Show that $\phi_1(x) = |x|^{r_1}$ and $\phi_2(x) = |x|^{r_2}$ are the solutions of the Euler equation $x^2y'' + a\,x\,y' + b\,y = 0, a, b$ are constants and r_1, r_2 are distinct roots of $q(r) = r(r-1) + ar + b = 0$. Discuss the case when $r_1 = r_2$ [10]

(B) Find a solution of the equation $2x^2y'' + (x^2 - x)y' + y = 0$ in the form of

$$\phi(x) = x^r\sum_{k=0}^{\infty}c_kx^k, x > 0$$ [10]

OR

(C) Consider the equation $x^2y'' + a(x)x\,y' + b(x)y = 0$, where $a(x), b(x)$ have power series expansions convergent for $|x| < r_0, r_0 > 0$. Let r_1, r_2 are roots of indicial polynomial $q(r) = 0$. Discuss the solutions for the cases (i) $r_1 = r_2$ and (ii) $r_1 - r_2$ is positive integer. [20]

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M.Sc. (Mathematics) Semester III

MID SEMESTER EXAMINATION WINTER-2022

PROBABILITY AND DISTRIBUTION THEORY

[Time: 02Hrs] [Max.Marks:40]

Que 1	**A)**	State the p.m.f. of Binomial distribution.Obtain the M. G. F. and P. G. F. and hence the mean and variance of the binomial distribution.	[10]
	B)	Give the p.m.f of Poisson distribution. Show that Poisson distribution can be obtained as a limiting case of Binomial distribution.	[10]
		OR	
	C)	Define Geometric distribution. State and prove the lack of memory property of Geometric distribution.	[10]
	D)	Define (a) Joint probability density function (b) Marginal density function (c) Conditional density function	[06]
	E)	The joint p.d.f of two continuous r.vs X and Y is given by $f(x,y) = (2x+y)\ e^{-x-y}$ for $x > 0$ and $y > 0$ then find the marginal densities of X and Y	[04]
Que 2	**A)**	Define univariate Normal distribution. Obtain its M. G. F and hence the first two moments of it.	[10]
	B)	Define Cauchy distribution.Derive its p.d.f. Show that the moments of order ≥ 1, do not exist.	[10]
		OR	
	C)	Define Gamma Distribution. Derive its M.G.F. State and prove the additive property of Gamma distribution.	[10]
	D)	Define the Beta distribution of first kind and Beta distribution of second kind. For the first kind of Beta distribution obtain the rth raw moments and hence the mean . Also compute the harmonic mean of Beta distribution of first kind.	[10]

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M.Sc. (Mathematics) Semester I

MID SEMESTER EXAMINATION WINTER-2022

REAL ANALYSIS-I

[Time: 02Hrs] [Max.Marks:40]

Que 1 **A)** Suppose $\{f_n\}$ is a sequence of functions differentiable on $[a,b]$ and such that $\{f_n(x_0)\}$ converges for some point x_0 on $[a,b]$. If sequence $\{f_n'\}$ converges uniformly on $[a,b]$ then prove that $\{f_n\}$ converges uniformly on $[a,b]$ to a function f and $f'(x) = \lim_{n\to\infty} f_n'(x), x \in [a,b]$ [10]

B) Suppose K is compact, and

a) $\{f_n\}$ is a sequence of continuous functions on K,

b) $\{f_n\}$ converges pointwise to a continuous function f on K,

c) $f_n(x) \geq f_{n+1}(x)$ for all $x \in K, n = 1,2,...$

Then prove that $f_n \to f$ uniformly on K. [10]

OR

C) If f is continuous complex function on $[a,b]$, then prove that there exists a sequence of polynomials P_n such that $\lim_{n\to\infty} P_n(x) = f(x)$ uniformly on $[a,b]$. If f is real, then P_n may be taken real. [10]

D) Let α be monotonically increasing on $[a,b]$. Suppose $f_n \in R(\alpha)$ on $[a,b]$, for $n = 1,2,...$ and suppose $f_n \to f$ uniformly on $[a,b]$. Then prove that $f \in R(\alpha)$ on $[a,b]$ and $\int_a^b f\, d\alpha = \lim_{n\to\infty} \int_a^b f_n\, d\alpha$. [10]

Que 2 **A)** Prove the following

a) If $A \in L(R^n, R^m)$, then prove that $\|A\| < \infty$ and A is uniformly continuous mapping of R^n into R^m.

b) If $A, B \in L(R^n, R^m)$ and c is a scalar, then $\|A+B\| \leq \|A\| + \|B\|$ and $\|cA\| = |c|\|A\|$. With the distance between A and B defined as $\|A-B\|$, then $L(R^n, R^m)$ is a metric space. [10]

B) State and prove the chain rule. [10]

OR

C) Let Ω be the set of all invertible linear operators on R^n.

a) If $A \in \Omega, B \in L(R^n)$ and $\|B - A\| \cdot \|A^{-1}\| < 1$, then prove that $B \in \Omega$.

b) Ω be an open subset of $L(R^n)$ and the mapping A to A^{-1} is continuous on Ω. [10]

D) State and prove Contraction mapping theorem. [10]

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DEPARTMENT OF MATHEMATICS
M.Sc. (Mathematics) Semester I
MID SEMESTER EXAMINATION WINTER-2022
TOPOLOGY

[Time: 02Hrs] [Max.Marks: 40]

Que 1: A) Prove that

$Q_+ = \left\{ \frac{p}{q} \mid p,q \in Z_+ , q \neq 0 \right\}$ is countable. (8 Marks)

B) Prove that a finite product of countable set is countable. (12 Marks)

OR

C) Prove that (14 Marks)

(i) $N_0 N_0 = N_0$

(ii) $N_0\, c = c$

(iii) $c.c = c$

D) Prove that (6 Marks)

$N_0 + N_0 = N_0$

Que: 2 A) If (X,τ) is a topological space and $E \subseteq X$ then $c(E) = E \cup d(E)$. (10 Marks)

B) A family β of sets is a base for a topology for the set $X = \cup \{B \mid B \in \beta\}$ if and only if for any $B_1, B_2 \in \beta$ and for any $x \in B_1 \cap B_2$ their exist a set $B \in \beta$ such that $x \in B \subseteq B_1 \cap B_2$.

(10 Marks)

OR

C) Prove that $i(E) = \sim c(\sim E)$ (12 Marks)

D) Let $\{E_\alpha\}$ be a collection of sets E_α in a topological space (X,τ), then the closure operator in X has the following property.

i) $\bigcup_\alpha c(E_\alpha) \subseteq c\left(\bigcup_\alpha E_\alpha\right)$ ii) $\bigcap_\alpha c(E_\alpha) \supseteq c\left(\bigcap_\alpha E_\alpha\right)$ (8Marks)