Rashtrasant Tukadoji Maharaj Nagpur University
Department of Mathematics
M.Sc. (Mathematics) Semester I
End Semester Examination Winter-2022
Fundamentals of Computer and C-Programming

[Time: Three hour] Maximum Marks: 60

Q.1 (A) Explain the characteristics of computer. 5

(B) Explain the terms (i) Machine Language (ii) Assembly language. 5

OR

(C) Explain the computer block diagram system in detail. 10

Q.2 (A) According to Gregorian calendar it was Monday on 1st January 01 (i.e.01/01/01). If any year is input through the keyboard, write a C program using **decision control instruction** to find out what is day on 1st January of this year ? 5

(B) What is Armstrong number ? Write a program in C, to check entered number is Armstrong or not. 5

OR

(C) Explain the following terms 4

(i) printf( ) (ii) scanf( ) (iii) break statement (iv) continue statement

(D) Explain decision control instruction if-else by giving suitable example. 6

Q.3 (A) Write a Menu Driven Program which has following options: 10

1. Factorial of a number. 2.Number is prime or not
3. Number is even or odd 4. Exit

OR

(C) Write a program in C for addition of 2 Dimensional array. 10

Q.4 (A) What is the important features of MS Power point ? 5

(B) What is the important features of MS Word ? 5

OR

(C) Explain the use of MS Excel. 10

Q.5 Choose the correct option and write it in the answer sheet.

(A) Which of the following is not a logical operator ? 1

(a) & (b) && (c) || (d) !

(B) In mathematics and computer programming, which is the correct order of mathematical operators ? 1

(a) Addition, Subtraction, Multiplication, Division
(b) Division, Multiplication, Addition, Subtraction
(c) Multiplication, Addition, Division, Subtraction
(d) Addition, Division, Modulus, Subtraction

D M A S

(C) Who is the father of C language? 1

(a) Steve Jobs (b) James Gosling (c) Dennis Ritchie (d) Rasmus Lerdorf

(D) scanf( ) is a predefined function in ______ header file. 1

a) stdlib. h b) ctype. h c) stdio. h d) stdarg. h

(E) Choose correct statement about functions in C language. 1

(a) A function is a group of c statements which can be reused any number of times
(b) Every function has a return type
(c) Every function may not return a value
(d) All the above.

(F) Choose the correct syntax for C arithmetic compound assignments operators. 1

(a) p+=q is (p= p+ q)
p-=q is (p= p-q)

(b) p*=q is (p=p*q)
p/=q is (p = p/q)

(c) p%=q is (p=p%q)

(d) All the above.

(G) What is the output of C program ? 1

```
int main( )
{
        int k, j;
        for (k=1, j=10; k<=5; k++)
        {
                printf("%d", (k+j) );
        }
        return 0;
}
```

(a) compiler error
(b) 10 10 10 10 10
(c) 11 12 13 14 15
(d) None of the above.

(H) Identify wrong keywords below. 1
(a) auto, double, int, struct
(b) break, else, long, switch
(c) case, enum, register, typedef
(d) char, extern, intern, return

(I) Range of signed char and unsigned char are 1
(a) -128 to +127
0 to 255
(b) 0 to 255
-128 to +127
(c) -128 to -1
0 to +127
(d) 0 to +127
-128 to -1

(J) What is the full form of COBOL? 1
a) Common Business Oriented Language
b) Common Business Object Language
c) Common Beneficial Oriented Language
d) Common Beneficial Object Language

Q.6 Fill in the blanks
(A) The symbol -------- is used as a statement terminator in C. 1
(B) The format specifier ------ is used to print the values of double type variable 1
(C) The central processing unit is located in the ---------unit. 1
(D) Bold. Italin, Regular are known as ---------styles 1
(E) Microsoft Word, Microsoft Excel, and Microsoft PowerPoint are the part of -----------------suite. 1

Q.7 Answer the questions in short
(A) What is variable in C language? 1
(B) What is function in C? 1
(C) What is mean by array? 1
(D) Write the working of break statement. 1
(E) What is central processing unit? 1

Rashtrasant Tukadoji Maharaj Nagpur University

Department of Mathematics

M.Sc. (Mathematics) Semester I

End Semester Examination Winter-2022

Algebra-I

[Time: Three hours] [Maximum Marks: 60]

Q.1 (A) Let $S = \{1, i, -1, -i\}$. Show that $(S, \cdot)$ is an abelian group. Is S cyclic? If yes, then find the generators of S. 5

(B) Prove that every permutation on a finite set is either a cycle or it can be expressed as a product of disjoint cycles. 5

OR

(C) Let $S = \{1, 2, 3, 4, 5\}$ and $f = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 3 & 4 & 2 \end{pmatrix}, g = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{pmatrix}, h = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 2 & 1 & 3 \end{pmatrix}$ 5
(1) Find $fg,\ gh,\ f^{-1}$ (2) Show that $f(gh) = (fg)h$ and $f^{-1}f = i$ (identity permutation)

(D) Let H be a subgroup of a group G and $[G: H] = 2$. Then prove that H is a normal subgroup of group G. 5

Q.2 (A) If the group G be the internal direct product of its subgroups H and K, then prove that G is isomorphic to the external direct product $H \times K$. 5

(B) Find the primary decomposition and the corresponding invariant factor decompositions for all abelian groups of order 360. 5

OR

(C) State and prove first Sylow theorem. 5

(D) If G be a group of order pq for some primes p and q such that $p > q$ and q does not divides $(p-1)$ then prove that $G \simeq Z_{pq}$. 5

Q.3 (A) Define simple group. Prove that A_5 is simple. 5

(B) Define nilpotent groups. Prove that every abelian group is nilpotent. 5

OR

(C) Prove that symmetric group of order 3 is solvable. 5

(D) Let A_n be the alternating group of degree n, then prove that $|A_n| = \frac{n!}{2}$ 5

Q.4 (A) Prove that in a ring $(R, +, \cdot)$, the following results are true: 5
(i) $a \cdot 0 = 0 \cdot a = 0$, for all $a \in R$, 0 being the zero element in R.
(ii) $a \cdot (-b) = (-a) \cdot b = -(ab)$, for all $a,\ b \in R$
(iii) $(-a)(-b) = ab$ for all $a,\ b \in R$

(B) Prove that the set of all units in a ring R with unity forms a group with respect to multiplication. 5

OR

(C) Show that the ring of matrices $\left\{\begin{bmatrix} 2a & 0 \\ 0 & 2b \end{bmatrix} : a,\ b \in \mathbb{Z}\right\}$ contains divisors of zero and does not contain the unity. 5

(D) If a is the unit in a ring R with unity, then prove that a is not a divisor of zero. 5

Q.5 Choose the correct option and write it in the answer sheet.

(A) The group is said to be abelian if it satisfy ______________ property. 1

(a) Associativity (b) Commutativity (c) Inverse (d) Identity

(B) Let $(G, \cdot)$ be a group $a \in G$, and e be the identity element in G, then the order of a in G is the least positive integer n such that 1

(a) $a^n = e$ (b) $na = e$ (c) $ne = a$ (d) $ae = n$

(C) Let $G = \{5,\ 15,\ 25,\ 35\}_{mod\ (40)}$ then the identity element of group $(G, mod\ (40))$ is 1

(a) 5 (b) 15 (c) 25 (d) 35

(D) In dihedral group D_6, the number of elements of order 2 =__________. 1

(a)3 (b) 5 (c) 7 (d) 9

(E) The order of $(1,\ 2)$ in the group $Z_3 \times Z_4$ is __________. 1

(a) 3 (b) 4 (c) 5 (d) 6

(F) The invariant factor decomposition of the group $Z_4 \times Z_{12} \times Z_{18}$ is __________. 1

(a) $Z_2 \times Z_{24} \times Z_{18}$ (b) $Z_4 \times Z_{12} \times Z_{18}$ (c) $Z_6 \times Z_8 \times Z_{18}$ (d) $Z_2 \times Z_{12} \times Z_{36}$

(G) Let $|G| = 1225$ then G has a 5-Sylow subgroup of order______. 1

(a) 5 (b) 25 (c) 125 (d) 625

(H) Which of the following is a commutative ring without unity ? 1

(a) $\mathbb{Z}$ (Set of integers) (b) $\mathbb{R}$ (Set of reals)

(c) $2\mathbb{Z}$ (Set of even numbers) (d) none of the above

(I) Which of the following is the example of ring with zero divisions ? 1

(a) $\mathbb{Z}$ (Set of integers) (b) $\mathbb{R}$ (Set of reals) (c) $\mathbb{Z}_6$ (d) $\mathbb{Z}_5$

(J) The characteristic of the ring $(Z_5, +, \cdot)$ is ______. 1

(a) 1 (b)3 (c) 5 (d) 7

Q.6 Fill in the blanks

(A) The semigroup $(G,*)$ satisfies ______ and ________ property. 1

(B) The number of elements in a group G is called ______ of group G. 1

3)(4

(C) Let $f = (1\ \ 2\ \ 5)(3\ \ 5)$ then $f^{-1} =$ ________. 1

(D) The order of an element 2 in $\mathbb{Z}_5$ with respect to addition is _________. 1

(E) Let $(G, \cdot)$ and $(G',*)$ be two groups. A mapping $\phi: G \longrightarrow G'$ is said to be a homomorphism if ____________. 1

Answer the questions in short

Q.7 (A) Define ring and give its examples. 1

(B) Define normal subgroup and give its examples. 1

(C) Define field and give its one example. 1

(D) Define unit in a ring. Find the uints in the ring $(\mathbb{Z}_5,\ +,\ \cdot)$. 1

(E) Define integral domain and give its two examples. 1

ESE/W22/MMT1T02

Rashtrasant Tukadoji Maharaj Nagpur University

Department of Mathematics

M.Sc. (Mathematics) Semester I

End Semester Examination Winter-2022

Real Analysis-I

[Time: Three hour] Maximum Marks: 60

Q.1 (A) Suppose $f_n \to f$ uniformly on a set E in a metric space. Let x be a limit point of E and suppose that $\lim_{t \to x} f_n(t) = A_n$, $n = 1,2,....$ Then prove that $\{A_n\}$ converges and $\lim_{t\to x} f(t) = \lim_{n\to\infty} A_n$. 5

(B) If $\{f_n\}$ is a sequence of continuous functions on E and if $f_n \to f$ uniformly on E then prove that f is continuous on E. 5

OR

(C) If $\{f_n\}$ is a sequence of continuous function on E and if $f_n \to f$ uniformly on E then prove that f is continuous on E. 5

if k is compact, $f_n \in C(K)$ for $n=1,\cdots$ if $\{f_n\}$ is pointwise bounded & equicontinuous on K then prove that is uniformly bounded on K

(D) Show that $\lim_{m\to\infty} \lim_{n\to\infty} (cos m!\pi x)^{2n}$; $m = 1,2, \dots ....$ is everywhere discontinuous function which is not R-integrable. 5

Q.2 (A) Suppose X is a vector space and $dimX = n$. Then show that a set E of n vectors in X spans X if and only if E is independent. 5

(B) Suppose f maps a convex open set $E \subset \mathbb{R}^n$ into $\mathbb{R}^m$, f is differentiable in E and there is a real number M such that $\|f'(x)\| \le M, \forall x \in E$. Then prove that $|f(b) - f(a)| \le M|b - a|, \forall a, b \in E$. 5

OR

(C) Suppose K is a compact subset of R^n and $\{V_\alpha\}$ is an open cover of K. Then prove that there exists functions $\psi_1, \psi_2, \dots . \psi_s \in C(R^n)$ such that $0 \le \psi_i \le 1$ for $1 \le i \le s$. 5

(D) Suppose E is an open set in R^n, $f: E \to R^m$, f is differentiable at $x_0 \in E$, g maps an open set containing $f(E)$ into R^k and g is differentiable at $f(x_0)$. Then prove that the mapping $F: E \to R^k$ defined by $F(x) = g(f(x))$ is differentiable at x_0 and $F'(x_0) = g'(f(x_0)).f'(x_0)$. 5

Q.3 (A) An equivalence relation ~ on X is open, π is an open mapping and X has a countable basis of open sets then show that $X/\sim$ has a countable basis of open sets. 5

(B) Prove that a topological manifold M is locally compact. 5

OR

(C) Let ~ be an equivalence relation on a topological space X. Also $R \subseteq X \times X$ be defined as $R = \{(x, y) \mid x \sim y\}$. Then show that R is a closed subset of the space $X \times X$ if the quotient space $X/\sim$ is Hausdorff. 5

(D) Prove that S^1 is a $\mathbb{C}^\infty$-one dimensional differentiable manifold. 5

Q.4 (A) Show that F is an immersion of the following: 5

i. $F: R \to R^3$, given by $F(t) = (cos2\pi t, sin2\pi t, t)$

ii. $F: (1, \infty) \to R^2$, given by $F(t) = \left(\frac{1}{t}\cos 2\pi t, \frac{1}{t}\sin 2\pi t\right)$

(B) If $F: N \to M$ is a one to one immersion and N is compact then prove that F is an imbedding and $\tilde{N} = F(N)$ is a regular submanifold of M. 5

OR

(C) If G_1 and G_2 are lie groups then show that $G_1 \times G_2$ is a lie group. 5

(D) Prove that $GL(n, \mathbb{R})$ is a lie group. 5

Q.5 **Choose the correct option and write it in the answer sheet.**

(A) If K is a compact metric space, if $f_n \in C(K)$ for $n = 1,2, \dots$ and if $\{f_n\}$ converges uniformly on K, then $\{f_n\}$ is .... on K. 1

(a) Continuous (b) Equicontinuous (c) Differentiable (d) Uniformly continuous

(B) Suppose K is compact and : 1

i. $\{f_n\}$ is a sequence of continuous functions on K,

ii. $\{f_n\}$ converges pointwise to a continuous function f on K,

iii. $f_n(x) \leq f_{n+1}(x)$ for all $x \in K, n = 1,2,3 ....$

Then $f_n \rightarrow f$ uniformly on K.

Which of these statement **is/are correct?**

(a) (i), (iii) (b) (i), (ii) (c) (i),(ii) and (iii) (d) only (iii)

(C) Suppose f is a ______ of an open set $E \subset R^n$ into R^n, $f'(a)$ is invertible for some $a \in E$ and $b = f(a)$. Then there exist open sets U and V in R^n such that $a \in U, b \in V, f$ is one-one on U and $f(U) = V$. 1

(a) C'-mapping (b) open mapping (c) uniformly convergent (d) convergent

(D) If $A \in L(R^{n+m}, R^n)$ and if A is invertible, then there corresponds to every $k \in R^m$ a unique $h \in R^n$ such that ________. 1

(a) $A(h,k) = 1$ (b) $A(h,k) \geq 1$ (c) $A(h,k) \neq 0$ (d) $A(h,k) = 0$

(E) If X is a complete metric space and if $\emptyset$ is a contraction of X into X, then there exists oneand only one $x \in X$ such that $\emptyset(x) =$______. 1

(a) x^2 (b) x (c) 1 (d) 0

(F) Let X be a vector space. An operator $P \in L(X)$ is said to be a projection in X if $P^2 =$______. 1

(a)0 (b) 1 (c) P (d) ∞

(G) A topological space is called______ if each point is contained in a compact neighborhood. 1

(a) locally compact (b) σ −compact (c) locally connected (d) compact subset

(H) A space is called__________if every open cover has countable subcover. 1

(a) locally compact (b) compact subset (c) σ −compact (d) Lindelof

(I) Let $F: N \rightarrow M$ be an immersion. Then each $P \in N$ has a neighbourhood U such that______ is an imbedding of U in M. 1

(a) U/F (b) F/U (c) $F.U$ (d) F

(J) If $F: N \rightarrow R$ is a C^∞ manifolds, dimension of R is one and dimension of N is n. If rank of F isone at every point of $A = F^{-1}(a)$, then A is a closed, regular submanifold of N of ________. 1

(a) One-dimension (b) dimension 2 (c) dimension n (d) dimension $n - 1$

Q.6 **Fill in the blanks**

(A) There exists a real continuous function on the real line which is nowhere__________ 1

(B) If $A \in L(R^n, R^m)$, then $\|A\| < \infty$ and A is a __________ mapping of R^n into R^m. 1

(C) The real projective space $P^2(R)$ is a differentiable manifold of________. 1

(D) Let $F: N' \rightarrow M$ be an imbedding of a c^∞-manifold N' of dimension n in a c^∞- manifold M of dimension m. Then $N = F(N')$ has the n-submanifold property and that F is a __________. 1

(E) A linear operator A on a_______vector space X is one-to-one if and only ifthe range of A is all of X. 1

Q.7 **Answer the questions in short**

(A) Define the term equicontinuity. 1

(B) State the Stone-Weierstrass theorem. 1

(C) State Implicit Function Theorem. 1

(D) Define Topological manifold. 1

(E) Define regular submanifold. 1

Rashtrasant Tukadoji Maharaj Nagpur University

Department of Mathematics

M.Sc. (Mathematics) Semester I

End Semester Examination Winter-2022

Topology-I

[Time: Three hour] Maximum Marks: 60

Q.1 (A) Prove that a finite product of countable set is countable. 10

OR

(B) Prove that i) $N_0 N_0 = N_0$ ii) $N_0 c = c$ 10

Q.2 (A) If (X,τ) is a topological space and $E \subseteq X$ then prove that $C(E) = E \cup d(E)$ 10

OR

(B) Let $\{E_\alpha\}$ be a collection of sets E_α in a topological space (X,τ), then prove that the closure operator in X has the following property 5

i) $\bigcup_\alpha C(E_\alpha) \subseteq C\left(\bigcup_\alpha E_\alpha\right)$ ii) $\bigcap_\alpha C(E_\alpha) \subseteq C\left(\bigcap_\alpha E_\alpha\right)$

(C) Prove that $i(E) = \sim C(\sim E)$ 5

Q.3 (A) Prove that the union of any family $\{C_\alpha\}$ of connected sets having a nonempty intersection is a connected set. 5

(B) Prove that every compact subset of a topological space is countably compact. 5

OR

(C) Prove that every open subset of a locally connected topological space is itself locally connected. 5

(D) Show that a mapping f on X into X^* is open iff $f(i(E)) \subseteq i^* f(E)$ 5

Q.4 (A) Prove that a topological space is a T_1-space iff every singleton set is closed. 5

(B) Let X be a T_1-space and $E \subseteq X$. Then prove that x is a limit point of E iff every open set containing x contains infinite many points of E. 5

OR

(C) Prove that T_2- property is both topological and hereditary. 10

Q.5 Choose the correct option and write it in the answer sheet.

(A) If N_0 and c are the cardinal numbers of Z_+ and R respectively, then 1

(a) $2^{N_0} = 1$ (b) $2^{N_0} = 0$ (c) $2^{N_0} = N_0$ (d) $2^{N_0} = c$

(B) Which of the following statement is/are necessarily true? 1

(a) Every countable set contains an uncountable set.
(b) Every subset of uncountable set is uncountable.
(c) Every subset of countable set is countable.
(d) All of these

(C) If A and B are two subsets of a topological space X, then 1

(a) $d(A \cup B) = d(A) \cap d(B)$ (b) $d(A \cap B) = d(A) \cup d(B)$

(c) $d(A \cup B) = d(A) \cup d(B)$ (d) Both (a) & (b)

(D) Let (X,τ) be a topological space and E be any subset of X. A point $x \in X$ is said to be a limit point of a set E if and only if each open set $G \in \tau$, containing a point x, ... 1

(a) $E \cap G - \{x\} = \phi$ (b) $E \cap G - \{x\} \neq \phi$ (c) $E \cap G = \{x\}$ (d) None of these

(E) A topological space X is said to be countably compact if and only if 1
(a) X is countable
(b) X compact and countable
(c) every infinite subset of it has a limit point in it
(d) each point x inX, has a neighbourhood which is compact.

(F) If C is a connected set and if $C \subseteq E \subseteq c(C)$, then E is a.......... 1
(a) closed set (b) complete set c) disconnected set (d) connected set

(G) Which of the following statement is necessarily correct: 1
(a) subset of compact topological space is compact.
(b) closed subset of countably compact topological space is countably compact.
(c) image of connected topological space is connected
(d) None of these

(H) Let (X,τ) be a topological space and for every pair (x,y) of distinct elements $x,y \in X$ there exists an open sets $G_1, G_2 \in \tau$ such that $x \in G_1$ but $y \notin G_1$ and $y \in G_2$ but $x \notin G_2$. Then (X,τ) is called 1
(a) T_0 space
(b) T_1 space
(c) Hausdorff space
(d) None of these

(I) A topological space (X,τ) is T_1 space if 1
(a) (X,τ) is T_0 space, because every T_0 space is T_1 space
(b) to each point $x \in X$, $\{x\}$ is open in X
(c) to each point $x \in X$, $\{x\}$ is closed in X
(d) None of these

(J) Which of the following statement is not correct? 1
(a) Every second axiom space is a first axiom space.
(b) Every first axiom space is a second axiom space.
(c) Subspace of T_2 space is T_0 space.
(d) None of these

Q.6 Fill in the blanks
(A) If A is finite set, then its power set is 1
(B) Interior of E is the ....open subset of E. 1
(C) Closure of E is always 1
(D) .... of a compact topological space is always compact 1
(E) The components of distinct points are 1

Q.7 Answer the questions in short
(A) Define power set. 1
(B) Define Interior of a set. 1
(C) Define separation of a set. 1
(D) Define Locally Connected set. 1
(E) Define Hausdorff space. 1

RashtrasantTukadojiMaharaj Nagpur University

Department of Mathematics

M.Sc. (Mathematics) Semester I

End Semester Examination Winter-2022

Ordinary Differential Equations

Time: Three hour **Maximum Marks: 60**

Q.1 (A) Let $\phi_1, \phi_2, \dots, \phi_n$ are n solutions of $L(y) = y^{(n)} + a_1(x)y^{(n-1)} + \dots + a_n(x)y = 0$ on an interval I, satisfying $\phi_i^{(i-1)}(x_0) = 1, \phi_i^{(j-1)}(x_0) = 0, j \neq i$ for some $x_0 \in I$. If ϕ is any solution of $L(y) = 0$, on I then prove that there are n constants $c_1, c_2, \dots, c_n$ such that $\phi = c_1\phi_1 + c_2\phi_2 + \dots + c_n\phi_n$ for all x in I. 5

(B) Verify that $\phi_1(x) = x^3, x > 0$ satisfies the equation $x^2y'' - 7xy' + 15y = 0$ hence find a second independent solution. 5

OR

(C) Find two linearly independent power series solutions of the equation $y'' - xy = 0$. For what values of x do the series converge? 10

Q.2 (A) Prove that a basis for the solutions of the Euler equation $x^2y'' + axy' + by = 0$, a and b constats, is $\phi_1(x) = |x|^{r_1}, \phi_2(x) = |x|^{r_2}$ in case of r_1, r_2 distinct roots of polynomial $q(r) = r(r-1) + ar + b$. Also prove that basis of solutions is given by $\phi_1(x) = |x|^{r_1}, \phi_2(x) = |x|^{r_1}\ln x$ in case r_1 is a root of $q(r)$ with multiplicity two. 10

OR

(B) Determine the Bessel's functions of the first and second kind as solution of the differential equation $x^2y'' + xy' + (x^2 - \alpha^2)y = 0, \alpha$ constat. 10

Q.3 (A) Suppose R is a rectangle $|x - x_0| \le a$, $|y - y_0| \le b$ $(a, b > 0)$ and that f is a real-valued function defined on R such that $|f(x,y)| \le M$, for all (x,y) in R, further assume that f satisfies a Lipschitz condition on R with Lipschitz constant K. Prove that the successive approximations $\phi_{k+1}(x) = y_0 + \int_{x_0}^{x} f(t, \phi_k(t))dt, \phi_0(x) = y_0$ 10

OR

(B) Compute the first four successive approximations $\phi_0, \phi_1, \phi_2, \phi_3$ for the initial value problem $y' = 1 + xy$, $y(0) = 1$. 5

(C) Prove that a function ϕ is a solution of the initial value problem $y' = f(x,y)$, $y(x_0) = y_0$ on an interval I if and only if it is a solution of the integral equation $y = y_0 + \int_{x_0}^{x} f(t, y)dt$ on I. 5

Q.4 (A) Find the solution ϕ of $y'' = 1 + (y')^2$ which satisfies $\phi(0) = 0$, $\phi'(0) = 0$. 5

(B) Let f be a continuous vector valued function defined on S: $|x - x_0| \le a$, $|y| < \infty$, $(a > 0)$ and satisfy their Lipschitz condition. Then prove that the successive approximations $\{\phi_k\}$ for the problem $y' = f(x,y)$, $y(x_0) = y_0$, $(|y_0| < \infty)$ exist on $|x - x_0| \le a$. 5

OR

(C) Let f be a vector valued function defined on the region R: $|x| \le 1$, $|y| \le 1$ (y in R_2) by $f(x,y) = (y_2^2 + 1, x + y_1^2)$. Then find an upper bound M for $|f(x,y)|$ for (x,y) in R and compute a Lipschitz constant K for f on R. 5

(D) Let f be a continuous vector valued function defined on S: $|x - x_0| \le a$, $|y| < \infty$, $(a > 0)$ and satisfy their Lipschitz condition. Then prove that the successive approximations $\{\phi_k\}$ for the problem $y' = f(x,y), y(x_0) = y_0, (|y_0| < \infty)$ on $|x - x_0| \le a$ converge there to a solution ϕ of this problem. 5

Q.5 **Choose the correct option and write it in the answer sheet.**

(A) Let x_0 be in I and let $\alpha_1, \dots, \alpha_n$ be any n constants. If there are two solutions ϕ, ψ of $L(y) = y^{(n)} + a_1(x)y^{(n-1)} + \dots + a_n(x)y = 0$ on I satisfying $\phi(x_0) = \alpha_1, \phi'(x_0) = \alpha_2, \dots, \phi^{(n-1)}(x_0) = \alpha_n$ then 1

(a) $\phi(x) \neq \psi(x)$ for all x in I.
(b) $\phi(x) = \psi(x)$ for some x in I.
(c) $\phi(x) = \psi(x)$ for all x in I
(d) None of these

(B) If ϕ_1 is a solution of $y'' + a_1(x)y' + a_2(x)y = 0$ for $x > 0$ on an interval I then second solution ϕ_2 is 1

(a) $\phi_1(x)\int_{x_0}^{x}\frac{1}{[\phi_1(s)]^2}e^{-\int_{x_0}^{s}a_1(t)dt}ds.$ (b) $\phi_1(x)e^{-\int_{x_0}^{x}a_1(t)dt}$.

(c) $\int_{x_0}^{x}\frac{1}{[\phi_1(s)]^2}e^{-\int_{x_0}^{x}a_1(t)dt}$ (d) None of these

(C) If $\phi_1(x)$ is a solution of $(1-x^2)y'' - 2xy' + 2y = 0$ for $0 < x < 1$ then second independent solution ϕ_2 is 1

(a) $\frac{1}{x}$ (b) x^2 (c) $\ln\left(\frac{x+1}{x-1}\right)$. (d) $-1+\frac{x}{2}\ln\left(\frac{x+1}{x-1}\right)$.

(D) What is indicial polynomial for linear equation $x^2y'' + 2xy' - 6y = 0$? 1

(a) $q(r) = r(r-1)$ (b) $q(r) = (r+3)(r-2)$

(c) $q(r) = r(r-1)+2$ (d) None of these.

(E) General solution of the linear equation $x^2y'' - (2+i)xy' + 3iy = 0$, for $|x| > 0$ is 1

(a) $\phi(x) = c_1|x|^i + c_2|x|^3$, where c_1, c_2 are constants.

(b) $\phi(x) = c_1|x|^{2+i} + c_2|x|^i$, where c_1, c_2 are constants.

(c) $\phi(x) = c_1|x|^{2+i} + c_2|x|^{3i}$, where c_1, c_2 are constants.

(d) None of these.

(F) What are the roots of the indicial polynomial for the equation $x^2y'' + xe^xy' + y = 0$? 1

(a) $r = \pm i$ (b) $r = \pm 1$ (c) $r = 1, 2$ (d) $r = 0, 1$

(G) If a function ϕ is a solution of the integral equation $y = y_0 + \int_{x_0}^{x} f(t,y)dt$ on I then it is a solution of the initial value problem 1

(a) $y' = f(x),\ \ y(x_0) = y_0$ on I (b) $y' = f(x,y),\ \ y(x_0) = y_0$ on I

(c) $y' = f(x,y),\ \ y(y_0) = x_0$ on I (d) None of these.

(H) The first three successive approximations for the equation $y' = y^2, y(0) = 1$ are 1

(a) $\phi_0(x) = 1, \phi_1(x) = 1+\frac{x}{2}, \phi_2(x) = 1+x+\frac{x^2}{2}$.

(b) $\phi_0(x) = 1, \phi_1(x) = 1+x, \phi_2(x) = 1+x+\frac{x^2}{2}$.

(c) $\phi_0(x) = 1, \phi_1(x) = 1+x, \phi_2(x) = 1+x+x^2+\frac{x^3}{3}$.

(d) None of these.

(I) The function $f(x,y) = xy^2$ satisfies a Lipschitz condition on 1

(a) $R: |x| \le 1, |y| \le 1$ (b) $R: |x| \le 1, |y| < \infty$ (c) $R: |x| \le 10, |y| < \infty$ (d) None of these

(J) What is a solution ϕ of the system $y_1' = y_1;\ y_2' = y_1 + y_2$ satisfying condition $\phi(0) = (1,2)$?

(a) $\phi(x) = (e^x, (x+1)e^x)$ (b) $\phi(x) = (e^x, (x+2)e^x)$

(c) $\phi(x) = (e^x, xe^x)$ (d) $\phi(x) = (xe^x, (x+2)e^x)$

Q.6 Fill in the blanks

(A) The equation $L(y) = (1-x^2)y'' - 2xy' + \alpha(\alpha+1)y = 0$, where α is a constant, is known as ____. 1

(B) The polynomial solution P_n of degree n of $(1-x^2)y'' - 2xy' + n(n+1)y = 0$ satisfying $P_n(1) = 1$ is called ________. 1

(C) An upper bound M for the function $f(x,y) = 1 - 2xy$ on $R: |x| \le \frac{1}{2}$, $|y| \le 1$ is ________. 1

(D) The solution of the equation $yy'' - (y')^2 = 0$ which satisfies $\phi(0) = 1$, $\phi'(0) = 2$, is $\phi(x) =$ ____. 1

(E) For the equation $x^2y'' + a(x)xy' + b(x)y = 0$, where a, b have convergent power series expansion for $|x| < r_0$, $r_0 > 0$, the indicial polynomial is $q(r) =$ ________. 1

Q.7 Answer the questions in short

(A) State Lipschitz condition. 1

(B) Find the regular singular point of the equation $x^2y'' + (\sin x)y' + (\cos x)y = 0$ 1

(C) State Lipschitz condition Define Rodrigue's formula 1

(D) For the vector valued function f defined on $R: |x| \le a, |y| \le \infty$, ($x$ real, y in R_n) by $f(x,y) = (3y_1 + xy_3,\ y_2 + x^3y_3,\ 2xy_1 - y_2 + e^xy_3)$, find the Lipschitz constant K. 1

(E) Find the regular singular point for the linear equation $(x^2+x-2)^2y'' + 3(x+2)y' + (x-1)y = 0$ 1

Rashtrasant Tukadoji Maharaj Nagpur University

Department of Mathematics

M.Sc. (Mathematics) Semester I

End Semester Examination Winter-2022

Probability and Distributions

Time: Three hours] **[Maximum Marks: 60]**

Q.1 (A) Give the axiomatic definition of probability and describe it's any three properties. 5

(B) Define independence of events. Consider a random experiment of selecting a card randomly from a pack of cards. Let A be the event of drawing a queen and B be the event of drawing a spade. Does the occurrence or non-occurrence of A affect the occurrence or non-occurrence of B ? 5

OR

(C) State and prove Borel-Cantelli lemma. 5

(D) Define random variable, discrete random variable and continuous random variable. Consider a random experiment of two tosses of coin, let X be the number of heads then show that X is a random variable. 5

Q.2 (A) Define Poisson distribution. Show that Poisson distribution is a limiting case of Binomial distribution. 5

(B) Define Binomial distribution. Obtain the M.G.F. and P.G.F. and hence obtain the mean and variance. 5

OR

(C) Define Geometric distribution. State and prove lack of memory property of Geometric distribution. 5

(D) Give the p.m.f. of the negative binomial distribution. Obtain an expression for its M.G.F. Hence find mean and variance. 5

Q.3 (A) Prove that a linear combination of independent normal variate is also a normal variate. 5

(B) Define the exponential distribution and obtain first two raw moments and the variance of exponential distribution. 5

OR

(C) Define Beta distribution of second kind and obtain it's first two raw moments and the harmonic mean. 5

(D) Define Cauchy distribution and show that in general the mean of Cauchy distribution does not exist. 5

Q.4 (A) Define Fischer's t-distribution and obtain it's p.d.f. 10

OR

(B) Define F distribution. Obtain mode of F distribution. 5

(C) Give applications of t distribution and F distribution. Also write the relation between t and F distribution. 5

Q.5 **Choose the correct option and write it in the answer sheet.**

(A) The mode of Poisson distribution is _____ 1

(a) $(\lambda+1)/(x+1)$ (b) $(\lambda-1)/(x+1)$ (c) λ/x (d) $(\lambda-1)/(x)$

(B) A random variable that assumes infinite or uncountable number of values is called_____ 1

(a) discrete random variable (b) continuous random variable
(c) irregular random variable (d) none of the above

(C) Two fair dice are thrown then the probability that two fives occur, if it is known that the total is divisible by 5 is _____ 1

(a) 1/36 (b) 7/36 (c) 1/7 (d) 2/7

(D) If A and B are two independent events then, 1

(a) A^c and B are independent (b) A and B^c are independent

(c) A^c and B^c are independent (d) All of the above

(E) Which of the following is not true for an exponential distribution with parameter λ? 1

(a) Mean = $1/\lambda$ (b) The area under the curve is equal to one.

(c) Standard deviation = $1/\lambda$ (d) None of the above.

(F) If X is a continuous random variable, then the expected value is given by 1

(a) $\int X\,P(x)$ (b) $\sum X\,P(x)$ (c) $P(x)$ (d) None of these.

(G) Suppose that the probability of event A is 0.2 and the probability of event B is 0.4. Also, suppose that the two events are independent. Then $P(A|B)$ is ____ 1

(a) $P(A) = 0.2$ (b) $P(A)/P(B) = ½$ (c) $P(A) \times P(B) = 0.08$ (d) None of these

(H) For a discrete uniform distribution the expectation of random variable X is ____ 1

(a) $(n+1)/4$ (b) $(n+1)/2$ (c) $n^2/4$ (d) $(n+1)(2n+1)/6$

(I) Which of the following is an example of Poisson distribution? 1

(a) Number of printing mistakes on each page of a book.

(b) Number of deaths from a disease like heart attack.

(c) Both of these (d) None of these

(J) If $f(x, y) = 3x(x+y)$; where $0<x<1$, $0<y<2$ then marginal distribution of Y is ____ 1

(a) $3(y/2 + 1/3)$ (b) $3(y + 1)$ (c) $3/5\,(y + 1)$ (d) $6(y + 2)$

Q.6 **Fill in the blanks**

(A) The coefficients of skewness and kurtosis for a Poisson distribution are ____ and ____ 1

(B) The p.m.f. of hypergeometric distribution is ____ 1

(C) If X and Y are two independent Chi Square variates with m and n degrees off freedom then by using transformation of variable technique, the ratio X/Y follows ____ distribution with degrees of freedom m/2 and n/2. 1

(D) The conditional probability is defined as ____ 1

(E) The M.G.F. of exponential distribution is ____ 1

Q.7 **Answer the questions in short**

(A) Define marginal distribution function and conditional distribution function. 1

(B) State Bayes theorem. 1

(C) Give the properties of probability. 1

(D) Give the characteristics of normal distribution. 1

(E) Give any two applications of Beta distributions. 1