# Solutions Manual

for

# Satellite Communications

Second edition

Timothy Pratt, Charles Bostian, Jeremy Allnutt

Virginia Tech, Blacksburg, 2003

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# Chapter 2  Solutions to Problems

Question 1.

Explain what the terms centrifugal and centripetal mean with regard to a satellite in orbit around the earth.

A satellite is in a circular orbit around the earth. The altitude of the satellite's orbit above the surface of the earth is 1,400 km. (i) What are the centripetal and centrifugal accelerations acting on the satellite in its orbit? Give your answer in m/s$^2$. (ii) What is the velocity of the satellite in this orbit? Give your answer in km/s. (iii) What is the orbital period of the satellite in this orbit? Give your answer in hours, minutes, and seconds. Note: assume the average radius of the earth is 6,378.137 km and Kepler's constant has the value $3.986004418 \times 10^5$ km$^3$/s$^2$

**Solution to question 1:**

In the case of a satellite orbiting the earth, the centrifugal force on the satellite is a force on the satellite that is directly away from the center of gravity of the earth ($F_{OUT}$ in Fig. 2.1) and the centripetal force is one directly towards the center of gravity of the earth ($F_{IN}$ in Fig. 2.1). The centrifugal force on a satellite will therefore try to fling the satellite away from the earth while the centripetal force will try to bring the satellite down towards the earth.

(i) From equation (2.1) centripetal acceleration $a = m/r^2$, where $m$ is Kepler's constant. The value of $r$ = 6,378.137 + 1,400 = 7,778.137 km, thus $a = 3.986004418 \times 10^5$ / $(7,778.137)^2$ = 0.0065885 km/s$^2$ = 6.5885007 m/s$^2$. From equation (2.3), the centrifugal acceleration is given by $a = v^2/r$, where $v$ = the velocity of the satellite in a circular orbit.. From equation (2.5) $v = (m/r)^{1/2} = (3.986004418 \times 10^5 / 7,778.137)^{1/2}$ = 7.1586494 km/s and so $a$ = 0.0065885007 km/s$^2$ = 6.5885007 m/s$^2$. **NOTE:** since the satellite was in stable orbit, the centrifugal acceleration must be equal to the centripetal acceleration, which we have found to be true here (but we needed only to calculate one of them).

(ii) We have already found out the velocity of the satellite in orbit in part (i) (using equation (2.5)) to be 7.1586494 km/s

(iii) From equation (2.6), the orbital period $T = (2pr^{3/2})/(m^{1/2}) = (2p7,778.137^{3/2})/(3.986004418 \times 10^5)^{1/2}$ = (4,310,158.598)/(631.3481146) = 6,826.912916 s = 1 hour 53 minutes 46.92 seconds

Question 2

A satellite is in a 322 km high circular orbit. Determine:
a. The orbital angular velocity in radians per second;
b. The orbital period in minutes; and

c. The orbital velocity in meters per second.
Note: assume the average radius of the earth is 6,378.137 km and Kepler's constant has the value $3.986004418 \times 10^5$ km$^3$/s$^2$.

**Solution to question 2:**

It is actually easier to answer the three parts of this question backwards, beginning with the orbital velocity, then calculating the period, and hence the orbital angular velocity. First we will find the total radius of the orbit $r$ = 322 + 6,378.137 km = 6700.137 km

(c) From eqn. (2.5), the orbital velocity $v = (m/r)^{1/2} = (3.986004418 \times 10^5 / 6700.137)^{1/2}$ = 7.713066 km/s = 7,713.066 m/s.

(b) From eqn. (2.6), $T = (2pr^{3/2})/(m^{1/2}) = (2p6{,}700.137^{3/2})/(\ 3.986004418 \times 10^5)^{1/2}$ = (3,445,921.604)/(631.3481146) = 5,458.037372 seconds = 90.9672895 minutes = 90.97 minutes.

(a) The orbital period from above is 5,458.037372 seconds. One revolution of the earth covers 360$^o$ or 2$\pi$ radians. Hence 2$\pi$ radians are covered in 5,458.037372 seconds, giving the orbital angular velocity as 2$\pi$/5,458.037372 radians/s = 0.0011512 radians/s. An alternative calculation procedure would calculate the distance traveled in one orbit ($2pr = 2p6700.137$ = 42,098.20236 km). This distance is equivalent to 2$\pi$ radians and so 1 km is equivalent to 2$\pi$/42,098.20236 radians = 0.0001493 radians. From above, the orbital velocity was 7.713066 km/s = 7.713066 $\times$ 0.0001493 radians/s = 0.0011512 radians/s.

Question 3.

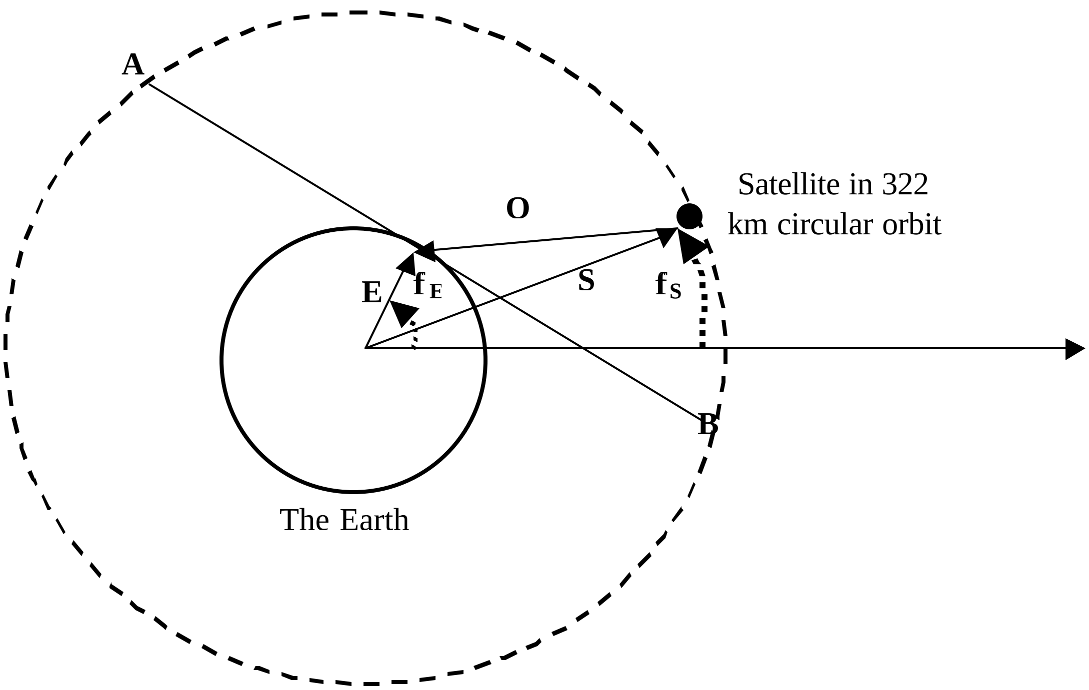
The same satellite in question 2 above (322 km circular orbit) carries a 300 MHz transmitter.
a. Determine the maximum frequency range over which the received signal would shift due to Doppler effects if received by a stationary observer suitably located in space. Note: the frequency can be shifted both up and down, depending on whether the satellite is moving towards or away from the observer. You need to determine the maximum possible change in frequency due to Doppler (i.e. $2\Delta_f$).
b. If an earth station on the surface of the earth at mean sea level, 6,370 km from the center of the earth, can receive the 300 MHz transmissions down to an elevation angle of 0$^o$, calculate the maximum Doppler shift that this station will observe. Note: Include the earth's rotation and be sure you consider the *maximum possible* Doppler shift for a 322 km circular orbit.

**Solution to question 3**

a. The highest Doppler shift would be observed in the plane of the satellite at the orbital height of the satellite: the satellite would be coming directly *at* the observer or directly *away* from the observer. The maximum Doppler shift would therefore be the sum of these two values. The orbital velocity was calculated in question 2 as 7,713.066 m/s. Using equation (2.44a), $Df / f_T = V_T / v_p$ , where $Df$ is the Doppler frequency, $f_T$ is the frequency of the transmitter at rest, $V_T$ is the component of the transmitter's velocity directed at the observer, and $v_p$ is the phase velocity of light. Since the observer is at orbital height, the component of the transmitter's velocity towards the observer is the actual velocity of the satellite. Thus $Df = (7{,}713.066 \times 300{,}000{,}000) / 2.9979 \times 10^8 =$ 7,718.468928 Hz. The maximum Doppler shift therefore = $7{,}718.468928 \times 2 =$ 15,436.93786 Hz = 15,436.94 Hz.

b. It is best to draw a diagram to see what the set up looks like. Below is a view from above the orbit of the satellite (orthogonal to the orbital plane).

The important element in this part of the question is the component of the satellite's velocity towards the earth station.




**O** = vector from the satellite to the earth station
**S =** vector from the origin to the satellite
**E** = vector from the origin to the earth station
$\mathbf{f_S}$ = angular coordinate of the satellite
$\mathbf{f_E}$ = angular coordinate of the earth station
**A** and **B** are the points in the satellite's orbit when the elevation angle at the earth station is zero, and hence (if the plane of the orbit takes the satellite directly over the earth station at zenith) the point where the Doppler shift is highest – either positive or negative.

Using the law of cosines,

$\mathbf{O}^2 = \mathbf{E}^2 + \mathbf{S}^2 - 2\mathbf{ES}\cos(f_E - f_S)$

For an orbital height of $h$, $\mathbf{S} = \mathbf{E} + h$, giving

$\mathbf{O}^2 = \mathbf{E}^2 + (\mathbf{E} + h)^2 - 2\mathbf{E}(\mathbf{E} + \mathrm{h})\cos(f_E - f_S)$

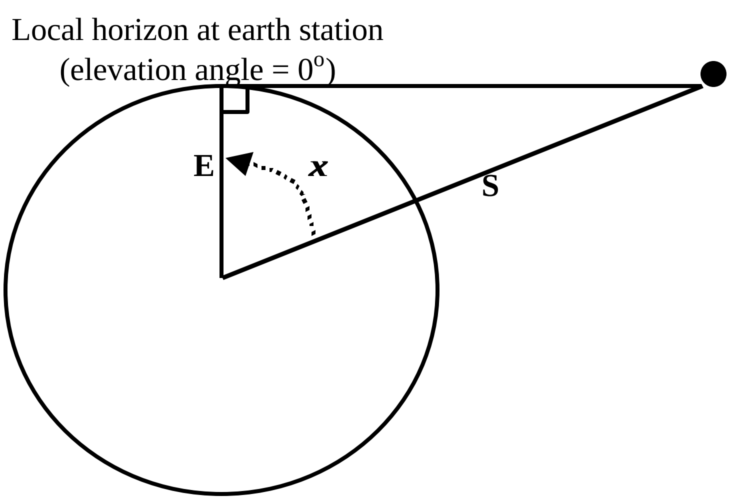
The component of the satellite's velocity towards the earth station is d**O**/dt and we can obtain it by differentiating the above equation (remembering the **E**, *h*, and **S** may be assumed to be constant values), term by term, thus:

$2\mathbf{O}(d\mathbf{O}/dt) = 2\mathbf{E}(\mathbf{E} + h)\sin(f_E - f_S)(df_E/dt - df_S/dt)$ , giving

$(d\mathbf{O}/dt) = (\mathbf{E}(\mathbf{E} + h)\sin(f_E - f_S)(df_E/dt - df_S/dt))/(\mathbf{E}^2 + \mathbf{S}^2 - 2\mathbf{ES}\cos(f_E - f_S))^{1/2}$

$df_E/dt$ is the earth's rotational angular velocity = $2\pi$ radians/24hours = $7.2722052 \times 10^{-5}$ radians/second

$df_S/dt = \pm n$, where $n$ is the orbital angular velocity = 0.0011512 radians/s from question 2 part (a). The sign depends on whether the orbital motion and the earth's rotation are in the same direction. To find the maximum Doppler shift, we need to find $(f_E - f_S)$ when the elevation angle is $0^o$. This is drawn below for one of the geometries (the other being the mirror image on the other side).




In this figure $x = (f_E - f_S)$

$\cos x = \mathbf{E}/\mathbf{S} = \mathbf{E}/(\mathbf{E} + h) = (6378.137)/(6378.137 + 322) = 0.9519413$

$(f_E - f_S) = \cos^{-1}(0.9519413) = 17.8352236^o$

Thus

$$\frac{d\mathbf{O}}{dt}=\frac{(6378.137)(6378.137+322)\sin(17.835)(7.272\times10^{-5}\mp1.15\times10^{-3})}{((6378.137)^2+(6378.137+322)^2-2(6378.137)(6378.137+322)\cos(17.835))^{1/2}}$$

$$=\frac{-14{,}100.03935,+16{,}003.6389}{2052.096993}$$

= -6.8710394 or +7.7986757 km/s

The satellite may be rotating in the same direction as the angular rotation of the earth or against it. Thus, for the same direction rotation,

$$2\Delta f=\frac{2\times7.7986757\times10^3\times300\times10^6}{2.9979\times10^8} = 15{,}608.27719 = 15.6\text{ kHz}$$

For the opposite direction rotation

$$2\Delta f=\frac{2\times6.8710394\times10^3\times300\times10^6}{2.9979\times10^8} = 13{,}752.41469 = 13.75\text{ kHz}$$

Question 4

What are Kepler's three laws of planetary motion? Give the mathematical formulation of Kepler's third law of planetary motion. What do the terms perigee and apogee mean when used to describe the orbit of a satellite orbiting the earth?

A satellite in an elliptical orbit around the earth has an apogee of 39,152 km and a perigee of 500 km. What is the orbital period of this satellite? Give your answer in hours. Note: assume the average radius of the earth is 6,378.137 km and Kepler's constant has the value $3.986004418\times10^5$ km$^3$/s$^2$.

**Solution to question 4**

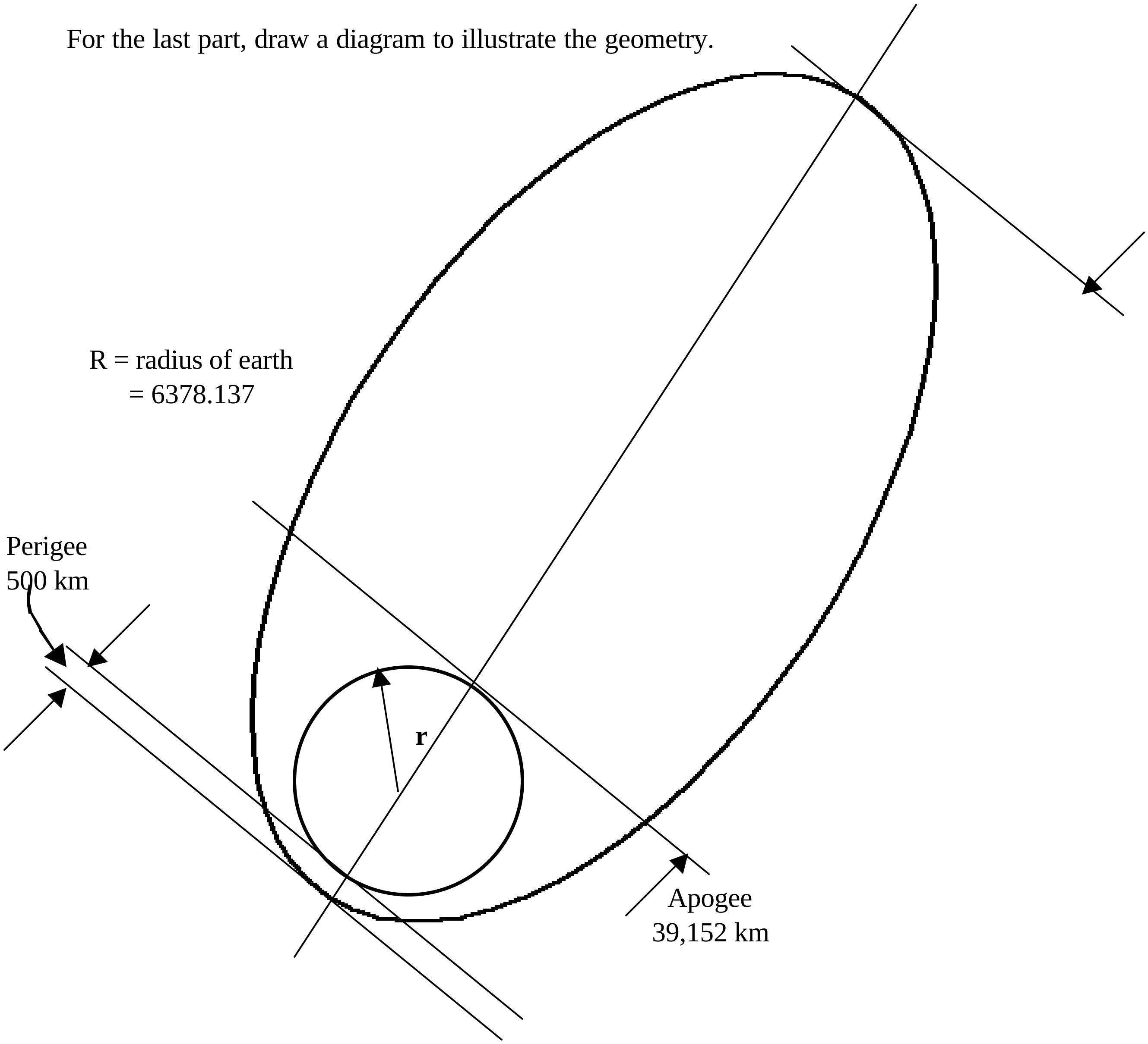
Kepler's three laws of planetary motion are (see page 22)

1. The orbit of any smaller body about a larger body is always an ellipse, with the center of mass of the larger body as one of the two foci.
2. The orbit of the smaller body sweeps out equal areas in time (see Fig. 2.5).
3. The square of the period of revolution of the smaller body about the larger body equals a constant multiplied by the third power of the semimajor axis of the orbital ellipse.

The mathematical formulation of the third law is $T^2 = (4p^2a^3)/\mu$, where $T$ is the orbital period, $a$ is the semimajor axis of the orbital ellipse, and $m$ is Kepler's constant.

The perigee of a satellite is the closest distance in the orbit to the earth; the apogee of a satellite is the furthest distance in the orbit from the earth.

For the last part, draw a diagram to illustrate the geometry.




The semimajor axis of the ellipse = (39,152 + (2 × 6378.137) + 500)/2 = 26,204.137 km
The orbital period is

$T^2 = (4p^2a^3)/\mu = (4p^2(26{,}204.137)^3)/\ 3.986004418 \times 10^5 = 1{,}782{,}097{,}845.0$

Therefore, $T$ = 42,214.90075 seconds = 11 hours 43 minutes 34.9 seconds

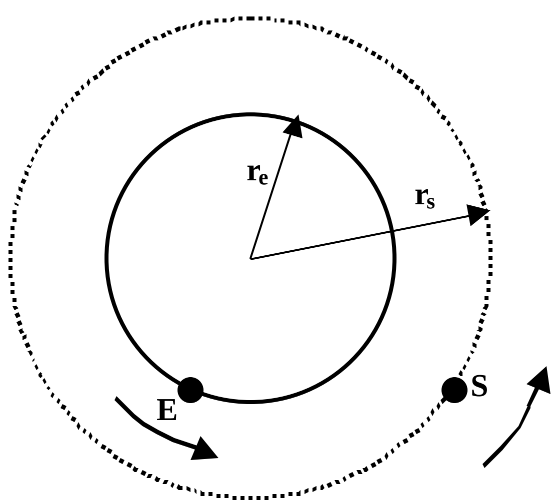
Question 5

An observation satellite is to be placed into a circular equatorial orbit so that it moves in the same direction as the earth's rotation. Using a synthetic aperture radar system, the satellite will store data on surface barometric pressure, and other weather related parameters, as it flies overhead. These data will later be played back to a controlling earth station after each trip around the world. The orbit is to be designed so that the satellite is directly above the controlling earth station, which is located on the equator, once every 4 hours. The controlling earth station's antenna is unable to operate below an elevation angle of $10^{o}$ to the horizontal in any direction. Taking the earth's rotational period to be exactly 24 hours, find the following quantities:

a. The satellite's angular velocity in radians per second.
b. The orbital period in hours.
c. The orbital radius in kilometers.
d. The orbital height in kilometers.
e. The satellite's linear velocity in meters per second.
f. The time interval in minutes for which the controlling earth station can communicate with the satellite on each pass.

**Solution to question 5**

First, draw a schematic diagram to illustrate the question. The diagram is from the North Pole, above the earth, with the satellite's orbit and the equator in the plane of the paper.




$\mathbf{r_e}$ = earth radius, 6378.137 km
$\mathbf{r_s}$ = orbital radius, km
**S** = satellite
**E** = earth station
Arrows above E and S show the direction of motion of the earth station and satellite, respectively

a. Let $\boldsymbol{h}_s$ = satellite angular velocity and $\boldsymbol{h}_e$ = earth's rotational angular velocity. This gives

$$h_e = \frac{2\boldsymbol{p} radians}{24 hours} = \frac{2\boldsymbol{p}}{86{,}400} = 7.2722052 \times 10^{-5} \, radians / s$$

If the satellite was directly over the earth station at time t = 0, their angular separation at time t will be $\boldsymbol{D}_\phi = (\boldsymbol{h}_s - \boldsymbol{h}_e) \times t$. The first overhead pass occurs at $\boldsymbol{D}_\phi = 0$ and the second occurs at $\boldsymbol{D}_\phi = \pm 2\pi$. We want $\boldsymbol{D}_\phi = \pm 2\pi$ to occur at t = 4 hours = 14,400 seconds. Thus $\pm 2\pi = (\boldsymbol{h}_s - \boldsymbol{h}_e) \times 14{,}400$, which gives $(\boldsymbol{h}_s - \boldsymbol{h}_e) = \pm 2\pi/14{,}400 = \pm 0.0004363$. The possible values are (i) $\boldsymbol{h}_s = \boldsymbol{h}_e + 0.0004363 = 7.2722052 \times 10^{-5} + 0.0004363 = 0.0005090221$ radians/s and (ii) $\boldsymbol{h}_e - 0.0004363 = 7.2722052 \times 10^{-5} - 0.0004363 = -0.0003635779$ radians/s.

The first value corresponds to an orbital period of $T = 2\pi/0.0005090221 = 12{,}343.64194$ seconds = 3 hours 25 minutes 43.64 seconds. The second value corresponds to an orbital period of $T = 2\pi/0.0003635779 = 17{,}281.58082$ = 4 hours 48 minutes 1.58 seconds. The second value corresponds to the satellite orbiting in the opposite direction to the earth's rotation. The question stated that the satellite rotated in the same direction as the earth and so the satellite's angular rotation is 0.0005090221 radians/s.

A simple way to check this answer is to note that in 4 hours the earth rotates $60^o$ or $\pi/3$ radians. A satellite going in the direction of the earth's rotation must cover $(2\pi + \pi/3)$ in 4 hours to catch up with the earth station (that is, one full rotation plus the additional distance the earth has rotated). The angular velocity is therefore $(2\pi + \pi/3)/4$ hours $= (2\pi + \pi/3)/14{,}400 = 0.000509$ radians/s (assuming answer (i) above).

b. The orbital period, T, is given by the number of radians in one orbit ($2\pi$) divided by the angular velocity (0.0005090221 radians/s) = 12,343.64194 seconds = 3.43 hours (=3 hours 25 minutes 43.64).

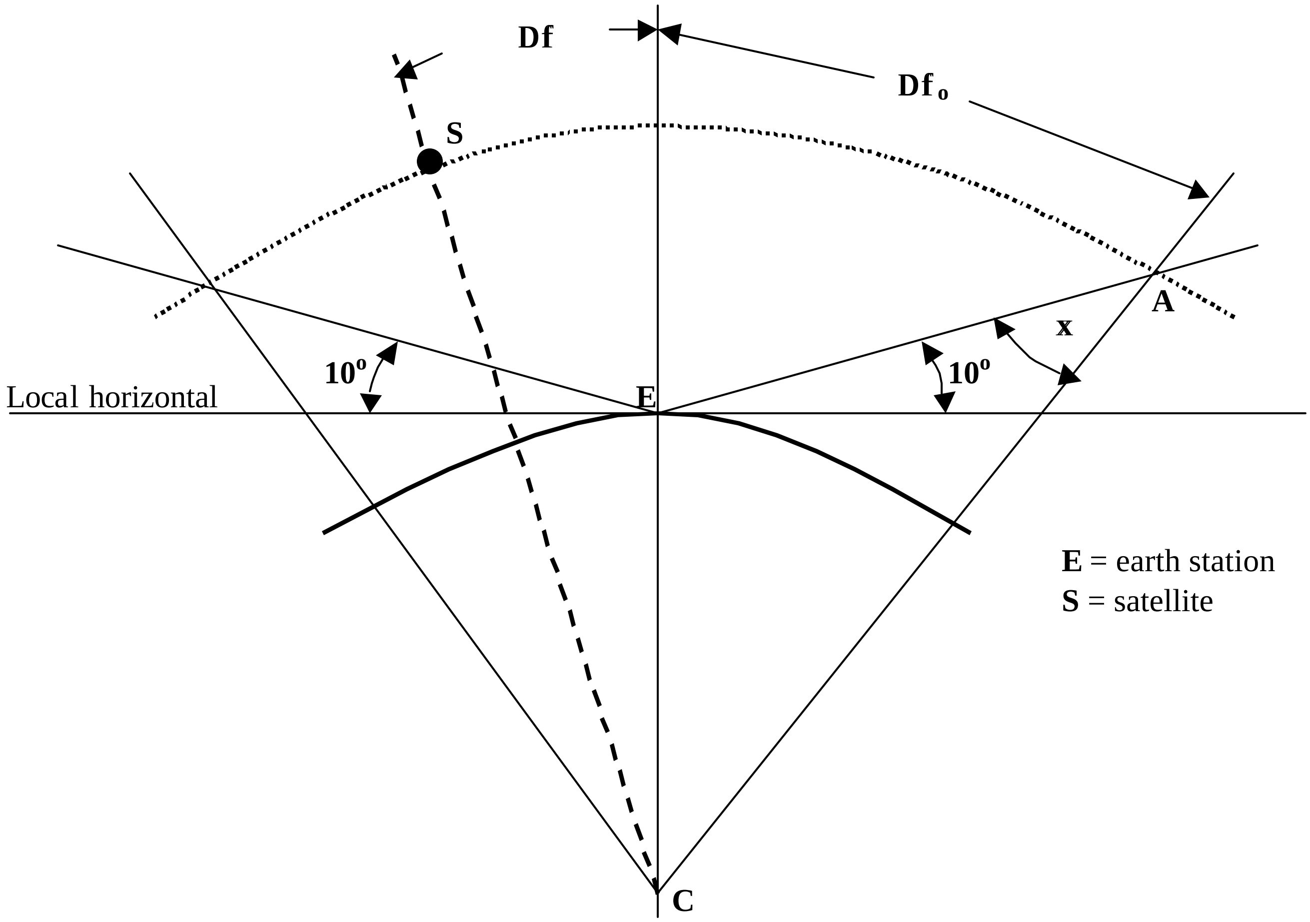
c. From equation (2.25) orbital angular velocity = $(\mu^{1/2})/(a^{3/2})$, where $\mu$ is Kepler's constant = $3.986004418 \times 10^5$ km$^3$/s$^2$ and a is the semimajor axis of the orbit. The orbit is circular in this question and so the radius of the orbit is a. We know $\mu$ and the orbital angular velocity, thus
$a = (3.986004418 \times 10^5)^{1/3}/(0.0005090221)^{2/3}$ = 11,543.96203 km = 11,543.96 km

d. The orbital height is the radius of the orbit (from the earth's center) minus the earth's radius = 11,543.96203 – 6378.137 km = 5,165.825030 = 5,165.83 km

e. The linear velocity of the satellite can be found in two ways. From equation (2.5), the orbital velocity = $(\mu/r)^{1/2} = ((3.986004418 \times 10^5)/(11{,}543.96203))^{1/2}$ = 5.8761306 km/s = 5.876 km/s. Alternatively, orbital velocity = radius of orbit ×

orbital angular velocity = 11,543.96203 × 0.0005090221 = 5.8761306 km/s.

f. For this part it is best to draw a diagram illustrating the visibility geometry.




Communications are possible if $|\Delta\phi| \le \Delta\phi_o$. We can find $\Delta\phi_o$ by trigonometry. Angle AEC = 100° and, by the law of sines, a/sin(100) = $r_e$/sin$\xi$, where a is the radius of the orbit. From this, sin$\xi$ = $r_e$ sin(100)/a = (6,378.137 × 0.9848078)/11,543.96203 = 0.5441146 giving $\xi$ = 32.9641821 =32.96°.

Given $\xi$ = 32.96°, angle ACE = 180 – 100 – 32.96 = 47.04°. The time the satellite goes from overhead to 10° elevation angle is the angle traveled/relative angular velocity of the satellite = 47.04° in radians/(0.000509 radians/s – 0.000072722052) = 0.8209299/0.0004363 = 1,881.572083 seconds = 31.359347 minutes.

The total time from one extreme to the other of the visible orbit is twice this value = 62.72 minutes.

Question 6

What is the difference, or are the differences, between a *geosynchronous* satellite and a *geostationary* satellite orbit? What is the period of a geostationary satellite? What is the name given to this orbital period? What is the velocity of a geostationary satellite in its orbit? Give your answer in km/s.

A particular shuttle mission released a TDRSS satellite into a circular low orbit, with an orbital height of 270 km. The shuttle orbit was inclined to the earth's equator by approximately $28^{o}$. The TDRSS satellite needed to be placed into a geostationary transfer orbit (GTO) once released from the shuttle cargo bay, with the apogee of the GTO at geostationary altitude and the perigee at the height of the shuttle's orbit. (i) What was the eccentricity of the GTO? (ii) What was the period of the GTO? (iii) What was the difference in velocity of the satellite in GTO between when it was at apogee and when it was at perigee? Note: assume the average radius of the earth is 6,378.137 km and Kepler's constant has the value $3.986004418 \times 10^5$ $km^3/s^2$.

**Solution to question 6**

A *geostationary* satellite orbit is one that has zero inclination to the equatorial plane, is perfectly circular (eccentricity is zero), and is at the correct orbital height to remain apparently stationary in orbit as viewed from the surface of the earth. A *geosynchronous* satellite orbit has most of the attributes of a geostationary orbit, but is either not exactly circular, not in the equatorial plane, or not at exactly the correct orbital height.

From Table 2.1, the orbital period of a geostationary satellite is 23 hours, 56 minutes, and 4.1 seconds.

The orbital period of a geostationary satellite is called a sidereal day.

From Table 2.1, the velocity of a geostationary satellite is 3.0747 km/s.

(i) The Geostationary Transfer Orbit (GTO) will have an apogee of 35,786.03 km (the geostationary altitude) and a perigee of 270 km (the release altitude of the TDRSS).

The semimajor axis $a = (2r_e + h_p + h_a)/2 = (2 \times 6378.137 + 270 + 35{,}786.03)/2 =$ 24,406.152 km

From equation (2.27) and example 2.1.3, $r_o = r_e + h_p$ and the eccentric anomaly $E = 0$ when the satellite is at perigee. From equation (2.27) $r_o = a(1 - e\cos E)$, with $\cos E = 1$. Therefore, $r_e + h_p = a(1 - e)$ and, rearranging the equation, $e = 1 - (r_e + h_p)/a = 1 -$ $(6378.137 + 270)/24{,}406.152 = 0.727604$. The eccentricity of the GTO is 0.728.

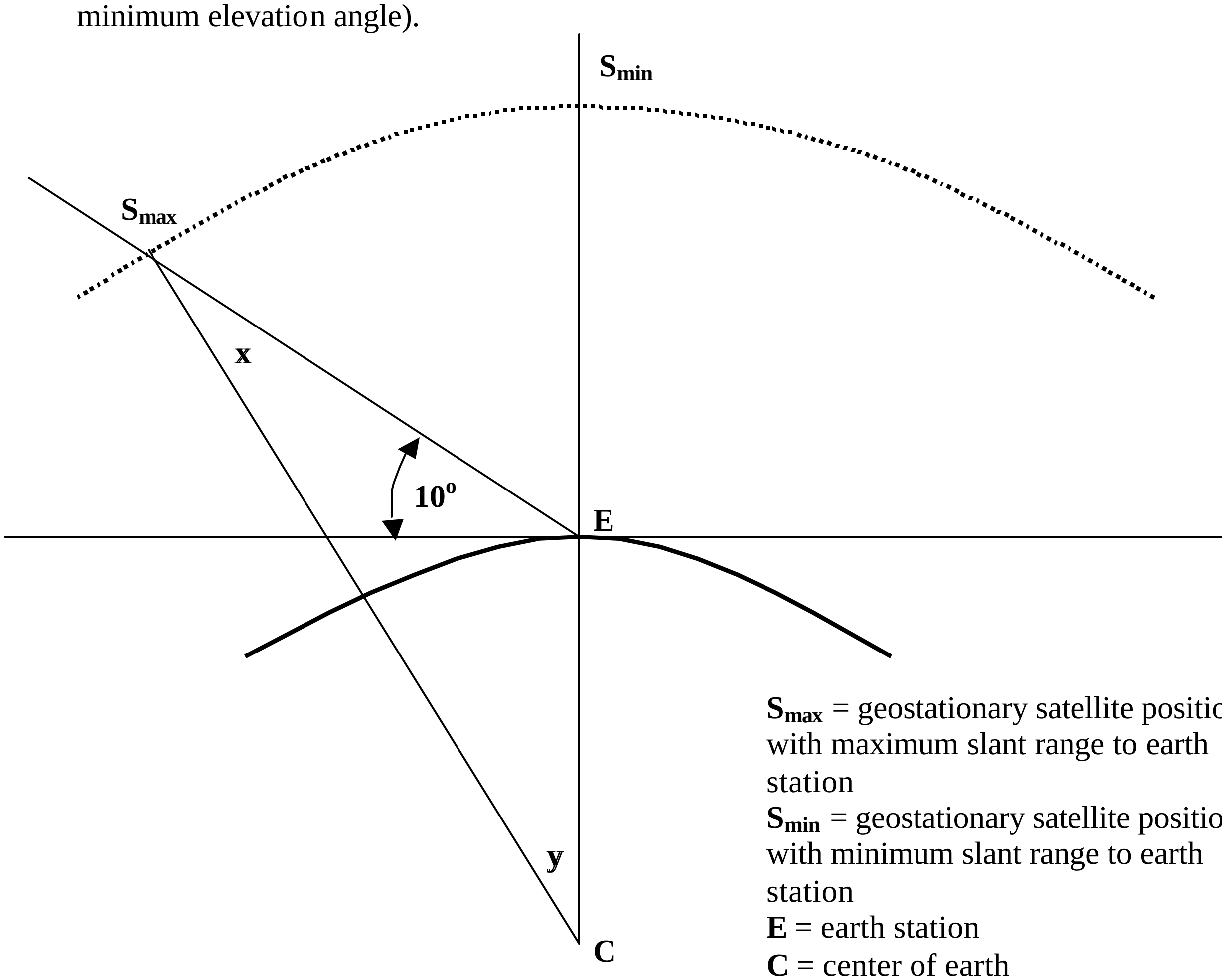
(ii) The orbital period $T = ((4\pi^2 a^3)/\mu)^{1/2} = ((4\pi^2 \times 24{,}406.152^3)/\ 3.986004418 \times 10^5)^{1/2} =$ 37,945.47102 seconds = 10 hours 32 minutes 25.47 seconds.

Question 7

For a variety of reasons, typical minimum elevation angles used by earth stations operating in the commercial Fixed Services using Satellites (FSS) communications bands are as follows:
C-Band $5^o$; Ku-Band $10^o$; and Ka-Band $20^o$.
(i) Determine the maximum and minimum range in kilometers from an earth station to a geostationary satellite in the three bands. (ii) To what round-trip signal propagation times do these ranges correspond? You may assume the signal propagates with the velocity of light in a vacuum even when in the earth's lower atmosphere.

**Solution to question 7**

Below is a schematic of one of the three frequency situations, in this case Ku-band ($10^o$ minimum elevation angle).




$\mathbf{S_{max}}$ = geostationary satellite position with maximum slant range to earth station
$\mathbf{S_{min}}$ = geostationary satellite position with minimum slant range to earth station
**E** = earth station
**C** = center of earth
$\mathbf{r_e}$ = earth radius = 6378.137 km = **CE**
$\mathbf{h_s}$ = height of satellite = $\mathbf{ES_{min}}$ = 35,786.03 km

Part (i)
By the law of sines, $CS_{max}/\sin 100^o = CE/\text{sine}\xi$ giving
$(35{,}786.03 + 6378.137)/\sin 100^o = 6378.137/\sin\xi$ and thus
$\sin\xi = (\sin 100^o \times 6378.137)/42{,}164.167 = 0.1489710$ and so $\xi = 8.5673001$

Angle $\psi = 180 - 100 - \xi = 71.4326999$

Again, by the law of sines, $ES_{max}/\sin\psi = CS_{max}/\sin 100^o$ giving
$ES_{max} = (42{,}164.167 \times \sin 71.4326999)/\sin 100 = 40{,}586.12894$ km

For a minimum elevation angle of $5^o$, $\xi = 8.6671185$, $\psi = 180 - 95 - 8.86671185 = 76.3328815^o$, and $ES_{max} = 41.126.78334$ km

For a minimum elevation angle of $20^o$, $\xi = 8.1720740$, $\psi = 180 - 110 - 8.1720740 = 61.8279266^o$, and $ES_{max} = 39{,}554.56520$ km

Maximum ranges are 41,126.78 km (C-band), 40,586.13 km (Ku-band), and 39,554.57 km (Ka-band).

The minimum range will be the same for all three frequencies of operation = 35,786.03 km, which assumes the earth station is on the equator and the satellite is directly above the earth station.

Part (ii)
The round trip propagation times, assuming the velocity of light is $2.997 \times 10^8$ m/s, are given by distance/velocity = $2 \times 41{,}126.78$ km/$2.997 \times 10^8$ m/s (C-band), $2 \times 40{,}586.13$ km/$2.997 \times 10^8$ m/s (Ku-band), and $2 \times 39{,}554.57$ km/$2.997 \times 10^8$ m/s (Ka-band) = 274.5, 270.8, and 264 milliseconds, respectively.

Question 8

Most commercial geostationary communications satellites must maintain their orbital positions to within $\pm 0.05^o$ of arc. If a geostationary satellite meets this condition (i.e. it has an apparent motion $\pm 0.05^o$ of arc N-S and $\pm 0.05^o$ of arc E-W, as measured from the center of the earth), calculate the maximum range variation to this satellite from an earth station with a mean elevation angle to the center of the satellite's apparent motion of $5^o$. You may assume that the equatorial and polar diameters of the earth are the same.

**Solution to question 8**

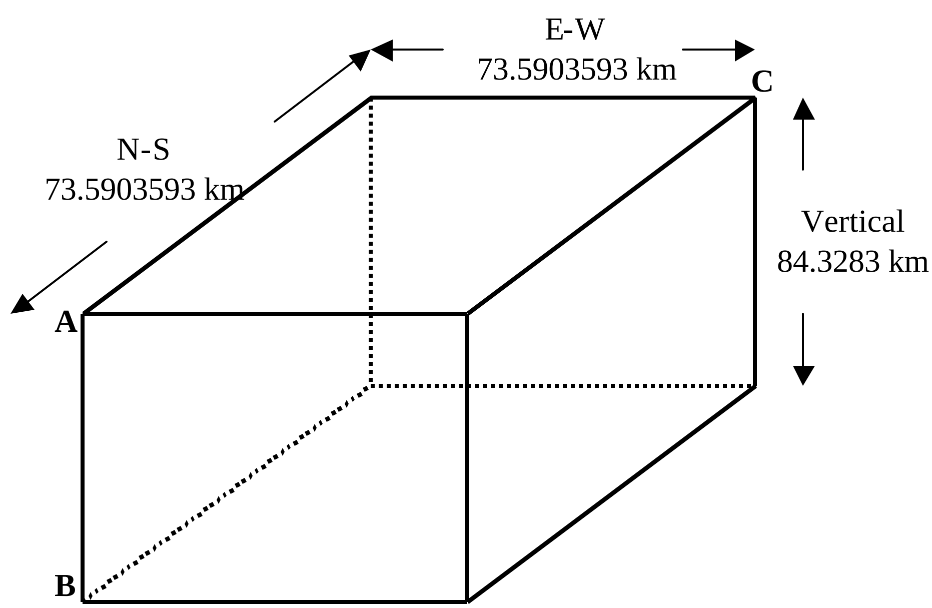
No information was given on eccentricity in this question, so we will assume that it is 0.001.

We can treat the station keeping variation of the satellite as if it were in a box with sides given by (in vertical extent) the eccentricity of the orbit and (in longitudinal and latitudinal extent) the inclination of the orbit.

If $\Delta r$ is the difference between the maximum orbit radius and the average radius = the difference between the minimum orbit radius and the average radius, then the total vertical movement is $2\Delta r$. The total vertical motion $2\Delta r = r_{max} - r_{min} = a(1 + e) - a(1 - e) = 2ae$, where $a$ is the average orbit radius and $e$ is the eccentricity of the orbit. Thus $2\Delta r = 2 \times 42{,}164.17 \times 0.001 = 84.3283$ km. Alternatively, using equation (10.4), $\Delta r = \pm eR_{av}$, where $R_{av}$ = the average orbit radius, giving $\Delta r = \pm(0.001) \times 42{,}164.17 = \pm 42.16417$, and so the total vertical motion = $2 \times 42.16417 = 84.3283$ km.

The lateral movement of the satellite N-S (longitude) and E-W (latitude) in its orbit is obtained by multiplying the orbit radius by the angular movement in radians. The angular movement is $\pm\ 0.05^{o} = 0.1^{o}$ total $\Rightarrow$ 0.0017453 radians and so the physical movement = $0.0017453 \times 42{,}164.17 = 73.5903593$ km.

The movement "box" is shown below.

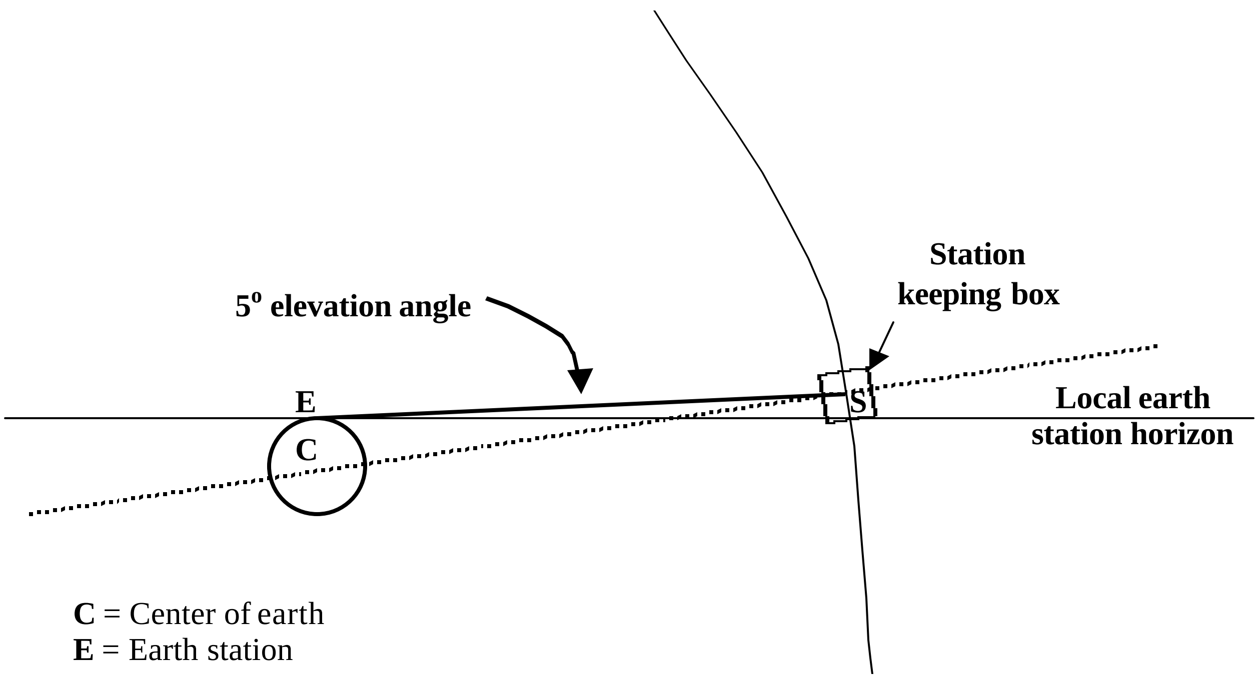



The center of the "box" is the nominal position of the satellite, with excursions up and down given by the eccentricity of the orbit and N-S and E-W given by the inclination of the orbit. Diagonal BC across the box is the maximum movement of the satellite (but note that there are also three other similar lengths within the box, created by joining the relevant opposing corners of the box.). Using Pythagorean geometry, we have

$BC^2 = AC^2 + AB^2$ and $AC^2 = 73.5903593^2 + 73.5903593^2 = 10{,}831.08196$

Thus $BC^2 = 10{,}831.08196 + 84.3283^2 = 10{,}831.08196 + 7{,}111.262181 = 17{,}942.34414$, which gives BC = 133.9490356 km = 133.95 km.

The range variation observed by an earth station will depend on the precise coordinates of the earth station. While the elevation angle ($5^o$) was given, the earth station coordinates were not. At this elevation angle, the situation looks like that below.



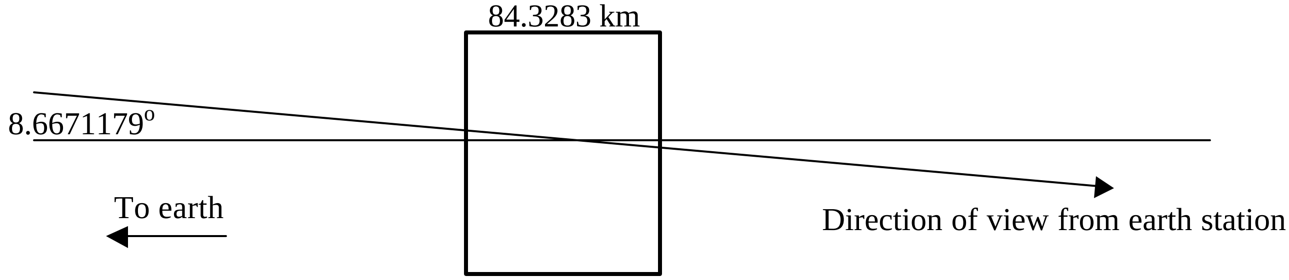

**C** = Center of earth
**E** = Earth station
**S** = Satellite

For an elevation angle of $5^o$, Angle CES = $95^o$. By the law of sines, we have

$$\sin ESC = \frac{6378.137 \times \sin 95}{42164.17} = 0.1506935$$

Giving angle ESC = $8.6671179^o$

The situation inside the station-keeping box is as below



The maximum range variation in this case is 84.3283 × (cos8.6671179°) = 83.3653157 = 83.365 km.

Question 9

An interactive experiment is being set up between the University of York, England (approximately 359.5°E, 53.5°N) and the Technical University of Graz, Austria (approximately 15°E, 47.5°N) that will make use of a geostationary satellite. The earth stations at both universities are constrained to work only above elevation angles of 20° due to buildings, etc., near their locations. The groups at the two universities need to find a geostationary satellite that will be visible to both universities simultaneously, with both earth stations operating at, or above, an elevation angle of 20°. What is the range of sub-satellite points between which the selected geostationary satellite must lie?

**Solution to question 9**

From equation (2.38), where *El* = elevation angle,

$$\cos(El) = \frac{\sin(g)}{\left[1.02288235 - 0.30253825\cos(g)\right]^{1/2}}$$

Let cos(γ) = X, C = cos (*El*), A = 1.02288235, B = 0.30253825, and so $(1 - X^2)^{1/2} = \sin(\gamma)$

The equation relating *El* to γ can be written as a quadratic in X and solved by the quadratic formula as follows:

$1 - X^2 = C^2(A - BX)$

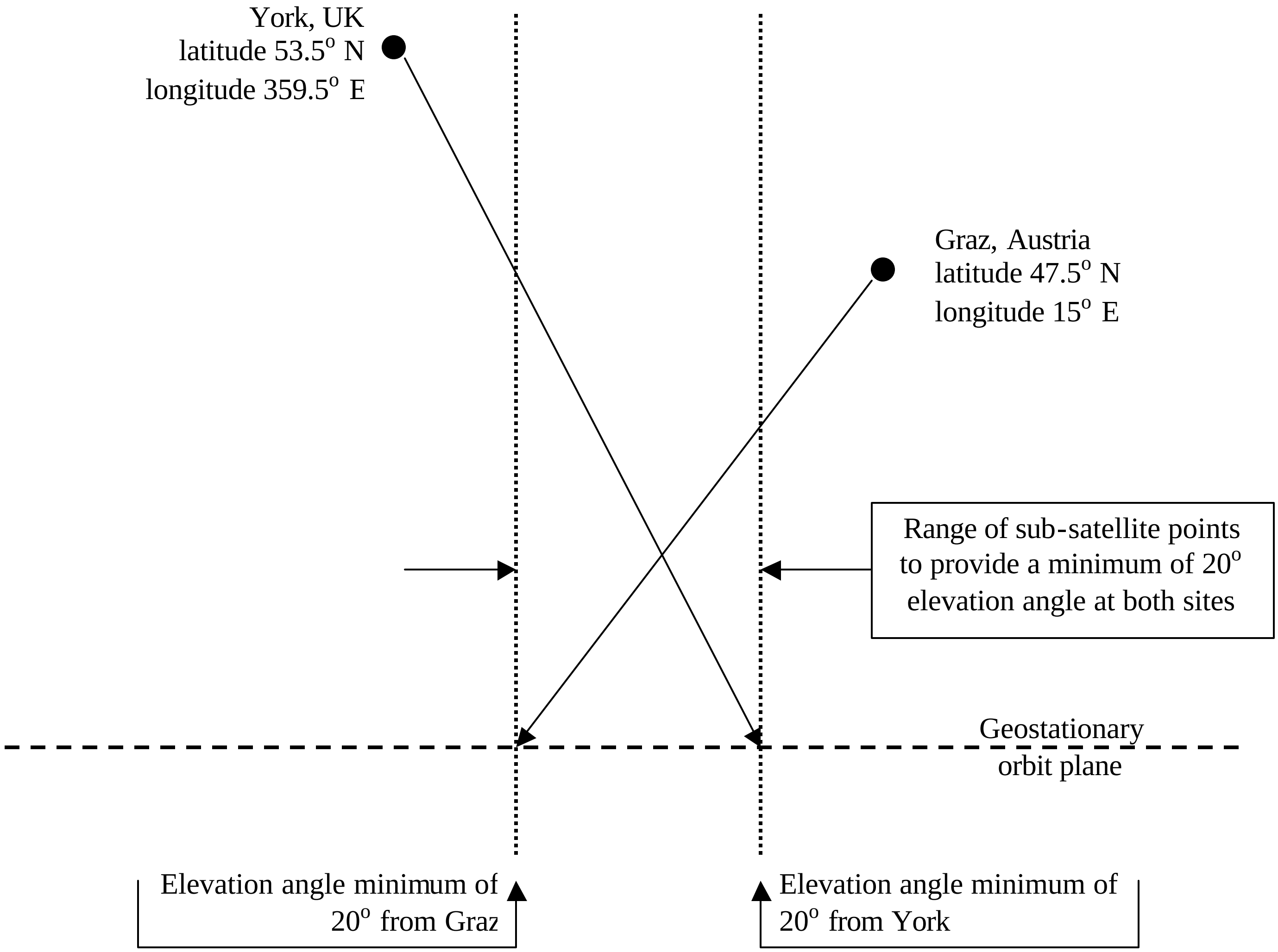
(NOTE: this is the cos (*El*) equation written with the new parameters, and the subject of the equation changed)

Solving gives

$$X = \frac{C^2 B \pm (C^4 B^2 - 4(C^2 A - 1))^{1/2}}{2}$$

The correct solution is the positive root. With a minimum elevation angle of 20°, C = 0.9396926 giving X = 0.4721212, and thus $\gamma = \cos^{-1}(0.4721212) = 61.8279251°$.

The geometry of the linked pair of earth stations is as shown in the schematic below.




From equation (2.36)

$\text{Cos}(\gamma) = \cos(Le)\cos(ls - le)$

For York: $\cos(61.8279251) = \cos(53.5)\cos(ls - le)$ giving $\cos(ls - le) = 0.7937174$. The difference between the York longitude and the sub-satellite point is $\cos^{-1}(0.7937174) = 37.4657212^o$. Since York is at a longitude of $359.5^o$E, there are two solutions for the placement of the satellite (one east of York and one west of York). Only the placement east of York will be visible to Graz and so the east most placement of the satellite as far as York is concerned is $36.9657212^o$E = $36.97^o$E.

For Graz: $\cos(61.8279251) = \cos(47.5)\cos(ls - le)$ giving $\cos(ls - le) = 0.6988278$. The difference between the Graz longitude and the sub-satellite point is $\cos^{-1}(0.6988278) = 45.6669685^o$. Since Graz is at a longitude of $15^o$E, there are two solutions for the placement of the satellite (one east of Graz and one west of Graz). Only the placement west of Graz will be visible to York and so the west most placement of the satellite as far as Graz is concerned is $329.3330315^{o}$E = $329.33^{o}$E

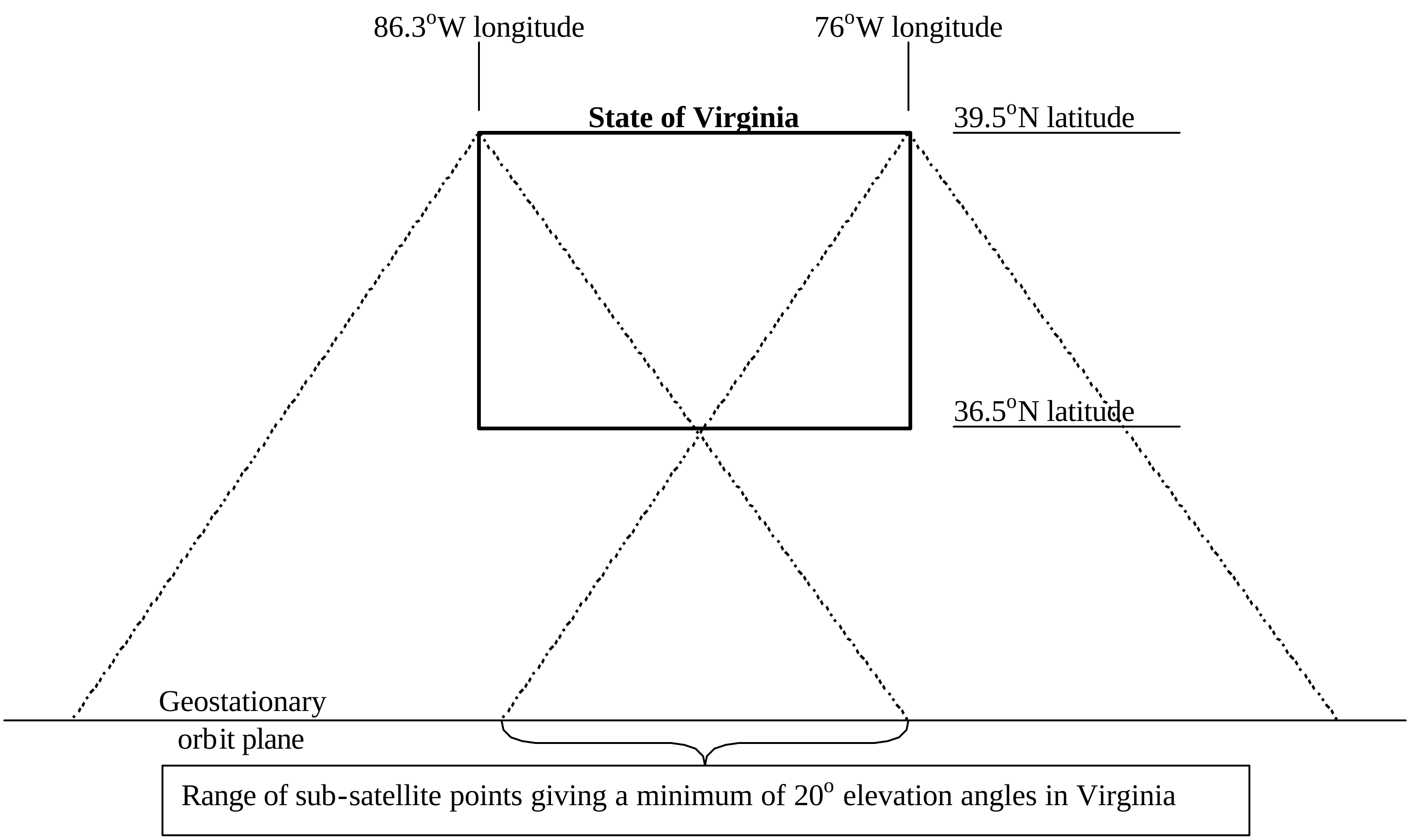
The range of sub-satellite points that will provide an elevation angle of at least $20^{o}$ to both York and Graz is from $329.33^{o}$E to $36.97^{o}$E.

Question 10

The state of Virginia may be represented roughly as a rectangle bounded by $39.5^{o}$ N latitude on the north, $36.5^{o}$N latitude on the south, $76.0^{o}$ W longitude on the east, and $86.3^{o}$W longitude on the west. If a geostationary satellite must be visible throughout Virginia at an elevation angle no lower than $20^{o}$, what is the range of longitudes within which the sub-satellite point of the satellite must lie?

**Solution to question 10**

This question is similar in concept to the previous question, and the same quadratic solution may be applied. The schematic below illustrates the geometry of the question.



Using the same quadratic equation in the solution to question 9 gives $\gamma = 61.8279251^o$.

The lowest elevation angles will be at earth stations located at the north most corners of Virginia. That is at latitude $39.5^o$N, with one at $76^o$W and the other at $86.3^o$W. The factor cos(Le) will be the same in both cases, cos(39.5) = 0.7716246, as will cos(ls – le), which will be cos(61.8279251)/(0.7716246) = 0.6118535, which yields a separation between ls and le of $52.2763585^o$.

Thus the satellite may be approximately $52.3^o$ east or west of the earth station to remain at an elevation angle of $20^o$. However, to enable the other northern corner of Virginia to still "see" the satellite at an elevation angle of at least $20^o$, the satellite must be east of the west most earth station and west of the east most earth station. The east most northern earth station site is at $76^o$W and the west most earth station site is at $86.3^o$W. The sub-satellite points must therefore be between $76^o$W + 52.2763585 = $128.2763585^o$W and $86.3^o$W – 52.2763585 = $34.0236415^o$W

## Question 11

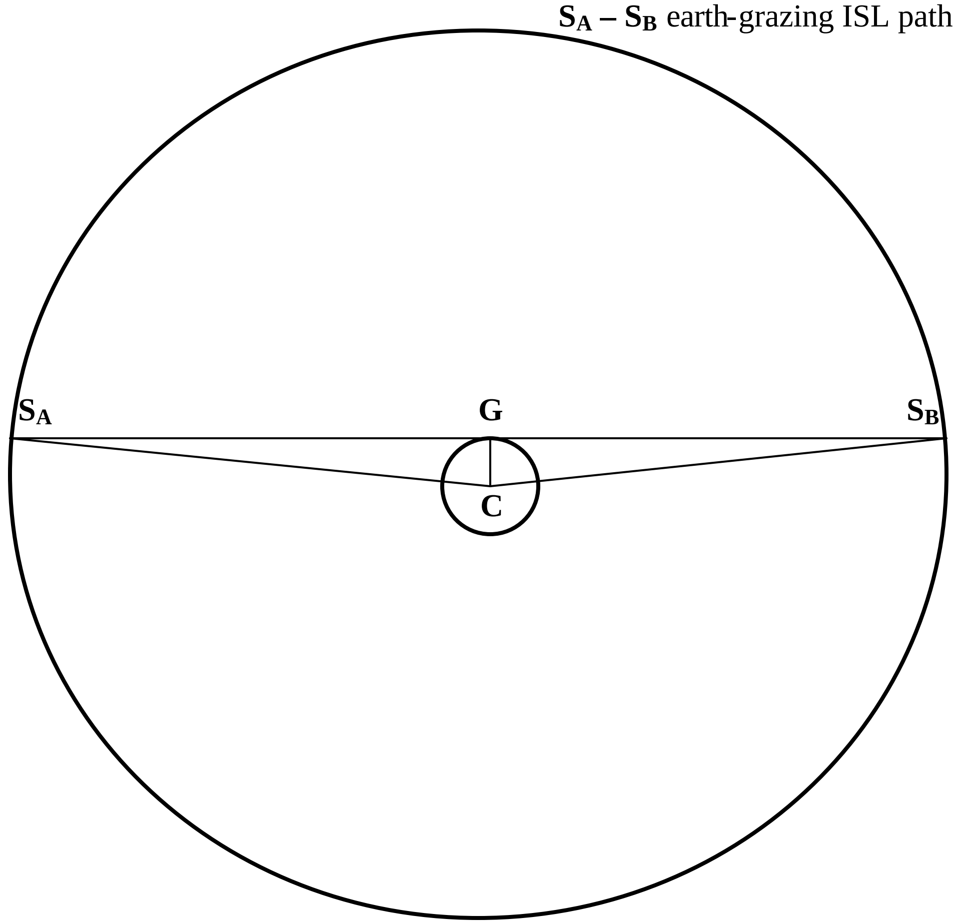
A geostationary satellite system is being built which incorporates inter-satellite links (ISLs) between the satellites. This permits the transfer of information between two earth stations on the surface of the earth, which are not simultaneously visible to any single satellite in the system, by using the ISL equipment to link up the satellites. In this question, the effects of ray bending in the atmosphere may be ignored, processing delays on the satellites may initially be assumed to be zero, the earth may be assumed to be perfectly circular with a flat (i.e. not hilly) surface, and the velocity of the signals in free space (whether in the earth's lower atmosphere or essentially in a vacuum) may be assumed to be the velocity of light in a vacuum.

(i) What is the furthest apart two geostationary satellites may be so that they can still communicate with each other without the path between the two satellites being interrupted by the surface of the earth? Give your answer in degrees longitude between the sub-satellite points. (ii) If the longest, one-way delay permitted by the ITU between two earth stations communicating via a space system is 400 ms, what is the furthest apart two geostationary satellites may be before the transmission delay of the signal from one earth station to the other, when connected through the ISL system of the two satellites, equals 400 ms? The slant path distance between each earth station and the geostationary satellite it is communicating with may be assumed to be 40,000 km. (iii) If the satellites in part (ii) employ on-board processing, which adds an additional delay of 35 ms in each satellite, what is the maximum distance between the ISL-linked geostationary satellites now? (iv) If both of the two earth stations used in parts (ii) and (iii) must additionally now send the signals over a 2,500 km optical fiber line to the end-user on the ground, with an associated transmission delay in the fiber at each end of the link, what is the maximum distance between the ISL-linked geostationary satellites now? You may assume a refractive index of 1.5 for the optical fiber and zero processing delay in the earth station equipment and end-user equipment.

**Solution to question 11**

A schematic of part (i) is shown below.

**$S_A$** = satellite A; **$S_B$** = satellite B; **C** = Center of earth
**G** = point where ISL path is tangential to earth
**$CS_A$** = GEO radius = 42,164.17 km
**CG** = radius of earth = 6378.137 km
**$S_A$ – $S_B$** earth-grazing ISL path between satellites



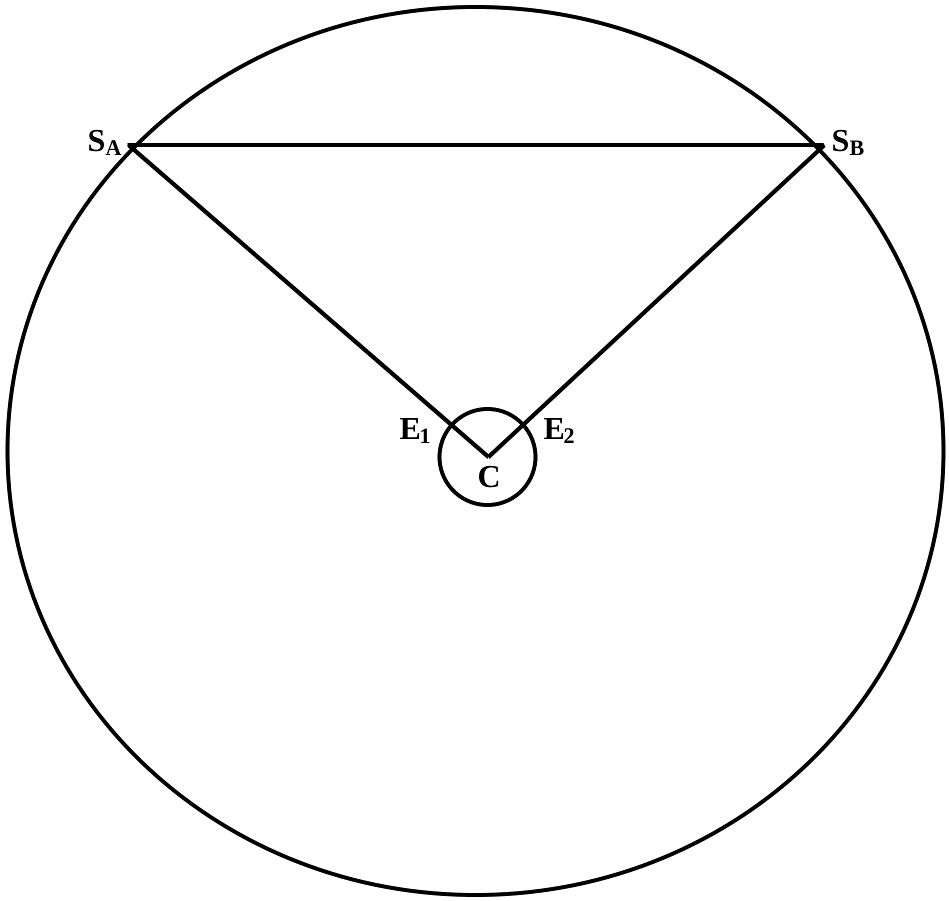

$(GS_A)^2 = (CS_A)^2 – (GC)^2$ = 1,777,817,232 – 40,680,699.86 = 1,737136,562,
Giving $GS_A$ = 41,678.97026 km

By the law of sines, $\text{Sin}(S_ACG) = (S_AG \times \text{Sin}90^o)/(CS_A)$ = 41,678.97026/42,164.17 = 0.9884926, hence angle $S_ACG = 81.2995147^o$.

The difference in sub-satellite point between the two satellites with this geometry is twice angle $S_ACG = 162.6^o$

Part (ii)
A schematic of part (ii) is shown below.

$\mathbf{S_A}$ = satellite A; $\mathbf{S_B}$ = satellite B; **C** = Center of earth
$\mathbf{E_1}$ = Earth station 1 and $\mathbf{E_2}$ **=** earth station 2
$\mathbf{CS_A}$ = $\mathbf{CS_B}$ = GEO radius = 42,164.17 km
$\mathbf{CE_1}$ = $\mathbf{CE_2}$ = radius of earth = 6378.137 km
$\mathbf{S_A}$ **–** $\mathbf{S_B}$ ISL path between satellites




For a 400 ms delay and a signal velocity of $2.997 \times 10^8$ m/s, the total path length traversed must be velocity $\times$ time = $(2.997 \times 10^8$ m/s$) \times (400 \times 10^{-3})$ = 119,880,000 m = 119,880 km.

$E_1S_A = E_1S_B$ = 42,164.17 km – 6378.137 = 35786.033 therefore $S_AS_B$ = 119,880 – $(35786.033 \times 2)$ = 48,307.934 km.

The question, however, stipulates that the slant path distances between the earth stations and their respective satellites (i.e. distances $E_1S_A$ and $E_1S_B$) are 40,000 km. Thus, $S_AS_B$ = 119,880 – 80,000 = 39,880 km. The maximum physical separation between satellites to keep the end-to-end delay below 400 ms is 39,880 km, which translates into a separation between sub-satellite points of $56.4471538^o = 56.45^o$.

Part (iii)

In this part, an additional total delay of 70 ms occurs in the satellites (35 ms at each satellite), which reduces the total path length for the free space part of the link to $(2.997 \times 10^8$ m/s$) \times ((400 - 70) \times 10^{-3}) = 98{,}901{,}000$ m = 98,901 km. With this reduced free space path length available, the maximum physical separation of the two satellites is 98,901 – 80,000 = 18,901 km, which translates into a separation between sub-satellite points of 25.9°.

Part (iv)

In this part, an additional transmission line exists at both ends of the link, totaling 5,000 km. The velocity of light in the fiber link is $(2.997 \times 10^8$ m/s)/refractive index = $1.998 \times 10^8$ m/s, which leads to an additional delay due to the fiber links of $5{,}000{,}000/1.998 \times 10^8 = 0.0250250$ s $= 25 \times 10^{-3}$ s. The available free space path length is therefore reduced to $(2.997 \times 10^8$ m/s$) \times ((400 - 70 - 25) \times 10^{-3}) = 91{,}408{,}500$ m = 91,408.5 km. With the on board processing delay and extended fiber optic transmission line at each end, the maximum physical separation of the two satellites is 91,408.5 – 80,000 = 11,408.5 km, which translates into a separation between sub-satellite points of 7.78°.

Summarizing, we have found the angular separation of satellites connected via an Inter Satellite Link to be as follows:

(i) Maximum separation = 41,678.9702683,357.9 km ≡ 162.6° between satellites

(ii) With a free space delay of 400 ms, separation = 39,880 km ≡ 56.45° between satellites

(iii) With a free space delay of 400 ms and 70 ms of total on board processing delay, separation = 18,901 km ≡ 25.9° between satellites

(iv) With a free space delay of 400 ms, 70 ms of total on board processing delay, and 25 ms total back haul delay at each end of the link, separation = 11,408.5 km ≡ 7.7°.

A 2,500 km back haul link is a typical design requirement, thus on board processing delays need to be kept to a minimum if ISLs are to operate successfully in real time links.

# Chapter 3 Solution to Problems

**1.** The telemetry system of a geostationary communications satellite samples 100 sensors on the spacecraft in sequence. Each sample is transmitted to earth as an eight-bit word in a TDM frame. An additional 200 bits are added to the frame for synchronization and status information. The data are then transmitted at a rate of 1 kilobit per second using BPSK modulation of a low-power carrier.

**a.** How long does it take to send a complete set of samples to earth from the satellite?

**Answer:** The TDM frame consists of $8 \times 100 = 800$ bits of data, plus 200 bits of sync and status information transmitted at 1 kbps. Thus a frame is 1000 bits transmitted in exactly one second.

**b.** Including the propagation delay, what is the longest time the earth station operator must wait between a change in a parameter occurring at the spacecraft and the new value of that parameter being received via the telemetry link? (Assume a path length of 40,000 km.)

**Answer:** Using velocity of EM waves, $c = 3 \times 10^8$ m/s and time delay = R / c,

$$T = 40 \times 10^6 / 3 \times 10^8 = 0.1333 \text{ s.}$$

Longest delay to receive a complete frame is $1.0 + 0.133 = 1.1333$ s.

**2.** A spinner satellite has solar cells wrapped round a cylindrical drum 3.00 m in diameter, with a height of 5.0 m on station. The drum is rotated at 60 rpm to spin-stabilize the satellite. At the end of life, the solar cells are required to deliver 4.0 kW of electrical power.

**a.** Calculate the efficiency of the solar cells at end of life. Assume an incident solar power of 1.39 kW/m$^2$, and that the effective solar radiation absorbing area of the solar cells is equal to the cross sectional area of the drum.

**Answer:** Area of solar cells absorbing sunlight is equivalent to cross sectional area of drum.

$$A = 3.0 \times 5.0 = 15.0 \text{ m}^2.$$

At the end of life the solar cells are producing 4000 watts of electrical power.

Hence efficiency at end of life is

$$\eta_{EoL} = 4000 / (15 \times 1390) = 19.2\%$$

**b.** If the solar cells degrade by 15 percent over the lifetime of the satellite, so that the end-of-life output power is 85% of the beginning-of-life output power, what is the output of the solar cells immediately after launch?

**Answer:** The beginning of life output of the cells is $\eta_{BoL}$ where

$$\eta_{BoL} = 4000 / 0.85 = 4706 \text{ watts}$$

**c.** If the drum covered in solar cells of the spinner design had been replaced by solar sails that rotated to face the sun at all times, what area of solar sails would have been needed? Assume that cells on solar sails generate only 90% percent of the power of cells on a spinner due to their higher operating temperature.

**Answer:** Using the end of life output of 4000 W, the efficiency of the solar cells on the sails is

$$\eta_{ss} = 0.9 \times 19.18 = 17.26 \%$$

The area of solar sails required is A where

$$A = 4000 / (0.1726 \times 1390) = 16.67 \text{ m}^2$$

**3.** A Direct Broadcast Television (DBS-TV) satellite is in geostationary orbit. The electrical power required to operate the satellite and its transmitters is 4 kW. Two designs of satellite can be used: three axis stabilized with solar cells and a spinner.

**a.** A three axis stabilized satellite has two solar sails of equal area that rotate to face the sun at all times. The efficiency of the solar cells at end of life is predicted to be 15%. Calculate the area of cells required by the GEO satellite, and the length of each sail if its width is 2.0 m.

**Answer:** $A_{ss} = 4000 / (0.15 \times 1390) = 19.18 \text{ m}^2$

Each sail is 2 m wide so total length needed = 19.18 / 2 = 9.60 m

Each sail is 4.8 m long.

**b.** A spinner design of DBS-TV satellite is made up from a drum coated in solar cells. The drum has a diameter of 3.5 m. The efficiency of the solar cells is predicted to be 18 % at end of life. Since half the solar cells are in darkness, and some are weakly illuminated by the sun, the effective area of solar cells on a spinner is equal to the diameter of the satellite multiplied by the height of the solar cells on the drum. Calculate the height of the drum to provide the required

4 kW of electrical power.

**Answer:** Effective solar cell area for spinner is $A_{eff} = W \times H$

$$A_{eff} = 4000 / (0.18 \times 1390) = 16.0 \text{ m}^2$$

Height of drum $= A_{eff} / W = 16.0 / 3.5 = 4.57$ m.

**4.** Batteries make up a significant part of the in-orbit weight of a communi

Battgh.

**5**. A geostationary satellite provides service to a region which can be covered by the beam of an antenna on the satellite with a beamwidth of $1.8^o$. The satellite carries transponders for Ku band and Ka band, with separate antennas for transmit and receive. For center frequencies of 14.0/11.5 GHz and 30.0/20.0 GHz, determine the diameters of the four antennas on the satellite.

**a.** Find the diameters of the two transmitting antennas. Specify the diameter and calculate the gain at each frequency.

**Answer**: Use the approximate relationship for beamwidth: $\theta_{3\text{ dB}} = 75\,\lambda / D$.

Hence $D = 75\,\lambda / \theta_{3\text{ dB}}$. Gain is approximately $33{,}000 / (\theta_{3\text{ dB}})^2$

The transmitting antennas on the satellite operate at the lower frequency (downlink) in each band.

For 11.5 GHz: $\lambda = 0.02609$ m, $D = 75 \times 0.02609 / 1.8 = 1.087$ m

For 20 GHz: $\lambda = 0.015$ m, $D = 75 \times 0.015 / 1.8 = 0.625$ m

$G = 33{,}000 / 1.8^2 = 10.185$ or 40.1 dB

**b.** Find the diameters of the two receiving antennas. Specify the diameter and calculate the gain at each frequency.

The receiving antennas on the satellite operate at the higher frequency (uplink) in each band.

For 14.0 GHz: $\lambda = 0.02143$ m, $D = 75 \times 0.02413 / 1.8 = 0.893$ m

For 30.0 GHz: $\lambda = 0.010$ m, $D = 75 \times 0.010 / 1.8 = 0.417$ m

Because the beamwidth of each antenna is the same, the gains are all the same: $G = 40.1$ dB

**6.** A geostationary satellite provides communications within the United States at Ku band. The antennas on the satellite have beamwidths of $6^o$ in the E-W direction and $3^o$ in the N-S direction. A separate antenna is used for transmitting in the 11 GHz band and receiving in the 14 GHz band.

**a.** Find the dimensions and estimate the gain of the transmitting antenna in the N-S and E-W directions.

**Answer:** Using the approximations $\theta_{3\text{ dB}} = 75\,\lambda / D$, and $G = 33{,}000 / (\theta_{3\text{ dB}})^2$:

N-S dimension $= 75\,\lambda / 3.0 = 25\,\lambda$ E-W dimension $= 75\,\lambda / 6 = 12.5\,\lambda$

For transmit antenna at 11.0 GHz, $\lambda = 0.02727$ m

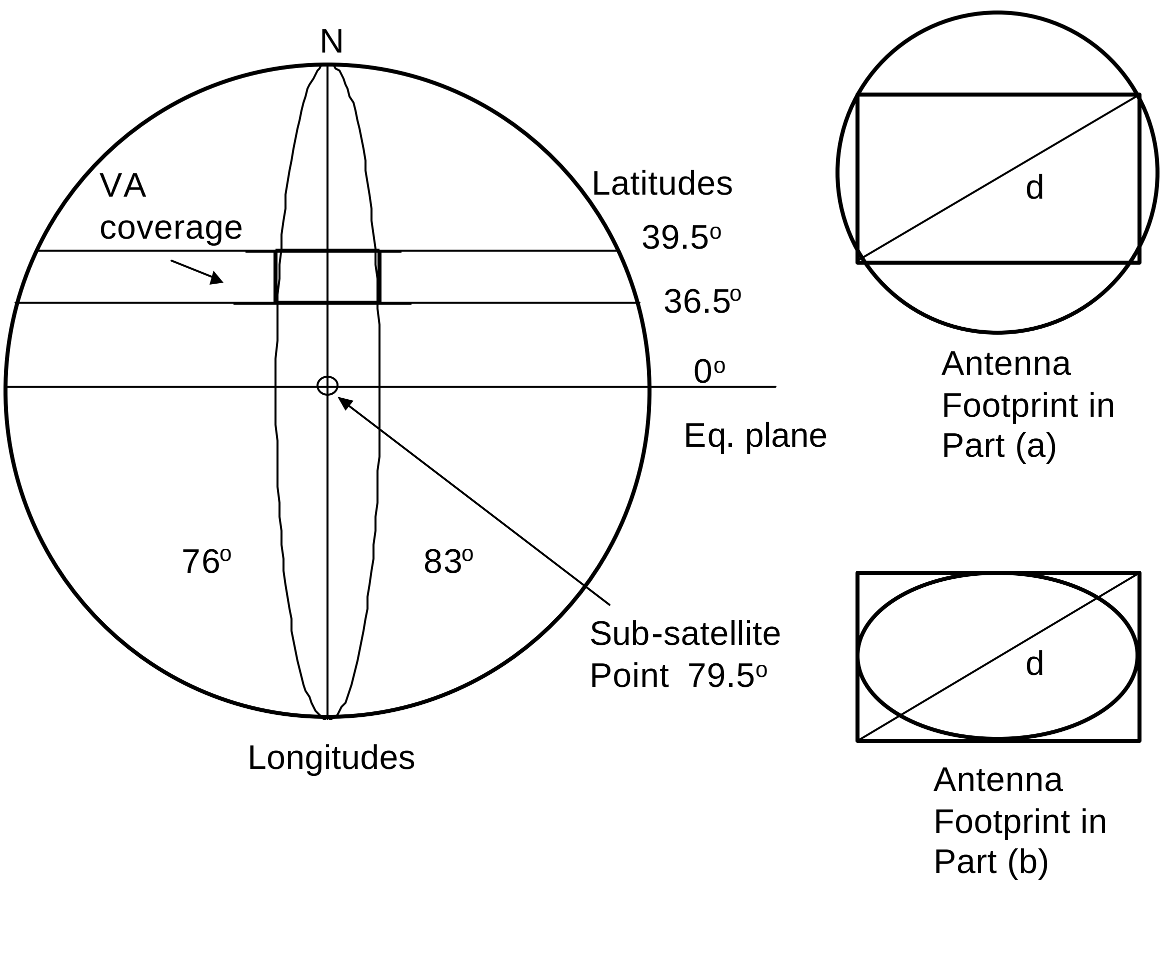
$D_{N-S} = 25 \times 0.02727 = 0.681$ m     $D_{E-W} = 12.3 \times 0.02727 = 0.341$ m

**b.** Find the dimensions and estimate the gain of the receiving antenna in the N-S and E-W directions.

**Answer:** For receiving antenna at 14.0 GHz, $\lambda = 0.02143$ m

$D_{N-S} = 25 \times 0.02143 = 0.536$ m     $D_{E-W} = 12.5 \times 0.02143 = 0.268$ m

**7.** The State of Virginia can be represented approximately on a map as an area bounded by 39.5° N latitude, 36.5° N latitude, 76.0° W longitude, and 83.0° W longitude. A geostationary satellite located at 79.5° W longitude has an antenna with a spot beam that covers all of Virginia at a downlink center frequency of 11.155 MHz. In this problem you will estimate the antenna dimensions subject to two different assumptions. In both cases use an aperture efficiency of 65 percent.



**a.** The antenna is a circular parabolic reflector generating a circular beam with a 3 dB beamwidth equal to the diagonal of the area bounding the State of Virginia. Estimate the length of the diagonal by measuring the distance on a map of the US, and calculate the beamwidth of the antenna from simple geometry. Hence determine the diameter of the antenna on the satellite in meters and its approximate gain in decibels.

**Answer:** The diagram above shows the boundaries of the rectangle enclosing the State of Virginia. Measurement on a map shows that the diagonal pf the rectangle is approximately 800 km. The diagonal can be calculated from the lengths of the sides of the rectangle using the given altitude and longitude boundaries.

$$D_{N\text{-}S} = 6378 \times 3.0/57.3 = 334 \text{ km}$$

$$D_{E\text{-}W} = 6378 \times 7.0/57.3 = 779 \text{ km}$$

$$\text{Diagonal } d = [\,D_{E\text{-}W}^2 + D_{N\text{-}S}^2]^{1/2} = 848 \text{ km}$$

Working in the N-S longitude plane at 79.5$^o$ W, the distance from the satellite to the center of the rectangle at Lat 37.5$^o$ N, Long 79.5$^o$ W is s where a = GEO radius, $r_e$ = earth radius

$$s^2 = a^2 + r_e^2 - 2\, a\, r_e \cos 37.5^o = 1.3918 \times 10^9 \qquad \text{hence} \quad s = 37{,}307 \text{ km}$$

The beamwidth of the antenna beam at the satellite can be found as a first approximation from the length of the diagonal of the rectangle. Using the map value of d = 800 km:

$$\theta_{3\,dB} = d / s \text{ radians} = 57.3 \times 800 / 37{,}307 = 1.23^o.$$

For an antenna operating at 11.155 GHz, $\lambda$ = 0.02689 m, the antenna diameter is 75 $\lambda$/D giving

$$D = 75 \times 0.02689 / 1.23 = 1.640 \text{ m.}$$

The gain of the antenna, with aperture efficiency of 65%, can be found from

$$G = \eta_A \times (\pi\, D / \lambda)^2 = 0.65 \times (\pi \times 1.640 / 0.02689)^2 = 23{,}862 \text{ or } 43.8 \text{ dB}$$

**b.** The antenna is an elliptical parabolic reflector with 3 dB beamwidths in the N-S and E-W directions are equal to the height and the width of the area bounding the State of Virginia. Find the N-S and E-W dimensions from a map of the US, and use geometry to calculate the required 3 dB beamwidths of the satellite antenna. Calculate the approximate gain of the antenna.

**Answer:** Dimensions of the rectangle from a map are approximately 330 km in the N-S direction and 750 km in the E-W direction. Similar dimensions based on the latitudes and longitudes are given in part (a) above. In this case, the dimensions of the antenna must create an elliptical footprint that fits inside the rectangle. Ignoring earth curvature and using the distance from the center of the rectangle to the satellite, s = 37,345 km, the N-S and E-W beamwidths are approximately

$$\theta_{N\text{-}S} = y_{N\text{-}S} / s \text{ radians} = 57.3 \times 330 / 37{,}307 = 0.51^o$$

$$\theta_{E\text{-}W} = x_{E\text{-}W} / s \text{ radians} = 57.3 \times 750 / 37{,}307 = 1.15^o$$

The antenna dimensions are

$$D_{N\text{-}S} = 75 \times 0.02689 / 0.51 = 3.95 \text{ m}$$

$$D_{E\text{-}W} = 75 \times 0.02689 / 1.15 = 1.75 \text{ m}$$

The gain of the antenna can be found from

$$G = 33{,}000 / (0.51 \times 1.15) = 56{,}273 \text{ or } 47.5 \text{ dB}$$

Curvature of the earth in the N-S direction makes the N-S angle at the satellite smaller than the result in the calculation above where earth curvature is ignored. A more accurate result can be obtained by recalculating the distance from the satellite to the upper and lower edges of the rectangle and then using the rule of sines to find the angle at the satellite.

$$s_1^2 = a^2 + r_e^2 - 2\, a\, r_e \cos 39.5^o = 1.4034 \times 10^9 \qquad \text{hence} \quad s = 37{,}423 \text{ km}$$

$$s_2^2 = a^2 + r_e^2 - 2\, a\, r_e \cos 36.5^o = 1.3861 \times 10^9 \qquad \text{hence} \quad s = 37{,}230 \text{ km}$$

The angle between the line from the satellite to the earth's surface and from the satellite to the center of the earth is $\alpha$ where

$$\sin \alpha / r_e = \sin \text{Lat} / s \text{ or } \sin \alpha = r_e \times \sin \text{Lat} / s$$

For the two latitudes $39.5^o$ and $36.5^o$ we have

$$\alpha_1 = 6378 / 37{,}423 \times \sin 39.5^o = 0.10841 \quad \text{and} \quad \alpha_1 = 6.223^o$$

$$\alpha_2 = 6378 / 37{,}230 \times \sin 36.5^o = 0.10191 \quad \text{and} \quad \alpha_2 = 5.849^o$$

The satellite antenna beamwidth in the N-S direction is $\theta_{3\text{ dB}} = \alpha_1 - \alpha_2 = 0.374^o$

The difference from the approximate result is significant because it makes the antenna even larger in the N-S direction. The approximation of a flat earth is reasonable for the E-W direction. Using the new N-S beamwidth, the N-S dimension of the antenna is

$$D_{N\text{-}S} = 75 \times 0.02689 / 0.374 = 5.392 \text{ m}$$

The gain of the antenna increases to $33{,}000 / (0.374 \times 1.15) = 76{,}726$ or 48.8 dB

**8.** The state of Pennsylvania is approximately one degree wide (E-W) by one half degree high (N-S) when viewed from geostationary orbit at a longitude of 75 degrees west. Calculate:

**a.** The dimensions of a downlink Ku-band antenna on a geostationary satellite with 3 dB beamwidths equal to the width and height of Pennsylvania. Use a frequency of 11.0 GHz. Identify the dimensions as E-W and N-S.

**Answer:** The wavelength for 11.0 GHz is 0.02727 m, so the antenna dimensions are

$$D_{N-S} = 75 \times 0.02727 / 0.50 = 4.010 \text{ m}$$

$$D_{E-W} = 75 \times 0.02727 / 1.00 = 2.005 \text{ m}$$

The gain of the antenna can be found from

$$G = 33{,}000 / (0.50 \times 1.0) = 66{,}000 \text{ or } 48.2 \text{ dB}$$

**b.** The dimensions of an uplink Ka-band antenna on a geostationary satellite with 3 dB beamwidths equal to the width and height of Pennsylvania. Use a frequency of 30.0 GHz. Identify the dimensions as E-W and N-S.

**Answer:** The wavelength for 30.0 GHz is 0.010 m, so the antenna dimensions are

$$D_{N-S} = 75 \times 0.010 / 0.50 = 1.500 \text{ m}$$

$$D_{E-W} = 75 \times 0.010 / 1.00 = 0.750 \text{ m}$$

The gain of the antenna does not change because the beamwidths are the same as for 11 GHz

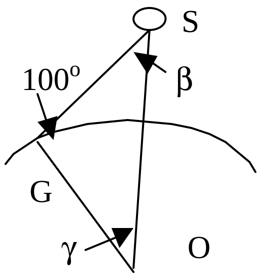
$$G = 33{,}000 / (0.50 \times 1.0) = 66{,}000 \text{ or } 48.2 \text{ dB}$$

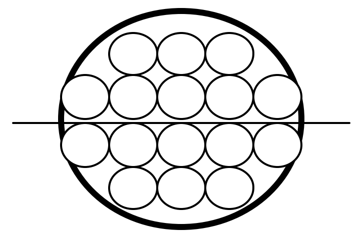
**c.** Suppose that the maximum dimension of the satellite at launch is 3 m wide, determined by the shroud of the ELV. Describe in a paragraph how you would launch the satellites in (a) and (b) above carrying: (a) the Ku band antenna, and (b) the Ka band antenna.

**Answer:** The dimensions of the antenna for 30 GHz in part (b) are below the diameter of the ELV shroud, so the antenna will fit inside the shroud. The antenna might need to be folded down against the satellite body for launch. In (a), the 11 GHz antenna dimension exceeds the shroud diameter, so the reflector of the antenna would have to be folded. This is difficult to do mechanically and is avoided whenever possible. Various methods have been used to make reflectors that can be folded or collapsed for launch – read Chapter 3 for the details.

**9.** A constellation of low earth orbit satellites has an altitude of 1000 km. Each satellite has two multiple beam antennas that generate 16 beams. One antenna is used to transmit at 2.4 GHz and the other antenna receives at 1.6 GHz.

**a.** Using simple geometry, find the coverage angle of the satellite antenna when the lowest elevation angle for an earth station is $10^o$. (Hint: draw a diagram of the earth and the satellite and use the law of sines to solve the angles in a triangle.)



Geometry to find angle at satellite.

O is the center of the earth, G is the earth station, and S is the satellite.

Coverage zone with 16 beams

**Answer:** The tiangle SGO in the above figure is used to solve for the angle $\beta$, which is one half of the coverage angle of the satellite. The angle OGS is $90^o + 10^o = 100^o$.

Lengths in triangle SGO are: OG = $r_e$ = 6378 km, SO = $r_e + h$ = 7378 km

From the law of sines: $\sin 100^o / (R_e + h) = \sin \beta / r_e$

Hence $\sin \beta = r_e / (r_e + h) \times \sin 100^o = 0.8513$; $\beta = 58.35^o$.

The coverage angle is $116.7^o$.

**b.** Estimate the coverage area over the surface of the earth, in km.

**Answer:** The arc length across the coverage zone is given by $d = 2\, r_e\, \gamma$ with $\gamma$ in radians.

$\gamma = 180^o - 100^o - 58.35^o = 21.65^o$

Hence $d = 2 \times 6378 \times 21.65 / 57.3 = 4820$ km

**c.** Assuming that all 16 beams from the satellite antennas have equal beamwidths, determine the beamwidth of one beam. (Hint: draw a circle representing the coverage area and fit 16 circles representing the 3 dB beamwidths of the beams inside the first circle.)

**Answer:** Between four and five of the multiple beams must fit across the coverage circle.

Hence the multiple beams have a beamwidth in the range 116.7 /4 to 116.7 / 5 = $29^o$ to $23^o$.

**d.** Find the gain and the dimensions of each antenna on the satellite.

**Answer:** At the uplink frequency of 1.6 GHz, $\lambda = 0.1875$ m, the diameter of the satellite antenna is in the range $75 \times 0.1875$ ($29^o$ to $23^o$) $= 0.485$ m to $0.611$ m.

At the downlink frequency of 2.4 GHz, $\lambda = 0.125$ m, the diameter of the satellite antenna is in the range $75 \times 0.125$ ($29^o$ to $23^o$) $= 0.323$ m to $0.408$ m.

At both frequencies the gain is the same because the beamwidths are the same.

Using $G = 33000 / \theta_{3\,dB}^2$, gain is 39.2 to 62.4 or 15.9 to 18.0 dB

**10.** A geostationary satellite carries a C-band transponder which transmits 15 watts into an antenna with an on-axis gain of 32 dB. An earth station is in the center of the antenna beam from the satellite, at a distance of 38,500 km. For a frequency of 4.2 GHz:

**a.** Calculate the incident flux density at the earth station in watts per square meter and in $dBW/m^2$.

**Answer:** $F = P_t G_t / 4\pi R^2 \ \ W/m^2 = 15 + 32 - 11 - 20 \log 38.50 \times 10^6 = -115.7 \ dBW / m^2$

Converting to ratios

$F = 10^{-11.57} = 2.69 \times 10^{-12} \ W / m^2$

**b.** The earth station has an antenna with a circular aperture 3 m in diameter and an aperture efficiency of 62%. Calculate the received power level in watts and in dBW at the antenna output port.

**Answer:** Received power is $F \times A_{eff} = F \times \eta \times \pi r^2 = 2.69 \text{ x } 10^{-12} \times 0.62 \times \pi \times 1.5^2$

$= 1.179 \times 10^{-11}$ watts or $-109.3$ dBW

**c.** Calculate the on-axis gain of the antenna in decibels.

**Answer:** At 4.2 GHz, $\lambda = 0.07143$:

$G = \eta (\pi D / \lambda)^2 = 0.62 \times (\pi \times 3.0 / 0.07143)^2 = 10{,}793$ or $40.3$ dB

**d.** Calculate the free space path loss between the satellite and the earth station.

Calculate the power received, Pr, at the earth station using the link equation:

Pr = Pt Gt Gr / Lp where Pt Gt is the EIRP of the satellite transponder and Lp is the path loss. Make your calculation in dB units and give your answer in dBW.

**Answer:** Path loss = 20 log ( 4 $\pi$ R / $\lambda$) = 20 log ( 4 $\pi$ 38.5 $\times 10^6$ / 0.07143) = 196.6 dB

The received power at the earth station is calculated from

$$P_r = P_t G_t G_r / L_p$$

Ignoring any losses. In dB units

$$P_r = \text{EIRP} + G_r - \text{path loss} = 15 + 32 + 40.3 - 196.6 = -109.3 \text{ dBW}$$

This is the same answer as obtained in part (b) using flux density.

**12.** Calculate the total power radiated by the sun in watts and in dBW.

Hint: The sun is 93 million miles (about 150 million kilometers) from the earth. At that distance, the sun produces a flux density of 1.39 kW/m$^2$. This power density is present over all of a sphere with a radius of 150 million km.

**Answer:** The entire light output of the sun passes through the surface of an imaginary sphere with a radius of 150 x $10^6$ km. At that distance, the flux density crossing the sphere is $1.39 \times 10^3$ W/m$^2$. Hence the output of the sun is $1.39 \times 10^3 \times 4\pi R^2$ where R = $1.5 \times 10^{11}$ m.

Hence P = $1.39 \times 10^3 \times 4\pi \times (1.5 \times 10^{11})^2 = 3.93 \times 10^{26}$ watts or 266 dBW.

If calculations show any transmitter power to approach 266 dBW, the calculations are wrong.

# Chapter 4 Solution to Problems

**Question #1.**

A C-band earth station has an antenna with a transmit gain of 54 dB. The transmitter output power is set to 100 W at a frequency of 6.100 GHz. The signal is received by a satellite at a distance of 37,500 km by an antenna with a gain of 26 dB. The signal is then routed to a transponder with a noise temperature of 500 K, a bandwidth of 36 MHz, and a gain of 110 dB.

a. Calculate the path loss at 6.1 GHz. Wavelength is 0.04918 m.

**Answer:** Path loss = $20 \log (4 \pi R / \lambda) = 20 \log (4 \pi \times 37{,}500 \times 10^3 / 0.04918)$ dB

$$L_p = 199.6 \text{ dB}$$

b. Calculate the power at the output port (sometimes called the output waveguide flange) of the satellite antenna, in dBW.

**Answer:** Uplink power budget gives

$$P_r = P_t + G_t + G_r - L_p \text{ dBW}$$

$$= 20 + 54 + 26 - 199.6 = -99.6 \text{ dBW}$$

c. Calculate the noise power at the transponder input, in dBW, in a bandwidth of 36 MHz.

**Answer**: $N = k\, T_s\, B_N = -228.6 + 27 + 75.6 = -126.0$ dBW

d. Calculate the C/N ratio, in dB, in the transponder.

**Answer:** $C/N = P_r - N = -99.6 + 126.0 = 26.4$ dB

e. Calculate the carrier power, in dBW and in watts, at the transponder output.

**Answer:** The gain of the transponder is 110 dB. Output power is

$$P_t = P_r + G = -99.6 + 110 = 10.4 \text{ dBW or } 10^{1.04} = 11.0 \text{ W.}$$

**2.** The satellite in Question #1 above serves the 48 contiguous states of the US. The antenna on the satellite transmits at a frequency of 3875 MHz to an earth station at a distance of 39,000 km. The antenna has a 6° E-W beamwidth and a 3° N-S beamwidth. The receiving earth station has an antenna with a gain of 53 dB and a system noise temperature of 100 K and is located at the edge of the coverage zone of the satellite antenna. (Assume antenna gain is 3 dB lower than in the center of the beam)

Ignore your result for transponder output power in Question 1 above. Assume the transponder carrier power is 10 W at the input port of the transmit antenna on the satellite.

a. Calculate the gain of the satellite antenna in the direction of the receiving earth station. [Use the approximate formula G = 33,000/(product of beamwidths).]

**Answer:** G = 33,000 / ( 6 x 3) = 1833 or 32.6 dB on axis.

Hence satellite antenna gain towards earth station is 32.6 – 3 = 29.6 dB.

b. Calculate the carrier power received by the earth station, in dBW.

**Answer:** Calculate the path loss at 3.875GHz. Wavelength is 0.07742 m.

Path loss = $20 \log ( 4 \pi R / \lambda) = 20 \log ( 4 \pi \times 39{,}000 \times 10^3 / 0.07742)$ dB

$L_p = 196.0$ dB

Downlink power budget gives

$P_r = P_t + G_t + G_r - L_p$ dBW

$= 10 + 29.6 + 53 – 196.0 = -103.4$ dBW

c. Calculate the noise power of the earth station in 36 MHz bandwidth.

**Answer**: $N = k T_s B_N = -228.6 + 20 + 75.6 = -133.0$ dBW

d. Hence find the C/N in dB for the earth station.

**Answer:** $C/N = P_r – N = -103.4 + 133.0 = 29.6$ dB

**3.** A 14/11 GHz satellite communication link has a transponder with a bandwidth of 52 MHz which is operated at an output power level of 20W. The satellite transmit antenna gain at 11 GHz is 30 dB towards a particular earth station. Path loss to this station is 206 dB , including clear air atmospheric loss.

The transponder is used in FDMA mode to send 500 BPSK voice channels with half rate FEC coding. Each coded BPSK signal has a symbol rate of 50 kbps and requires a receiver with a noise bandwidth of 50 kHz per channel. The earth stations used to receive the voice signals have antennas with a gain of 40 dB (1m diameter) and a receiver with $T_{system}$ = 150K in clear air, and IF noise bandwidth 50 kHz.

a. Calculate the power transmitted by the satellite in one voice channel.

**Answer:** In FDMA, the output power of the transmitter is divided equally between the channels. For $P_t$ = 20 W and 500 channels, power per channels is 20 / 500 = 40 mW/ch.

b. Calculate the C/N in clear air for an earth station receiving one BPSK voice signal.

**Answer:** Each channel receiver has a noise bandwidth of 50 kHz or 47 dBHz.

Path loss at 11GHz is 206.0 dB, including atmospheric loss..

Downlink power budget for one FDMA channel gives

$$P_r = P_t + G_t + G_r - L_p \text{ dBW}$$

$$= -14.0 + 30.0 + 40.0 - 206.0 = -150.0 \text{ dBW}$$

The noise power at the input to the receiver is

$$N = k\, T_s\, B_N = -228.6 + 21.8 + 47.0 = -159.8 \text{ dBW}$$

Hence $C/N = P_r - N = -150.0 + 159.8 = 9.8$ dB.

c. What is the margin over a coded BPSK threshold of 6 dB?

**Answer:** Margin is receiver C/N – minimum permitted C/N, in dB

Margin = 9.8 – 6.0 = 3.8 dB.

**4.** Geostationary satellites use L, C, Ku and Ka bands. The path length from an earth station to the GEO satellite is 38,500 km. For this range, calculate the path loss in decibels for the following frequencies:

**Note:** Round all results to nearest 0.1 dB.

**a.** 1.6 GHz, 1.5 GHz

Wavelengths are: 1.6 GHz, $\lambda$ = 0.1875 m; 1.5 GHz, $\lambda$ = 0.200 m.

**Answer:** Path loss = $20 \log ( 4 \pi R / \lambda)$

For 1.6 GHz, $L_p = 20 \log ( 4 \pi \times 38{,}500 \times 10^3 / 0.1875) = 188.2$ dB

For 1.5 GHz, $L_p = 20 \log ( 4 \pi \times 38{,}500 \times 10^3 / 0.200) = 187.7$ dB

Path loss at frequency $f_2$ can be found from path loss at frequency $f_1$ by scaling:

$L_p(f_2) = L_p(f_1) + 20 \log (f_2 / f_1)$. Using the result for 1.6 GHz, $L_p = 188.2$ dB:

**b.** 6.2 GHz, 4.0 GHz

**Answer:** At 6.2 GHz, $L_p = 188.2 \text{ dB} + 20 \log (6.2 / 1.6) = 200.0$ dB

At 4.0 GHz, $L_p = 188.2 \text{ dB} + 20 \log (4.0 / 1.6) = 196.2$ dB

**c.** 14.2 GHz, 12.0 GHz

**Answer:** At 146.2 GHz, $L_p = 188.2 \text{ dB} + 20 \log (14.2 / 1.6) = 207.2$ dB

At 12.0 GHz, $L_p = 188.2 \text{ dB} + 20 \log (12.0 / 1.6) = 205.7$ dB

**d.** 30.0 GHz 20.0 GHz

**Answer:** At 30.0 GHz, $L_p = 188.2 \text{ dB} + 20 \log (30 / 1.6) = 213.7$ dB

At 20.0 GHz, $L_p = 188.2 \text{ dB} + 20 \log (20 / 1.6) = 210.1$ dB

**Note:** All commercial satellite systems have path losses that fall within the above range, excepting any in the vhf and uhf bands, and above 40 GHz.

**5.** Low earth orbit satellites use mainly L band, with ranges varying from 1000 km to 2,500 km. Calculate the maximum and minimum path loss from earth to a satellite, in dB, for the uplink frequency of 1.6 GHz, and the downlink frequency of 1.5 GHz.

**Answer:** Wavelengths are: 1.6 GHz, $\lambda = 0.1875$ m; 1.5 GHz, $\lambda = 0.200$ m.

Path loss $= 20 \log (4 \pi R / \lambda)$

For 1.6 GHz, Maximum $L_p = 20 \log (4 \pi \times 2{,}500 \times 10^3 / 0.1875) = 64.5$ dB

For 1.5 GHz, $L_p = 20 \log (4 \pi \times 38{,}500 \times 10^3 / 0.200) = 187.7$ dB

**6.** A geostationary satellite carries a transponder with a 20 watt transmitter at 4 GHz. The transmitter is operated at an output power of 10 watts and drives an antenna with a gain of 30 dB. An earth station is at the center of the coverage zone of the satellite, at a range of 38,500 km. Using decibels for all calculations, find:

**a.** The flux density at the earth station in $dBW/m^2$

**Answer:** Flux density is given by $F = 20 \log [ P_t G_t / (4 \pi R^2) ]$ $dBW/m^2$

Hence for $R = 38{,}500$ km, $f = 4$ GHz, $\lambda = 0.075$ m

$$F = 10 \log P_t + G_t - 10 \log (4\pi) - 20 \log (38{,}500 \times 10^3) \ \ dBW / m^2$$

$$= 10.0 + 30.0 - 11.0 - 151.7 = -122.7 \ dBW / m^2$$

**b.** The power received by an antenna with a gain of 39 dB, in dBW.

**Answer:** Received power can be calculated from the effective area of the antenna aperture and the incident flux density, but since the antenna gain is given in dB, it is better to use path loss and the link budget.

Path loss $L_p = 20 \log (4\pi R / \lambda) = 10 \log (4\pi \times 38{,}500 \times 10^3 / 0.075) = 196.2$ dB

Downlink power budget gives

$$P_r = P_t + G_t + G_r - L_p \ \ dBW$$

$$= 10.0 + 30.0 + 39.0 – 196.2 = -117.2 \ dBW$$

Alternatively, the received power can be found from

$P_r = F \times A_{eff}$ where $A_{eff}$ is the effective aperture area of the antenna.

Given $G = 4\pi A_{eff} / \lambda^2 = 39$ dB, we can find $A_{eff}$ from

$$A_{eff} = G + 20 \log \lambda - 11.0 \ dB = 39.0 – 22.5 –11.0 = 5.5 \ dB \ m^2$$

$$P_r = -122.7 + 5.5 = - 117.2 \ dBW / m^2$$

**c.** The EIRP of the transponder in dBW.

**Answer:** Transponder EIRP $= P_t + G_t = 10 + 30 = 40$ dBW

**7.** A LEO satellite has a multi-beam antenna with a gain of 18 dB in each beam. A transponder with transmitter output power of 0.5 watts at 2.5 GHz is connected to one antenna beam. An earth station is located at the edge of the coverage zone of this beam, where the received power is 3 dB below that at the center of the beam, and at a range of 2,000 km from the satellite. Using decibels for all calculations, find:

**a.** The power received by an antenna with a gain of +1 dB, in dBW.

**Answer:** Find the path loss, $L_p$ , first, for a wavelength of $\lambda = 0.120$ m:

Path loss $L_p = 20 \log (4 \pi R / \lambda) = 10 \log (4 \pi \times 2000 \times 10^3 / 0.120) = 166.4$ dB

Downlink power budget gives

$$P_r = P_t + G_t + G_r - L_p - \text{losses dBW}$$

$$= -3.0 + 18.0 + 1.0 – 166.4 – 3.0 = -153.4 \text{ dBW}$$

**b.** The noise power of the earth station receiver for a noise temperature of 260K and an RF channel bandwidth of 20 kHz.

**Answer:** The noise power at the input to the receiver is

$$N = k\, T_s\, B_N = -228.6 + 24.1 + 43.0 = -161.5 \text{ dBW}$$

**c.** The C/N ratio in dB for the LEO signal at the receiver output.

**Answer:** $C/N = P_r – N = -153.4 + 161.5 = 8.1$ dB.

**8.** A satellite in GEO orbit is a distance of 39,000 km from an earth station. The required flux density at the satellite to saturate one transponder at a frequency of 14.3 GHz is -90.0 dBW/m$^2$. The earth station has a transmitting antenna with a gain of 52 dB at 14.3 GHz.

Find:

**a.** The EIRP of the earth station

**Answer:** $\text{EIRP} = P_t + G_t = P_t + 52$ dBW

Flux density is given by $F = 20 \log [\text{EIRP} / (4 \pi R^2)]$ dBW/m$^2$

Hence for $R = 39{,}000$ km, $f = 14.3$ GHz, $\lambda = 0.02010$ m

$$F = -90.0 = \text{EIRP} - 10 \log (4 \pi) - 20 \log (39{,}000 \times 10^3) \text{ dBW / m}^2$$

$$-90.0 = \text{EIRP} - 11.0 - 151.8 \text{ dBW / m}^2$$

$$\text{EIRP} = -90.0 + 162.8 = 72.8 \text{ dBW}$$

**b.** The output power of the earth station transmitter.

**Answer:** $\text{EIRP} = P_t + G_t = 72.8$ dBW. Hence $P_t = 72.8 – 52.0 = 20.8$ dBW.

**9.** A 12 GHz earth station receiving system has an antenna with a noise temperature of 50K, a LNA with a noise temperature of 100 K and a gain of 40 dB, and a mixer with a noise temperature of 1000 K. Find the system noise temperature.

**Answer:** System noise temperature is calculated from

$$T_s = T_{antenna} + T_{LNA} + T_{mixer} / G_{LNA} + \ldots$$

Hence for $G_{LNA}$ = 40 dB = 10,000 as a ratio

$$T_s = 50 + 100 + 1000 / 10{,}000 = 150.1 \text{ K}$$

**10.** A geostationary satellite carries a C-band transponder which transmits 20 watts into an antenna with an on-axis gain of 30 dB. An earth station is in the center of the antenna beam from the satellite, at a distance of 38,000 km. For a frequency of 4.0 GHz:

**a.** Calculate the incident flux density at the earth station in watts per square meter and in $dBW/m^2$.

**Answer:** Flux density is given by $F = 20 \log [ \text{EIRP} / (4 \pi R^2) ]$ $dBW/m^2$

Hence for R = 38,000 km, f = 4.0 GHz, $\lambda$ = 0.0750 m, EIRP = 13.0 + 30.0 = 43.0 dBW

$$F = 43.0 - 10 \log (4 \pi) - 20 \log (38{,}000 \times 10^3) \text{ dBW / m}^2$$

$$= 43.0 - 11.0 - 151.6 = -119.6 \text{ dBW / m}^2$$

**b.** The earth station has an antenna with a circular aperture 2 m in diameter and an aperture efficiency of 65%. Calculate the received power level in watts and in dBW at the antenna output port.

**Answer:** The effective area of the antenna is

$$A_{eff} = \eta_A \pi r^2 = 0.65 \times \pi \times 1 = 2.042 \text{ m or } 3.1 \text{ dBm}^2$$

For an incident flux density of $-119.6$ dBW / $m^2$ or $1.10 \times 10^{-12}$ W/$m^2$

$$P_r = 2.042 \times 1.10 \times 10^{-12} = 2.24 \times 10^{-12} \text{ W or } -116.5 \text{ dBW}$$

or $$P_r = -119.6 + 3.1 = -116.5 \text{ dBW}$$

**c.** Calculate the on-axis gain of the antenna in dB.

**Answer:** Antenna gain for a circular aperture is given by $G = \eta_A (\pi D / \lambda)^2$

$$G = 10 \log ( 0.65 \times (\pi \times 2 / 0.0750)^2) = 36.6 \text{ dB}$$

**d.** Calculate the free space path loss between the satellite and the earth station.

Calculate the power received, Pr, at the earth station using the link equation:

$$P_r = P_t G_t G_r / L_p$$

where $P_t G_t$ is the EIRP of the satellite transponder and $L_p$ is the path loss.

Make your calculation in dB units and give your answer in dBW.

**Answer:** At a frequency of 4.0 GHz, $\lambda = 0.075$ m.

Path loss $= 20 \log ( 4 \pi R / \lambda) = 20 \log ( 4 \pi \times 38{,}000 \times 10^3 / 0.075)$ dB

$$L_p = 196.1 \text{ dB}$$

Downlink power budget gives

$$P_r = P_t + G_t + G_r - L_p \text{ dBW}$$

$$= 13 + 30.0 + 36.6 – 196.1 = -116.5 \text{ dBW}$$

**11.** Repeat parts (a) through (d) of Question #10 for a Ka band transponder transmitting at a frequency of 20.0 GHz.

**a.** Calculate the incident flux density at the earth station in watts per square meter and in $\text{dBW/m}^2$.

**Answer:** Flux density is independent of frequency, so the result is the same as in Problem 10 above.

$$F = -119.6 \text{ dBW / m}^2$$

**b.** The earth station has an antenna with a circular aperture 2 m in diameter and an aperture efficiency of 65%. Calculate the received power level in watts and in dBW at the antenna output port.

**Answer:** Given a flux density at the earth's surface, an antenna of area $A_{eff}$ $\text{m}^2$ collects the same power at any frequency. Hence the result is the same as in question 10 above.

$$P_r = 2.042 \times 1.10 \times 10^{-12} = 2.24 \times 10^{-12} \text{ W or } –116.5 \text{ dBW}$$

**c.** Calculate the on-axis gain of the antenna in dB.

**Answer:** At $f = 20$ GHz, $\lambda = 0.0150$ m:

Antenna gain for a circular aperture is given by $G = \eta_A (\pi D / \lambda)^2$

$$G = 10 \log ( 0.65 \times (\pi \times 2 / 0.0150)^2) = 50.6 \text{ dB}$$

**d.** Calculate the free space path loss between the satellite and the earth station.

and the power received, Pr, at the earth station using the link equation:

**Answer:** At a frequency of 20.0 GHz, $\lambda = 0.015$ m.

Path loss $= 20 \log ( 4 \pi R / \lambda) = 20 \log ( 4 \pi \times 38{,}000 \times 10^3 / 0.015)$ dB

$$L_p = 210.1 \text{ dB}$$

Downlink power budget gives

$$P_r = P_t + G_t + G_r - L_p \text{ dBW}$$

$$= 13 + 30.0 + 50.6 - 210.1 = -116.5 \text{ dBW}$$

The answer is the same as in Question 10 for the 4 GHz satellite because the incident flux density on the earth's surface is the same, and the antenna effective area is the same.

**Note to instructor:** Questions 12, 13 and 14 are based on problems from Take Home exams. A significant length of time is needed to complete the solutions.

**12.** This sequence of questions requires you to design a communication link through a geostationary satellite to meet a C/N and link margin specification.

Use these constants:

Boltzmann's constant $k = -228.6$ dBW/K/Hz

Path length to satellite $= 38{,}500$ km

**Satellite**: Geostationary at 73° W longitude.

24 C band transponders, 28 Ku band transponders

3.2 kW RF power output

| | | |
|---|---|---|
| Antenna gain, on axis, C-band and Ku-band (transmit and receive) | | = 31 dB |
| Receive system noise temperature (C-band and Ku-band) | | = 500 K |
| Transponder saturated output power: C-band | | = 40 W |
| Transponder bandwidth: | C-band | = 36 MHz |

Transponder saturated output power: Ku-band = 80 W

Transponder bandwidth: Ku-band = 54 MHz

**Signals**: FM-TV analog signal to be received in a bandwidth of 27 MHz

Multiplexed digital TV signals transmitted as QPSK with symbol rate 27 Msps using half rate FEC with coding gain 5.5 dB

Minimum permitted C/N overall = 9.5 dB

**Question #12.1**

Design a transmitting earth station to provide a clear air C/N of 26 dB in a C-band transponder at a frequency of 6.285 GHz. Use an uplink antenna with a diameter of 9 m and an aperture efficiency of 68%, and find the uplink transmitter power required to achieve the required C/N. The uplink station is located on the 2 dB contour of the satellite footprint. Allow 0.5 dB for clear air atmospheric attenuation and other losses.

**Answer:** An uplink power budget and noise budget are required to find C/N at the satellite transponder input. Calculate the transmitting earth station antenna gain and path loss at 6.285 GHz first.

$$G_t = \eta_A (\pi D / \lambda)^2 = 10 \log ( 0.68 \times (\pi \times 9 / 0.04773)^2) = 53.8 \text{ dB}$$

$$L_p = 20 \log ( 4 \pi R / \lambda) = 20 \log ( 4 \pi \times 38{,}500 \times 10^3 / 0.04773) = 200.1 \text{ dB}$$

Uplink power budget

| | |
|---|---|
| $P_t$ | TBD |
| $G_t$ | 53.9 dB |
| $G_r$ 31 dB – 2 dB off axis loss | 29.0 dB |
| $L_p$ | -200.1 dB |
| Atmospheric loss and other losses | -0.5 dB |
| Receiver power $P_r$ | $P_t$ – 117.7 dBW |

Transponder input noise power budget

| | |
|---|---|
| k Boltzmann's constant | -228.6 dBW/K/Hz |
| $T_s$ 500 K | 27.0 dBK |
| $B_N$ 27 MHz (FM-TV) | 74.3 dBHz |
| N | -127.3 dBW |

We require overall $(C/N)_o = 13$ dB.

From the reciprocal formula for combining C/N ratios

$$1 / (C/N)_o = 1 / (C/N)_{up} + 1 / (C/N)_{dn}$$

$(C/N)_o = 13$ dB $=$ ratio 20.0. $(C/N)_{up} = 26.0 =$ ratio 400. Hence

$$(C/N)_{dn} = 1 / ( 1/20 – 1/400) = 21.05 \text{ or } 13.2 \text{ dB}$$

Then $(C/N)_{dn} = 13.2 \text{ dB} = G_r - 18.3 \text{ dB}$

$$G_r = 31.5 \text{ dB or } 1413 \text{ as a ratio}$$

The diameter of the receiving antenna is $D_r$ meters where

$$G_r = 1413 = \eta_A (\pi D / \lambda)^2 = 0.65 \times (\pi \times D_r / 0.07389)^2$$

$$D_r = (1413 / 0.65)^{0.5} \times 0.07389 / \pi = 1.096 \text{ m}$$

**Question #12.3.**

**a.** Under conditions of heavy rain, the C-band path from the transmitting station suffers an attenuation of 2dB. Calculate the overall C/N at the earth station in a bandwidth of 27 MHz under these conditions, and find the uplink link margin.

**Reminder:** The uplink margin is the number of dB of attenuation that can occur on the uplink before the receiver overall C/N reaches the limit of 9.5 dB.

**Answer:** The earth station overall C/N ratio will be reduced when the uplink is attenuated by rain. If we assume linear transponder operation, $(C/N)_{up}$ is reduced by 2 dB and $P_t$ from the satellite is reduced by 2 dB leading to a similar reduction of 2 dB in $(C/N)_{dn}$. When both $(C/N)_{up}$ and $(C/N)_{dn}$ fall by the same amount, the reduction in overall C/N is equal to the uplink attenuation. Hence $(C/N)_o = 13.0 – 2.0 = 11.0$ dB.

The minimum permitted overall C/N ratio is 9.5 dB. We must find the lowest value of $(C/N)_{up}$ which results in $(C/N)_o = 9.5$ dB to determine the uplink margin.

Uplink margin $=$ $(C/N)_o$ in clear air $-$ $(C/N)_o$ minimum $= 13.0 – 9.5 = 3.5$ dB.

**Check:** When the uplink is attenuated by 3.5 dB

$(C/N)_{up} = 26.0 – 3.5 = 22.5$ dB or 178 as a ratio.

$(C/N)_{dn} = 13.2 – 3.5$ dB $= 9.7$ dB or 9.33 as a ratio

Using the reciprocal formula $1 / (C/N)_o = 1 / (C/N)_{up} + 1 / (C/N)_{dn}$

$$1 / (C/N)_o = 1 / 178 + 1 / 9.33 = 0.1128$$

$$(C/N)_o = 1 / 0.11288.86 \text{ or } 9.5 \text{ dB}$$

**b.** Under conditions of heavy rain, the C-band path to the receive station suffers an attenuation of 1.5 dB. Assuming 100% coupling of sky noise into antenna noise, and 0.3 dB clear air gaseous attenuation, calculate the overall C/N under these conditions, and find the downlink margin.

**Hint:** You need to find the sky noise temperature that results from a total excess path attenuation of 1.8 dB (clear air attenuation plus rain attenuation); this is the antenna temperature. Then compute the new C/N in rain, using the new $T_{system}$ and received power values.

**Answer:** Under clear air conditions, the gaseous attenuation of 0.3 dB gives a sky noise temperature of approximately $0.3 \times 6.7 = 20.1$ K, which is why sky noise temperature is given as 20 K in Problem 12.2. When rain affects the downlink and rain attenuation is 1.5 dB and there is still 0.3 dB of atmospheric gaseous attenuation, giving a total attenuation on the downlink of 1.8 dB. The sky noise temperature increases because the downlink attenuation has increased . For a path attenuation of 1.8 dB, a gain with a ratio of 0.661, the sky noise temperature can be found from equation 4.19, since the rain is treated as a lossy medium. With an assumed medium temperature of 290 K, sky noise temperature in rain is

$$T_{sky\ rain} = 290\ (1 - 0.661) = 98.4 \text{ K}$$

Hence the new system noise temperature in rain is

$$T_s = 98.4 + 55 = 153.4 \text{ K}$$

The increase in system noise power is proportional to the increase in system noise temperature. The system noise temperature in clear air conditions was 75 K. Hence

$$\Delta N = 10 \log (153.4 / 75) = 3.1 \text{ dB.}$$

The carrier power has fallen by 1.5 dB because of the rain attenuation, so the total effect of C/N is a reduction of $1.5 + 3.1 = 4.6$ dB.

Hence the downlink C/N ratio with 1.5 dB of rain attenuation in the downlink path is

$$(C/N)_{dn\ rain} = 13.2 - 4.6 = 8.6 \text{ dB}$$

The overall C/N ratio is found from the reciprocal formula with the uplink clear air C/N value of 26 dB.

$$(C/N)_{o\ rain} = 1 / (1 / 7.24 + 1 / 400) = 7.11 \text{ or } 8.5 \text{ dB}$$

The signal is below the permitted overall C/N ratio, so the downlink margin is negative.

The lowest permitted C/N down for $(C/N)_o = 9.5$ dB is found from

$$(C/N)_{o\ min} = 1 / (1 / 8.91 - 1 / 400) = 9.11 \text{ or } 9.6 \text{ dB}$$

The downlink margin under the stated rain conditions is –1.1 dB.

Note that the increase in sky noise temperature has a contributes a larger effect on the overall C/N than the attenuation. This is because sky noise temperature is low under clear air conditions and the LNA has a low noise temperature. The increase of 78.4 K in sky noise temperature doubles the noise power at the receiver input, whereas the rain attenuation on the downlink reduces the received power by only 1.5 dB. The combined effect is disastrous to the link, which goes into outage. A larger receiving antenna, with 1.1 dB additional gain, would be needed to combat the stated downlink rain attenuation. Fortunately, rain attenuation of 1.5 dB at 4 GHz is rare on a GEO satellite link.

**Question #12.4**. Design a transmitting earth station to provide a clear air C/N of 30 dB in a Ku-band transponder at a frequency of 14.15 GHz. Use an uplink antenna with a diameter of 5m and an aperture efficiency of 68%, and find the uplink transmitter power required to achieve the required C/N. The uplink station is located on the 2 dB contour of the satellite footprint.

Allow 1.0 dB on the uplink for miscellaneous and clear air losses.

**Answer:** An uplink power budget and noise budget are required to find C/N at the satellite transponder input. Calculate the transmitting earth station antenna gain and path loss at 14.15 GHz first.

$$G_t = \eta_A (\pi D / \lambda)^2 = 10 \log ( 0.68 \times (\pi \times 5 / 0.0212)^2) = 53.8 \text{ dB}$$

$$L_p = 20 \log ( 4 \pi R / \lambda) = 20 \log ( 4 \pi \times 38{,}500 \times 10^3 / 0.02120) = 207.2 \text{ dB}$$

Uplink power budget

| | |
|---|---|
| $P_t$ | TBD |
| $G_t$ | 55.7 dB |
| $G_r$ 31 dB – 2 dB off axis loss | 29.0 dB |
| $L_p$ | -207.2 dB |
| Atmospheric loss and other losses | -1.0 dB |
| Receiver power $P_r$ | $P_t$ – 123.5 dBW |

The noise bandwidth of a 27 Msps digital signal is 27 MHz.

Uplink noise power budget:

| | |
|---|---|
| k Boltzmann's constant | -228.6 dBW/K/Hz |
| $T_s$ 500 K | 27.0 dBK |
| $B_N$ 27 MHz (27 Msps QPSK) | 74.3 dBHz |
| N | -127.3 dBW |

The C/N ratio at the transponder input is

$$C/N = C - N = P_t - 123.5 + 127.3 \text{ dB}$$

For C/N = 30 dB we require $P_t = C/N - 3.8 = 26.2$ dBW or 417 W

**Question #12.5**

Design a Ku-band receiving earth station to provide an overall clear air C/N of 17 dB in a 27 MHz IF noise bandwidth at a carrier frequency of 11.45 GHz. The antenna noise temperature is 30 K and the LNA noise temperature is 110 K. You may assume a high gain LNA and ignore the noise generated in other parts of the receiver. Determine the diameter of the receiving antenna. The receiving terminal is located on the 3 dB contour of the satellite footprint, and clear air attenuation on the path and other losses total 0.8 dB.

**Answer:** Set the receiving earth station antenna gain as $G_r$ dB.

Path loss at the downlink frequency of 11.45 GHz is

$$L_p = 207.2 + 20 \log (11.45/14.15) = 205.4 \text{ dB}$$

Transponder output power is $P_t$ saturated = 80 W or 19.0 dBW

No back off is quoted.

Downlink power budget

| | |
|---|---|
| $P_t$ | 19.0 dBW |
| $G_t$ 31 dB – 3 dB off axis loss | 28.0 dB |
| $G_r$ | TBD |
| $L_p$ | -205.4 dB |
| Atmospheric loss and other losses | -0.8 dB |
| Receiver power $P_r$ | $G_r$ – 159.2 dBW |

Earth station receiver input noise power budget

Earth station receiver input noise power budget

| | | |
|---|---|---|
| k | Boltzmann's constant | -228.6 dBW/K/Hz |
| $T_s$ | 30 K + 110 K = 140 K | 21.5 dBK |
| $B_N$ | 27 MHz (27 Msps QPSK) | 74.3 dBHz |
| N | | -132.8 dBW |

The downlink C/N ratio at the receiver input is

$$(C/N)_{dn} = C - N = G_r - 159.2 + 132.8 = G_r - 26.4 \text{ dB}$$

We require overall $(C/N)_o = 17$ dB.

From the reciprocal formula for combining C/N ratios

$$1 / (C/N)_o = 1 / (C/N)_{up} + 1 / (C/N)_{dn}$$

$(C/N)_o = 17$ dB = ratio 50.0. $(C/N)_{up} = 30.0$ = ratio 1000. Hence

$$(C/N)_{dn} = 1 / ( 1/50 - 1/1000) = 47.6 \text{ or } 16.8 \text{ dB}$$

Then $(C/N)_{dn} = 16.8 \text{ dB} = G_r - 26.4$ dB

$$G_r = 43.2 \text{ dB or a ratio of } 20{,}892$$

The diameter of the receiving antenna is $D_r$ meters where, for an aperture efficiency of 65%

$$G_r = 20{,}892 = \eta_A (\pi D / \lambda)^2 = 0.65 \times (\pi \times D_r / 0.02620)^2$$

$$D_r = (20{,}892 / 0.65)^{0.5} \times 0.02620 / \pi = 1.495 \text{ m}$$

**Question #12.6**

**a.** Under conditions of heavy rain, the Ku-band path to the satellite station suffers an attenuation of 6 dB. Calculate the overall C/N at the earth station in a bandwidth of 27 MHz under these conditions, and find the uplink link margin.

**Answer:** Overall C/N is 17 .0 dB in clear air. Using the same analysis as in Problem 12.3, the overall C/N ratio with 6 dB uplink rain attenuation is

$$(C/N)_{o \text{ uplink rain}} = 17.0 - 6.9 = 11.0 \text{ dB.}$$

**b.** Under conditions of heavy rain, the Ku-band path to the receive station suffers an attenuation of 5 dB. Assuming 100% coupling of sky noise into antenna noise, and 0.3 dB clear air attenuation, calculate the overall C/N under these conditions, and find the downlink margin.

**Answer:** The analysis must follow the method used in Problem 12.3, first finding the increase in system noise temperature and receiver noise power, then finding overall C/N in the downlink rain event.

Total path attenuation in the downlink in rain is 5.3 dB.

The sky noise temperature is then 290 ( 1 – 0.295) = 204 K.

System noise temperature is 204 + 110 = 304 K.

The increase in receiver noise power is

$$\Delta N = 10 \log (304 / 140) = 3.4 \text{ dB}$$

Hence the change in downlink C/N ratio is

$$\Delta (C/N)_{dn} = 5.0 + 3.4 = 8.4 \text{ dB.}$$

In clear air, the downlink C.N ratio was 17.0 dB, so with 5.0 dB rain attenuation it is

$$(C/N)_{dn} = 17.0 - 8.4 = 8.6 \text{ dB or a ratio of } 7.24$$

Overall C/N with uplink C/N = 30.0 dB is

$$(C/N)_{o\ rain} = 1 / ( 1 / 1000 + 1 / 7.24) = 7.19 \text{ or } 8.57 \text{ dB}$$

The overall C/N ratio is dominated by the low downlink C/N.

The downlink margin is again negative, at - 0.9 dB. However, with a FEC coding gain of 5.5 dB, the effective overall C/N ratio for the QPSK signal is 8.6 + 5.5 = 14.1 dB, which will provide a BER around $10^{-6}$ assuming a 0.5 dB receiver implementation margin.

**13**. A Direct Broadcast Television (DBS-TV) satellite is in geostationary orbit at 100 degrees west longitude. It carries 16 transponders, each with a saturated output power of 200 W and a bandwidth of 25 MHz. The antenna on the satellite has a gain (on axis) of 34 dB. The receiving terminals all use antennas with a circular aperture with a diameter of 18 inches and an aperture efficiency of 65%. The noise bandwidth of the digital TV receiver is 20 MHz.

Use a distance to the GEO satellite of 38,500 km in your calculations.

**a.** Calculate the free space path loss and the receiving terminal antenna gain at 12.2 GHz.

**Answer:** $L_p = 20 \log ( 4 \pi R / \lambda) = 20 \log ( 4 \pi \times 38{,}500 \times 10^3 / 0.02459) = 205.9$ dB

$$G_t = \eta_A (\pi D / \lambda)^2 = 10 \log ( 0.65 \times (\pi \times 0.5 / 0.02459)^2) = 34.2 \text{ dB}$$

**b.** Draw up a link budget for the downlink from the satellite to an earth station on the 3 dB contour of the satellite antenna beam. Assume that the satellite transmits at a power level of 180 W. Include a clear air atmospheric loss of 0.5 dB and miscellaneous losses of 0.2 dB in your downlink power budget.

**Answer:** Downlink power budget

| | |
|---|---|
| $P_t$ 180 W | 22.6 dBW |
| $G_t$ 34 dB – 3 dB off axis loss | 31.0 dB |
| $G_r$ | 34.2 dB |
| $L_p$ | -205.9 dB |
| Atmospheric loss and other losses | -0.7 dB |
| Receiver power $P_r$ | -118.8 dBW |

**c.** The receiving terminal has a system noise temperature of 110 K in clear air. Draw up a noise power budget for the receiver using the receiver's noise bandwidth.

**Answer:** Earth station receiver input noise power budget

| | |
|---|---|
| k Boltzmann's constant | -228.6 dBW/K/Hz |
| $T_s$ 110 K | 20.4 dBK |
| $B_N$ 20 MHz | 73.0 dBHz |
| N | -135.2 dBW |

**d.** Calculate the clear air C/N ratio for the receiver with a noise bandwidth of 20 MHz. The minimum permissible C/N ratio is 10.0 dB. What is the clear air link margin?

**Answer:** Pr = -118.8 dBW, N = -135.2 dBW.

Hence C/N for the downlink is –118.8 + 135.2 = 16.4 dB

**e.** For 0.3% of the time at the receiving location, heavy rain causes 2 dB excess path attenuation and the system noise temperature of the receiver increases to 210 K. Calculate the C/N under these rain conditions, and the link margin above the C/N threshold of 10.0 dB.

**Answer:** The increase in system noise temperature is from 110 K to 210 K.

The corresponding increase in receiver noise power is

$$\Delta N = 10 \log (210 / 110) = 2.8 \text{ dB}.$$

The carrier power falls by 2 dB, so the loss in C/N on the downlink is 2 + 2.8 = 4.8 dB.

The C/N ratio in the satellite transponder is assumed to be much greater than 16 dB, and can be ignored. Hence $(C/N)_{rain}$ = 16.4 – 4.8 = 11.6 dB.

**f.** Many of the DBS TV system customers live inside the 2 dB contour of the satellite beam. Calculate the clear air link margin and 0.3% time link margin for a receiver located on the 2 dB contour of the satellite footprint.

**Answer:** The previous results were for a receiving earth station located on the – 3 dB contour of the satellite antenna footprint. Customers on the 2 dB contour have a signal that is 1 dB greater.

Hence C/N in clear air = 16.4 + 1 = 17.4 dB

C/N in rain = 11.6 + 1 = 12.6 dB.

**g.** An uplink station for the DBS-TV satellite described in Question #1 is located in Utah, and transmits digital TV signals to 16 transponders on the satellite using QPSK with ¾rate forward error correction. The transmit earth station has a circular aperture antenna with diameter of 6 m and an aperture efficiency of 65%. Each transponder operates at a different carrier frequency in the 17 GHz band, and the RF channel noise bandwidth is 20 MHz. The noise temperature of the satellite receiver is 500 K (the satellite always looks toward the "hot" earth).

Use these values in the remaining parts of this question.

Calculate the uplink path loss and the uplink antenna gain at 17.5 GHz.

**Answer:** $L_p = 20 \log ( 4 \pi R / \lambda) = 20 \log ( 4 \pi \times 38{,}500 \times 10^3 / 0.01765 ) = 209.1$ dB

$G_t = \eta_A (\pi D / \lambda)^2 = 10 \log ( 0.65 \times (\pi \times 0.5 / 0.01765)^2 ) = 58.7$ dB

**h.** The gain of the receiving antenna on the satellite in the direction of Utah is 31 dB. Draw up a clear air uplink budget for the link from the earth station to a single transponder on the satellite using a transmit power of $P_t$ watts, and atmospheric and other losses of 1.0 dB.

**Answer:** Uplink power budget:

| | |
|---|---|
| $P_t$ | TBD |
| $G_t$ | 58.7 dB |
| $G_r$ | 31.0 dB |
| $L_p$ | -209.1 dB |
| Atmospheric loss and other losses | -1.0 dB |
| Receiver power $P_r$ | $P_t$ – 120.4 dBW |

**i.** Calculate the noise power at the input to the satellite receiver in a noise bandwidth of 20 MHz. Hence find the uplink transmitter power required to achieve a C/N of 28 dB in the satellite transponder.

**Answer:** Transponder input noise power budget

| | |
|---|---|
| k Boltzmann's constant | -228.6 dBW/K/Hz |
| $T_s$ 500 K | 27.0 dBK |
| $B_N$ 20 MHz | 73.0 dBHz |
| N | -128.6 dBW |

The uplink C/N ratio is required to be 28.0 dB. Hence the earth station transmitter power is given by $P_t$ where

$P_t$ – 120.4 dBW = -128.6 + 28 dB

$P_t$ = 120.4 -128.6 + 28 dBW = 19.8 dBW or 95.5 W.

**j.** The gain of the satellite transponder must be set to amplify the received signal at the transponder input to an output level of 180 watts. Calculate the gain of the transponder in decibels. (Ignore the change in frequency in the transponder.)
When designing RF equipment, a common rule to avoid oscillation is to make the amplification at any given frequency no higher than 60 dB. How would you design a bent-pipe DBS-TV transponder to provide the end to end gain that you calculated?

**Answer:** The power at the input to the transponder is $P_t$ – 120.4 dBW = 19.8 – 120.4 dBW

$P_r$ = -100.6 dBW, $P_t$ = 22.6 dBW

$G_{xp}$ = $P_t - P_r$ = 22.6 + 100.6 = 123.2 dB.

[Gain is $P_{out}/ P_{in}$ for any device, giving G = $P_{out}$ dBW – $P_{in}$ dBW]

RF and IF amplifiers should not be specified with G > 60 dB because of the risk of oscillation. In this transponder we need a gain of 123.2 dB. This should be distributed through the amplifer,. Typically, transponders are built with excess gain and include an RF or IF attenuater. So a typical arrangement would set these gains:

| | |
|---|---|
| RF input (17 GHz) | 30 dB |
| IF amplifier (~ 1 GHz) | 60 dB |
| RF amplifier (12 GHz) (LPA + HPA) | 60 dB |
| Attenuator (controlled from earth) | -6.8 dB |

**k.** The minimum permissible C/N in the transponder is 16.0 dB.

Calculate the clear air link margin for the uplink.

**Answer:** The received power at the transponder input is $P_r$ = -100.6 dBW.

The noise power at the transponder input is N = -128.6 dBW

Hence $(C/N)_{up}$ = 28.0 dB. This gives a margin over 16.0 dB of 12.0 dB.

**l.** Ignore the result you calculated for the downlink C/N in Question #1, and use a value of 15 dB in this question. Convert the clear air uplink and downlink C/N values to power ratios, and then find the overall C/N, in dB, in the earth station receiving terminal.

Use the following formula (where C/N values are ratios, not in dB) and give your answer in decibels:

$$1/(C/N)\ \text{overall} = 1/(C/N)\text{up} + 1/(C/N)\text{down}$$

$$1 / (C/N)_o = 1 / 631 + 1 / 31.6 = 0.03323$$

$$(C/N)_o = 1 / 0.03323 = 30.09 \text{ or } 14.8 \text{ dB.}$$

**Question #14** This is a multipart question.

All the questions are about the satellite communications system described below.

**Description of System**

A satellite communication system consists of 50 LEO satellites in 750 km orbits, several hubs stations operating in Ka-band, and many handheld transceivers operating in L-band. The handheld units transmit to transponders at 1600 MHz and receive from transponders at 2500 MHz. The system uses digital speech compressed into a transmission channel (RF) bandwidth of 16 kHz. Channels are spaced 20 kHz apart to allow a guard band between channels.

The Parameters of the system are given below: (You may not need all of these.)

**System Values**

| | |
|---|---|
| Uplink frequency for handheld transceiver | 1600 MHz |
| Downlink frequency for handheld transceiver | 2500 MHz |
| Uplink frequency for hub station | 29 GHz |
| Downlink frequency for hub station | 19 GHz |
| Maximum range to edge of coverage zone | 2000 km |

**Satellite Transponder**

| | | |
|---|---|---|
| Maximum output power | $P_t$ | 20 W |
| Transponder bandwidth | | 2 MHz |
| Transponder input noise temperature | $T_s$ | 500 K |

**Handheld Transceiver Parameters**

| | | |
|---|---|---|
| Transmitter output power | | 1.0 W |
| Antenna gain (transmit and receive) | G | 0 dB |
| Receiver system noise temperature | $T_s$ | 300 K |
| Receiver system noise bandwidth | $B_n$ | 10 kHz |

**Hub Station Parameters**

| | | |
|---|---|---|
| Maximum transmit power | $P_t$ | 100 W |
| Receiver system noise temperature (clear air) | $T_s$ | 250 K |
| Antenna gain at 29 GHz (transmit) | $G_t$ | 54 dB |
| Antenna gain at 19 GHz (receive) | $G_r$ | 52 dB |

**Constants:** Boltzmann's constant $k = 1.38 \times 10^{-23}$ J/K $= -228.6$ dBW/K/Hz

**Question #14.1 Preliminary calculations**

**a.** Calculate the path loss, in dB, for a 2000 km path at 1.6, 2.5, 19, and 29 GHz.

**Answer:** Path loss is calculated from $L_p = 20 \log ( 4 \pi R / \lambda )$ . For a worst case analysis, we must take the longest path length of 2000 km.

At 1.6 GHz, $\lambda = 0.1875$ m, $L_p = 20 \log ( 4 \pi \times 2000 \times 10^3 / 0.1875 ) = 162.5$ dB

At 2.5 GHz, $\lambda = 0.120$ m, $L_p = 20 \log ( 4 \pi \times 2000 \times 10^3 / 0.120 ) = 166.4$ dB

At 19 GHz, $\lambda = 0.01579$ m, $L_p = 20 \log ( 4 \pi \times 2000 \times 10^3 / 0.01579 ) = 184.0$ dB

At 29 GHz, $\lambda = 0.01344$ m, $L_p = 20 \log ( 4 \pi \times 2000 \times 10^3 / 0.010344 ) = 187.7$ dB

**b.** Calculate the noise power, in dBW, for the receiver in the transponder and for the receivers at the hub station and the handheld unit, in a single voice channel bandwidth of 10 kHz. (Note: use the bandwidth of one speech channel, 10 kHz for all the calculations, not 2 MHz.)

**Answer**: For the transponder, Ts = 500 K

$N_{xp} = k\,T_s\,B_N = -228.6 + 27.0 + 40.0 = -161.6$ dBW

For the hub station, $T_s = 250$ K

$N_H = k\,T_s\,B_N = -228.6 + 24.0 + 40.0 = -164.6$ dBW

For the handheld satellite phone, $T_s = 300$ K

$N_H = k\,T_s\,B_N = -228.6 + 24.8 + 40.0 = -163.8$ dBW

**c.** The satellite has broad coverage antennas at L-band and Ka-band with half-power beamwidths of 120 degrees. Estimate the gain, in dB, of the antennas at each frequency.

**Answer:** The gain of an antenna with a beamwidth of $\theta_{3\,dB}$ is $G = 33{,}000 / \theta_{3\,dB}^2$

This result is independent of frequency, so all antennas have the same gain:

$G = 33{,}000 / 120^2 = 2.29$ or 3.6 dB.

**Question #14.2 C/N ratios**

Use the values you obtained in Q#1 above, for path loss, antenna gain, and noise power in this question. Calculate C/N values for stations located at the edge of the coverage zone of the satellite, where the satellite antenna gain is 3 dB below its maximum value, and the range to the satellite is 2000 km. Take care to use the correct path loss and receiver noise power values for each frequency. Give your answers in decibels.

**a.** Calculate the C/N in the satellite transponder for the signal transmitted by one handheld transceiver located at the edge of the coverage zone (satellite antenna gain 3 dB below maximum) and at maximum range from the satellite (2000 km).

**Answer:** The EIRP of the satellite phone is 0 dBW (1 W transmitter, 0dB antenna gain)

$P_r = EIRP + G_r - L_p = 0 + 3.6 - 3.0 - 162.5 = -161.9$ dBW

The noise power at the input to the transponder is $N_{xp} = -161.6$ dBW

Hence $(C/N)_{up} = -0.3$ dB

**b.** Calculate the C/N in the satellite transponder for the signal transmitted by a hub station, using its full output power.

**Answer:** This calculation is to establish a reference case for a single 10 kbps channel uplink.

Hub station EIRP = 20 dBW + 54 dB = 74 dBW

At the – 3dB contour of the satellite antenna footprint $G_r$ = 3.6 – 3.0 = +0.6 dB

Uplink power budget:

| | |
|---|---|
| EIRP | 74.0 dBW |
| Path loss at 29 GHz | -187.7 dB |
| Receive antenna gain | 0.6 dB |
| $P_r$ | -113.1 dBW |
| Noise power at satellite transponder input | -161.6 dBW |
| C/N at transponder input | 48.5 dB |

**c.** Calculate the C/N in the hub station receiver for the signal transmitted by a satellite transponder using its full output power.

This calculation is to establish a reference case for a single 10 kbps channel downlink.

Satellite saturated EIRP = 13 dBW + 0.6 dB = 13.6 dBW at –3 dB contour.

Downlink power budget:

| | |
|---|---|
| EIRP | 13.6 dBW |
| Path loss at 19 GHz | -184.0 dB |
| Receive antenna gain | 52.0 dB |
| $P_r$ | -118.4 dBW |
| Noise power at hub receiver input | -164.6 dBW |
| C/N at hub receiver | 46.0 dB |

**d.** Calculate the C/N in the receiver of the handheld unit for the signal transmitted by a satellite transponder using its full output power.

The handheld phone has an antenna gain of 0 dB. With 20 W output from the satellite:

Downlink power budget:

| | |
|---|---|
| EIRP | 13.6 dBW |
| Path loss at 2.5 GHz | -166.4 dB |
| Receive antenna gain | 0 dB |
| $P_r$ | -152.8 dBW |
| Noise power at hub receiver input | -163.8 dBW |
| C/N at satellite phone receiver | 11.0 dB |

**e.** Calculate the overall C/N ratios at the hub station and at the handheld receiver.

Overall C/N values are calculated from the reciprocal formula; however, if the two C/N values differ by more than 25 dB, $(C/N)_o$ = lowest C/N ratio

For the hub station, $(C/N)_{up}$ = -0.3 dB, $(C/N)_{dn}$ = 46.0 dB

$(C/N)_o$ = -0.3 dB

For the handheld satellite telephone, $(C/N)_{up}$ = 48.5 dB, $(C/N)_{dn}$ = 18.0 dB

$(C/N)_o$ = 18.0 dB

**Question #14.3 Trade-off studies**

The link between the hub station and the satellite operating at Ka-band uses a high gain antenna at the hub station and achieves a high C/N. The transceiver operating in L-band uses a low gain, omni-directional antenna with low gain, which results in low C/N. For satisfactory operation under all weather conditions, the Ka-band links should have a minimum C/N of 20 dB in clear air, and the L-band links should have a minimum C/N of 10 dB.

The C/N of the handheld transceivers can be improved by using a multiple beam L-band antenna on the satellite, with higher gain and narrower beamwidth per beam. The high C/N of the hub station links can be traded for increased capacity. The hub station and transponder transmitter power can be shared among a group of voice channels.

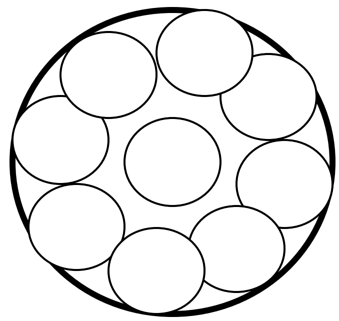
**a.** Determine the minimum gain required by the L-band antennas on the spacecraft to achieve a C/N of 10 dB at each L-band frequency. Using the higher of the two values, find the 3 dB beamwidth of one of the multiple beams. Estimate the number of beams that will be needed to serve the coverage zone of a single $120^o$ beamwidth antenna.

**Answer:** The link from the handheld satellite phone to the satellite has C/N = -0.3 dB. This C/N ratio is much too low for the demodulator at the earth station to recover the signal, so additional antenna gain must be included in the link. The minimum required C/N is 10 dB, so we must increase antenna gain at the satellite by 10.3 dB. We cannot increase the gain of the satellite telephone because it needs an omni-directional antenna, which has a gain of 0 dB y definition. The satellite antenna that services the whole coverage zone has G = 3.6 dB on axis, so we must provide an antenna with G = 13.9 dB ( = 24.4 as a ratio) to meet the uplink C/N objective of 10 dB. The beamwidth of an antenna with G = 13.9 dB is given by

$$G = 33{,}000 / \theta_{3\,dB}^{2} \text{ with the beamwidth in degrees.}$$

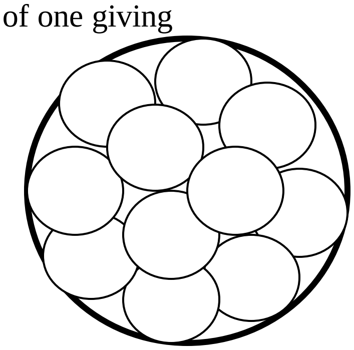
Hence $\theta_{3\,dB} = [33{,}000 / 24.4]^{0.5} = 36.8$ degrees.

We must now try to fit a number of these beams inside the coverage zone of $120^o$ at the satellite. We can fit approximately three beams of $36.8^o$ width across the coverage zone.
(120 / 36.8 = 3.26 so cross-over will be below –3 dB.)
The example at the right shows a 9 beam arrangement. The center beam could be expanded to provide a more even coverage. When a terminal is in the central part of the coverage area the path length to the satellite is much shorter allowing the use of a broader center beam with a lower gain.



9 beam pattern

We could increase the number of beams inside the coverage area by allowing more overlap. For example, three beams could be used in the central area instead of one giving a total of eleven beams.



11 beam pattern

**b.** Find the excess C/N available on the Ka-band links between the hub station and the satellite. By trading carrier power find the number of channels that the Ka-band links can carry with C/N = 20 dB. If the channel spacing is 20 kHz, can all of these channels fit into a 2 MHz bandwidth transponder?

**Answer:** The 29/19 GHz links have high C/N ratios, when operating with one channel:

$(C/N)_{up}$ = 48.5 dB, giving a margin of 28.5 dB over $(C/N)_{min}$ = 20 dB.

$(C/N)_{dn}$ = 46.0 dB, giving a margin of 26.0 dB over $(C/N)_{min}$ = 20 dB.

We can trade power per channel for additional bandwidth and increase the number of channels by a factor of 26 dB = $\times$ 400 before C/N = 20 dB in the downlink.

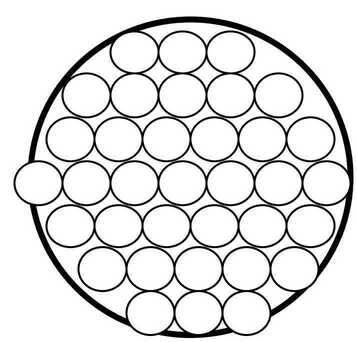
Since the available transponder bandwidth is 2.0 MHz and channels are to be spaced by 20 kHz, we can accommodate no more than 100 channels, and the 29/19 GHz links are bandwidth limited.

The 1.6/2.4 GHz links have relatively low C/N ratios. The uplink from the satellite phone was designed to achieve $(C/N)_{up}$ = 10.0 dB at the edge of one of the multiple beams. Any number of these channels can be added to the transponder until the bandwidth is fully occupied, since each uplink channel operates in SCPC mode. In the 2.4 GHz downlink to the handheld satellite telephone, $(C/N)_{dn}$ = 11.0 dB at the edge of the coverage zone with the single wide-angle antenna with G = 3.6 dB. The multiple beam antenna provides an additional 10.3 dB gain, so $(C/N)_{dn}$ = 21.3 dB with a single channel transmitted. We can reduce the power per channel until $(C/N)_{dn}$ = 10 dB, allowing 11.3 dB or 13 times as many channels.

**c.** Determine whether the transponders are power limited or bandwidth limited.
Give reasons for your answer.

**Answer:** As can be seen from the answers above in part (b), three of the links are bandwidth limited and could carry 100 channels each. The fourth link, from the satellite to the handheld satellite phone, is power limited. Additional antenna gain of 8.7 dB is needed at the satellite to allow 100 channels to be carried on the downlink, which implies narrower multiple beams and more of them. The antenna gain must be increased to 22.6 dB, giving a beamwidth of $13.5^{o}$ and a large number of beams – perhaps as many as 48.

The illustration shows and example with 35 beams.



35 beam pattern

**d.** The communication system described needs two transponders to permit two way voice communication between the hub station and the many transceivers. Based on your answers in Question #3, find the gain of each transponder from input port to output port.
Note: the transponder gain does **not** include antenna gain.)

**Answer:** Transponder Gain in dB is $G_{xp} = P_{out}$ in dBW – $P_{in}$ in dBW.

Let's assume that the satellite is redesigned with a different multiple beam antenna such that each transponder carries 100 channels.

For the inbound link from the telephone to the Hub station, $P_{in} = -161.9$ dBW, $P_{out} = 20$ W.

The transponder output of 20 W is divided between 100 channels, giving 0.2 W = -7.0 dBW per channel. Hence $G_{xp}$ inbound = 161.9 – 7.0 = 154.9 dB.

For the outbound link from the Hub station to the telephone, $P_{in} = -113.1$ dBW, $P_{out} = 20$ W.

Power per downlink channel is again –7.0 dBW for 100 channels.

Hence $G_{xp}$ inbound = 113.1 dBW –7.0 = 106.1 dB.

The situation in which the inbound link can carry 100 channels and the outbound link carries only 13 channels is impractical, because a pair of channels, one inbound and one outbound, is needed to make a telephone link.

**Question #14.4 Costs**

For any satellite system to be viable, the communication capacity must be sold at a price that is attractive to customers**.** This question looks at the cost of the system over its lifetime and calculates a minimum cost per voice circuit.

**a.** Each LEO satellite carries 20 transponders. What is the total number of speech channels that the satellite can support when fully loaded? How many telephone circuits – it takes two channels to make a telephone circuit?

**Answer:** We will assume that each transponder can carry 100 telephone channels, so the capacity of one satellite is 2000 one way channels, or 1000 two way telephone conversations, when fully loaded.

**b.** Each LEO satellite costs $40 M in orbit and the LEO system costs $100 M per year to run. The expected lifetime of the satellites is 10 years, and the system requires a total of 10 spare satellites to be launched over the 10 year period. Calculate the cost of operating the system for a ten year period. Add a 27% factor to cover interest payments and dividends, and calculate the 10 year cost of the entire system.

**Answer:** There are 60 satellites in the system at a cost of $40 M each, so the in-orbit part of the system has a capital cost of $2,400 M. Operating costs are $100 M per year, total $1000 M over ten years. Total cost over ten years is $3,400 M. This cost, plus a return on investment of 27%, or $918 M, for a grand total of $4,318 M, is required to make the system viable.

**c.** Calculate the cost per minute per voice circuit assuming that each satellite can be loaded to an average of 20% of its capacity over its lifetime.

**Answer:** The total cost of $4,318 M over 10 years must be recovered by selling satellite channels. With a 20% load factor (typical for GEO satellites), there are the equivalent of 200 telephone circuits available for 365 days per year on each of 50 satellites. There are 525,600 minutes in a year, so we can sell $50 \times 200 \times 10 \times 565{,}600$ circuit minutes in the 10 year period.

Hence cost per minute = $\$ 4{,}318{,}000{,}000 / 50 \times 1.0512 \times 10^{9}$ = \$0.082.

**d.** Write two paragraphs discussing the cost of the system and the cost of a voice circuit.

What price per minute would you set for a satellite voice circuit?

Would you expect customers to be willing to pay this amount for a satellite telephone connection?

How does the cost compare to terrestrial cellular telephone charges?

**Answer:** The cost per channel to use the satellite looks attractive, compared to a terrestrial cellular call costing $0.20 per minute (typical year 2000 charge). It should be possible to charge a customer more than $0.082 for the call via satellite and make additional profit.

However, there are a number of difficulties with the LEO satellite system that are not obvious in the above analysis.

If the satellites are in polar orbits, most of them are over the poles or the oceans and cannot be used. For example, if this system used polar orbits, only two satellites would be visible from the United States at any time, giving the entire population of the US a capacity of only 2000 telephone circuits. To reach a utilization factor of 20%, customers must be found in countries other than the United States. If each US customer were to use the two visible satellites with a 50% load factor for 10 minutes each day, during the hours 6 am – 10 pm, the available circuit minutes = 960,000, so there could be a maximum of 96,000 US customers. To recover the system cost of $431,800,000 per year, each customer must spend $4,497 on satellite telephone calls each year. Put this way, it is clear that a large number of overseas customers must be recruited to make the system pay.

One further factor that makes satellite telephones less attractive than cellular phones is that the weak signals will not penetrate buildings. You have to go outside to make and receive calls, - not pleasant in the middle of winter.

The difficulty for the satellite system owner is that the above calculation is misleading. Customers cannot be signed up until the system is operational, and then the customer base will grow only slowly. The owner of the system must pay interest on money borrowed to build the system ($2,400 M in this example) while revenue is low. This has crippled several of the LEO satellite telephone systems built in the late 1990s.

## Chapter 5 Solution to Problems

**1.** A C-band satellite link sends a single NTSC-TV signal through a 36 MHz transponder on a C-band GEO satellite. The NTSC video signal is modulated onto the carrier using wideband frequency modulation, and the bandwidth of the transmitted RF signal is 32 MHz. The baseband bandwidth of the TV signal is 4.2 MHz.

**a.** Calculate the peak frequency deviation of the FM carrier using Carson's rule.

**Answer:** Carson's rule gives the bandwidth of an FM signal in terms of the peak frequency deviation, $\Delta f_{pk,}$ and the maximum baseband frequency, $f_{max}$.

$$B = 2 ( \Delta f_{pk} + f_{max})$$

Hence the peak frequency deviation can be found as

$$\Delta f_{pk} = B/2 - f_{max}$$

For $B = 32$ MHz and $f_{max} = 4.2$ MHz

$$\Delta f_{pk} = 16 – 4.2 = 11.8 \text{ MHz}$$

**b.** Calculate the unweighted FM improvement factor for the video signal.

**Answer:** The unweighted FM improvement factor is given by

$$\text{Improvement in S/N} = 10 \log (B/f_{max}) + 20 \log (\Delta f_{pk} /f_{max}) + 1.8 \text{ dB}$$

Using the results from part (a) above,

$$\text{Improvement} = 10 \log (32 / 4.2) + 20 \log(11.8 / 4.2) + 1.8 \text{ dB}$$

$$= 8.8 + 9.0 + 1.8 = 19.6 \text{ dB}$$

**c.** The overall C/N in an earth station receiving the FM-TV transmission is 17 dB. What is the unweighted video S/N ratio at baseband?

**Answer:** The unweighted S/N ratio at baseband is the receiver C/N ratio plus the FM improvement

$$S/N_{unweighted} = C/N + \text{FM improvement}$$

Using the value for improvement obtained in part (b) above and C/N = 17 dB

$$S/N_{unweighted} = 17 + 19.6 = 36.6 \text{ dB}$$

**d.** De-emphasis and weighting factors improve the S/N of the baseband signal by a subjective factor of 17 dB. What is the weighted S/N of the baseband video signal?

**Answer:** Adding the weighting factors to the unweighted S/N ratio gives the weighted S/N

$$S/N_{weighted} = 36.6 + 17 = 53.6 \text{ dB}$$

**2.** When overall C/N is sufficiently high, it is possible to transmits two FM-TV signals in one 36 MHz transponder. The signal to noise ratio improvement is reduced when two TV signals are transmitted rather than one because the frequency deviation must be reduced. Two NTSC FM-TV signals are transmitted through a 36 MHz bandwidth transponder. The bandwidth of each signal is 16 MHz.

**a.** Calculate the peak frequency deviation of the FM signal using Carson's rule.

**Answer:** Carson's Rule gives the bandwidth required for transmission of FM signals as

$$B = 2 ( \Delta f_{pk} + f_{max})$$

The maximum baseband frequency for a NTSC TV signal is $f_{max} = 4.2$ MHz.

Hence the peak frequency deviation is found from

$$16 \text{ MHz} = 2 ( \Delta f_{pk} + 4.2 \text{ MHz})$$

$$\Delta f_{pk} = 8 - 4.2 = 3.8 \text{ MHz}$$

**b.** Calculate the unweighted S/N in the baseband video bandwidth of 4.2 MHz for an overall C/N ratio in the earth station receiver of $(C/N)_o$.

**Answer:** Unweighted S/N ratio for an FM TV signal is given by

$$S/N = (C/N)_o + 10 \log (B/f_{max}) + 20 \log (\Delta f_{pk} / f_{max}) + 1.8 \text{ dB}$$

$$= (C/N)_o + 10 \log (16 / 4.2) + 20 \log (3.8 / 4.2) + 1.8 \text{ dB}$$

$$= (C/N)_o + 5.8 - 0.9 + 1.8 \text{ dB}$$

$$= (C/N)_o + 6.7 \text{ dB}$$

**c.** What value must $(C/N)_o$ have to ensure that the unweighted (S/N) of the video signal is 33 dB?

**Answer:** The value of overall C/N at the earth station receiver must be

$$(C/N)_o = 33 - 6.7 = 26.3 \text{ dB}$$

**d.** Use the value of $(C/N)_o$ you found in part (c) above to find the baseband video S/N ratio in clear air conditions. The de-emphasis and subjective weighting factors for the video signal total 17 dB. If the value of $(C/N)_o$ at the earth station receiver falls by 4 dB because of rain in the downlink path, what is the weighted baseband video S/N? How would you rate the quality of the video signal?

**Answer:** Subjective improvement and pre/de-emphasis improvement adds 17 dB to the unweighted S/N to yield the weighted S/N.

$$\text{Weighted S/N} = 26.3 + 17 = 43.3 \text{ dB}$$

If the $(C/N)_o$ value falls by 4 dB, from 26.3 dB to 22.3 dB, we are still well above the FM threshold. S/N will fall by the same amount, so weighted S/N = 39.3 dB.

The video signal in clear air has S/N = 43.3 dB, which is a reasonable quality signal. There would be some perceptible noise in the TV picture. With S/N = 39 dB, noise would be visible, marginally to an annoying extent, and the TV picture would be rated as acceptable quality.

**3.** In Problem #2, two NTSC video signals are transmitted as FM carriers in a bandwidth of 36 MHz. Each FM carrier occupies a bandwidth of 16 MHz. A digital T1 carrier with a bandwidth of 2.0 MHz can be sent through the same transponder by using a gap between the two FM carriers. Some of the transponder power must be devoted to the T1 carrier, with the result that the FM-TV carriers have reduced C/N at the earth station and lower video S/N at baseband. This question asks you to determine the reduction in video S/N. You will need to solve problem #2 before attempting this problem.

**a.** The power at the output of the transponder must be shared between the three RF signals in proportion to bandwidth occupied by each signal. For convenience, assume that the transponder radiates a total power of 20 watts. Calculate the power allocated to each signal when only two FM-TV signals are transmitted, and when all three signals are transmitted.

**Answer:** When two TV signals are transmitted, power at the transponder output must be shared equally between them. Hence each carrier gets 10 W.

When an additional T1 signal is added, power must be shared in proportion to bandwidth occupied by each signal to keep PSD across the transponder constant.

Total bandwidth occupied = 34 MHz.

Total power radiated = 20 watts

Power per MHz = 20/34 W/MHz = 0.588 W/MHz.

Each TV signal gets 0.588 x 16 = 9.41 W

T1 carrier gets 0.588 x 2 = 1.18 W

**b.** Using the results from part (a) above, determine the reduction in C/N of the FM-TV signals. Hence find the reduction in baseband video S/N and the new value of unweighted video S/N ratio, based on results from part (d) of Problem #2.

**Answer:** The TV signals were transmitted at a power of 10 W with two signals in the transponder. When the third signal is added, transmitted power drops by 10 log (9.41/10) dB = 0.3 dB. Hence the unweighted S/N drops by 0.3 dB from 33.0 to 32.7 dB.

**c.** What is the overall C/N ratio at the earth station receiver for the T1 carrier?

**Answer:** The overall C/N for the T1 carrier is the same as for the TV carriers, 26.0 dB. The T1 carrier gets less power, but also is received against a lower noise background because of its narrower bandwidth. The effects are in proportion, so the C/N value is the same for both the carrier and the T1 signal.

**5.** A satellite telemetry link operating in S-band uses frequency modulation to transmit the value of an analog voltage on the satellite to a receiving earth station. The voltage has a range from –1.0 volts to + 1.0 volts, and a maximum frequency of 1000 Hz. The FM modulator on the satellite has a constant of 10,000 Hz per volt. At the receiving earth station the C/N ratio of this signal is 10 dB measured in the Carson’s rule bandwidth, and is 3 dB above the FM threshold of the FM demodulator.

**a.** What is the Carson's rule bandwidth for the FM signal?

**Answer:** We must first calculate the peak frequency deviation for this signal.

The frequency deviation of an FM signal is given by

$$\Delta f = k_f \, m(t)$$

where $k_f$ is the modulator constant and m(t) is the modulating signal. Hence the peak frequency deviation is given by

$$\Delta f_{pk} = k_f \, m(t)_{max}$$

The question gives $m(t)_{max}$ as $\pm$ 1 volt, and $k_f$ as 10 kHz/volt.

Hence $\Delta f_{pk} = 10$ kHz.

Applying Carson's rule with $f_{max} = 1$ kHz

$$B = 2(\Delta f_{pk} + f_{max}) = 2 \times (10 + 1) = 22 \text{ kHz}$$

**b.** What is the baseband S/N ratio at the earth station receiver output for the recovered analog signal?

**Answer:** The C/N at the receiver input is given as 10 dB, so the unweighted S/N is

$$S/N = C/N + 10 \log (B/f_{max}) + 20 \log (\Delta f_{pk} / f_{max}) + 1.8 \text{ dB}$$

$$= 10.0 + 10 \log (22/1) + 20 \log (10/1) + 1.8 \text{ dB}$$

$$= 10 + 13.4 + 20 + 1.8 = 45.2 \text{ dB}$$

**6.** A satellite link has an RF channel with a bandwidth 2.0 MHz. The transmitter and receiver have RRC filters with $\alpha = 0.5$. What is correct symbol rate (pulse rate) for this link?

**Answer:** For any RF channel, the bandwidth occupied by a digital signal with a symbol rate $R_s$ is $\quad B = R_s (1 + \alpha)$

Hence the symbol rate with a bandwidth of 2.0 MHz and $\alpha = 0.5$ is

$$R_s = B / 1.5 = 1.333 \text{ Mbaud (Msps)}$$

**7.** A Ku band satellite uplink has a carrier frequency of 14.125 GHz and carries a symbol stream at $R_s = 16$ Msps. The transmitter and receiver have ideal RRC filters with $\alpha = 0.25$. What is bandwidth occupied by RF signal, and what is the frequency range of the transmitted RF signal?

Note: There is a typo in the text in the first printing that gives the frequency as 14.125 MHz instead of GHz.

**Answer:** Using the same rule as in Q # 6 above

$$B = R_s (1 + \alpha) = 16.0 ( 1 + 0.25) = 20.0 \text{ MHz}$$

The signal occupies the frequency range 14.115 GHz to 14.135 GHz.

**8.** A T1 data transmission system transmits data at 1.544 Mbps over a GEO satellite link At the receiving terminal the clear air value of overall $(C/N)_o$ is 16.0 dB. The modulation used on the link is BPSK and the implementation margin of the BPSK demodulator is 0.5 dB.

**a.** Find the BER at the receiver output and the average time between errors.

**Answer:** For a BPSK link, the BER is given by Equation 5.65

$$P_e = Q [ \sqrt{(2\ C/N)_{effective}} ]$$

With C/N = 16 dB and an implementation margin of 0.5 dB,

$$(C/N)_{effective} = 16.0 – 0.5 = 15.5 \text{ dB} = 10^{15.5/10} = 35.5 \text{ as a ratio}$$

Hence the BER is given by

$$P_e = Q [ \sqrt{(2 \times 35.5)} ] = Q[\sqrt{71.0}] = Q[8.43]$$

Using the Q(z) table in Appendix C, BER < $10^{-16}$

Data is delivered at $1.544 \times 10^6$ bps, so an error occurs less frequently than $10^{-10}$ seconds.

There are $3600 \times 24 \times 365 = 3.1536 \times 10^7$ seconds in a year, so errors occur (theoretically) at a rate of about one every 300 years - which means there are no errors on this link.

**b.** Rain affects the downlink from the satellite and the overall C/N ratio in the receiver falls by 6.0 dB to 10.0 dB. What is the bit error rate now?

**Answer:** With the C/N at 10.0 dB, $(C/N)_{effective} = 9.5$ dB = 8.91 as a ratio.

Hence the probability of a bit error is

$$P_e = Q [ \sqrt{(2 \times 8.91)} ] = Q[\sqrt{17.82}] = Q[4.22]$$

Using the Q(z) table in Appendix C, and interpolating between entries for z = 4.2 and z = 4.3,

$$\text{BER} \approx 10^{-6}$$

The data rate is 1.544 Mbps, so there are 1.544 bit errors per second, on average.

**9**. A satellite data transmission system transmits data from two T1 carriers as a single 3.088 Mbps bit stream using QPSK . The symbol rate on the link is 1.544 Msps. The satellite link uses ideal RRC filters with $\alpha = 0.25$. At the receiving terminal the clear air value of overall $(C/N)_o$ is 16.0 dB and the implementation margin of the QPSK demodulator is 1 dB.

**a.** What is the bandwidth occupied by this signal, and the noise bandwidth of the receiver for this signal?

**Answer:** From equation 5.32, the bandwidth occupied by any digital signal with a symbol rate $R_s$ is

$$B_{occ} = R_s ( 1 + \alpha)$$

Hence for a symbol rate of 1.544 Msps and $\alpha = 0.25$

$$B_{occ} = 1.544 \times 10^6 \times ( 1 + 0.25) = 1.93 \text{ MHz}$$

In every case where RRC filters are used in the receiver, the noise bandwidth $B_N$ is equal to the symbol rate. Hence $B_N = 1.544$ MHz

**b.** Find the BER at the receiver output and the average time between errors.

**Answer:** For a QPSK link, the BER is given by Equation 5.68

$$P_e = Q [ \sqrt{( C/N)_{effective}} ]$$

With C/N = 16 dB and an implementation margin of 1.0 dB,

$$(C/N)_{effective} = 16.0 – 1.0 = 15.0 \text{ dB} = 10^{15.0/10} = 31.6 \text{ as a ratio}$$

Hence the BER is given by

$$P_e = Q [ \sqrt{(31.6)} ] = Q[5.62]$$

Using the Q(z) table in Appendix C, $\text{BER} \approx 1 \times 10^{-8}$

Data is delivered at $3.088 \times 10^6$ bps, so an error occurs once every 32.4 seconds.

**c.** Rain affects the downlink from the satellite and the overall C/N ratio in the receiver falls by 6.0 dB to 10.0 dB. What is the bit error rate now?

**Answer:** For the QPSK link, the BER is given by Equation 5.68

$$P_e = Q [ \sqrt{( C/N)_{effective}} ]$$

With C/N = 10 dB and an implementation margin of 1.0 dB,

$$(C/N)_{effective} = 10.0 - 1.0 = 9.0 \text{ dB} = 10^{9.0/10} = 7.94 \text{ as a ratio}$$

Hence the BER is given by

$$P_e = Q [ \sqrt{(7.94)} ] = Q[2.82]$$

Using the Q(z) table in Appendix C, and interpolating between entries for z = 2.8 and z = 2.9, BER $\approx 2.4 \times 10^{-3}$

Data is delivered at $3.088 \times 10^6$ bps, so there are $3.088 \times 2.4 \times 10^3 = 5488$ bit errors per second. FEC would be needed to maintain a more reasonable error rate on this link.

**10 a.** A 36 MHz bandwidth transponder is used to carry digital signals. A 20 MHz bandwidth in the transponder is occupied by a QPSK signal generated by a transmitter with ideal Nyquist filters with parameter alpha = 0.25. What is the symbol rate of the QPSK signal in Msps? What is the bit rate of the QPSK signal?

**Answer:** From equation 5.32, the bandwidth occupied by any digital signal with a symbol rate $R_s$ is $B_{occ} = R_s ( 1 + \alpha)$

Hence for an occupied bandwidth of 20 MHz, and $\alpha = 0.25$, the symbol rate is $R_s$ where

$$B_{occ} = R_s ( 1 + 0.25)$$

Hence $R_s = 16.0$ MHz.

Since we are using QPSK, there are two bits transmitted with every QPSK symbol, so

$$R_b = 2 \times 16 = 32 \text{ Mbps.}$$

**b.** Under clear air conditions, the overall $(C/N)_o$ ratio in the earth station receiver is 18.0 dB If the QPSK demodulator has an implementation margin of 1.5 dB, what is the Bit Error Rate of the baseband digital signal in clear air conditions? How often does a bit error occur. (Give your answer in days, hours, minutes, or seconds, as appropriate.)

**Answer:** For a QPSK link, the BER is given by Equation 5.68

$$P_e = Q [ \sqrt{( C/N)_{effective}} ]$$

With C/N = 18.0 dB and an implementation margin of 1.5 dB,

$(C/N)_{effective} = 18.0 – 1.5 = 16.5 \text{ dB} = 10^{16.5/10} = 44.7$ as a ratio

Hence the BER is given by

$P_e = Q [ \sqrt{(44.7)} ] = Q[6.68]$

Using the Q(z) table in Appendix C, BER $\approx 1.2 \times 10^{-12}$

Data is delivered at $32 \times 10^6$ bps, so an error occurs once every $32 \text{ x } 10^6 \times 1.2 \times 10^{-12}$

$= 3.84 \times 10^{-5}$ seconds.

Put another way, we have one error every 26042 seconds.

An error occurs on the link, on average, every 26042 seconds = 7 hours 14 minutes 2 seconds.

**c.** Under rain conditions, the overall $(C/N)_o$ ratio of the QPSK signal in part (a) above falls to 14.3 dB at a receiving station. What Bit Error Rate would you expect in the recovered bit stream? How often does a bit error occur?

**Answer:** For a QPSK link, the BER is given by Equation 5.68

$P_e = Q [ \sqrt{( C/N)_{effective}} ]$

With C/N = 14.3 dB and an implementation margin of 1.5 dB,

$(C/N)_{effective} = 14.3 – 1.5 = 12.8 \text{ dB} = 10^{1.28} = 19.05$ as a ratio

Hence the BER is given by

$P_e = Q [ \sqrt{(19.05)} ] = Q[4.37]$

Using the Q(z) table in Appendix C, BER $\approx 6.1 \times 10^{-6}$

Data is delivered at $32 \times 10^6$ bps, so an errors occurs at a rate of 195 per second.

**11.** A satellite communication system is built as a star network with one large hub station and many remote small earth stations. The system operates at Ka band using the K9 geostationary satellite, and carries digital signals which may be voice, data, or compressed video. The K9 satellite has transponders with a bandwidth of 60 MHz that can be operated in either of two modes: as a bent pipe or with a 40 Msps QPSK baseband processor.

The outbound link from the hub to the remote stations has an uplink from the hub station to the satellite that is the input of transponder #1. Signals from the hub are transmitted using a single TDM carrier and QPSK modulation with a symbol rate of 40 Msps. In the initial system design the remote earth stations use receivers capable of receiving 40 Msps QPSK signals. The transponder is operated in bent pipe mode with sufficient back-off to make it linear. The hub transmitter operates at an output power of 100 W, which gives C/N = 30 dB in the transponder in clear air, measured in the correct noise bandwidth of a 40 Msps QPSK receiver equipped with RRC filters having $\alpha = 0.4$. In clear air conditions, the resulting C/N of the earth station receiver, ignoring noise transmitted by the satellite, is 20 dB. The receiver has a QPSK demodulator with an implementation margin of 1.0 dB. For the purposes of this question you may assume that all transmitters and receivers in the network and on the K9 satellite have ideal RRC filters.

**a.** Find the overall C/N in the earth station receiver in clear air conditions and estimate the bit error rate for the recovered data signal, assuming that FEC is not used. What is the correct noise bandwidth for the earth station receiver that receives the QPSK signal, and what is the bit rate of the link?

**Answer:** The overall C/N must be found from the reciprocal formula

$1 / (C/N)_o = 1 / (C/N)_{up} + 1 / (C/N)_{dn}$ Convert the C/N and C/I values to ratios:

$(C/N)_{up} = 30$ dB or 1000, $(C/N)_{dn} = 20$ dB or 100, hence

$$1 / (C/N)_o = 1 / 1000 + 1 / 100 = 0.011$$

$$(C/N)_o = 90.9 \text{ or } 19.6 \text{ dB}$$

The symbol rate on the link is 40 Msps, so the noise bandwidth of the RRC bandpass filter in the receiver must be 40 MHz. The bandwidth occupied by the QPSK signal is $R_s \times (1 + \alpha)$. Hence

$$B_{occ} = 40 \times 10^6 \times (1 + 0.4) \text{ Hz} = 56 \text{ MHz.}$$

The bit error rate for QPSK signals is given by $P_e = Q[\sqrt{(C/N)_{eff}}]$.

$$(C/N)_{eff} = (C/N)_o - \text{implementation margin} = 19.6 - 1.0 = 18.6 \text{ dB or } 72.44$$

$$P_e = Q[\sqrt{(C/N)_{eff}}] = Q[8.51] \text{ which is a number less than } 10^{-16}.$$

There are no errors on this link in clear air conditions.

Bit rate on the link is $R_b = 2 \times R_s = 80$ Mbps.

**b.** An uplink fade occurs which causes an attenuation of 10 dB between the hub station and the satellite. The transponder is operated in bent pipe mode. Find the overall C/N in the remote earth station receiver and estimate the BER of the recovered data.

**Answer:** We will assume a linear transponder, so that the uplink attenuation of 10 dB causes $(C/N)_{up}$ to fall by 10 dB to 20 dB, and the output power from the transponder to fall by 10 dB also. Then $(C/N)_{dn}$ falls by 10 dB to a new value of 10 dB and $(C/N)_o = 9.6$ dB.

The bit error rate for the QPSK receiver is given by $P_e = Q[\sqrt{(C/N)_{eff}}]$.

$(C/N)_{eff} = (C/N)_o$ – implementation margin $= 9.6 – 1.0 = 8.6$ dB or 7.24

$P_e = Q[\sqrt{(C/N)_{eff}}] = Q[2.69] \approx 3.6 \times 10^{-3}$.

There are now thousands of errors every second and the link is in an outage.

(Note: The noise temperature of the satellite receiver is unaffected by uplink attenuation, so $(C/N)_{up}$ falls by 10 dB when there is 10 dB rain attenuation on the uplink.

**c.** An uplink fade occurs which causes an attenuation of 10 dB between the hub station and the satellite. Transponder #1 is switched to operate with the 40 Msps baseband processor.

The QPSK demodulator in transponder #1 has an implementation margin of 1 dB. Find the overall C/N in the remote earth station receiver and estimate the BER.

**Answer:** With a baseband processor on the satellite, the uplink and downlink are independent, and the bit error rates on the uplink and downlink add. (This assumes low error rates, say $< 10^{-4}$ on each link). With 10 dB rain attenuation on the uplink $(C/N)_{up} = 20$ dB, and $(C/N)_{dn} = 20$ dB also. On both links the effective C/N ratio is 19.6 dB, so the error rate is the same as in part (a): there are no errors on either link.

**d.** Rainfall statistics for the location of the hub station show that attenuation at the uplink frequency will exceed 20 dB for 0.01% of an average year. If the hub station uses uplink power control to mitigate the effects of uplink rain attenuation, determine the maximum uplink transmitter power (in watts) that must be transmitted to ensure that the link BER does not exceed $10^{-6}$ at the remote earth station receiver output for 99.99% of an average year when:

**(i)** A linear bent pipe transponder is used

**(ii)** A 40 Msps QPSK baseband processing transponder is used.

**Answer:** The requirement here is that the bit error rate at the earth station receiver output not exceed $10^{-6}$ for more than 0.01% of a year ( about 52 minutes per year).

**(i)** Linear bent pipe transponder.

The overall $(C/N)_o$ must not exceed the value that gives BER = $10^{-6}$.

From the Q(z) table in Appendix C, $Q(z) = 10^{-6}$ gives $z = 4.76$ and $z^2 = (C/N)_{eff}$ for QPSK. Hence $(C/N)_{eff} = 22.66$ or 13.6 dB and $(C/N)_o = 13.6 + 1.0 = 14.6$ dB. In an uplink fade due to rain attenuation, both $(C/N)_{up}$ and $(C/N)_{dn}$ are affected equally, causing $(C/N)_o$ to fall directly in proportion to the uplink attenuation. Under clear air conditions, $(C/N)_o$ was 19.6 dB, allowing 5.0 dB of uplink fading before the BER limit is reached.

Uplink power control (UPC) must compensate for 15 dB of rain fading to ensure that the overall C/N ratio at the earth station receiver does not fall below 14.6 dB with 20 dB rain attenuation in the uplink. This is a substantial increase in transmitter power – a factor of 31.6.

**Check:** The limiting condition is when there is 5.0 dB rain attenuation on the uplink. Then $(C/N)_{up} = 25.0$ dB, $(C/N)_{dn} = 15.0$ dB, $(C/N)_o = 28.75$ or 14.6 dB, as required.

**(ii)** 40 Msps QPSK baseband processing transponder.

A baseband processor on the satellite makes the uplink and downlink independent.

The downlink $(C/N)_{dn} = 20$ dB regardless of uplink fading., and BER for the downlink is zero. We can therefore allow the uplink C/N ratio to fall until $(C/N)_{up+} = 14.6$ dB, when BER for the uplink is $10^{-6}$. Thus rain attenuation of $30.0 – 14.6 = 15.4$ dB can be tolerated before UPC must be applied, and the dynamic range of the UPC system needs to be only 4.6 dB, a factor of 2.9.

**e.** Discuss the value of UPC at the hub station transmitter in this application. Would you recommend a linear transponder or a baseband processing transponder be used on the K9 satellite? Give reasons for your answer.

**Answer:** Uplink power control is effective at preventing the downlink from going into outage because of rain attenuation on the uplink, when a linear transponder is used. Two factors tend to make this happen more often than can be allowed in many Ku band satellite links: the uplink is always at a higher frequency and therefore suffers more rain attenuation than the downlink, and the downlink C/N ratio is often lower than the uplink C/N ratio. Without UPC, a typical Ku band link is likely to fail because of uplink rain attenuation. With UPC applied, this effect can be ameliorated and outages are then caused primarily by rain attenuation on the downlink.

With a linear transponder the dynamic range of the UPC may need to be large – 31.6 in the example above. If clear air conditions require a transmit power level $P_t = 100$ W, uplink rain attenuation of 20 dB requires an increase to 3,160 W. This happens for less than one hour per year, on average, so the expensive high power transmitter is run at a low power setting most of the time.

With a baseband processing transponder, the separation of rain attenuation effects between the uplink and downlink allows the downlink to operate as usual when uplink rain attenuation occurs. This places fewer demands on the uplink power control system and a dynamic range of 4.6 dB meets the specification. The transmitter maximum output power is only 290 W, when $P_t$ = 100 W in clear air conditions. The range of the uplink power control system is now 5 dB, a factor of 3.16. However, it is easier to install a powerful transmitter at an earth station than to build a highly reliable baseband processing transponder for the satellite, so linear transponders are still preferred in most GEO satellites.

**12.** A Ku-band VSAT station receives a TDM data stream at 1.544 Mbps from a GEO satellite. The modulation is QPSK and under clear air conditions the downlink C/N in the VSAT receiver is 20 dB (ignoring noise from the satellite). The C/N in the satellite transponder is 30 dB. A nearby terrestrial LOS link causes interference with the VSAT such that the carrier to interference ratio C/I in the VSAT receiver is 19.6 dB. All C/N and C/I values are quoted for the optimum noise bandwidth of the VSAT receiver. The receiver uses ideal RRC filters with $\alpha$ = 0.4 and its QPSK demodulator has an implementation margin of 1 dB.

**a.** What is the symbol rate of the QPSK signal and the noise bandwidth of the VSAT receiver?

**Answer:** Symbol rate $R_s = R_b/2 = 1.544/2 = 0.772$ MHz = 772 kHz

Receiver noise bandwidth with RRC filters always equals the symbol rate.

Hence $B_N$ = 772 kHz.

**b.** What is the clear air overall C/N ratio in the VSAT receiver, assuming that the interference can be considered AWGN? What BER would you expect at the data output of the VSAT receiver assuming no FEC is applied to the signal?

**Answer:** The overall C/N must be found from the reciprocal formula, treating the interfering signal as noise.

$$1 / (C/N)_o = 1 / (C/N)_{up} + 1 / (C/N)_{dn} + 1 / (C/I)$$

Convert the C/N and C/I values to ratios:

$(C/N)_{up}$ = 30 dB = 1000, $(C/N)_{dn}$ = 20 dB = 100, (C/I) = 19.6 dB = 91.2

Hence

$$1 / (C/N)_o = 1 / 1000 + 1 / 100 + 1 / 91.2 = 0.02196$$

$$(C/N)_o = 45.5 \text{ or } 16.6 \text{ dB}$$

Using QPSK with a 1 dB implementation margin, $(C/N)_{eff}$ = 15.6 dB = ratio 36.3

$$BER = Q(\sqrt{36.3}) = Q(6.03) \approx 8.3 \times 10^{-10}$$

**c.** The system is redesigned and half rate FEC is added to the signal so that the bit rate at the transmitter is doubled, but transmitter power is not increased. In the receiver, the FEC decoder has a coding gain of 6 dB. For the case when FEC is used, determine the overall C/N ratio and the expected BER during a rain fade that causes the C/N ratio of the received signal to fall by 5 dB but which does not attenuate the interfering signal.

**Answer:** With half rate FEC added to the signal, the bit rate increases to 3.088 Mbps, and the symbol rate is 1.544 Msps. This increases the receiver noise bandwidth by a factor of two, to 1.544 MHz, and lowers the transponder and earth station receiver C/N values by 3 dB.

Let's assume first that the interference is wide band and increases by 3 dB when the receiver bandwidth is doubled.

The new values for C/N on the uplink and downlink are $(C/N)_{up}$ = 30.0 – 3.0 = 27.0, (C/N) = 20.0 – 3.0 = 17.0, and the new overall C/N in clear air = 16.6 dB.

If the downlink signal is attenuated by rain and $(C/N)_{dn}$ falls by 5 dB, the $(C/N)_{dn}$ ratio in rain is $(C/N)_{dn\ rain}$ = 17.0 – 5.0 = 12.0 dB = ratio 15.84.

The C/I ratio for wideband interference is 19.6 – 3.0 = 16.6 dB = ratio 45.7

$$1 / (C/N)_o = 1 / 500 + 1 / 15.84 + 1 / 45.7 = 0.0870$$

$$(C/N)_o = 11.49 \text{ or } 10.6 \text{ dB}$$

[If the interference is narrowband and does not increase when the receiver bandwidth is doubled, C/I = 19.6 = ratio 91.2 and

$$1 / (C/N)_o = 1 / 500 + 1 / 15.84 + 1 / 91.2 = 0.0761$$

$$(C/N)_o = 13.14 \text{ or } 11.2 \text{ dB}].$$

The effective C/N with half rate FEC coding gain of 6 dB and an implementation margin of 1 dB is given by

$$(C/N)_{eff} = (C/N)_o - \text{Imp. Margin} + \text{coding gain}$$

$(C/N)_{eff}$ = 10.6 [or 11.2] - 1 + 6 = 15.6 dB [or 16.2 dB].

The corresponding bit error rate for a QPSK signal is given by $Q(\sqrt{C/N})$

BER = $Q(\sqrt{36.3})$ [or $Q(\sqrt{41.7})$] = Q(6.03) [or Q(6.45)]

The error rate is below $10^{-9}$ in both cases. The symbol rate is 40 Msps = $4 \times 10^7$ sps, so that a symbol error occurs no more often than once every 25 seconds, on average, in both cases.

We should assume the worst case, that the interference is wideband and that interference power into the earth station increases when we use a wider receiver bandwidth for the FEC encoded signal. For QPSK with Gray coding, there is one bit error for each symbol error, so the expected BER when $(C/N)_{dn}$ falls by 5 dB is approximately $10^{-9}$.

**d.** If the extra bandwidth to implement half rate FEC is available at the satellite, would you recommend that FEC be used in this case? Give reasons for your answer.

**Answer:** The implementation of half rate FEC on the link improves the performance from adequate in clear air conditions to essentially error free when the downlink C/N falls by 5 dB because of rain in the downlink path. This is a significant improvement in performance and justifies the use of half rate FEC, provided bandwidth is available in the transponder. Remember that adding half rate FEC to the transmitted signal doubled the bandwidth that was needed in the transponder.

It is useful to compare the link performance with and without forward error correction. No FEC, clear air, BER $\approx 8.5 \times 10^{-10}$. With a 5 dB reduction in $(C/N)_{dn}$ due to rain, $(C/N)_o$ = 22.93 or 13.6 dB. With 1 dB implementation margin, $(C/N)_{eff}$ = 12.6 dB or 18.2 and $P_e = Q(\sqrt{18.2}) \approx 10^{-5}$. This is at or below the lower limit for satisfactory operation in most links. With FEC encoding, in clear air, $(C/N)_o$ = 13.6 dB with wideband interference. The effective overall C/N ratio is $(C/N)_{eff}$ = 13.6 – 1.0 + 5.0 = 17.6 dB or 57.7 giving a symbol error probability of $P_e = 10^{-14}$ and essentially error free operation of the link in clear air. In a rain event that causes $(C/N)_{dn}$ to fall by 5 dB, BER $\approx 8 \times 10^{-10}$.

The link is error free in clear air conditions and almost error free in the stated downlink rain event when half rate FEC is employed. This is a big improvement over the performance without FEC.

**e.** What are the advantages and disadvantages of using forward error correction in satellite links? Illustrate your answer using the above example of a high data rate signal sent to a small earth terminal.

**Answer:** Advantages: Lower bit error rates (error free under all clear air conditions), and the ability to withstand fading on the uplink and downlink. In the case above, the link remains essentially error free when downlink C/N falls by 5 dB, corresponding to a typical rain attenuation of 2.5 to 3 dB on the downlink path. In east coast areas of the US, this attenuation occurs about 0.1% of the time (about 8 hours per year).

Disadvantages: More bandwidth is needed in the satellite transponder, which may increase the cost of leasing the satellite channel. Communication capacity is generally sold by the MHz, so doubling the transmission bandwidth will double the cost. Most VSAT systems seem to be willing to pay the price of extra bandwidth and use half rate FEC to improve BER performance in fading conditions.

**13.** A T1 data transmission system transmits data at 1.544 Mbps over a GEO satellite link At the receiving terminal the clear air value of overall $(C/N)_o$ is 16.0 dB. The modulation used on the link is BPSK and the implementation margin of the BPSK demodulator is 0.5 dB.

a. Find the BER at the receiver output, and the time that elapses, on average, between bit errors.

**Answer:** With BPSK modulation $R_b = R_s = 1.544$ Mbps in this link.

The effective overall C/N ratio at the input to the BPSK demodulator in the earth station receiver is 16.0 dB – 0.5 = 15.5 dB or a ratio of 35.48. The probability of a bit error for BPSK modulation is $P_e = Q[\sqrt{(2 \times C/N)_{eff}}] = Q[8.42]$, which is less than $10^{-16}$, so there are no errors on the link. There is no definable time between errors.

b. Rain affects the downlink from the satellite and the overall C/N ratio in the receiver falls by 6.0 dB to 10.0 dB. What is the bit error rate now? What is the average time between bit errors?

**Answer:** The effective C/N ratio is now 10.0 – 0.5 = 9.5 dB, a ratio of 8.91. Hence the bit error rate is

$$P_e = Q[\sqrt{(2 \times C/N)_{eff}}] = Q[4.22] \approx 10^{-5}.$$

The time between errors is given by $T_{error} = 1 / (R_b \times P_e) = 1 / (1.544 \times 10^6 \times 10^{-5}) = 0.065$s.

c. The modulation method is changed to QPSK and the bit rate is increased to $2 \times T1 = 3.088$ Mbps, and the symbol rate on the link is 1.544 Msps (Mbaud). What is the bit error rate now? What is the average time between bit errors?

**Answer:** The symbol rate on the link is unchanged, so the C/N ratios do not change. However, the BER is higher for QPSK than for BPSK at any given C/N ratio. We will assume that the implementation margin for the QPSK receiver is the same as the BPSK receiver, at 0.5 dB. In clear air conditions the effective C/N ratio is still 15.5 dB, a ratio of 35.48. For QPSK modulation, the bit error rate is

$$P_e = Q[\sqrt{(C/N)_{eff}}] = Q[5.96] \approx 1.4 \times 10^{-9}.$$

The average time between symbol errors is given by

$$T_{error} = 1 / (R_s \times P_e) = 1 / (1.544 \times 10^6 \times 1.4 \times 10^{-9}) = 462 \text{ seconds, or } 7.7 \text{ minutes.}$$

With Gray coding of the QPSK states, there will usually be one symbol error for each bit error, so BER $\approx 1.4 \times 10^{-9}$ and $T_{bit\ error} \approx 7.7$ minutes.

d. Rain affects the downlink from the satellite and the overall C/N ratio in the receiver falls by 6.0 dB to 10.0 dB. What is the bit error rate and average time between errors now for the QPSK link?

**Answer:** The effective C/N ratio is now $10.0 - 0.5 = 9.5$ dB, a ratio of 8.91. Hence the symbol error rate is

$$P_e = Q[\sqrt{(C/N)_{eff}}] = Q[2.98] \approx 1.4 \times 10^{-3}.$$

The time between symbol errors is given by

$T_{error} = 1 / (Rs \times P_e) = 1 / (1.544 \times 10^6 \times 1.4 \times 10^{-3}) = 0.000463$ s. Using Gray coding, one symbol error leads to one bit error so there are 2160 errors each second in the link.

e. Changing the modulation method to QPSK and increasing the bit rate to 3.0878 Mbps is likely to lead to an unacceptably high bit error rate when the satellite downlink was affected by rain because the receiver $(C/N)_o$ will fall by 6 dB. We could retain a bit rate on the link of

1.544 Mbps when using QPSK by changing the transmitter and receiver RRC filters to operate at a symbol rate $R_s = 1.544 / 2 = 0.772$ Msps. What is the bit error rate and average time between errors now for the QPSK link?

**Answer:** Reducing the bit rate on the link to 1.544 Mbps increases the C/N ratios by 3 dB because the receiver bandwidth is halved. In clear air conditions the effective C/N ratio is 15.5 + 3.0 = 18.5 dB, a ratio of 70.8. For QPSK modulation, the bit error rate is given by

$$P_e = Q[\sqrt{(C/N)_{eff}}] = Q[8.41] .$$

The BER is less than $10^{-16}$ so there are no errors on the link.

**f.** Rain affects the downlink from the satellite in part (e) above, and the overall C/N ratio in the receiver falls by 6.0 dB to 10.0 dB. What is the bit error rate now for the QPSK link?

**Answer:** The effective C/N ratio is now 13.0 – 0.5 = 12.5 dB, a ratio of 17.78. Hence the symbol error rate is

$$P_e = Q[\sqrt{(C/N)_{eff}}] = Q[4.22] \approx 10^{-5}.$$

The time between symbol errors is given by

$T_{error} = 1 / (R_s \times P_e) = 1 / (1.544 \times 10^6 \times 10^{-5}) = 0.065$ s. Using Gray coding, one symbol error leads to one bit error so there are 15 errors each second in the link, on average.

**14.** The baseband average S/N ratio for a probability of bit error $P_e$ (BER) with N-bit PCM is given by

$$S/N = 10 \log_{10} \left[ \frac{2^{2N}}{1+4P_e \times 2^{2N}} \right] \text{ dB}$$

a. The effective C/N in a digital receiver with QPSK modulation is 15.6 dB under clear air conditions. What is the baseband S/N for 8-bit PCM coded speech?

**Answer:** Quantization S/N = 6 N dB when there are no bit errors, where N = niumber of bits in PCM word. For N = 8, $(S/N)_Q = 48$ dB with linear encoding. The probability of error for QPSK with an effective C/N ratio of 15.6 = ratio 36.3 is $Q[\sqrt{36.3}] = Q[6.02] = 9 \times 10^{-10}$

In the absence of quantization noise, the signal to noise ratio from bit errors is $(S/N)_B = 1/(4 P_e)$.

For BER $= 9 \times 10^{-10}$, $(S/N)_B = 2.77 \times 10^8$ or 84 dB.

The quantization noise is dominant in this case and S/N = 48 dB.

b. In moderate rain conditions the effective C/N falls to 13.6 dB. What is the baseband S/N for the 8-bit PCM signal?

**Answer:** The effective C/N ratio is 13.6 dB, a ratio of 22.90, giving BER for the link as

$P_e = Q(\sqrt{22.90}) = Q(\sqrt{4.79}) = 10^{-6}$.

The bit errors on their own will cause $(S/N)_B = 1 / 4 \times 10^{-6} = 2.5 \times 10^5$ or 54.0 dB. Since the two S/N ratios are similar, we must use the formula at the beginning of the question to calculate their joint effect.

$$S/N = 10 \log_{10} \left[ \frac{2^{2N}}{1 + 4P_e \times 2^{2N}} \right] \text{ dB} = 10 \log [65{,}536 / [\ 1 + (4 \times 10^{-6} \times 65{,}536)\ ]]$$

$$= 10 \log [65{,}536 / 1.262] = 47.1 \text{ dB}$$

c. In heavy rain, the effective C/N falls to 11.6 dB. What is the baseband S/N for the 8-bit PCM signal?

**Answer:** : The effective C/N ratio is 11.6 dB, a ratio of 14.45, giving BER for the link as

$P_e = Q(\sqrt{14.45}) = Q(\sqrt{3.80}) = 7.25 \times 10^{-5}$.

The bit errors on their own will cause $(S/N)_B = 1 / (4 \times 7.25 \times 10^{-5}) = 3448$ or 35.4 dB, and will therefore be the dominant source of noise; S/N baseband ≈ 35.4 dB.

**Check:** putting $P_e = 7.25 \times 10^{-5}$ and $2^{2N} = 65{,}536$ in the S/N formula gives

$$S/N = 10 \log [65{,}536 / [\ 1 + ( 4 \times 7.25 \times 10^{-5} \times 65{,}536)\ ]] = 10 \log [65{,}536 / 19.0] = 35.4 \text{ dB}.$$

d. The minimum acceptable baseband S/N in a speech channel is usually set at 30 dB. What is the corresponding minimum allowable effective C/N for a QPSK link carrying 8-bit PCM coded speech?

**Answer:** The baseband S/N will be dominated by the bit errors at the demodulator output. Hence $S/N = 30 \text{ dB} = 1000 = 1 / 4 P_e$ and $P_e = 2.5 \times 10^{-4}$.

The probability of error in a QPSK link is $Q[\sqrt{(C/N)_{eff}}]$, so $\sqrt{(C/N)_{eff}} = 3.48$, and $(C/N)_{eff} = 12.11$ or 10.8 dB. Thus when the effective C/N at the QPSK demodulator input is 10.8 dB, the baseband S/N of the recovered PCM signal will be 30 dB. The minimum allowable QPSK effective C/N ratio is 10.8 dB.

**15.** Direct Broadcast Satellite TV

In this question you are asked to analyze the performance of the TV system using frequency modulation. The uplink station delivers a signal to the satellite which conforms to the following specification:

Transponder and satellite characteristics

| | |
|---|---|
| Transponder bandwidth | 25 MHz |
| $(C/N)_{up}$ in 20 MHz noise bandwidth | 24 dB |
| Saturated output power | 200 W |
| Downlink frequency | 12.5 GHz |
| Downlink antenna gain, on axis | 39.0 dB |
| Atmospheric clear air loss | 0.4 dB |
| All other losses | 0.5 dB |

Receive Station parameters

| | |
|---|---|
| Antenna diameter | 18 inches |
| Aperture efficiency | 70 % |
| Antenna noise temperature (clear air) | 40 K |
| Receiver noise temperature | 90 K |

a. The uplink master station transmits a NTSC video signal with a baseband bandwidth of 4.2 MHz to one transponder on the satellite using FM. The transponder is operated with 1 dB of output back-off and the FM signal occupies a bandwidth of 24 MHz. For an earth station with a high gain LNA, at a distance of 38,000 km from the satellite, on the – 4 dB contour of the satellite antenna beam. Find:

**Answer:**

i The peak frequency deviation of the FM signal.

Using Carson's rule: $B = 2 (\Delta f_{pk} + f_{max})$

Hence $\Delta f_{pk} = B/B - f_{max} = 24 / 2 - 4.2 = 7.8$ MHz

ii The power at the input to the earth station LNA.

Find the path loss and receive antenna gain first. Antenna diameter is 0.457 m.

$L_p = 20 \log ( 4 \pi R / \lambda)^2 = 20 \log ( 4 \pi \times 38 \times 10^6 / 0.025) = 205.6$ dB

$G_r = \eta_A \times ( \pi D / \lambda)^2 = 0.7 \times (\pi \times 0.457 / 0.025)^2 = 2309$ or $33.6$ dB

The downlink budget gives the received power. For 1 dB output back off of the 200 W transponder, atmospheric and miscellaneous loss of 0.9 dB, and the –4 dB contour:

$P_r = EIRP + G_r - L_p$ - losses dBW

$= 22 + 39 + 33.6 - 205.6 - 4.0 - 0.4 - 0.5 = -115.9$ dBW

iii The downlink $(C/N)_{dn}$ in a noise bandwidth of 24 MHz.

The noise power at the earth station receiver input is

$N = k\, T_s\, B_N$ where $T_s = 40 + 90 = 130$ K or 21.1 dBK

$N = -228.6 + 21.1 + 73.8 = -133.7$ dBW

Hence the downlink C/N ratio in the earth station receiver is

$C/N = P_r - N = -115.9 + 133.7 = 17.8$ dB

iv The overall $(C/N)_o$ in the earth station receiver.

The uplink C/N in the transponder is 24 dB. Using the reciprocal formula

$(C/N)_o = 1/( 1/251.2 + 1/60.25) = 1 / 0.0206 = 48.60$ or 16.9 dB

b. For the FM video signal in part (a) above:

**Answer:**

i The unweighted video S/N ratio at the baseband output of the receiver

$(S/N)_{unweighted} = C/N + 10 \log (B/f_{max}) + 20 \log (\Delta f_{pk} / f_{max} + 1.8$ dB

$= 16.9 + 10 \log (24.0 / 4.2) + 20 \log (7.8 / 4.2) + 1.8$

$= 31.7$ dB

ii The weighted S/N ratio after pre-emphasis and subjective improvements are added

The standard improvement factors for NTSC TV signals transmitted using FM modulation are P = 8 dB, Q = 9 dB giving a weighted S/N in clear air conditions

$(S/N)_{weighted} = 31.7 + 8 + 9 = 48.7$ dB

iii The link margin for the downlink given an FM threshold at 8.5 dB.

The clear air downlink C/N in clear air is 17.8 dB. $(C/N)_{up}$ is 24.0 dB.

The limiting value for downlink C/N ratio $(C/N)_{dnmin}$ is set by the FM threshold value of $(C/N)_{oth} = 8.5$ dB. Using the reciprocal formula

$$1 / (C/N)_{oth}) = 1 / (C/N)_{up} + 1 / (C/N)_{dnmin}$$

$$1 / 7.079 = 1 / 251.2 + 1 / (C/N)_{dnmin}$$

$$(C/N)_{dnmin} = 7.28 \text{ or } 8.6 \text{ dB}$$

The downlink margin is the difference between the clear air downlink C/N ratio of 17.8 dB and the minimum value of 8.6 dB.

Hence downlink margin is 17.8 - 8.6 = 9.2 dB

c. Heavy rain affects the uplink to the satellite causing the C/N in the transponder to fall to 18 dB. Assuming linear bent-pipe operation of the transponder, find:

**Answer:**

i The overall $(C/N)_o$ ratio in the earth station receiver

When uplink C/N ratio falls from 24 dB to 18 dB, the downlink C/N will also fall by 6 dB to 11.8 dB because we have a linear transponder. The overall C/N will also fall by 6 dB to 10.8 dB. Check using the reciprocal formula

$(C/N)_o = 1 / (1/63.10 + 1/15.14) = 12.21$ or 10.8 dB

ii The video S/N ratio. Is this an acceptable S/N for viewing a television picture?

In clear air conditions the weighted video S/N was 48.7 dB. S/N falls in direct proportion to overall C/N during rain attenuation events, so the weighted S/N will fall by 6 dB to 42.7 dB. The TV picture will have visible noise but would rate as acceptable.

d. Heavy rain affects the downlink from the satellite causing 4 dB of rain attenuation. The uplink is operating in clear air conditions.

Assuming a medium noise temperature of 270 K in rain and 100% coupling of sky noise into antenna noise temperature, find:

i The new value for $(C/N)_{dn}$ in the earth station receiver,

We must first find the increase in sky noise temperature caused by rain in the downlink path.

**Answer:** The sky temperature increases when rain is in the downlink path. Using the noise model for a lossy device, $T_{sky\ rain} = 270 \times (1 - 0.398) = 162.5$ K.

The new system noise temperature is $T_s = 90 + 162.5 = 252.5$ K

The increase in noise power in the earth station receive is $\Delta N$ where

$$\Delta N = 10 \log ( 252.5 / 130 ) = 2.9 \text{ dB}$$

The new downlink C/N in rain is

$$(C/N)_{dn\ rain} = 17.8 - 4.0 - 2.9 = 10.9 \text{ dB}$$

ii The corresponding overall $(C/N)_o$ ratio in the earth station receiver

Overall C/N is

$$(C/N)_o = 10 \log (1 / (1 / (C/N)_{up} + 1 / (C/N)_{dn}) = 10.7 \text{ dB}$$

iii The video S/N ratio. Is this an acceptable quality television picture?

Video S/N is proportional to the C/N ratio in the receiver. The weighted S/N ratio is given by

$$\text{S/N weighted} = (C/N)_o + \text{FM Improvement}$$

FM Improvement = 31.8 dB

Hence with C/N overall = 10.7 dB

$$\text{S/N weighted} = 10.7 + 31.8 = 42.5 \text{ dB}$$

e. Draw a block diagram for the earth station receiver, showing only the parts that relate to reception and output of the NTSC video signal. Your block diagram must show the center frequency, gain, and bandwidth of each block, as appropriate.

Do not specify filters with Q exceeding 50.

**Answer:** Block diagram not included in the Solutions Manual. See text for examples.

16. This problem examines the design and performance of a digital satellite communication link using a geostationary satellite with bent-pipe transponders, used to distribute digital TV signals from one central (hub) earth station to many receiving stations throughout the United States. The link uses QPSK digital transmission at 20 Msps with half rate forward error correction. The half rate FEC gives a coding gain of 5.5 dB.

The design requires that an overall C/N ratio of 9.5 dB be met in the earth station receiver to ensure that noise in the video signal on the TV screen is held to an acceptable level. The uplink transmitter power and the receiving antenna gain and diameter must be determined. The available link margins for each of the systems must be found and the performance of the system analyzed when rain attenuation occurs in the satellite – earth paths.

The system is specified in Table 1.

Table 1. System and Satellite Specification

| | |
|---|---|
| **Ku-band satellite parameters** | |
| Total RF output power | 3.2 kW |
| Antenna gain, on axis, Ku-band (transmit and receive) | 31 dB |
| Receive system noise temperature | 500 K |
| Transponder saturated output power: Ku-band | 80 W |
| Transponder bandwidth: Ku-band | 54 MHz |
| Earth station receiver IF noise bandwidth | 20 MHz |
| Minimum permitted overall C/N in receiver | 9.5 dB |
| Transponder HPA output back-off | 1 dB |
| **Transmitting Ku-band earth station** | |
| Antenna diameter | 5 m |
| Aperture efficiency | 68 % |
| Uplink frequency | 14.15 GHz |
| Required C/N in Ku band transponder | 30 dB |
| Miscellaneous uplink losses | 0.3 dB |
| Location: -2 dB contour of satellite receiving antenna | |
| **Receiving Ku-band earth station** | |
| Downlink frequency | 11.45 GHz. |
| Receiver IF bandwidth | 20 MHz |
| Aperture efficiency | 68 % |
| Antenna noise temperature | 30 K |
| LNA noise temperature | 110 K |
| Required overall $(C/N)_o$ in clear air | 17 dB |
| Miscellaneous downlink losses | 0.2 dB |
| Location: -3 dB contour of satellite transmitting antenna | |
| **Rain Attenuation and Propagation Factors at Ku-band** | |
| Clear air attenuation | |
| Uplink 14.15 GHz | 0.7 dB |

| | | |
|---|---|---|
| Downlink | 11.45 GHz | 0.5 dB |
| Rain attenuation | | |
| Uplink | 0.01 % of year | 6.0 dB |
| Downlink | 0.01 % of year | 5.0 dB |

a. Uplink design

Find the uplink transmitter power to achieve the required $(C/N)_{up} = 30$ dB in the transponder in clear air atmospheric conditions. Find the noise power in the transponder for a noise bandwidth of 20 MHz, and then add 30 dB to find the transponder input power level. Calculate the earth station transmit antenna gain, and the path loss at 14.15 GHz. Generate an uplink power budget and find the required power at the transponder input to meet the $(C/N)_{up} = 30$ dB objective in the transponder. Don't forget the various uplink losses.

**Answer :** Begin with the noise power in the transponder, and then create an uplink power budget. Transponder input noise temperature is 500 K, earth station receiver noise bandwidth is 20 MHz. Noise power is N watts where

$$N = k\,T_s\,B_N = -228.6 + 27.0 + 73.0 = -128.6 \text{ dBW}$$

Uplink earth station transmit antenna gain at 14.15 GHz ($\lambda = 0.02120$ m) is given by

$$G_t = 10 \log ( \eta_A \times (\pi D / \lambda)^2) = 10 \log ( 0.68 \times (\pi \times 5 / 0.02120)^2 ) = 55.7 \text{ dB}$$

Path loss at 14.15 GHz for a typical GEO path length of 38,500 km is given by

$$L_p = 20 \log (\pi R / \lambda) = 20 \log ( 4 \pi \times 38.5 \times 10^6 / 0.0212) = 207.2 \text{ dB}$$

The uplink power budget includes losses of 2 dB off-axis, 0.7 dB atmospheric, and 0.3 dB misc.

| | |
|---|---|
| $P_t$ | TBD |
| $G_t$ | 55.7 dB |
| $G_r$ | 31.0 dB |
| Path loss | -207.2 dB |
| Off-axis contour loss | -2.0 dB |
| Other losses | -1.0 dB |
| Receiver power | $P_t$ - 123.5 dBW |

We require $(C/N)_{up} = 30$ dB with N = -128.6 dBW, so the required transmitter power at the uplink earth station is

$$P_t = 30 + 123.5 - 128.6 = 24.9 \text{ dBW or } 309 \text{ W.}$$

b. Downlink design

Assume a high gain LNA and ignore the noise generated in other parts of the receiver. Calculate the downlink $(C/N)_{dn}$ to give overall $(C/N)_o$ = 17 dB when $(C/N)_{up}$ = 30 dB. Hence find the receiver input power to give the required $(C/N)_{dn}$ using a value of receiving antenna gain $G_r$.

Calculate the path loss at the downlink frequency of 11.15 GHz. Don't forget the downlink losses. The transponder is operated with 1 dB output back off. Find the transponder output power and then generate a downlink power budget .

Hence find the receiving antenna gain $G_r$ and diameter for a frequency of 11.45 GHz. This diameter antenna will provide the required $(C/N)_o$ in the earth station receiver under clear air conditions.

**Answer:** Find the path loss first by scaling from 14.15 GHz to 11.45 GHz

Path loss $L_p$ = 207.16 + 20 log (11.45 / 14.15) = 205.3 dB

The downlink power budget includes losses of 3 dB off-axis, 0.5 dB atmospheric, and 0.2 dB miscellaneous. Transponder output power is 80 W with 1 dB back-off, giving $P_t$ = 19.0 dBW.

| | | |
|---|---|---|
| $P_t$ | 80 W - 1 dB back-off | 19.0 dBW |
| $G_t$ | | 31.0 dB |
| $G_r$ | | TBD |
| Path loss | | -205.3 dB |
| Off-axis contour loss | | -3.0 dB |
| Other losses | | -0.7 dB |
| | Receiver power | $G_r$ - 159.0 dBW |

We require $(C/N)_o$ = 17.0 dB with $(C/N)_{up}$ = 30 dB. Using the reciprocal formula

$(C/N)_{dn} = 1 / ( 1/(C/N)_o - 1 / (C/N)_{up}) = 52.76$ or 17.2 dB

The noise power in the receiver is $N = k\, T_s\, B_N$ watts. $T_s$ = 30 K + 110 K = 140 K.

$N = -228.6 + 21.5 + 73.0 = -134.1$ dBW

Hence $G_r$ - 159.0 dBW = 17.2 - 134.1 dBW, $G_r$ = 42.1 dB

The gain or the receiving antenna is given by

$$G_r = 42.1 \text{ dB} = 10 \log ( \eta_A \times (\pi D / \lambda)^2) = 10 \log ( 0.68 \times (\pi \times D / 0.02620)^2 )$$

Hence $16{,}218 = 0.68 \times (\pi \times D / 0.02620)^2$ and D = 1.288 m.

c. Rain effects

A practical system must continue to operate under adverse weather conditions, so we need a margin for rain attenuation and increase in sky noise temperature during rain. In the following section you will determine the margins available on the uplink and downlink to combat rain attenuation and increase in sky noise temperature.

d. Uplink rain attenuation

Under conditions of heavy rain, the Ku-band path to the satellite suffers an attenuation of 6 dB for 0.01 % of the year. We must find the uplink attenuation margin and decide whether uplink power control would improve system performance at Ku-band.

The uplink C/N was 30 dB in clear air. With 6 dB uplink path attenuation, the C/N in the transponder falls to 24 dB. (Rain on the earth has no effect on the satellite transponder system noise temperature.) Assume linear transponder characteristic and no uplink power control. Find the transponder output power with 6 dB of rain attenuation in the uplink.

Hence find the overall $(C/N)_o$ in an uplink rain fade of 6 dB, and the link margin available on the uplink. Is this an adequate uplink margin, given the rain attenuation for most of the US?

**Answer:** We assume a linear transponder, so the output power of the transponder falls by 6 dB when rain attenuation of 6 dB occurs on the uplink. When both $(C/N)_{up}$ and $(C?N)_{dn}$ fall by the same amount, say X dB, the overall $(C/N)_o$ ratio also falls by X dB.

Hence, with an uplink fade of 6 dB, $(C/N)_o = 17.0 - 6.0 = 11.0$ dB. The system objectives require a minimum overall $(C/N)_o$ of 9.5 dB. This leaves an uplink margin of 1.5 dB with 6 dB uplink attenuation, i.e. maximum uplink attenuation of 7.5 dB. This attenuation is exceeded about 0.01% of an average year for a typical GEO satellite link at 11.45 GHz in the East Coast region of the US. If the location of the uplink station is the West Coast or central parts of the United States, an uplink margin of 7.5 dB at 11.45 GHz will ensure satisfactory operation of the receiving terminals for more than 99.99% of an average year.

e. Downlink attenuation and increase in sky noise in rain

The 11.45 GHz path between the satellite and the receive station suffers rain attenuation exceeding 5 dB for 0.01% of the year. Assuming 100% coupling of sky noise into antenna noise, and 0.5 dB clear air gaseous attenuation, calculate the overall C/N under these conditions.

Assume that the uplink station is operating in clear air. Calculate the available downlink fade margin.

Find the sky noise temperature that results from a total excess path attenuation of 5.5 dB (clear air attenuation plus rain attenuation); this is the new antenna temperature in rain, because we assumed 100 % coupling between sky noise temperature and antenna temperature. Evaluate the change in received power and increase in system noise temperature in order to calculate the change in C/N ratio for the downlink.

In clear air, the atmospheric attenuation on the downlink is 0.5 dB. The corresponding sky noise temperature is approximately 0.5 x 7 = 35 K, which leads to the antenna temperature of 30 K given in the Ku-band system specification. When the rain causes 5 dB attenuation, the total path attenuation from the atmosphere and the rain is 5.5 dB. The sky noise will be much higher in rain. Find the increase in noise power caused by the increase in sky temperature.

Hence find the new $(C/N)_{dn\ rain}$ value with 5.5 dB attenuation on the downlink path. Find the overall C/N by combining the clear air uplink $(C/N)_{up}$ of 30 dB with the rain faded downlink $(C/N)_{dn\ rain}$ to give overall $(C/N)_o$ in rain.

Is the downlink link margin acceptable? If not, calculate the gain and diameter of an earth station antenna that will ensure an overall C/N value that does meet the specification.

**Answer:** When there is 5 dB of rain attenuation on the downlink to the earth station, total path attenuation is 5.5 dB. Assuming a medium temperature of 290 K, sky noise temperature for 5.5 dB path loss is given by

$$T_{sky} = 290 \times (1 - 0.282) = 208 \text{ K.}$$

The corresponding system noise temperature is $T_{s\ rain} = 208 + 110 = 318$ K.

In clear sky conditions, the system noise temperature was 140 K. Hence the increase in earth station receiver noise power is

$$\Delta N = 10 \log (318 / 140) = 3.6 \text{ dB}$$

The downlink has 5 dB rain attenuation, so total reduction in $(C/N)_{dn}$ is 8.6 dB, giving $(C/N)_{dn\ rain} = 17.2 - 8.6 = 8.6$ dB. Overall C/N ratio with $(C/N)_{up} = 30$ dB is 8.6 dB, rounded to the nearest 0.1 dB. This is below the requirement of 9.5 dB for satisfactory signal quality at the video output of the earth station receiver. To improve the overall C/N ratio at the receiver output to 9.5 dB, we require $(C/N)_{dn} = 9.5$ dB. Receiving antenna gain must be increased by 0.9 dB to meet this objective, requiring an increase in antenna diameter by a factor of

$10^{0.9/20} = 1.109$ to $1.288 \times 1.134 = 1.429$ m.

Because there are many steps in the calculation here, it is wise to check that the system design meets the specification. Here are the revised downlink budget and noise power figures.

Antenna gain for a 1.46 m antenna with 68% aperture efficiency and diameter 1.429 m is 43.0 dB, an increase over the original gain of 0.9 dB, as required. Received power increases by 0.9 dB, giving $P_r = G_r - 159.0$ dBW $= -116.0$ dBW in clear air, and –121.0 dBW with 5 dB rain attenuation in the downlink. With $T_s = 318$ K, the noise power referred to the earth station receiver input is –130.6 dBW. This gives $(C/N)_{dn\ rain} = 9.6$ dB. Uplink $(C/N)_{up}$ is 30 dB, giving an overall C/N ratio in the earth station receiver of 9.56 dB. This is very close to the reuirement of 9.5 dB – the difference is due to round-off errors in all the many decibel values used in the calculation.

f. Summarize your design for the Ku band earth station and uplink and downlink.

Compare the earth station receiving antenna diameter for the Ku band system with the antenna in the notes for a similar C band system.

If the Ku band antenna is larger (and therefore has a much higher gain) explain why.

**Answer:** The receiving antenna at Ku band that meets the 9.5 dB overall $(C/N)_o$ requirement for 99.99% of an average year has a diameter of 1.429 m. The uplink will provide better than 99.99% availability of signals provided that it is located away from the east and south of the United States, in a region where heavy rainfall is less frequent. The downlink meets the 99.99% availability criterion over most of the United States. In the south eastern states, a larger antenna would be needed to guarantee 99.99% availability of the downlink. A system of this type could be used for distribution of a digital TV signal to cable TV companies; however, a compressed digital signal at a higher bit rate carrying multiple TV signals, and a wider bandwidth transponder would be used, requiring a receiving antenna about 2 m in diameter.

A typical C-band receiving antenna for FM-TV signals had a diameter of 3 m, but was used with satellites having much lower output power, in the 10 – 30 W range. The Ku-band antenna in this problem is larger than the antennas used for direct broadcast satellite television because of the lower satellite transponder output power and slightly lower satellite antenna gain assumed in this example. See Chapter 11 for details of DBS-TV systems.

**17.** A satellite communication system uses a single 54 MHz bandwidth Ku-band transponder to carry 400 two way telephone conversations (800 RF channels) using analog modulation with single channel per carrier frequency modulation (SCPC-FM). The parameters of any one channel are:

| | |
|---|---|
| Voice channel bandwidth : | 100 - 3,400 Hz |
| RF channel bandwidth: | 45 kHz |
| RF channel spacing: | 65 kHz |
| Downlink path loss (inc. atmos. loss) | 206.5 dB |
| Satellite downlink antenna gain (on axis) | 29 dB |
| Demodulator FM threshold: | 5 dB |

The transponder has a saturated power output of 40 watts, but is run with 3 dB output backoff to achieve near-linear operation. The uplink stations which transmit the SCPC-FM signals to the transponder achieve $(C/N)_{up}$ = 25 dB in the 45 kHz channel noise bandwidth of the earth station receiver. The system noise temperature of the receiving earth station is 120 K in clear air.

a. Calculate the power per RF channel at the transponder output.

**Answer:** Transponder saturated output power is 40 watts = 16 dBW. With 3 dB backoff, $P_t$ = 13 dBW or 20 W. Each voice channel in the satellite transponder occupies 65 kHz with its guard bands, so we can fit 54,000 / 65 = 831 channels into the transponder. Hence, downlink transmit power per channel is 20 W / 831 = 0.0241 W/channel = -16.2 dBW per channel.

b. The gain of the antenna at a receiving earth station that is located on the -3 dB contour of the satellite footprint which will provide an overall C/N = 10 dB in a receiver for a single RF channel with a noise bandwidth of 45 kHz, in clear air conditions.

**Answer:** We know the downlink path loss, so we can establish a downlink power budget using a receiving antenna gain of $G_r$ dB. The downlink power budget includes a losses of 3 dB for the receiving station location on the –3 dB contour of the satellite footprint.

The downlink budget is:

| | | |
|---|---|---|
| $P_t$ | 24 mW per channel | -16.2 dBW |
| $G_t$ | | 29.0 dB |
| $G_r$ | | TBD |
| Path loss | | -206.5 dB |
| Off-axis contour loss | | -3.0 dB |
| | Receiver power | $G_r$ - 196.7 dBW |

Noise power in a single earth station receiver noise bandwidth of 45 kHz, for one FM voice channel, with $T_s$ = 120 K in clear air is

$$N = k\,T_s\,B = -228.6 + 20.8 + 46.5 = -161.3 \text{ dBW}$$

We require $(C/N)_o$ = 10.0 dB with $(C/N)_{up}$ = 25 dB. Using the reciprocal formula

$$(C/N)_{dn} = 1 / (\,1/(C/N)_o - 1 / (C/N)_{up}) = 10.14 \text{ or } 10.1 \text{ dB.}$$

Hence the earth station receive antenna gain is

$$G_r = 196.7 + 10.1 - 161.3 = 45.5 \text{ dB.}$$

c. Calculate the diameter of the receiving antenna with a circular aperture having 65% aperture efficiency at a frequency of 11.5 GHz.

**Answer:** The gain or the receiving antenna with $\lambda$ = 0.02608 m is given by

$$G_r = 45.5 \text{ dB} = 10 \log (\,\eta_A \times (\pi\, D / \lambda)^2) = 10 \log (\,0.65 \times (\pi \times D / 0.02608)^2\,)$$

Hence $35{,}481 = 0.65 \times (\pi \times D / 0.02608)^2$ and $D = 1.940$ m.

d. The receiver applies a de-emphasis weighting of 6 dB to the recovered voice signal and a psophometric weighting of 2.5 dB.

Calculate the weighted S/N at the baseband output of the receiver.

**Answer:** The weighted baseband S/N for a single channel FM signal is given by Equation 5.18, with an added subjective improvement factor Q dB:

$$(S/N)_w = C/N + 10 \log (B_{RF} / f_{max}) + 20 \log (\Delta f_{pk} / f_{max}) + 1.8 + P + Q \text{ dB}$$

For the voice signal in this problem, $f_{max}$ = 3.4 kHz, $\Delta f_{pk}$ = 45/2 – 3.4 = 19.1 kHz, P = 6 dB and Q = 2.5 dB. The overall C/N ratio in the earth station receiver is 10.0 dB.

$$(S/N)_w = 10 + 10 \log (45 / 3.4) + 20 \log 19.1 / 3.4) + 1.8 + 6 + 2.5 = 46.5 \text{ dB.}$$

e. Comment on the performance of the system. Is the S/N adequate in clear air.
If the downlink fades by 5 dB because of rain, what is the S/N at baseband?
Is this acceptable for voice communications?

**Answer:** The baseband S/N ratio of 45.6 dB would be rated as good, slightly below the wireline telephone system objective of 50 dB. With a 5 dB reduction in earth station receiver overall C/N ratio, the overall C/N ratio in the earth station receiver will be 5 dB. This will be at the threshold of the FM demodulator, assuming a threshold extension design, so operation is marginal and we should deduct 1 dB from the baseband S/N ratio. Hence baseband S/N = 39.6 dB in this rain condition, at the lower limit for acceptable operation.

The antenna diameter, close to 2 m, is similar to many older VSAT systems that used lower power Ku-band transponders. Newer satellites have higher transponder powers in the 50 – 100 W range, and transmission is digital using QPSK, not analog using FM.

**18.** In problem #5, an analog voltage was transmitted from a satellite to earth using frequency modulation. The signal could have been sent digitally using a digital to analog converter and PSK modulation. This problem compares the performance of the digital link to the analog link of problem #5.

The digital link is allocated an RF bandwidth of 25 kHz, and uses BPSK modulation. At the receiving terminal, the C/N ratio is 10 dB. The link has ideal RRC filters with $\alpha = 0.25$ and the BPSK demodulator has an implementation margin of 0.5 dB.

**a.** The analog voltage is sampled at 2.5 kHz and converted to a series of digital words with an analog to digital converter. Determine the maximum number of bits in each word and the average quantization signal to noise ratio of the recovered analog signal.

**Answer:** The available bandwidth of the RF channel is 25 kHz. With RRC filters of $\alpha = 0.25$, the symbol rate in the channel is 25 / 1.25 = 20 ksps, and thus $R_b$ = 20 kbps for BPSK modulation. The analog signal is sampled at 2.5 kHz, so we can send 8 bits per sample as a 20 kbps bit stream. The average quantization signal to noise ratio is $(S/N)_Q$ = 6 N dB = 48 dB.

**b.** Find the BER for the recovered bit stream at the output of the BPSK demodulator, and hence calculate the average S/N ratio in the analog voltage due to bit errors.

**Answer:** BER = $Q[\sqrt{(2\ C/N)_{eff}}]$ for a BPSK signal. The C/N ratio in the receiver is 10 dB in a noise bandwidth of 20 kHz. The RRC filter in the BPSK receiver has a noise bandwidth equal to the symbol rate of 20 ksps.

The effective C/N is 10.4 – 0.5 = 9.5 dB or 8.91 as a ratio.

BER is then $Q[\sqrt{(2\ C/N)_{eff}}] = Q[\sqrt{2 \times 8.91}] = Q[4.22] = 1.2 \times 10^{-5}$.

The average baseband S/N for a PCM signal with a bit error rate $P_e$ is $1 / (4\ P_e)$, giving

$$S/N_{\text{bit errors}} = 20{,}833 \text{ or } 43.2 \text{ dB.}$$

The lower S/N ratio caused by the $10^{-5}$ bit error rate will dominate over the quantization S/N ratio, so the baseband S/N will be around 43 dB.

**c.** Solve problem #5 for the FM version of this link. Which link has the better performance? What changes should be made to the link with the poorer performance to make the S/N ratios approximately equal for the FM and BPSK links? If a RF bandwidth of 50 kHz could be used for the BPSK signal, would the addition of half rate forward error correction with a coding gain of 6 dB improve the performance of the BPSK link?

**Answer:** The FM version of the link was analyzed in problem #5. The peak deviation was found to be $\Delta f_{pk} = 10$ kHz giving a Carson's rule bandwidth with $f_{max} = 1$ kHz

$$B = 2(\Delta f_{pk} + f_{max}) = 2 \times (10 + 1) = 22 \text{ kHz}$$

Using this RF bandwidth as the FM receiver noise bandwidth, and a receiver C/N ratio of 10 dB, the baseband S/N ratio at the earth station receiver output for the recovered analog signal is

$$\begin{aligned} S/N &= C/N + 10 \log (B/f_{max}) + 20 \log (\Delta f_{pk} / f_{max}) + 1.8 \text{ dB} \\ &= 10.0 + 10 \log (22/1) + 20 \log (10/1) + 1.8 \text{ dB} \\ &= 10 + 13.4 + 20 + 1.8 = 45.2 \text{ dB} \end{aligned}$$

This is slightly better than the S/N for the BPSK signal transmitted in a bandwidth of 25 kHz, and would have more graceful degradation than the BPSK receiver output during a rain fade.

If we have a wider RF bandwidth of 5 kHz available and add half rate FEC with a coding gain of 6 dB, the receiver C/N falls by 3 dB because of the wider RF bandwidth and effective C/N ratio becomes

$$(C/N)_{eff} = 7.0 + 6.0 - 0.5 = 12.5 \text{ dB}$$

The bit error rate with BPSK modulation and $(C/N)_{eff}$ = 12.5 dB = ratio 17.78 is

$$BER = Q[\sqrt{2 \times 17.78}] = Q[5.96] \approx 10^{-10}.$$

Quantization noise will now dominate giving S/N = 48 dB, and the BPSK link has a downlink margin of several decibels before the S/N ratio due to bit errors equals the quantization signal to noise ratio of 48 dB. This happens when z = 4.49 and C/N effective = 10.0 dB, C/N = 4.5 dB, giving a downlink C/N ratio margin of 2.5 dB. Performance of the two links is therefore comparable, with the digital link giving better performance for 99% of the time. Because of the many advantages of digital transmission, the BPSK (or a QPSK equivalent) system would be preferred.

.

## Chapter 6 Solution to Problems

**1.** You are designing an FDM/FM/FDMA analog link that will occupy 36 MHz of an INTELSAT VI transponder. The uplink and downlink center frequencies of the occupied band are 5985.5 MHz and 3760.5 MHz. The distance from the satellite to your earth station is 40,000 km. The saturation uplink flux density for your uplink is - 75 dBW/m$^2$ and the satellite's G/T is = -11.6 dBK$^{-1}$. At saturation the transponder EIRP for your downlink is 29 dBW and the earth station's G/T is 41 dBK$^{-1}$. The transponder is linear in that the EIRP in dBW is BO dB below the saturation value when the uplink flux density is backed off BO dB below saturation. The intermodulation carrier to noise ratio, (C/N), in dB, is related to the back-off BO in dB by

$$(C/N)_I = 7.86 + 0.714 \times BO$$

In other words, at saturation the value of $(C/N)_I$ is 7.86 dB. Find the maximum overall carrier-to-noise ratio (C/N), in dB that this link can achieve. What back-off must be used to achieve it? (When you need a frequency in your calculations, use the uplink or downlink center frequency as appropriate.) Make your calculations for beam center.

**Answer:** This problem specifies receive system G/T ratio for the satellite and the receiving earth station. The C/N ratio in the receiver for either uplink or downlink is calculated in dB units as

$$(C/N) = P_t + G_t + G_r / T_s - L_p - 10 \log k - 10 \log B_N$$

Since the uplink specification is given in terms of saturation flux density we need to convert path loss, $L_p$ to flux density, F:

$$F = P_t / (4\pi R^2) = P_t \times [\lambda / (4\pi R)]^2 \times 4\pi / \lambda^2 = P_t \times L_p \times 4\pi / \lambda^2$$

In decibel units

$$F = P_t + L_p - 10 \log (4\pi / \lambda^2)$$

Analyze the uplink first. For saturation of the transponder at f = 5985.5 MHz, $\lambda = 0.050121$ m

$$(C/N)_{up\ sat} = F_{sat} - 10 \log (4\pi / \lambda^2) + G_r / T_s - 10 \log k - 10 \log B_N$$

$$= -75.0 - 36.99 - 11.6 + 228.6 - 75.56 = 29.45 \text{ dB}$$

The operating $(C/N)_{up}$ value of the transponder is set by the transponder input back off BO

$$(C/N)_{up\ op} = 29.45 - BO \text{ dB}$$

Repeating the same analysis for the downlink at a frequency of 3760.5 MHz, $\lambda = 0.07978$ m and the saturated output power of the transponder

Path loss, $L_p = 20 \log (4 \pi R / \lambda) = 196.0$ dB

$(C/N)_{dn\ sat} = 29.0 + 41.0 - 196.0 + 228.6 - 75.56 = 27.04$ dB

The operating value of the downlink C/N ratio is

$(C/N)_{dn\ op} = 27.04 - BO$ dB

The intermodulation $(C/N)_I$ ratio is given by

$(C/N)_I = 7.86 + 0.714\ BO$ dB

The overall C/N ratio in the earth station receiver is found from the reciprocal formula

$(C/N)_o = 1 / [ 1 / (C/N)_{up} + 1 / (C/N)_{dn} + 1 / (C/N)_I ]$

Tabulate the values of each C/N ratio and calculate overall C/N as a function of back-off BO. Some iteration is required to find the best value of $(C/N)_o$.

| BO dB | $(C/N)_{up}$ dB | $(C/N)_{dn}$ dB | $(C/N)_I$ dB | $(C/N)_o$ dB |
|---|---|---|---|---|
| 1.0 | 28.45 | 26.06 | 8.57 | 8.45 |
| 5.0 | 24.45 | 22.06 | 11.43 | 10.87 |
| 10.0 | 19.45 | 17.06 | 15.0 | 12.03 |
| 15.0 | 14.45 | 12.06 | 18.57 | 9.51 |
| 12.5 | 16.95 | 14.56 | 16.78 | 11.18 |
| 11.25 | 18.20 | 15.81 | 15.89 | 11.73 |
| 10.63 | 18.83 | 16.44 | 15.45 | 11.91 |

The maximum value of overall $(C/N)_o$ ratio of 11.9 dB in the earth station receiver occurs when the transponder backoff is set to 10.6 dB.

**Problems 2 through 5 all involve a satellite and earth stations with the same specifications.**

Five earth stations share one transponder of a 6/4 GHz satellite. The satellite and earth station characteristics are given below:

| | | |
|---|---|---|
| *Satellite* | Transponder BW | = 36 MHz |
| | Transponder gain | = 90 dB (max) |
| | Input noise temp. | = 550 K |
| | Saturated output power | = 20 W (max) |
| | 4 GHz antenna gain | = 20.0 dB |
| | 6 GHz antenna gain | = 22.0 dB |
| *Earth station* | 4 GHz antenna gain | = 60.0 dB |
| | 6 GHz antenna gain | = 63.0 dB |
| | Receive Sye B | =1050 K |

4 n =1 96 dB

6 GHz, Ln n =2006 dB

**3.** Assume that the TDMA system uses a 125 μs frame time. Find the number of channels that each earth station can send within the TDMA frame when:

a. No time is lost in overheads, preambles, and the like.

**Answer:** Each 125 μs frame sends a total of 40 Mbps $\times$ 125μs = 5,000 bits from multiple speech channels. Each digital speech channel sends 8 bits every 125 μs

N = 5,000 / 8 = 625 channels.

b. A 5 μs preamble is added to the beginning of each earth station's transmission.

**Answer:** Each speech channel in the 125 ms frame is now divided into two parts: a 5 μs preamble and 8/40 μs of speech bits, giving a total of $5 \times 40 + 8 = 208$ bits/ speech channel. . Each frame sends 40 Mbps $\times$ 125 μs = 5000 bits. Hence

N = 5000 / 208 = 24 channels.

c. A 5 μs preamble is added to each station’s transmission and 2 μs guardband is allowed between every transmission.

**Answer:** Each speech channel is preceded by a 5 μs preamble and followed by a 2 μs gap when there is no transmission (the guard band). Overhead around each speech channel requires 7 μs transmission time; at 40 Mbps, the link sends 40 bits in 1 μs, so each speech channel now requires the equivalent of $7 \times 40 + 8 = 288$ bits. Hence

N = 5000 / 288 = 17 channels

(rounded down from N = 17.36 – we cannot send partial channels).

**4.** A 750 μs frame time is used instead of a 125 μs frame. Find the new channel capacities of the earth stations for the cases in Problem 3 above.

**Answer:** The longer frame reduces the impact of overhead, but requires more bits per speech channel in each frame. In 750 ms, each speech channel delivers $750 / 125 \times 8 = 48$ bits. Each frame must contain 48 bits from each speech channel. Each 750 μs frame delivers a total of 750 μs $\times$ 40 Mbps = 30,000 bits.

With no overhead, N = 30,000 / 48 = 625 channels.

With 5 μs lost to a preamble before each 48 bit speech transmission, , the frame can carry 30,000 / 248 = 120 channels

With 2 μs guard bands between speech transmissions and a 5 μs preamble before each transmission, 80 bits are lost in guard times and 200 bits are used for preamble, so each speech channel requires the equivalent of 200 + 48 + 80 = 328 bits.

Hence N = 30,000 / 328 = 91 channels. (N = 91.46, so there is an extended guard time at the end of each frame of $2 + 0.54 \times 328 / 40 = 6.428$ μs. )

The reduced effect of overhead is well illustrated in the last value of N. The 125 μs frame delivered only 17 channels.

**5.** Find the earth station transmitter power and received (C/N) when the system is operated:

a. In TDMA with the transponder saturated by each earth station in turn.

**Answer:** We will assume that the transponder is set to its maximum gain of 90 dB.

The transponder is accessed by each TDMA earth station in sequence, and outputs its saturated output power of 20 W = 13 dBW.. Thus each earth station must achieve a transponder input power $P_r$ = 13 – 90 = -77.0 dBW.

The uplink power budget allows us to find transmitter power $P_t$ for the 6 GHz uplink. Assuming an earth station in the center of the satellite antenna footprint and path loss $L_p$ = 200 dB, with no other losses:

$$P_r = -77.0 = P_t + 63.0 + 22.0 - 200$$

$$P_t = 38 \text{ dBW or } 6310 \text{ W.}$$

Each of the transmitting stations must send bursts of signal at this high transmitter output power.

The transponder has a receiving system noise temperature of $T_s$ = 550 K. The 40 Mbps signals are sent using QPSK, so the symbol rate is 20 Msps, and assuming ideal RRC filters in the receivers, $B_N$ = 20 MHz. Hence

$$N_{xp} = k\, T_s\, B_N = -228.6 + 27.4 + 73 = -128.2 \text{ dBW.}$$

The received power level was – 77 dBW at the transponder input so

$$(C/N)_{up} = -77 + 128.2 = 51.2 \text{ dB.}$$

On the downlink, the transponder is operated at its saturated output power of 20 W or 13 dBW. The received power at the earth station input is found from the downlink power budget. Assuming again an earth station at the center of the satellite footprint

$P_r = 13.0 + 20.0 + 61.0 – 196.0 = -102.0$ dBW

The system noise temperature of the receiver is 100 K, hence

$N_{es} = -228.6 + 20.0 + 73.0 = -135.6$ dBW

$(C/N)_{dn} = -102.0 + 135.6 = 33.6$ dB.

Combining the uplink and downlink C/N ratios

$(C/N)_o = 1 / ( 1/(C/N)_{up} + 1/ (C/N)_{dn}) = 2252$ or 33.5 dB

b. In FDMA with 3-dB input and output back-off.

**Answer:** The frequency band is divided into five equal bands, each band allocated to a different earth station. The satellite transponder output power must also be divided equally between the five earth stations. Thus each earth station transmits a 40/5 = 8 Mbps bit stream using QPSK giving a 4 Msps symbol rate and receiver noise bandwidth of 4 MHz. With 3 dB output back off the transponder transmits a total of 10 W, or 2 W per earth station channel. With the same assumptions as for the TDMA system, the uplink earth station transmit power $P_t$ and uplink C/N ratio are given by

$P_r = 3.0 – 90.0 = -87.0 \text{ dBW} = P_t + 63.0 + 22.0 - 200.0$

$P_t = 28.0$ dBW or 631 W.

The noise power of a 4 MHz noise bandwidth channel on the satellite is 7 dB less than the noise power for a 20 MHz channel, so $N = -128.2 - 7.0 = -135.2$ dBW.

$(C/N)_{up} = -87.0 + 135.2 = 48.2$ dB.

The downlink has transponder output power of 2 W per earth station channel and noise bandwidth of 4 MHz giving $N_{es} = -135.6 – 7 = 142.6$ dBW. Hence received power and $(C/N)_{dn}$ are

$P_r = 3.0 + 20.0 + 60.0 - 196.0 = -113.0$ dBW

$(C/N)_{dn} = -113.0 + 142.6 = 29.6$ dB

Combining the uplink and downlink C/N ratios

$(C/N)_o = 1 / ( 1/(C/N)_{up} + 1/ (C/N)_{dn}) = 900$ or 29.5 dB

**6.** A digital communication system uses a satellite transponder with a bandwidth of 54 MHz. Several earth stations share the transponder using QPSK modulation using either FDMA or TDMA. Standard message data rates used in the system are 80 kbps and 2.0 Mbps. The transmitters and receivers in the system all use ideal RRC filters with $\alpha = 0.25$, and FDMA channels in the satellite are separated by 100 kHz guard bands. When TDMA is used, the TDMA frame is 125 μs in length, and a 2 μs guard time is required between each access. A preamble of 148 bits must be sent by each earth station at the start of each transmitted data burst.

a. What are the symbol rates for the 80 kbps and 2.0 Mbps QPSK signals sent using FDMA?

**Answer:** Symbol rates for QPSK are one half the bit rate.

For $R_b = 80$ kbps, $R_s = 40$ ksps

$R_b = 2.0$ Mbps, $R_s = 1.0$ Msps

b. What is the symbol rate of each earth station's transmitted data burst when TDMA is used?

**Answer:** Each earth station must transmit at a burst rate that occupies the transponder bandwidth of 54 MHz. Digital signals transmitted with RRC filters with $\alpha = 0.25$ have an occupied bandwidth $B_{occ} = R_s ( 1 + \alpha)$. Hence $R_s = B_{occ} / ( 1 + \alpha) = 54 / 1.25 = 43.2$ Msps. Each station must transmit QPSK signals at a symbol rate of 43.2 Msps, giving a bit rate of 86.4 Mbps.

c. Calculate the number of earth stations that can be served by the transponder when 80 kbps channels are sent using (i) FDMA and (ii) TDMA.

**Answer:** **FDMA:** Each station requires a RF bandwidth given by

$B_{RF} = 1.25 \times R_s + 100$ kHz

For $R_b = 80$ kbps, $R_s = 40$ ksps, $B_{RF} = 50 + 100 = 150$ kHz.

$N = 54.0 / 0.150 = 360$ channels

**TDMA:** All TDMA transmissions are made at a burst rate of 43.2 Msps, giving a bit rate of 86.4 Mbps. A 2 μs guard time is required after each transmission and 148 bits of preamble are sent at the start of each transmission. The 2 μs guard time is equivalent to $2 \times 86.4 = 172.8$ bits. The signal rate of 80 kbps requires $125\ \mu s \times 0.08$ Mbps $= 10$ bits of data per frame.

Each station burst sends the equivalent of $148 + 10 + 172.8 = 330.8$ bits.

Hence the number of channels in a 125 μs frame of $125 \times 86.4 = 10{,}800$ bits is

$N = 10{,}800 / 330.8 = 32$ stations.

d. Calculate the number of earth stations that can be served by the transponder when 2.0 Mbps channels are sent using (i) FDMA and (ii) TDMA.

**Answer: FDMA:** For $R_b = 2.0$ Mbps, $R_s = 1.0$ Msps. The RF bandwidth required for each channel is $B_{RF} = 1.0 + 0.1 = 1.1$ MHz. Hence the number of earth stations that can share the transponder is

$$N = 54.0 / 1.1 = 49 \text{ channels}$$

**TDMA:** When the bit rate is 2.0 Mbps per station, each earth station transmission sends $125 \times 2 = 250$ data bits. Each transmission contains the equivalent of $148 + 250 + 172.8 = 570.8$ bits. Hence the number of earth stations that can share the transponder is

$$N = 10{,}800 / 570.8 = 18 \text{ stations}$$

FDMA is clearly a much more effective method when the data transmission requirements of each station are small. The short frame time makes the system inefficient, even for the 2 Mbps data transmission rate.

**7.** The capacity of the TDMA system described in Problem 6 can be increased substantially by using satellite switched TDMA. In a group of earth stations, each station sends a 2.0 Mbps signal to every other earth station in every frame. It takes one microsecond to reposition the satellite antenna beam from one earth station to another. Only the downlink antenna beam is switched; the uplink uses a common zone beam. The frame length to be used is 1000 μs, with a 148 bit preamble and 2 μs guard times between transmissions arriving at the satellite The extra antenna gain at the satellite is traded for an increase in the data rate by using 16-QAM on the downlink. Other parameters of the system are unchanged.

a. Find the number of earth stations that can share the transponder.

**Answer:** Several parameters have changed from problem #6. The downlink is now using 16-QAM which sends 4 bits per symbol. Thus the bit rate for the downlinks is

$$R_b = 43.2 \times 4 = 172.8 \text{ Mbps.}$$

Downlink frames consist of 1000 μs frames at 172.8 Mbps, giving 172,800 bits/frame. Each earth station is transmitting at 2.0 Mbps, so it must send 2000 bits in each 1000 μs frame.

Transmissions consist of bursts from N uplink stations, with each burst containing (N-1) transmissions of 2000 / (N-1) bits to each of the other (N-1) earth stations. Each uplink burst requires a guard time of 2 μs and each downlink transmission requires 148 bits of preamble at the start of each burst and a 1 μs beam repositioning time for each transmission to (N - 1) other earth stations. Using equivalent bits, the 2 μs guard time is equivalent to 345.6 bits and the 1 μs beam repositioning time is equivalent to 172.8 bits. There are 2000 / (N-1) data bits transmitted to each of the other (N-1) earth stations by each of the N earth stations in each frame. The preamble must be sent to each of the receiving earth stations so that it can synchronize its symbol and bit clocks to the received signal.

Hence the frame of 172,800 bits is built up as N bursts from the N transmitting stations. Each burst contains (N-1) packets. Each packet contains a transmission to the (N - 1) other stations, with a preamble of 148 bits, and a data transmission of 2000/(N-1) bits followed by a beam repositioning time equivalent to 172.8 bits. After each of the N bursts there is a guard time equivalent to 345.6 bits. Hence the frame equation is

$$172{,}800 = N \times [\ (N-1) \times (\ 148 + (2000 / (N - 1) + 172.8) + 345.6)\ ]$$

$$172{,}800 = 2024.8\ N + 320.8\ N^2$$

$$320.8\ N^2 + 2024.8\ N - 172{,}800 = 0$$

or $$N_2 + 6.312\ N - 538.65 = 0$$

Solving the quadratic equation gives $N = 20\ (.266)$

There are 20 stations in the network.

**Check**: It is easy to make errors in such calculation and therefore wise to check that the frame works correctly with 20 stations.

Each station transmits 2000 data bits in each frame, split into packets of 105 bits for each of the other 19 earth stations. Each 105 data bit packet has the equivalent of 148 bits of preamble and 172.8 bits to reposition the beam added to give a packet of 425.8 bits in duration. There are 19 of these packets in each burst from each earth station, so the burst consists of the equivalent of $19 \times 425.8 = 8090.2$. bits. There are 20 bursts of 8090.2 bits (equivalent) followed by a 2 μs guard time, equivalent to 345.6 bits. Hence the total number of bits in a frame is X where

$$X = 20 \times (8090.2 + 345.6) = 168{,}716 \text{ bits.}$$

The remaining 4084 bits form an additional guard time of 23 μs at the end of the TDMA frame.

There is not exact agreement between this check calculation and the original calculation with regard to the time at the end of the frame because of the use of fractional bits.

b. Find the total data throughput of the transponder after all preamble bits have been removed.

**Answer:** The data bits transmitted in each frame add up to twenty signals at 2 Mbps. Hence the total data bits transmitted each second is $20 \times 2$ Mbps = 40 Mbps.

Given that the downlink operates at 172.8 Mbps, the link is inefficient.

**8**. A LEO satellite system transmits compressed digital voice signals to handheld terminals (satphones). The satphones work in groups of ten. The inbound bit stream from the satphone to the satellite is at 10 kbps. The data are sent as a BPSK signal. The outbound bit stream from the satellite is at a bit rate of 100 kbps, and consists of packets addressed to each of ten satphones. This signal is sent using QPSK, and all ten satphones receive the 100 kbps bit stream.

The system operates in L-band where rain fading can be ignored, but blockage from buildings and trees is a significant factor. The satellite uses on-board processing and multi-beam antennas. The links use square root raised cosine (RRC) filters with $\alpha = 0.5$. In this question we will be concerned only with the links between the satellite and the satphones, and ideal RRC filters will be assumed.

**a.** What is the noise bandwidth of the narrowest bandpass filter in:

(i) the satphone receiver (ii) the satellite receiver for the inbound link

**Answer: (i)** The inbound channel from the satphone to the satellite carries a 10 kbps signal with BPSK modulation. Hence the symbol rate is $R_s$ = 10 ksps and the noise bandwidth of the receiver RRC filter (in the satellite) is 10 kHz. This is a SCPC link.

**(ii)** The outbound channel from the satellite to the satphone carries a 100 kbps signal with QPSK modulation. Hence the symbol rate is $R_s$ = 50 ksps and the noise bandwidth of the receiver RRC filter (in the satphone) is 50 kHz. This is a TDM link.

**b.** What is occupied RF bandwidth of the radio signals of:

(i) the inbound link (phone to satellite) (ii) the outbound link (satellite to phone)

**Answer:** Occupied bandwidth is always equal to $R_s \times (1 + \alpha)$ when RRC filters are used.

Hence

(i) $B_{occ} = 10\ k \times 1.5 = 15$ kHz (ii) $B_{occ} = 50\ k \times 1.5 = 75$ kHz.

**c.** The inbound link has clear air $(C/N)_o = 18.0$ dB and the BPSK demodulator on the satellite has an implementation margin of 0.5 dB. What is the clear air BER in the baseband of the satellite receiver?

**Answer:** The bit error rate with BPSK modulation is given by $P_e = Q[\sqrt{(2\ C/N_{eff})}]$.

For C/N = 18.0 dB and an implementation margin of 0.5 dB, $(C/N)_{eff} = 17.5$ dB or 56.23.

Hence BER = $Q[\sqrt{112.46}] = Q[10.60] \ll 10^{-16}$. There are no errors on this link in clear air.

**d.** What is the available fade margin ( for $(C/N)_o$ on the uplink to the satellite) if the inbound link operating threshold is set at BER = $10^{-4}$?

**Answer:** Overall C/N ratio for the uplink (inbound) is the same as $(C/N)_{up}$ because we have a baseband processor on the satellite. For $P_e = 10^{-4}$ with a BPSK link we require

$Q[\sqrt{(2\ C/N_{eff})}] = 10^{-4}$. Hence from the Q(z) table, $[\sqrt{(2\ C/N_{eff})}] = 3.70$,

$C/N_{eff} = 6.85$ or 8.4 dB and $(C/N)_{up} = 8.9$ dB. The uplink fade margin is:

Fade margin = 18.0 – 8.9 = 9.1 dB.

**e.** The outbound link has clear air $(C/N)_o = 18.0$ dB and the QPSK demodulator in the satellite phone has an implementation margin of 0.8 dB. What is the clear air BER?

**Answer:** The bit error rate with QPSK modulation is given by $P_e = Q[\sqrt{(C/N_{eff})}]$.

For C/N = 18.0 dB and an implementation margin of 0.8 dB, $(C/N)_{eff} = 17.2$ dB or 52.48.

Hence BER = $Q[\sqrt{52.48}] = Q[7.24] \approx 2.2 \times 10^{-13}$. With a bit rate of 100 kbps, there are no errors on this link.

**f.** What is the available fade margin ( for the overall $(C/N)_o$ on the downlink to the satphone ) if the outbound link operating threshold is set at BER = $10^{-5}$ ?

Overall C/N ratio for the downlink (outbound) is the same as $(C/N)_{dn}$ because we have a baseband processor on the satellite. For $P_e = 10^{-5}$ with a baseband link we require

$Q[\sqrt{(C/N_{eff})}] = 10^{-5}$. Hence from the Q(z) table, $[\sqrt{(C/N_{eff})}] = 4.23$,

$C/N_{eff} = 17.89$ or 12.5 dB. Hence $(C/N)_{dn} = 13.3$ dB. The fade margin is

Fade margin = 18.0 – 13.3 = 4.7 dB.

**9**. A Ka band satellite broadcasts digital television signals over the United States. The nominal bit rate of the signal is 28 Mbps. The digital signal can convey up to ten pre-recorded NTSC video signals. QPSK modulation is used, and error mitigation techniques are employed that provide an effective coding gain of 6 dB. [Coding gain of 6 dB means that when the $(C/N)_o$ value of the received signal is X dB, the BER corresponds to C/N = (X + 6) dB.]

The QPSK demodulator in the receiver has an implementation margin of 1.6 dB. The transmitters and receivers use ideal RRC filters with $\alpha = 0.25$.

**a.** What is the occupied bandwidth of the RF TV signal?

**Answer:** The signal is transmitted at 28 Mbps (this rate includes error coding) using QPSK modulation. The symbol rate is $R_s = 28 / 2 = 14$ Msps. Hence the occupied bandwidth is

$$B_{occ} = R_s \times (1 + \alpha) = 14 \times 1.25 = 17.5 \text{ MHz}.$$

**b.** What is the symbol rate of the transmitted QPSK signal, and the noise bandwidth of the earth terminal receiver?

**Answer:** The signal is transmitted at 28 Mbps (this rate includes error coding) using QPSK modulation. The symbol rate is $R_s = 28 / 2 = 14$ Msps. The noise bandwidth of any digital signal when RRC filters are used is equal to the symbol rate. Hence $B_N = 14$ MHz.

**c.** The minimum permitted BER after error mitigation in the receiver is $10^{-6}$. What is the minimum permitted $(C/N)_o$ for the digital TV receiver?

**Answer:** The bit error rate for QPSK signals is $P_e = Q[\sqrt{(C/N_{eff})}]$ .

For BER = $10^{-6}$ we require $Q(z) = 10^{-6}$, $z = \sqrt{(C/N_{eff})} = 4.76$. Hence $C/N_{eff} = 22.26$ or 13.5 dB. With 1.6 dB implementation margin and 6 dB coding gain, the minimum overall C/N ratio is

$$C/N = 13.5 + 1.6 - 6.0 = 9.1 \text{ dB}.$$

**d.** The Ka-band link suffers rain attenuation that reduces $(C/N)_o$ in the receiver by 7 dB for 0.1% of the year. If the BER is $10^{-6}$ under the 0.1% year conditions, what is the clear air

$(C/N)_o$ value?

**Answer:** Clear air C/N = 9.1 + 7 = 16.1 dB.

$(C/N)_{eff}$ = 16.1 – 1.6 + 6.0 = 20.5 dB or 112.2.

The bit error rate is given by $P_e = Q[\sqrt{(C/N_{eff})}] = Q(10.59) << 10^{-16}$. There are no errors on this link in clear air conditions.

**e.** A new coding algorithm is developed that provides a coding gain of 7 dB with a bit rate that increases to 30 Mbps. Assuming that the RRC filters in the system can be changed to match the new symbol rate, does implementation of the new coding algorithm improve the system performance? If so, what is the new $(C/N)_o$ margin?

**Answer:** Increasing the bit rate to 30 MHz gives a new symbol rate $R_s$ = 15 Msps and requires a receiver noise bandwidth of 15 MHz. Noise power in the receiver increases by a factor 10 log (15/14) = 0.3 dB, and C/N is therefore reduced by 0.3 dB form 16.1 dB to 15.9 dB. The new coding gain is 7.0 dB, which exceeds the drop in C/N ratio by 0.7 dB. Hence the effective C/N ratio on the link improves by 0.7 dB and system performance improves. There are no errors in clear air, but for the 0.1% time condition when rain attenuation causes $(C/N)_o$ to fall by 7 dB, the new C/N value in rain is 15.9 – 7.0 = 8.9 dB.

$(C/N)_{eff}$ = 8.9 + 7 – 1.6 = 14.4 dB, BER = $Q[\sqrt{(C/N_{eff})}] = Q(\sqrt{26.91}) = Q(5.19)$

BER = $10^{-7}$. This is a worthwhile improvement on the previous value of $10^{-6}$.

$(C/N)_o$ margin is the difference between the minimum permitted value of $(C/N)_o$ and the clear air value. Hence $(C/N)_o$ margin = 16.1 – (13.5 + 1.6 – 7.0) = 8.0 dB.

**10.** This problem is about multiple access techniques in the inbound link of a VSAT network. This set of questions compares the operation of a Ku band satellite transponder in FDMA, in TDMA, and in FDMA-RA. There are three parts to the problem.

**Part 1**

100 VSAT stations in a star network share one 54 MHz transponder using FDMA. Each VSAT station has a solid state transmitter with an output power of 1 watt and an EIRP of 41 dBW from a 1.1 m diameter antenna. The transmitted data signals have a bit rate of 128 kbps and are transmitted using QPSK modulation and half rate FEC, giving a symbol rate of 128

ksps. At the hub station, the overall C/N ratio for each signal received from a VSAT station is 16 dB in clear air.

The $(C/N)_{up}$ ratio for one channel in the satellite transponder is 19.0 dB, and the $(C/N)_{dn}$ ratio for one channel in the hub receiver is 19.0 dB. The threshold C/N ratio in any hub station receiver for BER = $10^{-6}$ is 9.0 dB. This includes the receiver implementation margin of 0.5 dB.

The stations share the transponder using FDMA, with 51 kHz guard bands between the edges of the RF signals. The RRC filters used in the VSAT transmitters and the hub station receivers have a roll off factor $\alpha = 0.4$. To minimize intermodulation between signals, the transponder is operated with 3 dB output back-off.

**a.** Calculate the RF bandwidth occupied by each VSAT transmission.

**Answer:** The VSAT transmission rate is 128 kbps and modulation is QPSK, with half rate forward error correction encoding.. Symbol rate is 128 ksps, and RRC filters with $\alpha = 0.4$ are used. Hence the occupied bandwidth of the VSAT transmitted signal is

$$B_{occ} = R_s \times (1 + \alpha) = 1.4 \times 128 \text{ k} = 179.2 \text{ kHz.}$$

**b.** Calculate the maximum number of VSAT stations that can be included in the network if the transponder is bandwidth limited.

**Answer:** The transponder has a bandwidth of 54 MHz. If all this bandwidth is occupied by VSAT signals, with 51 kHz guard bands, each signal requires 179.2 + 51 = 230.2 kHz and the maximum possible number of VSAT channels in one transponder is given by

$$N = 54 \text{ MHz} / 230.2 \text{ kHz} = 236 \text{ (.6)}$$

**c.** Calculate the clear air C/N ratio for a received signal at the hub station, and the link margin, if the number of VSAT stations in the network is increased to the number you calculated in (b) above. Remember that the power available from the transponder is fixed. Adding more stations to the network lowers the power per channel at the transponder output.

**Answer:** The uplink $(C/N)_{up}$ ratio for any VSAT signal in the satellite transponder is 19.0 dB, independent of the number of signals transmitted to the transponder. Each uplink signal is operating in SCPC-FDMA mode. The satellite must share transponder output power between

234 signals. With 100 signals, $(C/N)_{dn}$ in the hub receivers was 19.0 dB. With 234 signals in the transponder, $(C/N)_{dn}$ is reduced by 10 log (100 / 234) or -3.7 dB. Hence $(C/N)_{dn}$ = 15.3 dB. Overall $(C/N)_o$ in the hub station receiver in clear air conditions is

$$(C/N)_o = 1 / [ 1 / (C/N)_{up} + 1 / (C/N_{dn} ] = 23.75 \text{ or } 13.8 \text{ dB.}$$

The threshold for $(C/N)_o$ in the hub receiver is 9.0 dB. There are two link margins, one for the uplink and one for the downlink.

The limiting condition is overall C/N ratio at the hub station receiver = 9.0 dB.

For the uplink, $(C/N)_{up}$ is given by

$$(C/N)_o = 1 / [ 1 / (C/N)_{up} + 1 / (C/N_{dn} ] = 9.0 \text{ dB} = 7.94$$

With linear transponder operation (backoff = 3 dB), $(C/N)_o$ falls in proportion to the reduction in $(C/N)_{up}$. Hence the uplink margin is 13.8 - 9.0 = 4.8 dB.

For the downlink, $(C/N)_{dn}$ is given by

$$(C/N)_o = 1 / [ 1 / (C/N)_{up} + 1 / (C/N_{dn} ] = 9.0 \text{ dB} = 7.94$$

$(C/N)_{up}$ = 19.0 dB because we assume the uplink operates in clear air.

Hence $1 / [ 1 / 79.43 + 1 / (C/N)_{dn} ] = 7.94$ and $(C/N)_{dn} = 8.82$ or 9.5 dB

Hence the downlink margin is 13.8 - 9.5 = 4.3 dB.

**Part 2**

The VSAT network described in Part1 is modified to be operate with TDMA on the VSAT uplinks instead of FDMA. There are 100 VSAT stations in the network.

The TDMA frame has a duration of 2 ms and is made up of 100 bursts from the 100 VSAT stations. There is a preamble of 100 symbols at the start of each VSAT station burst, and each burst is separated from the next burst by a guard time of 1.0 μs.

**a.** There are 100 VSAT station RF bursts in each frame of 2.0 ms, and 100 guard times of 1.0 μs. What is the duration of each station's burst?

**Answer:** The available time for VSAT transmissions in the 2 ms frame is

$$T_{VSATs} = 2 \text{ ms} – 100\ \mu\text{s} = 1900\ \mu\text{s}$$

100 VSAT stations share the frame, so each burst lasts for 1900 / 100 = 19 μs.

**b.** Each VSAT station must deliver 128 kbps of data, in the form of 128 k symbols, every second. How many data symbols are there in each RF burst, and what is the total number of symbols per burst after accounting for the 100 symbol preamble at the beginning of each burst? Hence find the burst rate for the VSAT transmissions in symbols per second.

**Answer:** There are 500 TDMA frames each second. The VSAT stations must deliver 128 kb each second, which requires 128,000 / 500 = 256 data bits per burst. There are 100 preamble symbols in each burst, corresponding to 200 preamble bits, giving a total of 456 bits/burst, or 228 symbols. Hence the symbol rate for each VSAT is 228 symbols in 19 $\mu$s = 12.0 Msps.

**c.** If all the VSAT stations, and the hub receiver, have RRC filters with roll off factor $\alpha = 0.4$, what is the RF bandwidth occupied in the transponder?

If the symbol rate of transmissions were increased until all 54 MHz bandwidth of the transponder were filled, what is the maximum number of VSAT stations in the network?

**Answer:** Each VSAT station transmits bursts at a symbol rate $R_s = 12.0$ Msps. Bandwidth occupied in the satellite transponder is $R_s \times (1 + \alpha) = 1.4 \times 12$ M = 16.8 MHz.

The transponder has a bandwidth of 54 MHz so we could increase the symbol rate to 54/1.4 = 38.57 Msps if we wanted to fill the bandwidth completely. This would allow $100 \times 38.57 / 16.8$ = 229 stations in the TDMA network.

**d.** The transponder can be operated with 1 dB output back-off when TDMA is used, and the implementation margin of the hub receiver is 1.5 dB . The EIRP of the VSAT stations must be increased because the noise bandwidth of the hub receiver has increased.

By comparing the symbol rate with 100 FDMA VSAT stations in Q#1 with the TDMA symbol rate for 100 VSAT stations in part (b) above, estimate the decibel increase in EIRP required from each VSAT transmitter.

Comment on the feasibility of transmitting this power level from a VSAT station.

**Answer:** Let's assume that we want to achieve the same clear air performance with the TDMA network that was available when the networks used FDMA, in Part 1.

The FDMA system has $(C/N)_{up}$ = 19.0 dB with a 128 kbps signal at 128 ksps. The RRC filter noise bandwidth for this signal is 128 kHz. If we increase the noise bandwidth to

38.57 MHz for the TDMA signal, the transmit EIRP of the VSAT must increase by a factor of 38.57 / 128 = 431.1. Either we must increase VSAT transmit power to 431 W, or increase antenna diameter to $\sqrt{431.1} \times 1.1 = 22.84$ m, or some combination that gives the same EIRP. None of these combinations would classify as a VSAT station and all are clearly impractical.

On the downlink to the hub station, when operated in FDMA, the satellite achieved $(C/N)_{dn}$ = 19.0 dB with 100 signals at 128 ksps sharing the transponder power. This is equivalent to one signal at 12.8 Mbps. We have a single 38.67 Msps TDMA signal transmitted through the satellite, so the receiver noise bandwidth must be 38.57 MHz, giving a reduction in $(C/N)_{dn}$ of 4.8 dB. Backoff at the transponder output is reduced by 2 dB, from 3 dB to 1 dB, so the actual reduction in $(C/N)_{dn}$ is 2.8 dB, from 19.0 dB to 16.2 dB. Overall $(C/N)_o$ in the hub station in clear air is then

$$(C/N)_o = 1 / [\, 1 / (C/N)_{up} + 1 / (C/N)_{dn} \,] = 27.34 \text{ or } 14.4 \text{ dB.}$$

The system is clearly impracticable because of the high VSAT transmitter EIRP required to operate the network in TDMA. VSAT networks operate their inbound uplinks in SCPC-FDMA, or occasionally, given sufficient link margin, MF-TDMA grouping a few VSAT stations into a small TDMA frame.

**Part 3**

The FDMA system described in Part1 is used with random access to serve a very large number of VSAT stations. All the parameters of Part1 are the same, except that each station has a small amount of data to send at varying intervals of time. The average message data rate for each VSAT station is 5.0 kbps and the maximum permitted channel loading is 12%.

**a.** How many VSAT stations can share each RF frequency?

**Answer:** The VSAT inbound channels have a bit rate of 128 kbps. At 12% loading the transmission rate averages $0.12 \times 128\text{k} = 15.36$ kbps. Each station must transmit an average of 5 kbps, so three stations can share one RF channel.

**b.** What is the maximum number of VSAT stations in the network when the number of RF channels is the value you calculated in Part 1 (c)?

**Answer:** There were a maximum of 234 VSAT stations in the network in part 1 c. With random access, this number increases to $3 \times 234 = 702$ stations.

**11.** This problem examines the use of a Ka-band satellite to provide connection to the Internet from a small two-way terminal. The problem is in three parts. The first part establishes the design of the communications links and terminals. The second part examines the capacity of the satellite. The third part looks at changes that must be made to support portable terminals.

**Part 1. Communication Links**

**Description of the Satellite Communication System**

A Ka-band GEO satellite is located at longitude $100^o$ W. Star networks can be built with a single hub station, two transponders on the GEO satellite, and a number of earth stations, identified here as VSATs. The major parameters of the system components are given below. You may not need all of these parameters to answer the questions, and additional parameters are given in the individual questions.

The Ka-band satellite serves the United States. Coverage of the 48 contiguous states is achieved by a regional beam, but the satellite also carries an advanced antenna system with satellite switched spot beams that allow data packets to be transmitted to small earth stations with a high EIRP. This allows high speed data transmission in the outbound link.

The system is designed primarily to support Internet access via satellite, with highly asymmetrical links. Requests for access to the Internet are made by users at a low data rate through the satellite's region beam. Replies from the Internet can be received at a high data rate using the satellite's spot beam.

**System Values**

| | |
|---|---|
| Uplink frequency for transponder #1 | 28.2 GHz |
| Downlink frequency for transponder #1 | 21.7 GHz |
| Uplink frequency for transponder #2 | 28.1 GHz |
| Downlink frequency for transponder #2 | 21.6 GHz |
| Range to satellite (all stations) | 38,000 km |

**Satellite Transponders**

| | |
|---|---|
| Saturated output power | 30 W |
| Transponder bandwidth | 54 MHz |
| Transponder input noise temperature | 500 K |
| Antenna gain, on axis, regional beam | 33 dB |
| Antenna gain, on axis, switched spot beam | 48 dB |

**VSAT Station Parameters**

| | |
|---|---|
| Transmitter output power | 1.0 W |
| Transmit frequency | 28.2 GHz |
| Receive frequency | 21.7 GHz |
| Antenna diameter | 0.5 m |
| Aperture efficiency | 65 % |
| Receiver system noise temperature (clear air) | 250 K |
| Receiver system noise bandwidth | TBD |

**Hub Station Parameters**

| | |
|---|---|
| Maximum transmit power | 100 W |
| Transmit frequency | 28.1 GHz |
| Receive frequency | 21.6 GHz |
| Receiver system noise temperature (clear air) | 250 K |
| Antenna diameter | 4.0 m |
| Aperture efficiency | 65 % |
| Receiver system noise bandwidth | TBD |

**Atmospheric Losses and Miscellaneous Losses**

| | |
|---|---|
| In clear air at 28 GHz | 2.0 dB |
| In clear air at 21 GHz | 2.0 dB |

**Constants:** Boltzmann's constant, k, = $1.38 \times 10^{-23}$ J/K = -228.6 dBW/K/Hz

**Part 1 Problems: C/N ratios in clear air conditions**

Make all calculations for the worst case of a VSAT station that is located on the – 3dB contour of the satellite antenna beam (regional or spot), and for a hub station on the –2 dB contour of the regional beam. The spot beam is used only for transmissions at 21GHz from the satellite to the customers' earth stations. All other links use the satellite's regional beam.

a. Calculate the free space path loss for a 38,000 km path at 28.2 GHz and 21.7 GHz.

**Answer:** At $f = 28.2$ GHz, $\lambda = 0.01064$ m

$$L_{p28} = 20 \log ( 4 \pi R / \lambda) = 213.0 \text{ dB}$$

Scaling to 21.7 GHz, $L_{p21} = 213.04 – 2.28 = 210.8$ dB

The same path loss values will be used for the 28.1 GHz and 21.6 GHz links.

b. Calculate the gains of the hub and VSAT antennas at frequencies of 28.2 GHz and 21.7 GHz.

**Answer:** The hub station antenna has a diameter of 4.0 m and an aperture efficiency of 65%. At 28.2 GHz the hub station antenna gain is

$$G_{28H} = \eta \times (\pi D / \lambda)^2 = 0.65 \times (\pi \times 4.0 / 0.01064) = 906{,}671 \text{ or } 59.57 \text{ dB} \approx 59.6 \text{ dB.}$$

Scaling to 21.7 GHz, $G_{21H} = 59.57 - 2.28 \approx 57.3$ dB. We will use the same antenna gains for links at 28.1 GHz and 21.6 GHz.

The VSAT station has an antenna diameter of 0.5 m and an aperture efficiency of 65%. Scaling the gain of the hub station antenna gives, for the VSAT antenna

$$G_{28V} = 59.57 \text{ dB} - 20 \log (4.0 / 0.5) = 59.57 - 18.06 = 41.51 \approx 41.5 \text{ dB}$$

$$G_{21V} = 57.29 \text{ dB} - 20 \log (4.0 / 0.5) = 59.29 - 18.06 = 39.23 \approx 39.2 \text{ dB}$$

**c.** Calculate the inbound overall C/N in the hub station receiver in a noise bandwidth of 128 kHz when the VSAT has a transmitter output power of 1 watt and accesses the regional beam on the satellite. Make the overall C/N calculation for a single QPSK signal which is transmitted by transponder #1 at an output power of 1 watt.

**Answer:** VSAT stations transmit to the satellite in SCPC-FDMA mode. The VSAT EIRP at 28.1 GHz is 1 watt + 41.5 dB = 41.5 dBW. Path loss to the satellite is $L_{p28} = 213.0$ dB. The uplink budget follows:

| | | |
|---|---|---|
| EIRP | | 41.5 dBW |
| $G_r$ | Regional beam, on axis | 33.0 dB |
| Path loss at 28.2 GHz | | -213.0 dB |
| Losses 3 dB off axis + 2 dB atmospheric | | -5.0 dB |
| | Receiver power $P_{rxp}$ | -143.5 dBW |

The satellite transponder input noise temperature is 500 K = 27 dBK. Channel noise bandwidth for the VSAT signal is 128 kHz. Hence $N_{xp} = -228.6 + 27.0 + 51.1 = -150.5$ dBW

Hence uplink (transponder) $(C/N)_{up}$ ratio is -143.5 + 150.5 = 7.0 dB.

The downlink $(C/N)_{dn}$ ratio in the hub station receiver is calculated for a trial value of 1 W transponder output power. This is to establish C/N ratios that can later be modified in the final system design. For a satellite transmit power of 1 W and a hub station on the 2 dB contour of the satellite regional beam footprint, the downlink budget at 21.7 GHz is

| | |
|---|---|
| EIRP | 33.0 dBW |
| $G_r$ Hub station at 21.7 GHz | 57.3 dB |
| Path loss at 21.7 GHz | -210.8 dB |
| Losses 2 dB off axis + 2 dB atmospheric | -4.0 dB |
| Receiver power $P_{rxp}$ | -124.5 dBW |

The hub station receiver system noise temperature is 250 K = 24 dBK. Receiver noise bandwidth for the hub station 128 kHz.

Hence $N_{xp}$ = -228.6 + 24.0 + 51.1 = -153.5 dBW and $(C/N)_{dn}$ = 29.0 dB.

Overall $(C/N)_o$ for the inbound link is

$$(C/N)_o = 1 / [\, 1 / (C/N)_{up} + 1 / (C/N)_{dn} \,] = 4.98 \approx 7.0 \text{ dB.}$$

**d.** Calculate the outbound overall C/N in transponder #2 with a hub station transmit power of 1 watt. Make your calculation in a receiver noise bandwidth of 1 MHz, for a single QPSK signal, with the output power of transponder #2 set at 1 watt and the spot beam of the satellite transmitting to the customers' terminals.

Estimate the beamwidth of the spot beam from the satellite. Using a map of the United States, estimate the minimum number of spot beam positions required to serve the entire US.

**Answer:** The hub station transmits to transponder #2 on the satellite in SCPC-FDMA mode. The EIRP of the hub station at 28.1 GHz is 1 watt + 59.6 dB = 59.6 dBW. Path loss to the satellite is $L_{p28}$ = 213.0 dB.

The uplink budget follows:

| | |
|---|---|
| EIRP | 59.6 dBW |
| $G_r$ Regional beam, on axis | 33.0 dB |
| Path loss at 28.1 GHz | -213.0 dB |
| Losses 2 dB off axis + 2 dB atmospheric | -4.0 dB |
| Receiver power $P_{rxp}$ | -124.4 dBW |

The satellite transponder input noise temperature is 500 K = 27 dBK. Channel noise bandwidth for the trial hub station signal is 1.0 MHz.

Hence $N_{xp}$ = -228.6 + 27.0 + 60.0 = -141.6 dBW. Uplink (transponder) $(C/N)_{up}$

$(C/N)_{up}$ = -124.4 + 141.6 = 17.2 dB.

The downlink $(C/N)_{dn}$ ratio in the VSAT station receiver is calculated for a trial value of 1 W transponder output power. This is to establish C/N ratios that can later be modified in the final system design. For a satellite transmit power of 1 W and a VSAT on the 3 dB contour of the satellite spot beam footprint, the downlink budget at 21.6 GHz is

| | |
|---|---|
| EIRP | 48.0 dBW |
| $G_r$ VSAT station at 21.7 GHz | 39.2 dB |
| Path loss at 21.7 GHz | -210.8 dB |
| Losses 3 dB off axis + 2 dB atmospheric | -5.0 dB |
| Receiver power $P_{rxp}$ | -128.6 dBW |

The VSAT receiver system noise temperature in clear air is 250 K = 24 dBK. Channel noise bandwidth for the signal is 1.0 MHz.

Hence $N_{xp}$ = -228.6 + 24.0 + 60.0 = -144.6 dBW and $(C/N)_{dn}$ = 16.0 dB.

Overall $(C/N)_o$ for the outbound link is

$(C/N)_o = 1 / [\ 1 / (C/N)_{up} + 1 / (C/N_{dn}\ ] = 22.63 = 13.5$ dB.

The spot beam has a beamwidth of 48 dB. Hence the beamwidth is approximately

$\theta_{3\ dB} = [33{,}000 / 63095]^{1/2} = 0.72^o$.

If the United States is assumed to subtend a rectangle of $6^o$ E-W by $3^o$ N-S, when viewed from GEO orbit, it takes roughly 32 beams to fill this area. Similar results can be found using a smaller number of beams to fill the area on a map. However, the distribution of spot beams in a practical system must take account of population densities and provide more channels to highly populated regions than sparsely populated areas.

**Part 2 System performance**

Connection to the Internet is achieved by the following procedure.

The customer sends a connection request, in the form of a data packet, to the hub station via the satellite and its regional beam. The hub station decodes the request and notes the location of the station. The connection between the Internet and the hub is established through an Internet

Service Provider (ISP) and the public switched telephone network (PSTN). A response from the ISP is sent back to the customer using the satellite's spot beam. Since the packet from the customer contain the VSAT station location, the hub station can send instructions to the satellite to point the spot beam in the correct direction when transmitting packets to the customer. Note that with a linear transponder (bent pipe) on the satellite, the beam pointing instructions must be sent the satellite at the same time as the data packet.

The links between the ISP and the customer in this system are highly asymmetric. The customer can send only short requests at a low data rate. The ISP can dump data to the customer at a high data rate, mainly because of the high EIRP of the satellite's spot beam transmissions. This mode of operation suits applications where the customer is browsing the Internet for information, or is requesting large files or video frames from the Internet. It works less well for sending files from the customer to the Internet – as is done with e-mail, for example. In this problem you are asked to design a VSAT network based on your results from Part 1.

Ka band links are subject to high attenuation in rain. The outbound link is required to achieve a 99.9% availability for a typical VSAT station for which slant path attenuation exceeds 7 dB at 21.7 GHz and 12 dB at 28.2 GHz, for 0.1 % of an average year. The inbound link is required to achieve a 99.7% availability for a typical VSAT station for which slant path attenuation exceeds 4 dB at 21.7 GHz and 7 dB at 28.2 GHz, for 0.3 % of an average year.

The link is declared unavailable if the BER exceeds $10^{-6}$ in the data stream supplied to the customer, or output by the hub station.

Begin your analysis by assuming that 20 active VSAT stations share the output power of transponder #1 equally at all times using QPSK-SCPC-FDMA. Half rate FEC coding is used in the inbound and the outbound link and provides a coding gain of 5 dB at a BER of $10^{-6}$ in the recovered data stream. The implementation margin of the QPSK demodulators in the hub receiver is 0.5 dB, and in the VSAT receiver implementation margin is 0.8 dB. Assume that there are always 20 active VSAT stations receiving data from the outbound link in packet form, using TDM and a single QPSK carrier. Assume linear operation of the transponders, but include the effect of increased sky noise when rain is present on the uplink.

Transponder #1 (inbound, SCPC-FDMA) is operated with 2 dB output back-off.

Transponder #2 (outbound, TDM) is operated with 1 dB back off.

**Part 2 Problems**

**a.** Determine the clear air overall C/N required on the inbound uplink and downlink for one VSAT transmission to meet the 99.7% availability criterion, and the corresponding clear air C/N in the hub station receiver with

(i) rain in the inbound uplink

(ii) rain in the inbound downlink. Remember to include the effect of increased sky noise.

**Answer: (i) Inbound link analysis with rain attenuation on the uplink.**

The 99.7% availability requirement translates to a slant path attenuation that exceeds 4 dB at 21.7 GHz on the downlink to the hub station and 7 dB at 28.2 GHz on the uplink to the satellite. Both of these links use the regional beam of the satellite. The minimum permitted BER on any link is $10^{-6}$, corresponding to an effective overall $(C/N)_o$ ratio of 13.6 dB for QPSK modulation. The implementation margin of the hub receiver is 0.5 dB, and of the VSAT receiver it is 0.8 dB. There is 5 dB coding gain from the use of half rate forward error correction coding. Hence the minimum overall $(C/N)_o$ ratio in the hub station receiver is

$(C/N)_o$ minimum = 13.6 – 5.0 + 0.5 = 9.1 dB

When rain occurs in the uplink, $(C/N)_o$ in the hub station receiver falls in direct proportion to rain attenuation on the uplink path. For 7 dB uplink rain attenuation we require

$(C/N)_o$ = 9.1 + 7.0 = 16.1 dB in clear air.

The value of $(C/N)_{up}$ in the trial calculation in Part 1 was 7.0 dB with 1 W transmitted by the VSAT station. $(C/N)_{dn}$ was 29.0 dB with 1 watt per channel transmitted by the satellite. With 20 VSAT signals and 2 dB backoff at the transponder output, the transponder transmits 30 W – 2 dB = 12.8 dBW. This power is shared between 20 signals, so $P_t$ per channel is 12.8 – 13.0 = -0.2 dBW. Hence in clear air conditions, $(C/N)_{dn}$ = 28.8 dB.

With these operating (C/N) values, $(C/N)_o$ = 7.0 dB in clear air.

To achieve $(C/N)_o$ = 16.1 dB in clear, air, we must reduce the receiver noise bandwidth by 9.1 dB, a factor of 8.13, to 128 kHz / 8.13 = 15.75 kHz. With QPSK modulation, the symbol rate of the VSAT transmission is 15.75 ksps. Because we have half rate FEC encoding in the links, the data rate is equal to QPSK symbol rate, and the bit rate is 15.75 kbps.

**(ii) Inbound link analysis with rain attenuation on the downlink**

The limiting condition for the downlink is $(C/N)_o$ = 16.1 dB with 4 dB rain attenuation on the downlink. Sky noise temperature will increase due to the rain in the path. For a medium temperature of 290 K

$$T_{sky\ rain} = 290\,(1 - 0.4) = 174\ \text{K}$$

In clear air, the atmospheric attenuation at 21.7 GHz is given as 2.0 dB. (Atmospheric attenuation is really a variable that depends on water vapor content of the atmosphere, i.e. humidity at the receiving site. A value of 2 dB attenuation corresponds to a low angle slant path and high humidity.)

The clear sky noise temperature is

$$T_{sky\ clear\ air} = 290\,(1 - 0.631) = 107\ \text{K}$$

The receiver LNA contribution is 250 – 107 = 143 K, giving $T_{s\ rain}$ = 174 + 143 = 317 K. The increase in system noise power is therefore

$$\Delta N = 10 \log (317 / 250) = 1.0\ \text{dB}$$

giving a reduction in $(C/N)_{dn}$ of 4.0 + 1.0 = 5.0 dB.

Based on a receiver noise bandwidth of 128 kHz, the clear air downlink C/N ratio was $(C/N)_{dn\ ca}$ = 28.8 dB, so in this bandwidth $(C/N)_{dn\ rain}$ = 28.8 dB - 5.0 dB = 23.8 dB. However, the poor uplink performance in rain required the bandwidth to be reduced to 15.75 kHz, giving $(C/N)_{up}$ = 16.1 and $(C/N)_{dn}$ = 23.8 + 9.1 = 32.9 dB in clear air.

In clear air conditions the overall $(C/N)_o$ ratio in the hub receiver for a 15.75 ksps transmission from a VSAT station with 4.0 dB rain attenuation on the downlink is

$$(C/N)_o = 1 / [\, 1 / (C/N)_{up} + 1 / (C/N)_{dn}\, ] = 16.0\ \text{dB}.$$

This is well above the minimum permitted value of 9.1 dB, showing that the uplink from the VSAT station is the limiting path.

**b.** Using the results you obtained in Part 1, and Part 2 question (a), determine the maximum data rate for the VSAT request packets to meet the 99.7% availability criterion with access to the transponder through the satellite's regional beam, with 20 active VSATs at any time.

**Answer**: The inbound uplink from the VSAT station to the satellite is the limiting factor in the inbound link, because of the low uplink C/N ratio in the transponder. In Part 2 (a), the uplink $(C/N)_{up}$ was found to be 16.1 dB in clear air to meet the 99.7% availability criterion, forcing a reduction in channel noise bandwidth to 15.75 kHz.. The symbol rate on the inbound link is 15.75 ksps, giving an inbound data bit rate of 15.75 kbps. This is the bit rate at which requests can be sent to the hub station, and on to the ISP.

c. Determine the clear air overall C/N in the VSAT station receiver for an outbound data rate of 1 Mbps using QPSK-TDM to meet the 99.9% availability criterion, for

(iii) rain in the outbound uplink

(iv) rain in the outbound downlink. Remember to include the effect of increased sky noise.

**Answer: (i) Outbound link analysis with rain attenuation on the uplink.**

The 99.9% availability requirement translates to a slant path attenuation that exceeds 7 dB at 21.7 GHz on the downlink to the VSAT station and 12 dB at 28.2 GHz on the uplink to the satellite from the hub station. The outbound uplink uses the regional beam of the satellite and the outbound downlink uses the spot beam. The minimum permitted BER on any link is $10^{-6}$, corresponding to an effective overall $(C/N)_o$ ratio of 13.6 dB for QPSK modulation. The implementation margin of the VSAT receiver it is 0.8 dB. There is 5 dB coding gain from the use of half rate forward error correction coding. Hence the minimum overall $(C/N)_o$ ratio in the hub station receiver is

$$(C/N)_o \text{ minimum} = 13.6 - 5.0 + 0.8 = 9.4 \text{ dB}$$

When rain occurs in the uplink, $(C/N)_o$ in the VSAT station receiver falls in direct proportion to rain attenuation on the up link path. For 12 dB uplink rain attenuation we require

$$(C/N)_o = 9.4 + 12.0 = 21.4 \text{ dB in clear air.}$$

The value of $(C/N)_{up}$ in the trial calculation in Part 1 was 17.2 dB with 1 W per channel transmitted by the hub station in a channel noise bandwidth of 1 MHz. $(C/N)_{dn}$ was 16.0 dB with 1 watt per channel transmitted by the satellite.

The 20 signals destined to the VSAT stations are transmitted as a TDM bit stream at 1 Msps. We should increase the hub station transmit power to 20W now that there are 20 signals to be sent by TDM, giving $(C/N)_{up} = 30.2$ dB in clear air in 1 MHz receiver noise bandwidth.

Transponder #2 has 1 dB backoff at the transponder output, and transmits 30 W – 1 dB = 13.8 dBW. Hence in clear air conditions, $(C/N)_{dn} = 16.0 + 13.8 = 29.8$ dB.

With these operating (C/N) values, $(C/N)_o$ = 27.0 dB in clear air. There is now a margin above the minimum permitted overall C/N ratio that can be used to increase the bit rate of the TDM signal.

**(ii) Outbound link analysis with rain attenuation on the downlink**

The limiting condition for the downlink is $(C/N)_o$ = 21.4 dB with 7 dB rain attenuation on the downlink. Sky noise temperature will increase due to the rain in the path. For a medium temperature of 290 K

$$T_{sky\ rain} = 290\ (1 - 0.2) = 232\ K$$

In clear air, the clear sky noise temperature is

$$T_{sky\ clear\ air} = 290\ (1 - 0.631) = 107\ K$$

The receiver LNA contribution is 250 – 107 = 143 K, giving $T_{s\ rain}$ = 232 + 143 = 375 K. The increase in system noise power is therefore

$$\Delta N = 10 \log (375 / 250) = 1.8\ dB$$

Hence $(C/N)_{dn}$ falls by 8.8 dB from its clear air value of 39.8 dB to 21.0 dB. Overall $(C/N)_o$ is now 20.5 dB with 7 dB rain attenuation on the downlink. The requirement for 99.9% availability is met, since $(C/N)_o$ exceeds the minimum value of 9.4 dB by a margin of more than 10 dB.

d. Using the results you obtained in Part 1, determine the maximum data rate that can be supplied to each VSAT station with 20 active stations in the network at the same time, for the 99.9% availability criterion. Note that for the small percentages of time used here, you may assume that rain never occurs simultaneously in both the uplink and downlink.

**Answer:** The data rate at which the hub station can transmit is 1.0 Mbps, based on the design criteria used in part (c) above. The bit stream is a TDM sequence of packets delivered to 20 VSAT stations, so each station has a data rate $R_b$ = 50 kbps. The bit rate can be increased because there is excess (C/N) ratio on the outbound link when we meet the 99.9% availability criterion.

**e.** If your results from parts (b) and (d) above show that either transponder #1 or #2 is not bandwidth limited, it is possible to optimize the system to transmit at higher bit rates. Redesign the VSAT and hub stations to increase the bit rates in either the inbound link, the outbound link, or both links, within the limits that the VSAT antenna diameter cannot exceed 1m, and the transmit power cannot exceed 2 watts. The hub station antenna diameter cannot exceed 5m and the transmit power cannot exceed 200 watts. You might also consider whether the number of simultaneous users can be increased. The satellite is leased and cannot be changed, except that the gain of the transponders can be adjusted to suit the earth stations used in the network.

**Answer:** The transponders have a bandwidth of 54 MHz and are carrying very small bandwidth signals in the current design. If we increase transmit power at the hub station to 100 W and at the VSAT station to 2 W, and also increase the VSAT antenna diameter to 1 m, we can achieve significant improvements in performance. The EIRP of the VSAT station is increased by 9 dB, and EIRP of the hub station is increased by 20 dB. We need to repeat the link analysis in rain under the new conditions.

**Inbound links.** In part (a) we found that the minimum overall $(C/N)_o$ ratio in the hub station receiver is

$$(C/N)_o \text{ minimum} = 13.6 - 5.0 + 0.5 = 9.1 \text{ dB}$$

When rain occurs in the uplink, $(C/N)_o$ in the hub station receiver falls in direct proportion to rain attenuation on the uplink path. For 7 dB uplink rain attenuation we require

$$(C/N)_o = 9.1 + 7.0 = 16.1 \text{ dB in clear air.}$$

The value of $(C/N)_{up}$ in the trial calculation in Part 1 was 7.0 dB with 1 W transmitted by the VSAT station. We can increase VSAT EIRP by 9 dB, giving $(C/N)_{up}$ = 16.0 dB with 7 dB of uplink rain attenuation. In clear air conditions, $(C/N)_{dn}$ = 29.0 dB, giving $(C/N)_o$ = 15.78 dB ≈ 15.8 dB. To achieve $(C/N)_o$ = 16.1 dB we must reduce the receiver noise bandwidth by 0.32 dB, a factor of 1.076, to 128 kHz / 1.076 = 119 kHz. The clear air uplink $(C/N)_{up}$ ratio is 16.3 dB in 119 kHz noise bandwidth. With QPSK modulation, the symbol rate of the VSAT transmission is 119 ksps and the bit rate is 119 kbps.

The limiting condition for the downlink is $(C/N)_o$ = 16.1 dB with 4 dB rain attenuation on the downlink. Taking account of sky noise temperature increase, we found that a reduction in $(C/N)_{dn}$ of 5.0 dB occurs when there is 4 dB rain attenuation on the downlink, giving $(C/N)_{dn\ rain}$ = 29.0 dB - 5.0 dB = 24.0 dB in a receiver noise bandwidth of 128 kHz. Reducing the receiver noise bandwidth to 119 kHz increases $(C/N)_{dn}$ to 24.3 dB. Uplink performance in rain required the bandwidth to be reduced to 119 kHz, giving $(C/N)_{up}$ = 16.3 dB in clear air.

Overall $(C/N)_o$ ratio in the hub receiver for a 119 ksps transmission from a VSAT station with 4.0 dB rain attenuation on the downlink is

$$(C/N)_o = 1 / [\, 1 / (C/N)_{up} + 1 / (C/N)_{dn} \,] = 15.7\text{dB}.$$

This is well above the minimum permitted value of 9.1 dB, showing that the uplink from the VSAT station is still the limiting path. However, we are now able to access the hub station from the VSAT at 119 kbps, a major improvement over the original bit rate of 15.75 kbps.

**Outbound links.** In the calculations in part (c), a hub transmit power of 1 W was used. Increasing the hub station transmit power to 100 W will improve outbound performance a great deal. We need to analyze the outbound link with 100 W hub transmit power, 12 dB uplink rain attenuation and 7 dB downlink attenuation, using a VSAT station with a 1 m antenna and 6 dB additional gain.

The clear air uplink $(C/N)_{up}$ with hub station $P_t$ = 20 W was 30.2 dB in a noise bandwidth of 1 MHz. Increasing $P_t$ to 100 W makes $(C/N)_{up}$ = 37.2 dB. $(C/N)_{dn}$ was 29.8 dB in a noise bandwidth of 1 MHz in clear air. Increasing the VSAT antenna diameter to 1 m improves $(C/N)_{dn}$ to 35.8 dB. Thus overall $(C/N)_o$ in clear air is 28.8 dB. With 12 dB uplink rain attenuation, $(C/N)_o$ falls to 16.8 dB, which is 7.7 dB above the minimum permitted value of 9.1 dB.

When there is 7 dB rain attenuation on the downlink, $(C/N)_{dn}$ falls by 8.8 dB to 21.0 dB. $(C/N)_{up}$ is 37.2 dB in clear air giving $(C/N)_o$ = 19.9 dB, which is 10.8 dB above the minimum permitted value of 9.1 dB.

We can expand the receiver noise bandwidth in the VSAT terminals, to increase the outbound bit rate, or we can add more VSATs to the network. Since the outbound bit rate was rather low, at 50 kbps per VSAT station, let's increase the TDM outbound bit rate by 7.7 dB to 5.88 Mbps. This gives a symbol rate of 5.88 Msps and a receiver noise bandwidth of 5.88 MHz. Each VSAT now receives a signal at a bit rate of 294 kbps from the hub station. Alternatively, we could increase the number of VSATs to 50, giving each station a bit rate of 118 kbps on the outbound link. Many combinations of bit rate and number of VSATs are possible within the availability criteria.

If we increase the number of active VSATs to 50, the power in transponder #1 will be divided 50 ways, instead of 20 ways, giving $P_t$ = -0.2 - 4.0 = -4.2 dBW. This reduces $(C/N)_{dn}$ in the hub station in clear air by 4.0 dB to 25.0 dB. With 4 dB of rain attenuation on the downlink, this is reduced to $(C/N)_{dn\ rain}$ = 20.0 dB. With the upgraded VSAT station (D = 1 m, $P_t$ = 2 W) the uplink $(C/N)_{up}$ ratio in clear air is 16.0 dB, so overall $(C/N)_o$ is 14.5 dB, well above the limiting value of 9.1 dB. Hence uplink rain attenuation is still the limiting condition for the inbound links and we can maintain the inbound bit rate of 119 kbps.

**Performance Summary**:

| | |
|---|---|
| Number of simultaneous active VSAT stations in network : | 50 |
| Outbound data rate from hub to VSAT | 228 kbps |
| Inbound data rate from VSAT to hub | 119 kbps |
| Bandwidth occupied in Transponder #1 ($\alpha$ = 0.25, ideal RRC filters) | 7.35 MHz |
| Bandwidth occupied in Transponder #2 ($\alpha$ = 0.25, ideal RRC filters) | 7.35 MHz |
| ($\alpha$ = 0.25, ideal RRC filters and 50 kHz guard bands) | 9.937 MHz |
| Power required from transponder #1 | 18.9W |
| Power required from transponder #2 | 23.8 W |

**Part 3 Portable terminals**

The one advantage of radio systems over wired communications links is portability. This question asks you to design a portable Ka-band terminal which can be used to connect to the Internet (provided the customer has a clear view of the southern sky). The critical element in a portable communications link is the antenna. A large antenna provides a high data rate, but is cumbersome and must be pointed accurately at Ka-band frequencies. A small antenna is easier to set up, but cannot provide a high data rate. Let's assume that the dimension of the antenna are limited to the dimensions of a typical laptop computer – 0.25 m $\times$ 0.2 m – with an aperture efficiency of 25 %, and that some method is provided that helps the customer point the antenna beam towards the satellite so that there is no more than 1 dB loss of gain due to antenna mispointing.

Because the portable terminals cannot achieve the same C/N ratios as the fixed terminals, separate transponders will be needed to service the portables. For convenience, we will call these transponders #3 (inbound) and #4 (outbound) and use the same frequencies as transponders #1 and #2. The ability of the system to operate during rain fades on the outbound link is relaxed with an availability of 99.7% required in each direction.

**a.** Calculate the gain and the beamwidth of the portable antenna at frequencies of 28.2GHz and 21.7 GHz.

**Answer:** The aperture are of the antenna is $0.25 \text{ m} \times 0.2 \text{ m} = 0.05 \text{ m}^2$. Aperture efficiency is 25% so transmit gain at 28.2 GHz is

$$G_t = \eta_A 4 \pi A / \lambda^2 = 0.25 \times 4 \pi \times 0.05 / (0.01064)^2 = 1387 = 31.42 \text{ dB}$$

Scaling to 21.7 GHz

$$G_r = 31.42 - 2.28 = 29.14 \text{ dB}$$

**b.** Using your results from Part 1, find the inbound and outbound overall C/N ratios in the hub station and portable receivers using the conditions in Part 1 in clear air conditions. Don't forget to allow an extra 1 dB of loss to account for antenna mispointing.

**Answer:** The results from Part 1 (a) for the inbound link, using a noise bandwidth of 128 kHz for the uplink and 1 MHz for the downlink, with 1 watt of transmit power at the earth station and at the satellite, and a single channel, were

Uplink (transponder) $(C/N)_{up}$ ratio = 7.0 dB

Downlink (hub receiver) $(C/N)_{dn}$ = 29.0 dB

For the outbound link, using the same bandwidths, the results were

Uplink $(C/N)_{up}$ = 17.2 dB

Downlink $(C/N)_{dn}$ = 16.0 dB

The VSAT station antenna gains in Part 1 were

$G_{28V}$ = 41.51 dB, $G_{21V}$ = 39.23 dB

Antenna gain has been reduced by 10.1 dB and an additional loss of 1 dB is present because of antenna mispointing. Hence, inbound $(C/N)_{up}$ and outbound $(C/N)_{dn}$ will be lower by 10.1 dB. Using the part 1 results given above:

Inbound link: Uplink (transponder) $(C/N)_{up}$ ratio = 7.0 - 10.1 = -3.1 dB

Downlink (hub receiver) $(C/N)_{dn}$ = 29.0 dB

Overall C/N = -3.1 dB

Outbound link: Uplink $(C/N)_{up}$ = 17.2 dB

Downlink $(C/N)_{dn}$ = 16.0 – 10.1 = 5.9 dB

Overall C/N = 5.7 dB

**c.** Assume that ten active stations share each transponder. Determine the maximum data rates that customers can achieve on the inbound and the outbound links with 99.7% availability of the inbound and outbound links.

**Answer**: **Inbound link.** The 99.7% availability criterion requires minimum C/N at the hub station of 9.1 dB in clear air, with the addition of 7 dB uplink rain attenuation and 4 dB downlink rain attenuation.

With ten active stations, 30 W transponder output power and 2 dB output back off, $(C/N)_{dn}$ is 3 dB higher than in Part 1. So $(C/N)_{dn}$ = 32.0 dB. To achieve $(C/N)_o$ = 9.1 + 7 .0 = 16.1dB with 7 dB of uplink rain attenuation we require $(C/N)_{up}$ = 9.08 ≈ 9.1 dB. Since the calculated $(C/N)_{up}$ is –3.1 dB in 128 kHz noise bandwidth we must reduce the uplink noise bandwidth by 19.2 dB or a factor of 83.17 to 1.54 kHz. Symbol rate on the QPSK uplink is 1.54 ksps and we use half rate FEC encoding, the data bit rate is 1.54 kbps. Rain on the downlink causes little change in the high value of $(C/N)_{dn}$ and can be ignored. The very low bit rate for the inbound channel is unsatisfactory.

**Outbound Link.** The downlink to the portable terminal will be the limiting link. From Part 1, or 30 W of transponder transmit power with 1 dB output back off and a noise bandwidth of 1 MHz

Downlink $(C/N)_{dn}$ = 16.0 – 10.1 = 5.9dB

In Part 2 we showed that with 4 dB of rain attenuation on the downlink, $(C/N)_{dn}$ fell by 5.0 dB.

Uplink C/N will be very high with 10 active terminals, and its impact on overall C/N can be ignored and we will use $(C/N)_o$ = $(C/N)_{dn}$. Therefore we require

$(C/N)_{dn}$ = 9.4 + 5.0 = 14.4 dB

on the downlink to achieve 99.7% availability. We would need to reduce the downlink noise bandwidth by 8.5 dB or a factor of 7.08 to 141.2 kHz.

The limiting case will be when the uplink from the hub station suffers 7 dB rain attenuation. Both the uplink and downlink C/N ratios will fall by 7 dB, which is a worse case than attenuation on the downlink. Hence overall $(C/N)_o$ will fall by 7 dB, to –1.1 dB, which will require a bandwidth reduction of 15.5 dB, or a factor of 35.48 to 28.18 kHz. Thus the outbound link can operate at 28.18 kps, giving a data rate of 28.18 kbps.

**d.** Transponders #3 and #4 can be switched into baseband processing mode. In this mode, the incoming QPSK signal is demodulated to baseband, the data bits are recovered and then remodulated onto a carrier for transmission as a new QPSK signal. This allows the transponder to transmit at its rated output power at all times despite uplink attenuation. The bit error rate for the link is then the sum of the BERs on the uplink and the downlink. Rework your solution to part (c) above using baseband processors for both inbound and outbound links and determine the new data rates for the inbound and outbound links.

**Answer:** Baseband processing does not alter uplink C/N ratios. It prevents downlink C/N ratios from failing during uplink rain attenuation events. Thus the use of a baseband processing transponder (#1) in the inbound link improves only the downlink $(C/N)_{dn}$ ratio, which is already very high. It does not change the uplink C/N which is the limiting factor in the link.

On the outbound link to the portable terminal, we can remove the effect of uplink attenuation from the downlink. The limiting case is now 5.0 dB drop in $(C/N)_{dn}$ when 4 dB of rain attenuation occurs on the downlink. The reduction in bandwidth required to meet the 99.7% availability requirement is 13.5 dB or a factor of 22,39. Hence the outbound link noise bandwidth is 44.66 kHz, the symbol rate is 44.66 ksps and the date rate is 44.66 kbps.

**e.** Draw a block diagram of transponder #3 when used in its baseband processing mode. Your block diagram should include all the filters, amplifiers, mixers, oscillators, modulators and demodulators, and all other important blocks. Label each filter and amplifier with a center frequency and bandwidth, and indicate the gain of each amplifier. Label all oscillators with their frequencies. (Attach the block diagram as the last page of your answer packet.)

**Answer:** Block diagrams are not included in the solutions manual.

**f.** Comment on the performance of the fixed and portable Ka-band Internet link system. If the transponders on the GEO satellite cost $1.5 M per year each to lease, and the service provider's costs to support the customer base that shares these transponders are $ 0.5M per year, what would you expect to have to charge the customer for access to the Internet when using the fixed terminal and the portable terminal? You can establish a charging structure made up of a monthly fee plus a per minute access charge. Assume that you can achieve a continuous level of activity of 20 fixed or 10 portable terminals for 12 hours per day.

Each user can be assumed to connect to the Internet for 15 minutes once each day, but is active (in the sense of data transfer over the satellite) for 1 minute per day. How do the data rates and the charges you propose for the portable Internet access service compare to typical charges for cable modem service?

**Answer:** The capacity of the Ka band Internet access system is rather low. We can support 50 active VSAT terminals ( 1 m diameter antenna, 2 W transmitter) in the fixed network with inbound and outbound data rates of 118 kbps. This is comparable to DSL rates, The portable terminals have even lower data rates. The best that can be achieved with linear transponders is an outbound bit rate of 28.18 kbps and an inbound data rate of 1.54 kbps. Inbound transmissions would have to be kept to very short messages to keep access times reasonable. The outbound data rate is comparable to data rates available with some cellular telephone systems, but the inbound rate is much lower. The portable terminal would be attractive only where there is no access to cellular telephone data service.

The cost of operating the Internet access system is \$3.5 M per year, and each terminal needs one minute of data transfer time per effective day (12 hours). That allows 720 terminals to share each active channel. The fixed network can support $20 \times 720 = 14{,}400$ users, giving an annual cost per user of \$3.5 M / 14,400 = \$2430 per year. This can be divided up between monthly charges and user fees - for example, \$100 per month user fee plus 8.5 cents per minute. These figures are higher than most DSL providers were charging in 2001.

The network can support 7200 portable terminals, giving an annual cost per terminal of \$4860. User fees would have to be much higher, at least \$200 per month, with 17 cents per minute access fee. Many other charging packages can be developed - extensive market research would be needed before such a system could be marketed. The very low data rate on the inbound link is a severe limitation in the portable terminal. The uplink fade margin of 4 dB could be reduced a little in order to improve access data rate, but the small antenna size, low antenna efficiency, and low transmit power remain major limitations.

# Chapter 7    Solution to Problems

**1.** Alphanumeric characters are transmitted as 7-bit ASCII words, with a single parity bit added, over a link with a transmission rate of 9.6 kbps.

**a**. How many characters are transmitted each second?

**Answer:** Each character (a letter or number, or symbol) is seven data bits plus one parity bit, giving eight bits per character.

Bit rate $R_b$ = 9.6 kbps, hence the link transmits 9.6 / 8 = 1200 characters per second.

**b.** If a typical page of text contains 500 words with an average of five characters per word and a space between words, how long does it take to transmit a page?

**Answer:** One average page is 500 (written) words averaging five characters, with a space between words. Hence one page contains $500 \times 5 + 500 = 3000$ characters = 24,000 bits. Time to send one page is 24,000 / 9,600 = 2.5 seconds.

**c.** If the bit error rate on the link is $10^{-5}$ , how many characters per page are detected as having errors? How many undetected errors are there?

**Answer:** The probability of a bit error is $10^{-5}$. We can detect one bit error in an 8 bit character, and also 3, 5, or 7 bit errors. We cannot detect 3, 4, 6 or 8 bit errors. The probability of k errors in a word of n bits is found from the binomial equation (equation 7.4).

$$P_e(k) = (n, k)\, p^k (1 - p)^{n-k} \quad \text{where } (n, k) = n! / [k!\,(n - k)!]$$

For n = 8 and k = 1, 2, 3, we find for $p = 10^{-5}$

$$P_e(1) = (8, 1) \times 10^{-5} \times (0.99999)^7 \quad \text{where } (8, 1) = 8! / [\,1 \times 7!] = 8$$

$$P_e(1) = 8 \times 10^{-5}$$

$$P_e(2) = 2.8 \times 10^{-9}$$

$$P_e(3) = 5.6 \times 10^{-14}$$

All higher order terms are negligible, and $P_e(3)$ is negligible in comparison with $P_e(1)$. Hence the probability of a detected error in a character is $8 \times 10^{-5}$. With 3000 characters per page, the probability that a character error is detected on any page is $3000 \times 8 \times 10^{-5} = 0.24$.

**d**. On average how many pages can be transmitted before

**(i)** a detected error occurs

**(ii)** an undetected error occurs?

**Answer:** (i) With a bit error rate of $10^{-5}$, there are 0.24 errors per page, so we should expect to find one character error every four pages, on average.

(ii) The probability of an undetected error is $P_e(2) = 2.8 \times 10^{-9}$. With 3000 characters per page, the probability of a character error on any page is $3000 \times 2.8 \times 10^{-9} = 8.4 \times 10^{-6}$. We would have to send 119,000 pages before we would expect to find an undetected error – equivalent to transmitting this text book 222 times. Since it takes 2.5 seconds to send one page at 9.6 kbps, it takes 22 minutes to transmit the whole text book, and 82 hours elapse before the undetected error occurs.

**e.** If the BER increases to $10^{-3}$, how many detected and undetected errors are there in a page of the text?

**Answer:** With $p = 10^{-3}$ the new probabilities for 1, 2 and 3 errors per character are

$P_e(1) = 8 \times 10^{-3}$

$P_e(2) = 2.8 \times 10^{-5}$

$P_e(3) = 5.6 \times 10^{-8}$

We can ignore the probability of three errors in the eight bit character.

(i) The probability of a detected character error is now $P_e(1) = 8 \times 10^{-3}$. With 3000 characters per page, there are 24 detected errors per page on average.

(ii) The probability of an undetected error is $P_e(2) = 2.8 \times 10^{-5}$, giving a probability of 0.084 errors per page, or one undetected character on every 12 pages, on average.

A BER of $10^{-3}$ is at the lower limit for transmission of text using single parity error checking.

**2.** A (6, 3) block code has a minimum distance of two.

**a.** How many errors can be detected in a codeword?

**Answer:** The minimum distance, $d_{min}$, of a codeword is the smallest difference between two non-zero codewords in the set. The number of errors that can be detected is $(d_{min} – 1)$. Hence a code set with minimum distance two can detect one error.

**b.** How many errors can be corrected in a codeword?

**Answer:** The number of errors that can be corrected by a code set is given by $(d_{min} - 1) / 2$, rounded down to the next integer value. If $d_{min} = 2$, $(d_{min} - 1) / 2 = ½$ and the code cannot correct errors. The smallest value of $d_{min}$ in any code that can correct errors is $d_{min} = 3$, when the code can correct one error.

**3.** A QPSK data link carries a bit stream at 1.544 Mbps and has an overall $(C/N)_o$ ratio of 16 dB in the receiver at the VSAT earth station in clear air. The QPSK demodulator at the VSAT station has an implementation margin of 1.0 dB. For 0.1% of the year rain attenuation causes 3.0 dB reduction in the receiver $(C/N)_o$ ratio. For 0.01% of the year rain attenuation causes 6.0 dB reduction in the receiver $(C/N)_o$ ratio.

**a.** Calculate the BER in clear air, and the BER exceeded for 0.1% and 0.01% of the year.

**Answer:** The symbol error rate a QPSK link is given by

$P_e = Q[\sqrt{(C/N)_{eff}}]$ where $(C/N)_{eff}$ is the effective C/N ratio at the input to the QPSK demodulator, taking account of implementation margin. When Gray coding of the data bits is employed (as is always the case) BER $\approx P_e$. Hence for $(C/N)_{eff} = 16.0 - 1.0 = 15.0$ dB, a power ratio of 31.62

$$\text{BER} \approx Q[\sqrt{31.62}] = 5.62$$

Using the Q[z] table of Appendix C, BER $\approx 10^{-8}$.

For 0.1% of the year rain attenuation causes 3.0 dB reduction in the receiver $(C/N)_o$ ratio. Hence $(C/N)_{eff} = 15.0 - 3.0 = 12.0$ dB or a ratio of 15.85.

$$\text{BER} \approx Q[\sqrt{15.85}] = 3.98, \quad \text{BER} \approx 3.5 \times 10^{-5}.$$

**b.** Repeat the calculation when data are transmitted using half rate FEC with a coding gain of 5.5 dB (at all BERs). The bit rate on the link remains at 1.544 Mbps when coding is added. What is the data rate with half rate FEC applied to the data?

**Answer:** With half rate error correction coding applied to the bit stream, the effective $(C/N)_{eff}$ ratio is increased by 5.5 dB.

Hence for clear air conditions, $(C/N)_{eff} = 16.0 - 1.0 + 5.5 = 20.5$ dB, a power ratio of 112.2.

$\text{BER} \approx Q[\sqrt{112.2}] = Q[10.59]$

The Q[z] table of Appendix C gives values for Q[z] only to Q[8] because BER ≈ 0 for any value of z larger than eight, indicating that there are no errors occurring on the link.

For 0.1% of the year rain attenuation causes 3.0 dB reduction in the receiver $(C/N)_o$ ratio. Hence $(C/N)_{eff} = 15.0 - 3.0 + 5.5 = 17.5$ dB, a ratio of 56.23.

$\text{BER} \approx Q[\sqrt{56.23}] = Q[7.50]$, $\text{BER} \approx 3.2 \times 10^{-14}$.

At a bit rate of 1.544 Mbps, there is one error every $1 / (1.544 \times 3.2 \times 10^{-14}) = 4.94 \times 10^{8}$ s, or one error every 15.7 years. The link is operating error free with 3 dB of C/N reduction, for 99.9% of the year.

For 0.01% of the year rain attenuation causes 6.0 dB reduction in the receiver $(C/N)_o$ ratio. Hence $(C/N)_{eff} = 15.0 - 6.0 + 5.5 = 14.5$ dB, a ratio of 28.18.

$\text{BER} \approx Q[\sqrt{28.18}] = Q[5.31]$, $\text{BER} \approx 5.5 \times 10^{-8}$.

There is now an error every 11.8 seconds on the link.

Only half of the bits in the transmitted bit stream are data bits; the other half are parity bits used in the EC decoder to correct errors. Hence the data rate at the output of the link is 1.544 Mbps / 2 = 772 kbps.

**c.** Repeat the calculation of part (b) above when the bit rate on the link is increased to 3.088 Mbps with no increase in transmitter power.

**Answer:** If the bit rate on the link is increases to 3.088 Mbps and half rate forward error correction is applied, the data rate at the link output is 1.544 Mbps. However, the bandwidth of the link must be doubled to allow the higher bit rate signal to be transmitted, and this doubles the noise power in the receiver and reduces overall C/N by 3.0 dB.

In this case, overall C/N in clear air is 13.0 dB, so the BER in clear air corresponds to

$(C/N)_{eff} = 13.0 - 1.0 + 5.5 = 17.5$ dB, a ratio of 56.23.

$\text{BER} \approx Q[\sqrt{56.23}] = Q[7.50]$, $\text{BER} \approx 3.2 \times 10^{-14}$ and there are no errors on the link.

For 0.1% of the year rain attenuation causes 3.0 dB reduction in the receiver $(C/N)_o$ ratio and

$(C/N)_{eff} = 12.0 - 3.0 + 5.5 = 14.5$ dB, a ratio of 28.18.

$\text{BER} \approx Q[\sqrt{28.18}] = Q[5.31]$, $\text{BER} \approx 5.5 \times 10^{-8}$.

There is now an error every 11.8 seconds on the link.

For 0.01% of the year with 6.0 dB reduction in the receiver $(C/N)_o$ ratio,

$(C/N)_{eff}$ = 12.0 – 6.0 + 5.5 = 11.5 dB, a ratio of 14.13.

BER ≈ Q[√13.13] = Q[3.76], BER ≈ $8.4 \times 10^{-5}$.

There are now 262 errors every second on the link. This is towards the lower margin for operation of the link with speech signals (S/N ≈ 35 dB for 8-bit digital speech) and below the threshold of BER = $10^{-6}$ that is usually applied for data transmission.

**4.** The Analysis of a 56-kbps data link shows that it suffers burst errors that corrupt several adjacent bits. The statistics for burst errors on this link are given in Table P.4.

**Table P.4**

**Statistics for Burst Errors on a Link in Problem 4.**

| No. Adjacent Bits Corrupted | Probability of Occurrence |
|---|---|
| 2 | $4 \times 10^{-2}$ |
| 3 | $2 \times 10^{-3}$ |
| 4 | $3 \times 10^{-4}$ |
| 5 | $1 \times 10^{-6}$ |
| 6 | $2 \times 10^{-9}$ |
| 7 | $5 \times 10^{-11}$ |
| 8 | $1 \times 10^{-12}$ |
| 9 | $3 \times 10^{-14}$ |
| 10 | $2 \times 10^{-17}$ |

**a.** Using Table P7.4, select a burst error correcting code that will reduce the probability of an uncorrected burst error below $10^{-10}$.

**Answer:** We want to ensure that the probability of an uncorrected error is below $10^{-10}$.
From Table P7.4, the probability of an uncorrected burst error is below $10^{-10}$ for codes with burst error correction capability of 7 and more adjacent bits. The (103,88) code in Table P7.4 meets the requirements, with a code rate 88/103.

**b.** Calculate the data rate for messages sent over the link using the code you selected.

**Answer:** For a transmitted bit rate of $R_b = 56$ kbps, the output of the FEC decoder is a message data rate of

$$R_b = 88/103 \times 56 = 47.84 \text{ kbps.}$$

**c.** Estimate the average bit error rate for the coded transmission.

**Answer:** The probability of an uncorrected burst error is the sum of the probabilities that 8, 9, 10 … adjacent bits are in corrupted in the received bit stream, and the errors cannot be corrected by the FEC decoder. From Table P 7.4

$$P(8) = 1 \times 10^{-12}\text{-}$$

$$P(9) = 3 \times 10^{-14}$$

$$P(10) = 1 \times 10^{-17}$$

Higher order terms are negligible. Hence the probability of uncorrected error bits reaching the decoder output is approximately

$$P_e \approx 8 \times P(8) + 9 \times P(9) = 8 \times 10^{-12} + 27 \times 10^{-14} = 8.27 \times 10^{-12}$$

At a transmission rate of 56 kbps, bursts of bit errors are occurring at the output of the FEC decoder with a probability of approximately $10^{-12}$. The time between burst errors is $1 / (56 \times 10^3 \times 10^{-12}) = 1.786 \times 10^7$ s

or 207 days. The link is essentially error free.

**5.** A satellite link carries packet data at a rate of 256 kbps. The data are sent in 255-bit blocks using a (255, 247) code that can detect three errors. The probability of a single bit error, *p*, varies from $10^{-6}$ under good conditions to $10^{-3}$ under poor conditions. The one way link delay is 250 ms.

**a.** If no error detection is used, what is the message data rate for the link?

**Answer:** Message data rate without error detection is $247/255 \times 256 = 247.969$ kbps.

**b.** For a link BER of $10^{-6}$, find the probability of detecting an error in a block of 255 bits when error detection is applied.. Hence find how often an error is detected.

**Answer:** The error detection capability of the (247, 255) code is three bits, so we can detect the presence of 1, 2, or 3 errors in the received bit stream. Hence the probability that we detect an error in any block is

$$P(\text{error detected}) = P(1) + P(2) + P(3)$$

Using equation 7.4

$$P_e(k) = (n, k)\, p^k\, (1 - p)^{n-k} \quad \text{where } (n, k) = n! / [\, k! \times (n-k)!\, ]$$

For k = 1, 2, and 3 with $p = 10^{-6}$ and a block length n = 255 bits

$$P_e(1) = 255 \times 10^{-6} \times (1 - 10^{-6})^{254} = 255 \times 10^{-6} \times 0.9948 = 2.549 \times 10^{-4}$$

$$P_e(2) = (255 \times 254/2) \times 10^{-12} \times (1 - 10^{-6})^{253} = 3.237 \times 10^{-8}$$

$$P_e(3) = (255 \times 254 \times 253 / 6) \times 10^{-18} \times (1 - 10^{-6})^{252} = 2.73 \times 10^{-12}$$

Thus the probability of a detected error is

$$P(\text{detected}) = P(1) + P(2) + P(3) \approx P(1) = 2.549 \times 10^{-4}$$

The link transmits bits at 256 kbps, and a block has 255 bits, so the block transmission rate is

$$R_{block} = 256 \times 10^3 / 255 = 1003.92 \text{ blocks per second.}$$

The average time between block errors is

$$T_{error} = 1 / (R_{block} \times P(\text{error})) = 1 / (1003.92 \times 2.549 \times 10^{-4}) = 3.908 \text{ seconds}$$

**c.** Estimate the probability that a block of 255 bits contains an undetected error when the link BER is $10^{-3}$ and error detection is applied.

**Answer:** Only three errors can be detected in any one block, so four or more errors are undetected. Hence, the probability that errors in the block are not detected is

$$P(\text{undetected error}) = P(4) + P(5) + P(6) + \ldots$$

With a probability of bit error $p = 10^{-3}$ , the probability of undetected errors in the block is

$$P_e(4) \approx 172 \times 10^{6} \times 10^{-12} \times (1 - 10^{-3})^{251} = 1.337 \times 10^{-4}$$

$$P_e(5) = 8.637 \times 10^{9} \times 10^{-15} \times (1 - 10^{-3})^{250} = 6.762 \times 10^{-6}$$

$$P_e(6) = 3.599 \times 10^{11} \times 10^{-18} \times (1 - 10^{-3})^{249} = 2.805 \times 10^{-7}$$

Hence $P_{undetected} \approx P(4) + P(5) + P(6) = 1.404 \times 10^{-4}$

**d.** Find the message data throughput when the link BER is $10^{-6}$ and a stop and-wait ARQ system is used, assuming one retransmission always corrects the block.

**Answer:** With stop-and-wait ARQ protocol, the transmitter must stop and wait until it receives an ACK or a NAK from the receive end of the link before it sends the next block. The transmission rate is dominated by the 500 ms waiting time for the block to reach the receiver and the receiver to respond with an ACK or NAK. The time to transmit one block is approximately 1 ms, so a single block transmission plus waiting time is $2 \times 250 + 1 = 501$ ms, and throughput averages 509 bits per second, and the message data rate is $247/255 \times 509 = 493$ message bits per second. After 1003.92 blocks have been transmitted, we encounter a detected error and must retransmit the corrupted block. That occurs after 9.88 hours, so the extra time to resend blocks is not significant. Stop-and-wait is the wrong protocol for GEO satellite links, because of the long round trip time. Go-back-N or continuous transmission should be used.

As a supplement to this question, let's look at throughput with go-back-N and continuous transmission ARQ protocols.

**Go-back-N.** With a go-back-N protocol, transmission is continuous until a NAK is returned by the receiver indicating that a corrupted block has been received. At that point, 501 ms worth of blocks has been transmitted since the now corrupted block was sent, and must be restransmitted. At 256 kbps bit rate, this corresponds to 502.96 blocks, so we must resend 503 blocks. On average, we can send 1003.92 blocks before an error occurs, and then we must resend 503 blocks, so the average message bit delivery rate is

$$R_b = 247/255 \times 1003.92 / 1506.92 \times 256 \text{ kbps} = 165.198 \text{ kbps.}$$

This result ignores any time associated with decoding a received packet at the transmitter to recover the NAK, so the message rate will be slightly lower in practice.

**Continuous transmission.** With continuous transmission, we resend only the corrupted block when a NAK is received. Thus, on average, once every 1003.92 blocks we must add an extra block into the bit stream. The message data rate is then

$$R_b = 247/255 \times 1003.92/1004.92 \times 256 \text{ kbps} = 247.472 \text{ kbps.}$$

**6.** Repeat Problem 5 using a block length of 1024 bits and a (1024, 923) code that can detect 22 errors in a block. The (1024, 923) code can correct 10 errors. Find the average number of blocks that can be transmitted before an uncorrected error occurs when the BER is $10^{-3}$. Repeat the analysis for a BER of $10^{-2}$.

Note: The probability of an unlikely event (11 or more errors in a block of 1024 data bits with $P_b = 10^{-3}$ in this case) can be calculated from the Poisson distribution more easily than from the binomial distribution. The Poisson distribution is given by

$$P(x = k) = \frac{l^k e^{-l}}{k!}$$

where $\lambda = N P_b$. $N$ is the block length, and $k$ is the number of bits in error.

**Answer:** The (1024, 923) code can correct 10 errors, so a detected but uncorrected block error occurs when there are 11 through 22 bit errors in the 1024 bit block. We use the Poisson formula because it works well for a small number of errors in a large number of bits.

**a.** If we take no action regarding errors, the message data throughput is

$$R_b = 923/1024 \times 256 \text{ kbps} = 230.75 \text{ kbps}$$

**b.** Errors are corrected when there are 1, 2, …10 errors present in the block of 1024 bits. Hence the probability that all the errors in the block are corrected is

$$P(\text{corrected}) = P(1) + P(2) + P(3) + \ldots.$$

From the Poisson formula

$$P_e(k) = \lambda^k e^{-\lambda} / k! \quad \text{with } \lambda = N \times p \text{ and } N = 1024,\ p = 10^{-3} \text{ giving } \lambda = 1{,}024.$$

For k = 1, 2, 3 4, 5

$$P_e(1) = 1.024 \times e^{-1.024} = 0.3677$$

$$P_e(2) = 1.024^2 / 2 \times e^{-1.024} = 0.1928$$

$$P_e(3) = 1.024^3 / 6 \times e^{-1.024} = 0.0643$$

$$P_e(4) = 1.024^4 / 24 \times e^{-1.024} = 0.0165$$

$$P_e(5) = 1.024^5 / 120 \times e^{-1.024} = 0.0033$$

Higher order terms can be neglected. Thus the probability that the block contains a corrected error is approximately

$$P(\text{corrected}) \approx P(1) + P(2) + P(3) + P(4) + P(5) = 0.665$$

We can correct 10 errors in the block, so a retransmission is required when 11 or more errors are detected. Hence the probability of a retransmission is

$$P(\text{retransmission}) = P(11) + P(12) + P(13) + P(14) + \ldots$$

$$P_e(11) = 1.024^{11} / 11! \times e^{-1.024} = 1.168 \times 10^{-8}$$

$$P_e(12) = 1.024^{12} / 12! \times e^{-1.024} = 9.967 \times 10^{-10}$$

$$P_e(13) = 1.024^{13} / 13! \times e^{-1.024} = 7.851 \times 10^{-11}$$

Hence the probability or a retransmission is approximately

$$P(\text{retransmission}) \approx 1.168 \times 10^{-8} + 9.967 \times 10^{-10} + 7.851 \times 10^{-11} = 1.269 \times 10^{-8}$$

Block length is 1024 bits and transmission rate 256 kbps, so there are 250 blocks transmitted every second, one every 4 ms. The time between uncorrected block errors is

$$T_{error} = 1 / (250 \times 1.269 \times 10^{-8})\text{ s} = 315{,}208\text{ s} = 87.6\text{ hours.}$$

This is a sufficiently long time that the 504 ms required to retransmit one or 125 corrupted blocks does not significantly reduce throughput in go back N and continuous transmission ARQ protocols. Stop and wait is still just as inefficient as in the previous example, of course.

What is surprising in this example is that the system can operate successfully sending long blocks at a bad error rate. The ability to correct 10 bits in the 1024 block renders the link close to error free even when the BER is $10^{-3}$. The probability of eleven or more errors occurring in a block of 1024 bits is sufficiently low that almost all blocks are delivered correctly after forward error correction.

**Bit error rate $10^{-2}$**

When the bit error rate is $10^{-2}$, $N \times \lambda = 1024 \times 10^{-2} = 10.24$.

**a.** Message data throughput with no error detection leading to ARQ is unchanged at 230.75 kbps.

**b.** The probability of a retransmission increases considerably from the case where $p = 10^{-3}$, because, on average, there will now be 10 errors per block. When 11 … 22 errors occur, a retransmission is needed. Hence the probability of retransmission is

$$P(\text{retransmission}) = P(11) + P(12) + P(13) \;\ldots\; P(22)$$

Individual probabilities are shown below.

$P_e(11) = 10.24^{11} / 11! \times e^{-10.24} \quad = 0.1161$

$P_e(12) = 10.24^{12} / 12! \times e^{-10.24} \quad = 0.0991$

$P_e(13) = 10.24^{13} / 13! \times e^{-10.24} \quad = 0.0781$

$P_e(14) = 10.24^{14} / 14! \times e^{-10.24} \quad = 0.0571$

$P_e(15) = 10.24^{15} / 15! \times e^{-10.24} \quad = 0.390$

$P_e(16) = 10.24^{16} / 16! \times e^{-10.24} \quad = 0.0249$

$P_e(17) = 10.24^{17} / 17! \times e^{-10.24} \quad = 0.0150$

$P_e(18) = 10.24^{18} / 18! \times e^{10.24} \quad = 0.0085$

Ignoring higher order terms

$P(\text{retransmission}) \approx 0.478$

On average, one in every 2.28 blocks contains more than 10 errors and must be retransmitted. We can transmit $2.28 \times 923 = 2108$ message bits, on average, before we must repeat a block. The time to transmit 2.28 blocks is 9.12 ms, so a go-back-N ARQ protocol would, on average, send 2.28 blocks and then have to repeat 504 ms of transmissions, so throughput would average about $2 \times 2.28 = 4.56$ blocks per second, or 4407 message bits per second.

With continuous transmission, we need to introduce a repeat block into the bit stream after every 2.28 blocks, so throughput would be $2.28/3.28 \times 230.75 = 160.4$ kbps for message bits.

When the BER increases to $10^{-2}$ there are too many errors for the go-back-N ARQ protocol to work efficiently. Continuous transmission ARQ slows down the bit stream, but is able to maintain a reasonable throughput.

The answers here assumed that a single retransmission always correctly replaced a corrupted block. With $p = 10^{-2}$ there is a significant probability that a retransmitted block is corrupted, requiring yet another retransmission, which may also be corrupted. A BER of $10^{-3}$ is usually regarded as the lower limit for most communication systems.

With Turbo coding, it is possible to operate at higher bit error rates because of the powerful correction capabilities of turbo codes.

# Chapter 8 Solutions to Problems

1. (a) For a typical satellite-to-ground link, give one propagation impairment that has less impact on the average signal level as the frequency increases from 1 GHz to 30 GHz and give one propagation impairment that has a greater impact on the average signal level as the frequency increases from 1 GHz to 30 GHz.

(b) Ionospheric scintillation effects are variable in both time and location. (i) What causes the time variability? (ii) Over what periodic interval of time do we expect to observe the worst effects repeating on a given satellite link? (iii) In any given year, when would the worst ionospheric effects be observed? (iv) On any given day, when would the worst ionospheric effects be observed? (v) Within what latitude range on the earth's surface are these effects the worst for geostationary satellite links?

**Solution to question 1**

(a) Ionospheric effects become less and less as the transmission frequency goes from 1 to 30 GHz. The various effects have at least an inverse relationship to frequency, as follows: Faraday rotation $1/(f)^2$; Propagation delay $1/(f)^2$; Refraction $1/(f)^2$; Variation in direction of arrival $1/(f)^2$; absorption (polar and auroral cap) $1/(f)^x$ where $1 < x \leq 2$; absorption (mid-latitude) $1/(f)^2$; Dispersion $1/(f)^3$. All of the other atmospheric propagation impairments increase in their effect as the frequency goes from 1 to 30 GHz. Interestingly, though, as the frequency goes from 1 to 30 GHz, the amount of depolarization per 1 dB of signal attenuation actually reduces. This is because the propagation impairments are transitioning from a depolarization dominant effect at around 4 GHz to an attenuation dominant effect at 30 GHz.

(b) (i) Ionization in the atmosphere is directly influenced by the amount of solar energy incident at the location in question. Thus there will be a build up in ionization when there is sun directly on the atmosphere and recombination (de-ionization) when the solar energy is removed from that part of the atmosphere. Ionospheric scintillation occurs mainly in the transition from a highly ionized state to a much less ionized state. Thus the rotation of the earth will lead to the time variability in the scintillation observed on a specific link on a daily basis. The spin axis of the earth is tilted $23.5^o$ from the plane of the earth's orbit around the sun (the ecliptic). Twice a year, in the equinox periods in late March and late September, the equator of the earth points directly at the sun and the equatorial regions of the earth receive a maximum dose of solar radiation. There will thus be an additional annual variability induced by the fact the spin axis of the earth is tilted away from the normal to the ecliptic, with maximum scintillation effects occurring in the equinox periods. Finally, the sun has an approximately 11-year variability in its sunspot cycle (or a 22 year Hale cycle with half period maxima). There are therefore three factors in the time variability of ionospheric scintillation on a link through the earth's atmosphere: the spin of the earth (a daily variability), the tilt of the earth in its orbit around the sun (leading to equinoctial maxima), and the solar sun spot cycle (leading to an 11-year variability).

(ii) From the information given in the solution to part (i) above, we would expect the worst effects on any given link to repeat about every 11 years with the sun spot cycle.
(iii) In any given year, the worst effects would be observed around the two equinoctial periods in late March and Late September.
(iv) On any given day, the worst effects would normally be observed just after local sunset on the earth-space link through the ionosphere.
(v) Because the equatorial region receives the most energy from the sun and the magnetic equator helps to trap the ionized plasma around the equator, the worst effects will tend to be observed in a region with about $\pm 20^{o}$ of the ***geomagnetic*** equator on geostationary satellite links. NOTE: there will also be severe ionospheric effects observable in the regions above the North and South Poles (the so-called auroral regions), but geostationary satellite links do not pass through these regions.

2. A DTH (Direct To Home) satellite ISP (Internet Service Provider) consortium is designing a system to provide digital multimedia service to CONUS (CONtinental United States) coverage. They are investigating where in the US to locate the uplink transmitter for their service. The uplink to the Internet/Multimedia satellite will be at 30 GHz and the downlink to the user terminals will be at 20 GHz.

(a) Using Table 8.2 and Fig. 8.15, determine the approximate region within CONUS where you would expect to obtain the lowest outage time due to rain for the uplink transmitter in the Internet/Multimedia satellite system.

(b) For time percentages of 0.01% and 0.001% of a year, what are the specific attenuations observed at the site you have selected in part (a), assuming horizontal polarization is employed? (Hint: you will need to refer to Table 8.3 for this part of the question.)

(c) If the elevation angle of the transmitting earth station is $50^{o}$, it is 200m above mean sea level, and the height of the rain $h'_R$ is 4.5 km, what are the predicted values of rain attenuation measured for time percentages 0.01% and 0.001% of a year for the uplink transmitter?

**Solution to question 2**

(a) From Fig. 8.15 and Table 8.2, it can be seen that there are three regions, regions B, C, and E, where the rainfall rate for 0.01% and 0.001% of an average year are fairly low. Region C, which covers parts of Alaska, is mostly above the 48th parallel, but it does dip down in the New England region of the continental US. Region B covers mainly the Rocky Mountains. Region E covers a large swathe of the mid-West states and most of California. At the 0.01% time point, the rainfall rates are 12, 15, and 22 mm/h. Since the rain attenuation observed on any given link is directly proportional to the rainfall rate, it would be normal to choose the region with the lowest rainfall rate. However, the further north the earth station is, the lower the elevation angle would be and path attenuation increases as the elevation angle reduces. This would tend to rule out Alaska and the New England regions. These regions have a lot of snow and so it would be necessary to have deicing equipment on the antenna, which can be somewhat expensive to install and operate. For similar reasons, the Rocky Mountain region would generally be ruled out, except for highly populated areas (like Denver or Boulder in Colorado) as the cost of the infrastructure would lead to an expensive installation, in addition to the adverse climate in winter. As a compromise between rainfall rate, harsh winters, and geographic remoteness, a suitable site for the uplink transmitter would be in climate E, towards the south, perhaps in Arizona or New Mexico. These more southern regions also have the advantage of a higher elevation angle. Propagation effects tend to decrease in their impact as the elevation angle increases.

(b) Climate E has annual average rainfall rates of 22mm/h for 0.01% of a year and 70 mm/h for 0.001% of a year. From equation (8.13), the specific attenuation $\gamma_R = k(R)^{\alpha}$, and from Table 8.3, $k$ = 0.0751 at 20 GHz and 0.187 at 30 GHz and $\alpha$ = 1.099 at 20 GHz and 1.021 at 30 GHz, all values for horizontal polarization. The specific attenuation values are therefore:

| Frequency | 0.01% | 0.001% |
|---|---|---|
| 20 GHz | 2.2437dB/km | 8.0058 dB/km |
| 30 GHz | 4.3899 dB/km | 19.9345 dB/km |

(c) Referring to the ITU-R rain attenuation prediction procedure starting on page 321:
Step 1 – we have been given $h'_R$ as 4.5 km

Step 2 – We have been given the height of the earth station above mean sea level as 200 m and the elevation angle of the link as 50$^o$ and so we can compute $L_S = \frac{(h'_R - h_S)}{\sin\theta}$ = (4.5 - 0.2)/sin50 = 5.6133 km

Step 3 – $L_G = L_s \cos\theta$ = 5.6133cos(50) = 3.6081 km

Step 4 – $R_{0.01}$ = 22 mm/h from part (b) above

Step 5 – $\gamma_R = k\,(R_{0.01})^{\alpha}$ = 4.3899 from part (b) above

$$\text{Step 6 – } r_{0.01} = \frac{1}{1 + 0.78\sqrt{\frac{L_G \gamma_R}{f}} - 0.38\left(1 - e^{-2L_G}\right)}$$
$$= 0.8424$$

Step 7 – $\boldsymbol{z} = \tan^{-1}\left(\frac{h'_R - h_s}{L_G r_{0.01}}\right)$

$= 54.7453^{o}$

Parameter $\boldsymbol{z}$ is greater than the elevation angle, and so $L_R = \frac{L_G r_{0.01}}{\cos \boldsymbol{q}}$

Giving $L_R$ = 4.7286 km

We will assume that the latitude of the earth station is below 36$^{o}$, say 35$^{o}$, and so $\chi$ = 36 - |$\boldsymbol{f}$| degrees = 1 and so the parameter used in calculating the effective pathlength is

$$v_{0.01} = \frac{1}{1 + \sqrt{\sin(\theta)}\left(31\left(1 - e^{-\left(\theta/(1+\chi)\right)}\right)\frac{\sqrt{L_R \gamma_R}}{f^2} - 0.45\right)}$$

$= 1.3450$

Step 8 – The effective pathlength is $L_E = L_R v_{0.01}$

Thus $L_E$ = 4.7286 × 1.3450 = 6.36 km

Step 9 – The predicted path attenuation exceeded for 0.01% of an average year is

$$A_{0.01} = \boldsymbol{g}_R L_E$$

Giving $A_{0.01}$ = 4.3899 × 6.36 = 27.9198 dB

Step 10 – To find the predicted path attenuation for 0.001% of an average year we need use the equation

$$\mathrm{A_p} = A_{0.01}\left(\frac{p}{0.01}\right)^{-(0.655 + 0.033\ln(p) - 0.045\ln(A_{0.01}) - b\,(1-p)\sin(q))}$$

where $p$ is the time percentage of interest and, since $p$ < 1% and |$\boldsymbol{f}$| < 36$^{o}$ and $\boldsymbol{q}$ ≥ 25 deg: $\boldsymbol{b}$ = -0.005(|$\boldsymbol{f}$| - 36), giving $\boldsymbol{b}$ = 0.005, and

$A_{0.001} = A_{0.01}(p/0.01)^{-0.2734}$ = (27.9198) × (0.001/0.01)$^{-0.2734}$ = 52.3977 = 52.4 dB

The attenuations measured on the link for 0.01% and o.001% of an average year are 27.9 and 52.4 dB, respectively. The 0.001% level of attenuation is unlikely to be met economically in a single earth station site and so site diversity will probably have to be employed if availabilities of 99.999% of a year (outage of 0.001%) are required for the ISP. It is feasible to provide 27.9 dB with a single earth station if uplink power control of about 10 dB is possible.

3. A simple procedure to calculate the effect of changing the elevation angle is to use the cosecant law (see equation (8.21)). To calculate the effect of changing the frequency, either a simple formula may be used (see equation (8.22)) or a more complicated procedure may be used (see equation (8.23) et seq.). A DTH satellite ISP consortium is investigating whether it should change location and frequency for its uplink transmitter. Using their currently allocated frequency (30 GHz) on the uplink, they have observed an attenuation of 30 dB at an elevation angle of $50^o$ at the time percentage of interest to their service. They have been notified that they would be permitted to use an uplink frequency at 18 GHz, but they would have to relocate their earth station. The rain climate is the same as their current earth station, but the elevation angle would now be $15^o$ instead of $50^o$.

(a) Using the simple frequency scaling formula (8.22), what is the attenuation that would be observed at a frequency of 18 GHz if an attenuation of 30 dB was observed at 30GHz (both at an elevation angle of $50^o$)?

(b) If the elevation angle is reduced from $50^o$ to $15^o$, what attenuation would be observed at a frequency of 18 GHz, using the value you calculated for 18 GHz for an elevation angle of $50^o$ in part (a) above as input?

(c) Based on the answer you obtained in part (b), would you recommend moving the uplink transmitter and changing the uplink frequency to 18 GHz?

(d) Would your answer change if you used the more complicated frequency scaling formula in (8.23) instead of the frequency-squared law in equation (8.22)?

**Solution to question 3**

It always helps to list out the information and sometimes draw a diagram of the situation. Here we will just list out the information.

| Current earth station data: | Possible new earth station data: |
|---|---|
| Frequency = 30 GHz | Frequency = 18 GHz |
| Attenuation = 30 dB | Attenuation = To Be Found |
| Elevation angle = $50^o$ | Elevation angle = $15^o$ |
| Rain Climate = X | Rain Climate = X (no change) |

(a) Using simple frequency scaling formula in equation (8.25) for an elevation angle of $50^o$ in both cases, we have

$$A_{18} = A_{30} \times ((18)^2/(30^2) = 30 \times (324/900) = 10.8 \text{ dB}$$

Using the more complicated frequency scaling formula in equation (8.26) for an elevation angle of $50^o$ in both cases, we have from equation (8.27)

$f(f) = (f^2)/(1 + 10^{-4} f^2)$, which gives $f(30) = 981$ and $f(18) = 334.4976$, and

$$H(f_1, f_2, A_1) = 1.12\times10^{-3}\left(f_2/f_1\right)^{0.5}\left(f_1 A_1\right)^{0.55}$$

Giving $H(f_1,\ f_2, A_1) = 1.12 \times 10^{-3}(334.4976/981)^{0.5}(981 \times 30)^{0.55} = 0.1877$

And $A_{18} = A_{30}\ (f(18)\ /\ f(30)\ )^{1-H(f_1,\ f_2,\ A_{30})} = 30\ (334.4976/981)^{1-0.1877} = 12.5182$ = 12.5 dB

The simple formula gives $A_{18}$ = 10.8 dB while the more complicated formula gives $A_{18}$ = 12.5 dB

(b) Using the simple cosecant scaling equation (equation (8.24)) we have

$A_{18(15^o)} = A_{18(50^o)} \times$ (cosecant (18$^o$)/cosecant (50$^o$) = 10.8 $\times$ (3.2361/1.3054) = 26.7729 = 26.8 dB if we use the simple frequency scaling formula and $A_{18(15^o)}$ = 12.5 $\times$ (3.2361/1.3054) = 31.0328 = 31 dB if we use the more complicated formula.

(c) We can now redo the data we listed at the start of the question as follows

| Current earth station data: | Possible new earth station data: |
|---|---|
| Frequency = 30 GHz | Frequency = 18 GHz |
| Attenuation = 30 dB | Attenuation = 26.8 dB (simple) |
| | Attenuation = 31 dB (more complex) |
| Elevation angle = 50$^o$ | Elevation angle = 15$^o$ |
| Rain Climate = X | Rain Climate = X (no change) |

Based on the simple frequency scaling formula, it would appear to be better to move the site as the rain attenuation margin is lower by more than 3 dB. However, there will be many other factors to take into account, not the least of which will be disruption of engineering and support staff from the original location for such a small reduction in path attenuation.

(d) Yes, the answer would change using the more complicated formula, since there is no gain, and even an apparent increase of about 1 dB in path attenuation.

4. A Cable TV network (CATV) downlink signal to a cable head-end terminal is at approximately 12.5 GHz. The attenuation measured on the CATV downlink for 0.01% of the time in the location of interest is 12 dB. The downlink is using dual-**polarization** frequency re-use to permit all of the channels to be sent through one satellite. In order to meet the QOS (Quality Of Service) guarantees established for the terminal, the XPD must be no lower than 15 dB.

(a) What is the predicted XPD for 0.01% of the time if the downlink signal is linearly polarized with a tilt angle of $0^o$?

(b) What is the predicted XPD for 0.01% of the time if the downlink signal is circularly polarized?

(c) Is the XPD minimum of 15 dB for the QOS interference criterion met in either of these cases?

**Solution to question 4**

(a) We will need to calculate the XPD for this link at the time percentage of interest, given the path attenuation measured (12 dB) and the polarization employed (linear polarization). The XPD prediction procedure is given, starting on page 330.

Step 1 – $C_f = 30 \log f = 30 \log (12.5) = 32.9073$

Step 2 – $C_A = V(f) \log A_p$ where $V(F) = 12.8 f^{0.19}$ for $8 \le f \le 20$ GHz.

Thus $C_A = [12.8\, (12.5)^{0.19} \times \log (12)] = 20.6834 \times \log 12 = 22.3212$

Step 3 – $C_\tau = -10 \log [\,1 - 0.484\,(1 + \cos 4\tau)\,] = -10 \log [1 - 0.484\,(1 + \cos (0))]$

$= 14.9485$

Step 4 – $C_\theta = -40 \log (\cos \theta)$ for $\theta \le 60^o$ where $\theta$ is not given (the elevation angle). We will assume three values: 20, 40, and $60^o$. At these three elevation angles, $C_\theta = 1.0806$, $= 4.6298$, and $= 12.0412$, respectively.

Step 5 – $C_\sigma = 0.0052\, \sigma^2$ and $\sigma = 10^o$ for 0.01% of the time, giving $C_\sigma = 0.52$

Step 6 – $XPD_{rain} = C_f - C_A + C_\tau + C_\theta + C_\sigma$ dB, and for an elevation angle of $20^o$,

$= 32.9073 - 22.3212 + 14.9485 + 1.0806 + 0.52 = 27.1352 = 27.1$ dB

and $= 30.7$ ($40^o$ elevation angle) and $= 38.1$ ($60^o$ elevation angle)

Step 7 – $C_{ice} = XPD_{rain} \times (0.3 + 0.1 \log p) / 2$ dB = 1.3550, 1.535, and 1.905 dB for 20, 40, and 60 degree elevation angles, respectively.

Step 8 – $XPD_p = XPD_{rain} - C_{ice}$ = 25.7, 29.2, and 36.2 dB for 20, 40, and 60 degree elevation angles, respectively.

(b) For circularly polarized signals, the only change from the above will be step 3, which now becomes

$C_t = -10 \log [\ 1 - 0.484\ (1 + \cos 4\tau\ )\ ] = -10 \log [1 - 0.484\ (\ 1 + \cos (180))]$

$= 0$, and so

$XPD_{rain}$ = 32.9073 – 22.3212 + 0 + 1.0806 + 0.52 = 12.2, 15.7, and 23.1 dB for 20, 40, and 60 degree elevation angles, respectively. Thus, the total XPD will be 12.2 – 1.3550, 15.7 – 1.535, and 23.1 – 1.905 = 10.9, 14.2, and 21.2 for 20, 40, and 60 degree elevation angles, respectively.

(c) The QOS minimum of 15 dB is met in all three linear polarization cases, i.e. for all elevation angles between 20 and 60 degrees. For circular polarization, the QOS minimum level of 15 dB was only met for the $60^o$ elevation angle example.

5. Tropospheric scintillation is not an absorptive effect and so will not lead to an increase in receiver noise temperature, as rain attenuation will. There will be a slight increase in noise temperature due to enhanced humidity levels that lead to tropospheric scintillation, but we will ignore that aspect for this question.

A Direct To Home (DTH) Ku-Band receiver has the following design specifications:
System Noise Temperature in Clear Sky: 100K
Clear Sky C/N = 11 dB
Performance Margin: 99% of a year with C/N > 10dB
Availability Margin: 99.9% of a year with C/N > 6 dB

The climate in which the DTH receiver is to operate has been predicted to have the following tropospheric scintillation and rain attenuation statistics:

| Annual %-age Time | Scintillation fade level | Rain Attenuation Fade Level |
|---|---|---|
| 10% | 0.5 dB | 0 dB |
| 1% | 1.5 dB | 1 dB |
| 0.1% | 2.5 dB | 3 dB |
| 0.01% | 3.5 dB | 10 dB |

(a) What is the reduction in C/N due only to tropospheric scintillations at the four percentage times?
(b) What is the reduction in C/N due only to rain attenuation at the four percentage

times (the *effective temperature* of the rain medium = 280K)?

(c) Is the tropospheric scintillation a performance-limiting phenomenon, when acting in isolation from other effects?

(d) Is the tropospheric scintillation an availability-limiting phenomenon when acting in isolation from other effects?

(e) Is rain attenuation a performance-limiting phenomenon, when acting in isolation from other effects?

(f) Is rain attenuation an availability-limiting phenomenon when acting in isolation from other effects?

(g) When combining tropospheric scintillation and rain attenuation effects for this DTH terminal, does the terminal meet both the performance and the availability specifications?

(h) If your answer is "no" to either the performance or availability questions in part (g) above, what additional C/N margin is required to meet the performance and availability specifications?

**Solution to question 5**

(a) The tropospheric scintillation will only affect the *C* of the *C/N* parameter, thus we can construct the following table for just tropospheric scintillation:

| Annual %-age Time | Scint. fade level | C/N reduction | Overall C/N level |
|---|---|---|---|
| 10 | 0.5 dB | 0.5 dB | 10.5 dB |
| 1.0 | 1.5 dB | 1.5 dB | 9.5 dB |
| 0.1 | 2.5 dB | 2.5 dB | 8.5 dB |
| 0.01 | 3.5 dB | 3.5 dB | 7.5 dB |

(b) When there is an absorbing medium in the path, the increase in sky noise is given by equation (8.40). For the four time percentage points with attenuations of 0, 1, 3, and 10 dB, the increase in sky noise temperature values as perceived by the antenna, $T_b$, are $280(1 - e^{-A/4.34})$ = 0, 57.6, 139.7, 252.0 K, respectively.

The noise power = $kTB$, where $k$ is Boltzmann's constant, $T$ is the system noise temperature. Therefore, the ratio of the noise power in clear sky to that in rainy conditions is $(kT_{\text{clear sky}}B)/(kT_{\text{rainy sky}}B)$, where $T_{rainy\ sky} = T_{clear\ sky} + T_b$. Therefore $(kT_{\text{clear sky}}B)/(kT_{\text{rainy sky}}B) = (T_{\text{clear sky}})/(T_{\text{rainy sky}})$ = 100/100, 100/157.6, 100/239.7, and 100/352.0, leading to dB changes in the noise power, $N$, of 0, 1.9756, 3.7967, 5.4654. A new table can therefore be constructed as follows (remembering the C/N reduction = rain fade + noise increase):

| Annual %-age Time | Rain fade level | C/N reduction | Overall C/N level |
|---|---|---|---|
| 10 | 0 dB | 0 dB | 11 dB |
| 1.0 | 1 dB | 3.0 dB | 8 dB |
| 0.1 | 3 dB | 6.8 dB | 4.2 dB |
| 0.01 | 10 dB | 15.5 dB | - 4.5 dB |

(c) Performance limiting means that the phenomenon in question makes the C/N go below the required value at 99% of the time (the performance time percentage point). The margin required for 99% of a year is >10 dB. At 99% (1.0% outage), the C/N due to just tropospheric scintillation is 9.5 dB. Tropospheric scintillation is therefore performance limiting on this link.

(d) Availability limiting means that the phenomenon in question makes the C/N go below the required value at 99.9% of a year (the availability time percentage point). The margin required for 99.9% of a year (0.1% outage) is 6 dB, At 99.9% (0.1% outage), the C/N due to just tropospheric scintillation is 8.5 dB. Tropospheric scintillation is therefore not availability limiting.

(e) By the same criterion used in (c) above, the C/N in rain-only conditions at 99% (1% outage) is 8 dB. Rain is therefore also performance limiting on this link

(f) By the same criterion used in (d) above, the C/N in rain-only conditions at 99.9% (0.1% outage) is 4.2 dB. Rain attenuation is therefore availability limiting on this link.

(g) Combining the two effects (we can either assume they are independent variables and add them as root summed squares, or treat them as directly combinative, and so add the C/N reductions linearly – it is more likely the RSS approach is closest to the real effect) prevents the link from meeting the performance requirements.

Adding the C/N reductions as a RSS combination we have the following:

| Time point | Trop. C/N reduction | Rain C/N reduction | RSS combination |
|---|---|---|---|
| 10 % | 0.5 | 0 | 0.5 |
| 1.0 % | 1.5 | 3.0 | 3.4 |
| 0.1 % | 2.5 | 6.8 | 7.2 |
| 0.01 % | 3.5 | 15.5 | 15.9 |

The required total C/N reductions to meet the performance (1% outage) and availability (0.1% outage) from clear sky are 1 dB and 5 dB, respectively. The terminal therefore does not meet either the performance or availability requirements.

(h) The system needs to "buy" back 2.4 dB to meet the performance margin and 2.2 dB to meet the availability requirement. This can be done in a number of ways: increasing the diameter of the antenna by a factor of 1.32 (2.4 dB is a ratio of 1.7378; square-rooting this gives 1.3183 as the gain increases as the diameter squared); increasing the power of the transmitting amplifier by 2.4 dB; increasing the level of coding, etc.

6. An earth station complex is being designed to provide high availability access at Ka-Band to a satellite system used in a DTH Internet/Multimedia service offering. The earth station must meet an annual 99.99% availability level, i.e. the maximum outage in any year must not exceed 0.01% of a year. The maximum EIRP from the earth station on the 30 GHz uplink is limited by interference considerations, resulting in a maximum rain fade margin of 30 dB on the uplink. For other reasons, the downlink rain fade margin (which includes the C/N reduction due to noise temperature increase) at a frequency of 20 GHz is limited to a maximum of 9 dB. The earth station complex is situated in a region where the rain attenuation at 30 GHz is 50 dB for 0.01% of a year. The approximate frequency scaling formula may be used to find the equivalent 20 GHz fade level for 0.01% of a year. As can be seen, a single earth station will not be able to meet the availability requirements in this location. To overcome the rain attenuation, consideration is being given to operating two earth stations as a site diversity pair. The earth stations will operate at an elevation angle of $40^{o}$. The baseline between the earth stations is $90^{o}$ to the satellite azimuth.

(a) Calculate the diversity gain achievable at 30 GHz and 20 GHz for site separations of 2, 3, 5, 10, 15, and 20 km in the above diversity system.

(b) If an additional margin of 3 dB must be allowed to provide a hysteresis band to assist switching between the earth stations on the 30 GHz uplink (i.e. the 50 dB rain margin is increased to 53 dB), which is the first site separation calculated in part (a) that will permit the 30 GHz uplink availability target to be met (going from the lowest separation to the highest)?

(c) If an additional margin of 0.5 dB must be allowed for inefficiencies in the diversity combining on the 20 GHz downlinks between the diversity pair of earth stations (i.e. the 9 dB rain margin is increased to 9.5 dB), which is the first site separation calculated in part (a) that will permit the 20 GHz downlink availability target to be met (going from the lowest separation to the highest)?

**Solution to question 6**

(a) We will use the diversity gain calculation procedure that starts on page 336. Equation (8.45) states

$$G_d = a\,(1 - e^{-bd})$$

where $d$ is the separation (km) between the two sites,

$a = 0.78A - 1.94(1 - e^{-0.11A})$

$b = 0.59(1 - e^{-0.1A})$

and $A$ = path attenuation (dB) for a single site

On the uplink, path attenuation is 50 dB at the 0.01% point. On the downlink, using the approximate frequency scaling formula (equation (8.25)), the path attenuation is $A_{30} \times (20^2/30^2) = A_{30} \times (0.4444) = 50 \times 0.4444 = 22.22$ dB. For an attenuation of 50 dB at a frequency of 30 GHz, the parameters $a$ and $b$ are $a = 37.0679$ and $b = 0.5860$. For an attenuation of 22.22dB at a frequency of 20 GHz, the parameters $a$ and $b$ are $a = 15.5617$ and $b = 0.5261$. The values of $G_d$ vs. site separation d are as follows:

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Site separation (km) | = | 2 | 3 | 5 | 10 | 15 | 20 |
| $G_d$ (30 GHz) | = | 25.59 | 30.67 | 35.09 | 36.96 | 37.06 | 37.07 |
| $G_d$ (20 GHz) | = | 10.13 | 12.35 | 14.44 | 15.48 | 15.56 | 15.56 |

From equation (8.46), $G_f$ = 0.4724 for 30 GHz and 0.6065 for 20 GHz.

From equation (8.47), $G_\theta$ = 1.24 for both frequencies.

From equation (8.480, $G_\psi$ = 1.18 for both frequencies.

Equation (8.49) gives the net diversity gain, $G$, as $G_d \times G_f \times G_\theta \times G_\psi$ dB. With frequency (30 and 20 GHz) and site separation (d) as parameters, we have the following tabulation of diversity gain vs. frequency and site separation.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Site separation (km) | = | 2 | 3 | 5 | 10 | 15 | 20 |
| $G$ (30 GHz) | = | 17.69 | 21.20 | 24.25 | 25.54 | 25.62 | 25.62 |
| $G$ (20 GHz) | = | 8.99 | 10.96 | 12.81 | 13.73 | 13.81 | 13.81 |

(b) At time percentage point of 0.01% for an average year, the uplink attenuation at 30 GHz is given as 50 dB. If a hysteresis band of 3 dB is included, this attenuation becomes an effective 53 dB. The maximum rain fade margin that can be tolerated for interference reasons is given as 30 dB. The difference between the effective rain fade level (53 dB) and the margin permitted (30 dB) means that a site diversity gain of 53 – 30 = 23 dB must be achieved. Based on the table calculated at the end of part (a) to this question, a diversity gain at 30 GHz of ≥ 23 dB can be obtained with a site separation of 5 km.

(c) On the downlink, the 9 dB rain attenuation margin must be increased to 9.5 dB to allow for inefficiencies in the combining of the two diversity downlinks. At 0.01% of an average year, the downlink rain attenuation is 22.2222 dB. The difference between 22.2222 and 9.5 dB is the required diversity gain of the site diversity set up. The required diversity gain is therefore 12.7222 = 12.7 dB, and this is first achieved at a site separation of 5 km.

NOTE: Many operators have decided to play it safe and have site separations of 20 km, or more. If the added cost of increasing the site separation from 5 km to 20 km is relatively small, it can be argued that going for a 20 km site separation is a ‘safer’ choice than opting for 5 km. Care must be taken, though, to ensure that the two sites are not oriented along a lien of hills, or similar feature, that leads to an enhanced probability that the rain cells will form simultaneously over the two sites.

7. The attenuation that would be observed on a satellite path may be estimated by measuring the increase in antenna noise temperature and calculating the associated attenuation *A* from equation (8.40). Instruments that do this are called radiometers. They are useful for attenuation values between 0 and about 10 dB, but not very good for larger values of inferred attenuation. (The effect is called radiometer saturation.) Using equation (8.37), show why this occurs.

**Solution to question 7**

Equation (8.40) states that the *increase* in antenna noise temperature due to rain, $T_b$, may be estimated by

$T_b = 280(1 - e^{-A/4.34})$ K, or more generally

$T_b = T_m (1 - e^{-A/4.34})$ K, where $T_m$ is the temperature of the attenuating medium (the rain, in other words).

An alternative way of expressing this equation, is

$$A = 10\log_{10}[T_m / (T_m - T_b)]$$

We can clearly see that, as the antenna noise temperature increase, $T_b$, approaches that of the temperature of the rain, the value of $(T_m - T_b)$ becomes increasingly small. We can tabulate A vs. $T_b$ in 10-degree increments of antenna temperature increase, as shown below. Clearly, as $T_b$ becomes larger, the increase in *A* becomes disproportionately large for the same (10-degree) increase in $T_b$. If the accuracy of either (a) the measurement of $T_b$, or (b) the precision with which the value of $T_m$ is a fixed amount, there will come a point when the accuracy of *A* comes into question. The effect is similar to that of saturation. Typically, a well-calibrated radiometer may be used to infer attenuation values on the order of 10 to 12 dB.

NOTE: because the attenuation measured at very low values of $T_b$ is very accurate; radiometers are generally excellent instruments to use with beacon receivers in that the radiometer can set the clear sky level of the beacon receiver very accurately.

**Table of inferred path attenuation *A* vs. antenna temperature $T_b$**

| $T_b$ | *A* $T_m$ = 270 | 280 | 290 | 300 |
|---|---|---|---|---|
| 10 | 0.2 dB | 0.2 dB | 0.2 dB | 0.1 dB |
| 20 | 0.3 dB | 0.3 dB | 0.3 dB | 0.3 dB |
| 30 | 0.5 dB | 0.5 dB | 0.5 dB | 0.6 dB |
| 40 | 0.7 dB | 0.7 dB | 0.6 dB | 0.6 dB |
| 50 | 0.9 dB | 0.9 dB | 0.8 dB | 0.8 dB |
| 100 | 2.0 dB | 1.9 dB | 1.8 dB | 1.8 dB |
| 200 | 5.9 dB | 5.4 dB | 5.1 dB | 4.8 dB |
| 250 | 11.3 dB | 9.7 dB | 8.6 dB | 7.8 dB |
| 260 | 14.3 dB | 11.5 dB | 9.9 dB | 8.8 dB |
| 270 | infinity | 14.5 dB | 11.6 dB | 10 dB |
| 280 | - | infinity | 14.6 dB | 11.8 dB |
| 290 | - | - | infinity | 14.8 dB |
| 300 | - | - | - | infinity |

NOTE: at very low levels of sky temperature increase, the error in choosing $T_m$ is very small. For example, for $T_b$ below 40 K, the inferred path attenuation varies by less than 0.1 dB for a range of physical medium temperature of 30 K. This is why radiometers are so good at setting the clear sky level of beacon receivers in propagation experiments.

8. Geostationary communications satellites that use linear transponders have both advantages and disadvantages when compared with satellites that use On Board Processing (OBP). One of the advantages of a linear transponder is its flexibility in usage – it can easily accommodate analog and digital traffic with varying capacity streams, provided they fit within the transponder bandwidth and power limitations. Another advantage is the ability to apportion outage to different parts of a given link. A typical two-way link is designed one way at a time. That is, for example, in a service between the US and Thailand, the link from Thailand to the US is designed separately from the return link from the US to Thailand. For high availability services, a one-way outage of 0.04% of a year is permitted. The 0.04% can be split equally, 0.02% on the uplink and 0.02% on the downlink. Thailand has a much more severe rain climate than the west coast of the US and it may make the overall link design easier if a non-symmetrical split is made of the total outage time. If the earth station in Thailand is in climate M and the earth station on the western seaboard of the US is in Climate D (see Table 8.2); the uplink frequency is 12 GHz in both cases and the downlink frequency is 10 GHz in both cases;

linear, vertical polarization is used both ways; the elevation angle is $5^o$ at both the Thailand and the US earth stations, find the following:

(a) How should the 0.04% outage time be divided up between the 12 GHz Thailand uplink and the 10GHz US downlink on the Thailand-USA one-way link so that the same rain attenuation is experienced on the Thailand uplink as it is on the US downlink?

(b) How should the 0.04% outage time be divided up between the 12 GHz US uplink and the 10GHz Thailand downlink on the USA-Thailand one-way link so that the same rain attenuation is experienced on the US uplink as it is on the Thailand downlink?

[NOTE: for both parts (a) and (b) in the above question, the answer should be rounded to increments of 0.005%, e.g. 0.03%:0.01% or 0.035%:0.005%]

**Solution to question 8**

First, we need to calculate the rain attenuation for 0.01% for both of these links. The following information is provided, or obtainable from tables or figures in chapter 8. Information that is not given is assumed as follows: height above mean sea level = 50 m and rain heights of 5, and 4 km for Thailand and the west coast of the USA, respectively.

| Site | 0.01% rain rate | Lat. (approx) | Height a.m.s.l. | Rain ht. |
|---|---|---|---|---|
| Thailand | 63 mm/h | $15^o$N | 50m | 5 km |
| USA | 19 mm/h | $45^o$N | 50m | 4 km |

Using the rain attenuation prediction procedure starting on page 322, we have:

$L_S = \dfrac{(h_R' - h_S)}{\sin\theta}$ = (5 – 0.05)/sin5 = 56.7949 for Thailand and = (4 – 0.05)/sin5 = 45.3212 for the USA

$L_G = L_S \cos\theta$, thus $L_G$ = 56.5788 (Thailand) and 45.1487 (USA)

$\gamma_R = k\,(R_{0.01})^{\alpha}$ and for linear, vertical polarization at 12 GHz, k = 0.0168 and $\alpha$ = 1.200; for 10 GHz, k = 0.00887 and $\alpha$ = 1.264. This yields

| | Thailand 12 GHz | Thailand 10 GHz | USA 12 GHz | USA 10 GHz |
|---|---|---|---|---|
| $\gamma_R$ = | 2.4239 dB/km | 1.6684 dB/km | 0.5752 dB/km | 0.3667 dB/km |

$$r_{0.01} = \frac{1}{1 + 0.78\sqrt{\dfrac{L_G \gamma_R}{f}} - 0.38\left(1 - e^{-2L_G}\right)}$$

And so, $r_{0.01}$ = 0.3070 (Thailand, 12 GHz); 0.3315 (Thailand, 10 GHz); 0.5658 (USA, 12 GHz); 0.6159 (USA, 10 GHz).

Next, we need to calculate the vertical adjustment factor, $\nu_{0.01}$, for Thailand and the USA. To do this we need some intermediate parameters.

Part (a) Calculate $\zeta$, where

$$\zeta = \tan^{-1}\left(\frac{h'_R - h_s}{L_G r_{0.01}}\right)$$

= 15.9064 (Thailand, 12 GHz); 14.7843 (Thailand, 10 GHz); 8.7899 (USA, 12 GHz); 8.0848 (USA, 10 GHz).

For $\zeta > \theta$, which is 5° in this question for both ends of the link, $L_R = \dfrac{L_G r_{0.01}}{\cos\theta}$ km

= 17.4360 (Thailand, 12 GHz); 18.8275 (Thailand, 10 GHz); 25.6427 (USA, 12 GHz); 27.9133 (USA, 10 GHz)

For latitudes of earth stations >36°, which is the USA site, $\chi = 0$, while for latitudes of earth stations ≤ 36°, which is the Thailand site, $\chi$ = 36 - |latitude| = 36 – 15 = 21°.

We can now calculate the parameter $\nu_{0.01}$ as follows

$$\nu_{0.01} = \frac{1}{1 + \sqrt{\sin(\theta)}\left(31\left(1 - e^{-\left(\theta/(1+\chi)\right)}\right)\frac{\sqrt{L_R\gamma_R}}{f^2} - 0.45\right)}$$

Giving $\nu_{0.01}$ = 1.0514 (Thailand, 12 GHz); 1.0294 (Thailand, 10 GHz); 0.9012 (USA, 12 GHz); 0.8636 (USA, 10 GHz).

The effective path length is given by equation 8.21):

$L_E = L_R \nu_{0.01}$ = 18.3322km (Thailand, 12 GHz); 19.3810 km (Thailand, 10 GHz); 23.1092 km (USA, 12 GHz); 24.1059 km (USA.

The predicted attenuation exceeded for 0.01% of an average year is obtained from:

$A_{0.01} = \gamma_R L_E$ dB = 44.4 dB (Thailand, 12 GHz); 32.3 dB (Thailand, 10 GHz); 13.3 dB (USA, 12 GHz); 8.8 dB (USA, 10 GHz).

It is clear that the Thailand link has much more attenuation than the USA link. The link designs usually calculate the path attenuations at the same time percentages (0.02%) on the uplink and the downlink. In this case, we will trade time percentages between the uplinks at 12 GHz with the corresponding downlinks at 10 GHz to find when the two path attenuations are close to each other. We need to find the path attenuations at different time percentages to carry out this comparison.

The formula for calculating the attenuation at different time percentages (equation (8.23)) is

$$A_p = A_{0.01}\left(\frac{p}{0.01}\right)^{-(0.655+0.033\ln(p)-0.045\ln(A_{0.01})-b(1-p)\sin(q))}$$

This formula works for low latitudes and elevation angles above 25°. It also works for latitudes above 36° at high time percentages. We will therefore assume that the factor ***b*** = 0 in the above formula in both cases.

We can construct a table of attenuation vs. time percentages as follows, using the above equation.

| %-time | Thailand 12 GHz | Thailand 10 GHz | USA 12 GHz | USA 10 GHz |
|---|---|---|---|---|
| 0.001 | 80.1 | 58.3 | 27.2 | 18.0 |
| 0.005 | 55.0 | 40.0 | 17.1 | 11.3 |
| 0.010 | 44.4 | 32.3 | 13.3 | 8.8 |
| 0.015 | 38.6 | 28.1 | 11.3 | 7.5 |
| 0.020 | 34.7 | 25.2 | 10.0 | 6.6 |
| 0.025 | 31.8 | 23.1 | 9.1 | 6.0 |
| 0.030 | 29.6 | 21.5 | 8.4 | 5.6 |
| 0.035 | 27.8 | 20.2 | 7.8 | 5.2 |
| 0.040 | 26.3 | 19.1 | 7.3 | 4.8 |

(a) The closest we can get with the uplink at 12 GHz from Thailand having the same path attenuation as the 10 GHz downlink to the USA is with 0.04% allocated to the Thailand link (26.3 dB) and 0.001% allocated to the USA link (18 dB). This will cause the overall link to exceed the 0.04% allocation. There is no real way we can equalize the two links without having fade countermeasures on one, or both, of the link.

(b) To provide the same path attenuation on the uplink at 12 GHz from the USA as that on the 10 GHz downlink from Thailand we can pick 0.035% on the Thailand downlink (20.2 dB) and 0.005 on the USA uplink (17.1 dB). These levels of attenuation are not usually economic and again it is likely that some form of fade countermeasures will be required on both links.

# Chapter 9 Solutions to Problems

1. (a) What does the acronym VSAT stand for?
   (b) What is the typical range, in meters, of the aperture diameter for a VSAT operating with a Ku-band satellite?
   (c) As a Direct Broadcast Satellite Service (DBSS) operator, you want to identify the appropriate receive antenna to use in the home market. Calculate, and set down in tabular form, the gain (in dB) and 1 dB beamwidth of the following antenna diameters: 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, and 1.0 m. Assume a frequency of 12 GHz, an antenna efficiency of 55%, and that the 1 dB beamwidth is half that of the 3 dB beamwidth.
   (d) If users are able to point their antennas to within $\pm 0.5^o$ and require a minimum gain of 30 dB, what antenna diameter range is available to the users?
   (e) Given this acceptable range of antenna diameters, which one of these antenna diameters would you choose, stating your reasons?

**Solution to question 1**

(a) VSAT stands for **V**ery **S**mall **A**perture **T**erminal

(b) Typical VSAT antenna diameters operating to Ku-band satellites range from those used in direct broadcast satellite systems, ~0.6m, to those used in point of sale systems, ~1.2 to 2.4m. The data rate often will determine the gain (and hence the diameter) of the VSAT antenna; the higher the required data rate, the larger the antenna diameter.

**(c)** Table of gain and beamwidth assuming a frequency of 12 GHz, an efficiency of 55%, and that the 1 dB beamwidth is approximately half that of the 3 dB beamwidth.

| Antenna diameter | Gain | 3 dB beamwidth | 1 dB beamwidth |
|---|---|---|---|
| 0.1m | 19.4 | $17.2^{Oo}$ | $8.6^o$ |
| 0.2m | 25.4 | $8.6^o$ | $4.3^o$ |
| 0.3m | 28.9 | $5.7^o$ | $2.9^o$ |
| 0.4m | 31.4 | $4.3^o$ | $2.1^o$ |
| 0.5m | 33.4 | $3.4^o$ | $1.7^o$ |
| 0.6m | 35.0 | $2.9^o$ | $1.4^o$ |
| 0.7m | 36.3 | $2.5^o$ | $1.2^o$ |
| 0.8m | 37.5 | $2.1^o$ | $1.1^o$ |
| 0.9m | 38.5 | $1.9^o$ | $1.0^o$ |
| 1.0m | 39.4 | $1.7^o$ | $0.9^o$ |

(d) If an antenna can be pointed to within $\pm 0.5^o$, this means that a beamwidth of less than this amount will not be possible. Thus, a 1 dB beamwidth of $1^o$ will permit a pointing error of $\pm 0.5^o$ in setting up the antenna. However, 1 dB of gain will be lost at the edge of the 1 dB beamwidth, so a gain of 31 dB minimum is required with a 1 dB beamwidth of $\geq 1^{o}$ to achieve the gain requirement of 30 dB for the system. This range of antennas is shown within the boxed area, indicating that the antenna diameter can be between 0.4 and 0.9 m in diameter. The lower antenna diameter is at the minimum gain point and the higher antenna diameter is at the pointing error maximum.

(e) When a range of values has been found for a particular element of a system, it is usually best to stay away from the extremes of the range. In this example, we can apparently select a diameter for the antenna between 0.4 and 0.9 m (15.75 to 35.4 inches, 1.3 to 3 feet). Most sub developments for housing have restrictions on certain sizes of antennas. Antennas with a diameter 2 feet (61 cm) or below do not usually need to have planning permission to be installed on customer/owner premises. Thus, in this example, it may be best to select an antenna with a diameter that is below this limit. If we select the 0.6 m diameter, we will have a maximum gain of 35 dB. The 1 dB pointing error permitted is $1.4^{o}$, well above the $\pm 0.5^{o}$ (total error of $1^{o}$). With a pointing error of 1 dB, the gain available is 34 dB.

2. (a) Explain in your own words what "leapfrog technology" is.
   (b) Give three examples of leapfrog technology.

   A country with an emerging economy is seeking to increase the communications capability in its interior, which, for the present, lacks a significant terrestrial communications infrastructure. They plan to do this, in part, with a VSAT/WLL architecture. A typical VSAT will handle a two-way T1 stream (1.544 Mbit/s), which is capable of incorporating 24, 64 kbit/s digital voice/data channels.

   (c) If a linear satellite transponder, SCPC approach is used, what RF bandwidth will this VSAT T1 stream require on the satellite? Assume no FEC is used, a root raised cosine filter roll-off factor á = 0.3, and QPSK modulation is employed.
   (d) If ½ rate FEC is used, what is the occupied satellite bandwidth now?
   (e) What is the noise bandwidth in cases (a) and (b) above?
   (f) If realistic guard bands on the satellite are assumed, and ignoring satellite power issues, how many T1 streams can be handled by a 72 MHz transponder in cases (a) and (b) above? [NOTE: state your guard band requirements clearly.]
   (g) If more than 24 channels are required by the WLL through the VSAT, how would you go about accomplishing this if (i) these are all voice channels; (ii) these are a mix of voice and data channels; (iii) these are all data channels?

**Solution to question 2**

(a) Leapfrog technology is the use of an advanced technology to move more than one generation of technology forwards from an older technology, rather than moving one step of technology at a time. In aviation this would be like moving from propeller driven biplanes straight to jet driven monoplanes, missing out propeller driven monoplanes altogether.

(b) Examples in telecommunications of leapfrog technology could be:
(i) Going from analog TV straight to digital HDTV
(ii) Going from analog mobile telephone networks to digital 3G mobile networks
(iii) Going to VSAT/fixed wireless access for rural communications rather than building an infrastructure of terrestrial fiber or fixed services systems
(iv) Going directly to wideband wireless (so-called *wi-fi*) networks for computer LAN networks without using cable LANs initially

(c) The occupied bandwidth $B$ of the 1.544 Mbit/s VSAT channel is given by equation (9.1), thus

$B = R_s(1 + \alpha) / R_c$ Hz, where $R_s$ is the symbol rate and $R_c$ is the code rate.

In this part of the question, we are told that QPSK modulation is used (2 bits per symbol), there is no coding ($R_c = 1$), and the filter roll of is 0.3 ($\alpha = 0.3$). If the bit rate is 1.544 Mbit/s, the symbol rate will be half this, since there are 2 bits per symbol, giving $R_s = 772$ ksymbols/s. Therefore $B = 772(1 + 0.3)/1 = 1{,}003.6$ kHz = 1.0036 MHz.

On the satellite, we need to provide guard bands to prevent accidental overlap of channels due to frequency instability in the VSAT uplink frequency synthesizers. Typical guard bands can be 10 to 100 kHz, depending on the stability of the frequency. Assuming that we allow guard bands of 30 kHz either side of the channel, then the bandwidth required on the satellite is 30 + 1,003.6 + 30 kHz = 1.0636 MHz.

(d) If half rate encoding is used, $B = 1{,}003.6/(1/2)$ kHz = 2,007.2 kHz. With the same guard bands of 30 kHz either side, the bandwidth required on the satellite is 30 + 2,007.2 + 30 = 2.0672 MHz.

(e) The noise bandwidth is equal to the overall symbol rate. In case (a), the bit rate was 1.544 Mbit/s and with no coding, the symbol rate was 722 ksymbols/s, and so the noise bandwidth is 772 kHz. In case (b), the bit rate is doubled due to ½ rate FEC, thus the symbol rate is 1.544 Msymbols/s, thus the noise bandwidth is 1.544 MHz.

(f) For the two cases, one with no coding and the other with ½ rate encoding, the satellite bandwidth was found to be 1.0636 MHz and 2.0672 MHz, respectively. If a 72 MHz transponder bandwidth is available, and ignoring power issues, the number of T1 streams that can be handled is 72/1.0636 and 72/2.0672 = 67.69462 and 34.8297 = 67 and 34 rounding down to the nearest whole number.

(g) A T1 data stream of 1.544 Mbit/s can accommodate 24, 64 kbit/s individual data streams. For case (i), the individual data streams are all voice channels. If the number of individual voice channels needs to be higher than 24, we have a variety of ways to accomplish this. The first is to use low rate encoding. Individual voice streams, using LD-CELP (low delay – code excited linear prediction), can use 16 kbit/s rather than 64 kbit/s, thus increasing the number of available voice channels to 96 from 24. With 96 voice streams, it is now possible to use statistical multiplexing techniques such as DSI – digital speech interpolation. Gains of around 2 to 2.5 can be achieved, although it usually requires around 1000 voice channels tom achieve multiplications of 2.5. With 96 voice channels, at least 1.5 multiples could be achieved, giving a total of 144 voice channels. For case (ii) where there is a mix of voice and data, only half of the individual channels are available for LD-CELP and DSI techniques. Even so, the 12, 64 kbit/s voice channels could be increased to 48, 16 kbit/s LD-CELP voice channels, and possibly to 60 available voice channels with DSI. For case (iii), data channels are not normally amenable to data compression, although additional source encoding might be achievable. The only options are either to increase the modulation index (going from 2 bits per symbol of QPSK to some higher M-ary modulation) or remove the coding, both of which will require a higher transmitted power level for the same BER.

3. (a) Explain what MESH and STAR architectures are in a VSAT network.
   (b) Give two advantages and disadvantages of the MESH and STAR architectures.
   (c) Give the three major types of multiple access schemes that are used in satellite systems.
   (d) Which one of the three multiple access schemes identified in (c) is the closest to the ALOHA multiple access scheme?
   (e) What are the advantages and disadvantages of an MF-TDMA access scheme from a system perspective (i.e. how does this access scheme affect the earth terminal and the satellite payload design)?
   (f) What does MCDDD mean and how will it affect the design of the satellite payload?
   (g) Why has a TDM approach been adopted for most downlink applications for digital VSAT and Internet applications to small terminals?

**Solution to question 3**

(a) A MESH VSAT architecture allows each of the VSATs in the network to communicate directly with any of the other VSATs through the satellite. A STAR VSAT architecture only allows the VSATs to communicate with each other via the hub earth station.

(b) Two MESH network advantages: direct communication between VSATs; no double hops between VSATs. Two MESH disadvantages: low data rates in both direction (no large hub to compensate for small antennas at each end of the loop); low satellite transponder utilization leading to high space segment costs. Two STAR network advantages: relatively high data rates overall; high space segment utilization and thus lower space segment costs. Two STAR disadvantages: double hops required between VSATs; VSATs unable to communicate directly with each other.

(c) The three major types of multiple access schemes used in satellite systems are: FDMA, TDMA, and CDMA.

(d) ALOHA is closest to TDMA in its design, particularly constrained ALOHA.

(e) MF-TDMA is a combination of FDMA (which is the lowest cost and most flexible of all the access schemes) and TDMA (which is the most spectrally efficient access scheme in a non-interference limited environment). By using a relatively narrow band for the uplink, the terminal costs are kept reasonably low, and by having a narrow band TDMA scheme within the FDMA allocation, there is an added flexibility to the system by allowing a number of terminals to access the satellite simultaneously. The disadvantage of an MF-TDMA scheme is mainly due to the generic TDMA requirement to have the burst rate of every terminal much higher than the individual stream from the terminal would require on its own. There are also synchronization issues as with all TDMA schemes and the need to have timing control closely monitored. Using MF-TDMA on the uplink permits a relatively small terminal to be employed. If the satellite uses on board processing, then the satellite receivers are generally less complicated than wide band TDMA receivers, but the more individual data streams there are, the more complicated the baseband switching system on the satellite becomes. As with all telecommunications designs, a lot of iterative analysis is required to optimize a given satellite system architecture, which will include the earth terminal and payload design trade-offs.

(f) MCDDD is **M**ulti-**C**arrier **D**emodulation **D**emultiplexing and **D**ecoding. Sometimes decoding will take place before Demultiplexing, depending on how the code was added (i.e., if the coding was added to the individual streams before multiplexing, then decoding will take place after Demultiplexing; if coding was added to the multiplexed stream, then decoding will need to take place prior to demultiplexing.

(g) TDM – **T**ime **D**ivision **M**ultiplexing – has been adopted for most downlink applications involving relatively small terminals because no burst time synchronization is required (as it is with TDMA) making the approach a lot simpler and lower in cost; a frame grabber can act essentially as a demultiplexer, pulling the required section of bits out of the continuous stream of traffic; and the content of the TDM stream can consist of variable individual rates (as it does for direct broadcast applications with multiple programs in one TDM stream).

4. (a) What do the symbols ACK and NAK mean when applied to a packet switched communications system?
   (b) What is meant by the term "window" when used with packet communications systems?

(c) What does the term "spoofing" mean when applied to the interface between dissimilar networks.

An Internet Service Provider (ISP) is evaluating a couple of Non-Geostationary Satellite Orbit (NGSO) constellations that are being designed to provide global Internet access directly to Small Office/Home Office (SOHO) terminals located on the SOHO premises: Skybridge and Teledesic (see Table 10.7). Both of the NGSO systems have opted for orbital altitudes of approximately 1400 km, but there the similarities end. Teledesic employs an on board processing payload and an end-to-end system architecture using Inter Satellite Links (ISLs) between the satellites to complete the network. Skybridge employs a bent pipe approach on the satellite with the long distance component being carried over the terrestrial network, and just the end elements of the network employing the satellites (similar to Fig. 10.28). In the question below, the information is for illustrative purposes only for use in this example and does not reflect in any way the true system parameters of either of the two systems.

Assume the following:
- Altitude of satellites is 1,400 km
- Minimum operational elevation angle is $40^{o}$
- Delay introduced by bent pipe satellite is 0 ms
- Delay introduced by any switching element (ground or space) is 15 ms
- All antennas track accurately (earth terminal and ISL antennas)
- Gateway earth station located relatively close to SOHO terminals

(d) What is the propagation path delay from a terminal on the ground to one of the NGSO satellites when the satellite is viewed at the lowest permissible elevation angle?
(e) If a typical trans-Atlantic link via Teledesic requires three satellites in the link (up to the first satellite, across to the second, over to the third satellite and down to the end user terminal from the third satellite) and the user terminals at both ends operate at $40^{o}$ elevation angle, what is the total one-way signal delay if the satellite-satellite-satellite path is 6,000 km?
(f) What is the one-way signal delay for the Skybridge system between the same two terminals if the terrestrial link is 8,000 km?
(g) If no "spoofing" is performed at the satellite segment/earth segment interface, what is the minimum window size required for the Teledesic system in (e) and the Skybridge system in (f), ignoring the length of the messages being sent?

**Solution to question 4**

(a) ACK means "acknowledged" and NACK means "not acknowledged", the former indicating successful reception and the latter indicating errored reception.

(b) The *window* of a packet communications system usually means the length of time, or the number of individual packets, that may exist in the transmission portion of the

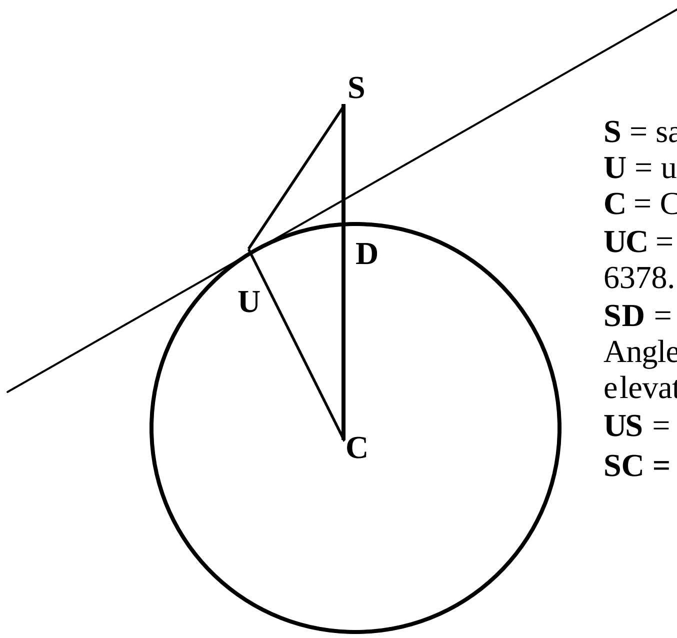
Skybridge

C D E

A B

t

fiber optic long haul link. Teledesic users send their signals to the nearest satellite, which then sends the signals via ISL links to the appropriate satellite at the far end. The longest path to either satellite system occurs with the minimum elevation angle, $40^o$. The pathlength can be calculated by simple geometry, as shown below.




**S** = satellite
**U** = user
**C** = Center of earth
**UC** = **DC** = radius of earth = 6378.137 km
**SD** = satellite height = 1,400 km
Angle **SUC** = $130^o$ (minimum elevation angle of $40^o + 90^o$)
**US** = longest path
**SC = SD** + **DC** = 7,778.137 km

By the law of sines, sin (USC) = sin(130) $\times$ 6378.137/7778.137 = 0.6282, giving angle (USC) = $38.9147^o$. Angle (UCD) = 180 – 130 – 38.9147 = $11.0853^0$.

Again by the law of sines, SU/sin(UCD) = SC/sin(SUC), giving SU = 7,778.137 $\times$ sin(11.0853)/sin(130) = 7,778.137 $\times$ 0.1923/0.7660 = 1,952.2417 km

For a path length of 1,952.2417 km, the path delay = distance/velocity = 1,952,241.7/2.997 $\times$ $10^8$ = 0.0065139864 seconds = 6,513.9864 microseconds = 6,514 μs.

(e) The Teledesic user-to-user path = uplink + cross-link + cross-link + downlink = 1952.2417 + 6,000 + 1952.2417 = 9,904.4834 km, leading to a total, one-way path delay of 9,904,483.4/2.997$\times$ $10^8$ = 0.0330 seconds = 33 ms.

(f) The Skybridge user-to-user path = uplink (from user) + downlink (to gateway) + 8,000 + uplink (from gateway) + downlink (to user) = 1952.2417 + 1952.2417 +8,000 + 1952.2417 + 1952.2417 = 15,808.9668 km = 15,808,966.8 m, leading to a total, one way path delay of 15,808,966.8/2.997$\times$ $10^8$ = 0.0527 seconds = 52.7 ms. In this example, we have assumed that the gateway and user links are both at the minimum elevation angle of $40^o$. Other assumptions may be used for the gateway link(s).

(g) A window has to be long enough to permit an ACK signal to be returned to the sender terminal so that no time-out interrupts the message. The minimum window time is therefore double the one-way delay time, which = 66 ms for the Teledesic system and 105.2 ms for the Skybridge system.

5. When power and bandwidth issues have been optimized, the fundamental limitation for most wireless systems is nearly always interference. Interference can be caused deliberately, as in the jamming of an opposing entity's signals, or unintentionally, as in a miss-pointed antenna (due to high wind) or an inadvertent increase in amplifier power (due to operator error). These are usually classified as short-term interferers and there is usually no way to protect a system against such interferers other than to clear them down. Of more interest to commercial systems is the potential for long-term interference caused by nearby systems.

   A Ku-band VSAT system is being designed for a new service to be offered to two-way SOHO terminals close to a major urban center. One equation used to determine the maximum EIRP permitted in any 40 kHz band at an angle $\phi$ off the main-beam axis between $2.5^o$ and $7^o$ is given by $33 - 25 \log \phi$ dBW. This equation is generally used for satellite spacings of $3^o$.

   (a) Using the above off-axis equation, what is the maximum off-axis EIRP permitted $3^o$ from the antenna main beam axis?

   There is a range of antennas being considered for the SOHO 14 GHz uplink. Assuming an antenna efficiency of 55%,
   (b) What is the on-axis gain value, in dB, for antennas having antenna aperture diameters of 1, 0.8, 0.6, 0.4, and 0.2 m at a frequency of 14 GHz?
   (c) What are the 3 dB beamwidths of the antennas in part (b) above?

   If we assume that the 6 dB beamwidth is 1.5 $\times$ the 3 dB beamwidth, the 10 dB beamwidth is 2 $\times$ the 3 dB beamwidth, and reasonable interpolations can be used for beamwidth values close to these,
   (d) What is the gain of each of the antennas in part (b) $3^o$ away from the main beam axis?
   (e) If there are three choices of output amplifier, 1, 0.5, and 0.1 W, and assuming no losses between the output amplifier and the antenna, what is the EIRP value on-axis for each of the antennas in part (b) using these three possible output amplifiers?
   (f) What is the EIRP value of each of the antenna plus amplifier combinations in part (e) $3^o$ away from the main beam axis?
   (g) Which of the antenna and amplifier combinations above meet the off-axis

limitations of part (a)?

(h) What additional EIRP allowance needs to be made if the system must now meet the interference requirements for satellites spaced $2^o$ apart rather than $3^o$ apart as in parts (a) through (g)?

**Solution to question 5**

(a) Theoff-axis emission equation quoted in the question is $33 - 25 \log \phi$ dBW. We are asked to find the EIRP $3^o$ from the main beam axis. Substituting $\phi = 3^o$ in the equation yields the off-axis EIRP = 33 – 25log(3) = 21.0720 = 21.1 dBW.

(a) We can construct a table giving on-axis gain (with 55% efficiency) at a frequency of 14 GHz. This is shown below (using a wavelength of 0.025 m).

| **Antenna diameter (m)** | **0.2** | **0.4** | **0.6** | **0.8** | **1.0** |
|---|---|---|---|---|---|
| **Antenna on-axis gain (dB)** | **25.4** | **31.4** | **35.0** | **37.5** | **39.4** |

(b) We can construct a table giving the 3 dB beamwidths (and 1 dB and 6 dB beamwidths), again using a wavelength of 0.025 m, as shown below. (NOTE: in calculating the values, four places of decimals were used before rounding.)

| **Antenna diameter (m)** | **0.2** | **0.4** | **0.6** | **0.8** | **1.0** |
|---|---|---|---|---|---|
| **1 dB beamwidth ($^o$)** | **4.3** | **2.1** | **1.4** | **1.1** | **0.9** |
| **3 dB beamwidth ($^o$)** | **8.6** | **4.3** | **2.9** | **2.1** | **1.7** |
| **6 dB beamwidth ($^o$)** | **12.9** | **6.4** | **4.3** | **3.2** | **2.6** |

(c) We can construct a table of off-axis gain $3^o$ away from boresight for the antennas shown in part (b), as shown below. In constructing this table, approximate interpolated values were obtained between the calculated values, or beyond the calculated values.

| **Antenna diameter (m)** | **0.2** | **0.4** | **0.6** | **0.8** | **1.0** |
|---|---|---|---|---|---|
| **Gain reduction $3^o$ off-axis (dB)** | **0** | **2** | **3** | **6** | **8** |
| **Antenna $3^o$ off-axis gain (dB)** | **25.4** | **29.4** | **32.0** | **31.5** | **31.4** |

(d) Again, we can construct a table giving on-axis EIRP vs. antenna diameter, using the gain values calculated in part (b).

| **Antenna diameter (m)** | **0.2** | **0.4** | **0.6** | **0.8** | **1.0** |
|---|---|---|---|---|---|
| **Antenna on-axis EIRP (dBW)** | | | | | |
| **0.1W amplifier** | **15.4** | **18.4** | **25.0** | **27.4** | **29.4** |
| **0.5 W amplifier** | **22.4** | **25.4** | **31.9** | **34.4** | **36.4** |
| **1 W amplifier** | **25.4** | **28.4** | **35.0** | **37.4** | **39.4** |

(e) As before, we can construct a table giving the EIRP $3^o$ away from the main beam axis in all of the cases shown in part (e), with the antenna gain reductions given in table (d), as shown below.

| | | | | | |
|---|---|---|---|---|---|
| **Antenna diameter (m)** | **0.2** | **0.4** | **0.6** | **0.8** | **1.0** |
| **Gain reduction $3^o$ off-axis (dB)** | **0** | **2** | **3** | **6** | **8** |
| **Antenna $3^o$ off-axis EIRP (dBW)** | | | | | |
| **0.1W amplifier** | **15.4** | **16.4** | **22.0** | **21.4** | **21.4** |
| **0.5 W amplifier** | **22.4** | **23.4** | **28.9** | **28.4** | **28.4** |
| **1 W amplifier** | **25.4** | **26.4** | **32.0** | **31.4** | **31.4** |

(f) The antenna + amplifier combinations that meet the off-axis limitations of 21.1 dBW calculated in part (a) are only the 0.1 W amplifier with the 0.2 and 0.4 m diameter antennas. The 0.1W amplifier may be increase 6 dB in the 0.2 m diameter antenna case and 5 dB in the 0.4 m diameter case and still meet the 21.4 dBW maximum off-axis limitation. These two amplifiers would therefore be ~ 400 mW (0.4 W) and 315 mW (0.315 W), respectively.

However, it is interesting to note that, as the antenna diameter increases above 0.8 m, the gain $3^o$ away from boresight is starting to reduce. This is because the beamwidth becomes much smaller with increasing diameter. There is also an optimum that may be found for increase in gain vs. antenna diameter vs. antenna tracking costs associated with a very small beamwidth.

(g) Using the data calculated earlier, a table of approximate off-axis gain reduction can be developed for $2^o$ of axis, as shown below. The data for $3^o$ off axis calculated before are shown for comparison.

| | | | | | |
|---|---|---|---|---|---|
| **Antenna diameter (m)** | **0.2** | **0.4** | **0.6** | **0.8** | **1.0** |
| **Gain reduction $3^o$ off-axis (dB)** | **0** | **2** | **3** | **6** | **8** |
| **Gain reduction $2^o$ off-axis (dB)** | **0** | **1** | **2** | **3** | **4** |

In this case, the same antenna-amplifier combinations (0.2 and 0.4 m diameter with 0.1W amplifiers) meet the $2^o$ off-axis limitations.

If a higher clear sky EIRP cannot be permitted, it may be possible to implement Up-Link Power Control (ULPC). For example, a 1 W amplifier could be backed off to 0.1 W in clear sky, but increased in power to 1 W under rain fade conditions. This is examined in the next question.

6. Many VSAT systems will operate close to acceptable long-term interference limits, both in terms of the interference they can tolerate from similar nearby systems and the interference they cause to similar systems nearby. It is also very possible that many of these VSAT systems will operate close to the performance

and availability minima acceptable to the services being offered. There are a number of techniques that may be used to increase the margin available in such cases, one of them being Uplink Power Control (ULPC or UPC). The amplifier power is increased during those periods when rain attenuation occurs in the path so that the received C/N remains the same. Increasing the EIRP incorrectly may violate agreed interference limits.

A Ka-band VSAT SOHO terminal employs a fixed increment of uplink power control under rain fading conditions. In clear sky, the EIRP is at its nominal level; under rain fade conditions, 7 dB of additional EIRP is switched in as a single-step command.
(a) If the VSAT uplink EIRP in clear sky conditions operates 3 dB below the agreed interference threshold, what is the smallest rain fade level at which the ULPC may be switched on to provide the fixed increment of 7dB of additional EIRP without violating the interference limit? In this part of the question, assume no error in measuring the rain fade level or setting the EIRP level.
(b) If the rain fade measurement accuracy is $\pm$ 0.5 dB, what is the revised answer to part (a)?
(c) If the rain fade measurement accuracy is $\pm$ 0.5 dB and the EIRP level may only be set to an accuracy of $\pm$ 0.5 dB, what is the revised answer to part (a)?
(d) Some ULPC systems require the uplink signal to be detected and measured on the satellite and a downlink data channel from the satellite contains the required ULPC information for the VSAT SOHO terminal. It the round trip delay, which includes all of the propagation, processing, and routing delays, is 2 s and the maximum rain fade is 1 dB/s, how would this change your answers to parts (a), (b), and (c)?
(e) Interference rules permit the EIRP commanded by ULPC systems to exceed the long-term interference limits for small time intervals. If (i) this interval is 60 seconds; (ii) we assume a worst case scenario of the path attenuation going from a very large value to zero attenuation in zero seconds (which happens occasionally due to intermittent accidental blockage of the antenna aperture); (iii) the limit of the path attenuation measuring equipment has been set so as to accommodate the rain attenuation measurement accuracy, the ULPC level measurement accuracy, and the ULPC amount (7 dB), what is the minimum time constant of the path attenuation measuring equipment that is required to enable the interference criteria to be met?

**Solution to question 6**

(a) It is nearly always assumed in ULPC calculations that the rain will attenuate the additional EIRP switched into the on-axis uplink by the same amount within the 2 and $3^{o}$ off-axis pointing. That is, if there is 3 dB of rain attenuation on axis, there will also be 3 dB of attenuation $2^{o}$ and $3^{o}$ away from boresight. In the system considered for this question, since the additional EIRP afforded by the single-increment ULPC system is 7dB, and the VSAT operates only 3 dB below clear sky, the ULPC system cannot be switched in until the on axis rain attenuation is 4 dB. Thus, for a 4 dB rain attenuation level on the on-axis path, when the single-increment (7 dB) additional EIRP is switched in, the uplink power will be 3 dB above clear sky and so will still (just) meet the interference limitations.

(b) If the measurement accuracy of the rain fade is ±0.5 dB, then the ULPC system cannot be switched in until a measured rain attenuation level of 4.5 dB is reached to compensate for the possible erroneous measurement of 0.5 dB rain fade.

(c) If the combined measurement accuracy (rain fade) and ULPC system setting (EIRP level) amounts to ±1 dB, then the single-increment (7 dB) ULPC system should not be switched in until a perceived rain fade of 5 dB is measured is reached to compensate for the combined rain fade measurement and ULPC setting accuracy. It could be argued that the rain fade measurement accuracy and the ULPC level setting accuracy are uncorrelated, and so a root summed square (RSS) approach might be used. The maximum combined error would therefore be $((0.5)^2 + (0.5)^2)^{1/2} = 0.7071$. Thus, if an RSS approach for the two errors is used, the ULPC system could be switched in at a perceived rain fade level of 4.7 dB, rather than 5 dB.

(d) If the maximum rain fade rate is 1 dB/s and the possible delay in implementing the ULPC is 2 seconds, the point at which the ULPC system should be switched in becomes much more complicated to calculate.

In parts (a), (b), and (c) we have calculated the point at which the ULPC system should be switched in as being at a rain fade of 4 dB, 4.5 dB, and 5 dB, respectively. If the rain attenuation level can change by up to 2 dB before the ULPC sensing system has time to react then we are faced with two separate decision points: when should we switch the ULPC in when the fade is increasing; and when should we switch the ULPC out when the rain fade is decreasing?

For the rain fade increasing situation, a number of strategies could be devised. These would be influenced if a predictive algorithm was being used that attempted to predict the maximum rain fade that is likely to occur from a measure of the rain fade rate. In a conditioning of increasing rain fade on the path, the timing of when to switch in the ULPC will generally only affect the performance and availability of the given link. It will not cause any increase in unacceptable interference to other systems, unless the maximum rain fade predicted from the rate of change of the rain fade is incorrect and the ULPC is commanded on in a rain fade that is below that which would permit interference levels to be exceeded.

For a rain fade that is decreasing, when to switch off the ULPC will be more of an interference decision rather than an availability decision, i.e. at what point should power be reduced to avoid interfering with other systems.

Because of the relatively low potential for interference (assuming the attenuation levels are reasonably accurate) in this VSAT system, coupled with a likely timing problem due to the 2-second delay, it may be a suitable strategy to ignore the timing problem. From the user's perspective, what is lost on a fade increase is gained on a fade decrease (assuming a symmetrical distribution of rain fade rates). From the other users' points of view (i.e. the interference potential to other systems), there will only be unacceptable interference when a decrease in rain fade is not detected quickly and/or accurately enough to switch off the ULPC. In most cases, this additional interference has been allowed for in the specification of ULPC systems by the ITU-R.

(e) In the system we have in this question, there were two extremes when the ULPC was switched in: a rain fade of 4 dB (no errors) and a rain fade of 5 dB (measurement and ULPC accuracy errors). If the rain attenuation sensing equipment has 60 seconds to switch off the ULPC, then the minimum time constant of the rain fade measuring equipment will 5 dB/60 seconds = 0.0833 dB/s. A ratio of 0.0833 dB is an unrealistic value taken in isolation, but since it is an average value measured over 60 seconds, it has not been rounded to a realistic dB value.

# Chapter 10 Solutions to Problems

1. What is the preferred orbit [approximate orbital height (max. and min., or average if close to circular), approximate orbital inclination, range of sub-satellite points on the equator if geostationary, and approximate orbit eccentricity] for a satellite that needs to do the following:
   (a) Observe the polar ice caps at least every two hours;
   (b) Observe the Falkland Islands (*Islas Malvinas*) with an elevation angle to the satellite from the surface of the islands of at least $60^o$ for six hours every 24 hours;
   (c) Observe the development of Tropical Cyclones in the southern Pacific ocean and Hurricanes in the northern Pacific ocean for 24 hours each day;
   (d) Observe swathes of the earth below the satellite illuminated by sunlight from directly behind the satellite on every pass the satellite makes on the sunlit side of the earth; and
   (e) Make observations of inter-stellar X-Rays for one hour per day when more than 40,000 miles from the earth, but be able to relay information back to earth once per orbit when less than 1000 miles above the earth;
   **NOTE:** The satellite in question need only do one of the above five missions at a time, not all of them at the same time.

**Solution to question 1**

(a) The ideal is to have a circular polar orbit at such an altitude that the orbital period is $\leq$ 2 hours. The orbit should be $\geq$ 500 km in altitude to be stable (i.e. it will not rapidly decay into the earth's atmosphere) and < about 2,300 km so that the orbital period does not exceed 2 hours. Hence
- orbital height in the range 500 – 2,300 km
- orbit inclination close to $90^o$
- range of sub-satellite points is not applicable as it is not a GEO satellite
- orbit eccentricity should be small, preferably < 0.001, although a larger eccentricity can be tolerated provided the orbital period stays under 2 hours

(b) To carry out this requirement will require the orbit to be highly elliptical. No bounds were given on time delay and so a range of highly eccentric orbits could accomplish the mission. While no bounds were given as to the repeatability of the ground track of the orbit, other than the observability should be 6 hours in every 24, the stated requirement will be satisfied more easily with a *Molniya* type orbit. Hence
- orbital height range between about 500 km (min.) to about 39,150 km (max.)
- orbit inclination around $63^o$, although this can vary as there is no ground track repeatability requirement
- range of sub-satellite points is not applicable as it is not a GEO satellite
- orbit eccentricity will be around 0.74

(c) The two requirements – location and 24 hours per day – can only be achieved using a GEO orbit. Hence

- Orbital height is GEO (35,786.03 km – see Table 2.1)
- orbit inclination is close to zero
- range of sub-satellite points is an arc over the Pacific Ocean about 150 – 190$^o$E
- eccentricity must be 0.001, or less

(d) This will require a sun-synchronous orbit. Thus
- Orbital height around 750 to 1,000 km, although it may be below or above, depending on the inclination to phase the orbit with the sun
- orbit inclination will be retrograde, generally around 98$^o$ from due east
- range of sub-satellite points is not applicable as it is not a GEO satellite
- orbit eccentricity should be small (0.001 or less) although this is not essential

An example of a sun-synchronous satellite series is TIROS, with typical inclinations of 98.7$^o$, periods on the order of 100 minutes, and orbital heights about 830 km.

(e) This is a specific orbit with high eccentricity. The X-ray satellite *Chandra* has an orbit that is like this in some respects (although Chandra has an orbit with a 133,000 km apogee and 16,000 perigee, and an orbital period of 64 h 18 min). In this question
- Orbital heights have been given (40,000 miles = 64,000 km, max.; 1,000 miles = 1,600 km, min.)
- This can be within quite a large range. It is unlikely to be higher than 80$^o$ as this would require an excessively energetic launch. However, if the apogee were above the earth's poles, this may reduce the X-Ray contributions from the ground. A likely range of inclinations would be between 0$^o$ and 65$^o$
- range of sub-satellite points is not applicable as it is not a GEO satellite
- Eccentricity will be approximately 0.8

2. Why is it optimum (in terms of launch energy requirements) to do the following:
   (a) Launch a satellite towards the east
   (b) Launch a satellite from the equator

   A fully steerable earth station antenna is on the equator and it observes a satellite that is in a circular equatorial orbit moving in an easterly direction. It can observe the satellite down to the horizon, which is at an effective elevation angle of 0$^o$ in every direction. What is the apparent orbital period of the satellite (i.e. the time between successive passes when the satellite is directly over the earth station) if the true orbital period is
   (c) 2 hours
   (d) 6 hours
   (e) 12 hours
   (f) Did anything look strange at first sight with your answer to (e) above? If 'yes', what was it and can you explain the answer you arrived at for (e), which looks counter-intuitive at first sight?
   **NOTE:** (1) For this question, do not use a sidereal day in developing your answers; assume 24 hours for the earth's rotational period. (2) If you saw nothing strange at first sight in your answer to (e) you are either an exceptional orbital

mechanic or not very curious!

**Solution to question 2**

(a) The spin of the Earth about its axis is towards the east and so a rocket launched towards the east will have this eastward rotational component added to its velocity prior to launch. The total energy required by the rocket to launch a satellite into an orbit towards the east will therefore be less than in any other direction.

(b) The rotational (spin) velocity of the earth is highest at the equator, thus launching a rocket from the equator will require less energy to reach orbit than at any other latitude.

(c) The apparent orbital period is given by equation (10.1) and is

Apparent orbital period = $24T / (24 - T)$ hours, where $T$ is the real orbital period. In this case, $T = 2$ hours, and so the apparent orbital period = 48/22 = 2.1818 h

(d) For $T = 6$ hours, the apparent orbital period = 144/18 = 8 h

(e) For $T = 12$ hours, the apparent orbital period = 24 hours

(f) The answer to part (e) says that the apparent orbital period is 24 hours. Looked at quickly, 24 hours might look like the satellite is in a GEO orbit (a period close to 24 hours) so how can a period of 12 hours ($T$) give a GEO satellite. The answer is that the satellite is moving in the same direction as the observer, but at an angular rotation that is faster than the observer on the equator. The satellite will stay above the horizon for 24 hours, unlike a GEO where it stays above the horizon forever.

3. The International Space Station (ISS) has an experimental package that will be located in the Free Flying Module (FFM). The experiment in the FFM was located there to avoid unnecessary vibrational tremors and thruster accelerations from the ISS impacting the gravity sensitive biological experiments on board the FFM. The experiments require continuous communications with a main research laboratory located in New Mexico, USA, so that real time monitoring and adjustment to the experiments can be made. This requires huge quantities of data and high-resolution video to be transmitted 24 hours per day to New Mexico. An average, one-way data rate of OC-192 (approximately 10 Gbit/s) is needed for the data/video link to New Mexico. A return link of OC-1 (51.84 Mbit/s) suffices for the uplink control path to the FFM.

   (a) What is your preliminary outline system design solution for the data link between the FFM and the earth station in New Mexico? Give justifications for your choice of system architecture and frequency bands (see the note at the end of this question regarding frequency bands).

(b) If 8-phase PSK modulation with ¾ rate FEC is employed for the OC-192 downlink from the FFM, approximately what instantaneous bandwidth will be required for the link?

(c) If the instantaneous bandwidth of a given transponder is limited to about 10% of the carrier frequency, what is the lowest downlink carrier frequency that can be contemplated for this link in the FSS bands?

(d) If you selected a preliminary answer to part (a) before you looked at parts (b) and (c), did you change your mind once you knew the tentative answers to parts (b) and (c)?

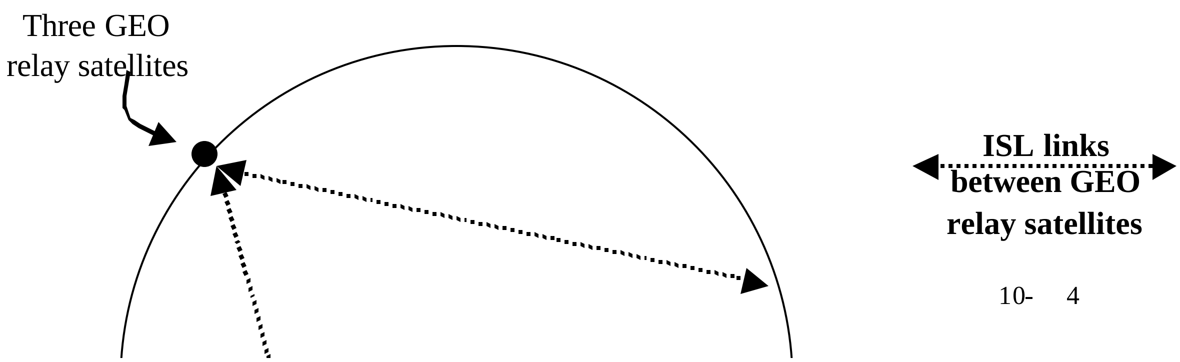
(e) If occasional shuttle blockages of the transmission path, or other small interruptions occur in the link, that lead to outage intervals of up to 5 minutes per day, what is the maximum data storage required for the transmission buffers?

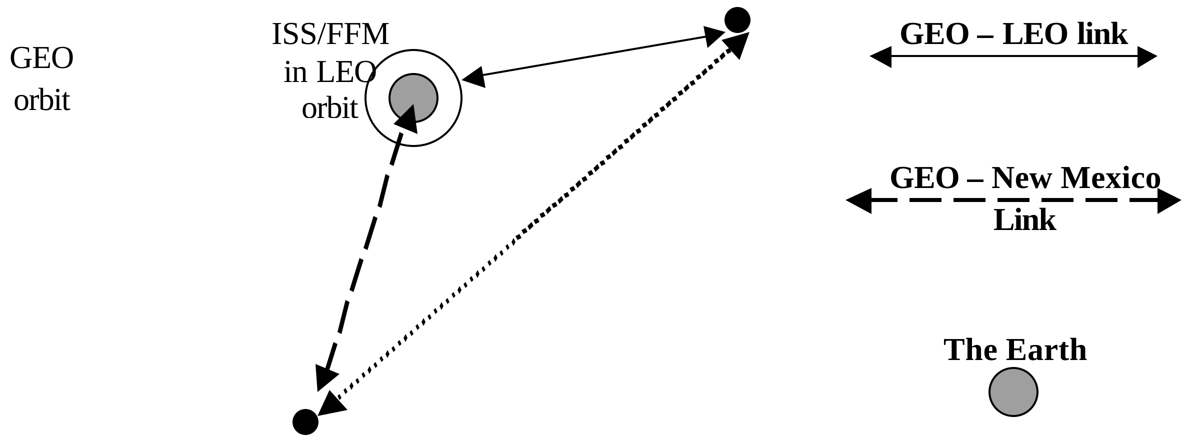
(f) The OC-192 data link has been designed to have some redundancy in the information capacity. If there is a redundancy margin of 0.5% in the data transmission requirement (that is, for every 1000 bits actually sent, normally only 995 bits are required so there is spare capacity to send 5 bits more of information within those 1000 bits, if the need arises), will the 5 minutes per day of anticipated outage be handled by the OC-192 link transmission rate?

**NOTE:** Four FSS bands are 6/4 GHz (C-Band), 14/11 GHz (Ku-Band), 30/20 GHz (Ka-Band), and 50/40 GHz (V/Q-Band). Remember that the uplink carrier frequency is given first in each paired band.

**Solution to question 3**

(a) There are several key design requirements. The first is that the link to the New Mexico earth station, and processing center, be continuous – i.e. 24 hours per day. Since the ISS and its attendant FFM are in low earth orbit, there are only two ways to establish a 24 hours per day communications link: either build a continuous chain of ground stations around the earth so that the ISS/FFM combination are never out of contact with one of these earth stations; or establish a link via GEO satellites from the ISS/FFM to the New Mexico site. Since the ISS/FFM combination will be moving continuously around the earth, three GEO satellites will be required. Thus there will be an ISS/FFM link to one of the three GEO satellites in view at the time; there will be an Inter-satellite Link (ISL) between the GEO satellites so that the information can be transferred to the GEO satellite that is 'over' the New Mexico site; and there will be a GEO to New Mexico link. The figure below is a schematic of the set up.






The GEO relay satellites with ISL communications between them and the ground and/or the ISS/FFM provide a 24-hour real-time link between the New Mexico research site and the payload in the ISS/FFM. It is expensive to accomplish, but does not require the construction and maintenance of at least half a dozen earth station sites around the world as was used for the Mercury, Gemini, and Apollo missions, with all of their attendant landing rights issues (political and security), plus remote location of telecommunications personnel.

To carry an OC-192 link, we will need to have at least 10 GHz of instantaneous bandwidth available if QPSK modulation is used with ½ rate FEC encoding. C-, Ku-, and Ka-band links do not have 10 GHz of instantaneous bandwidth available. It is possible that Ka-band could be used, but the order of the modulation (the modulation index) would be so high that the C/N requirements then become difficult to meet. The next paired band above Ka-band is V/Q (depending on which table is consulted) with a carrier frequency ~ 50 GHz on the uplink and ~40 GHz on the downlink.

(b) With 8-phase PSK (8-PSK) and ¾ rate FEC, the uncoded symbol rate is 10/3 Gsymbols/s (8-PSK has 3 bits per symbol) and the encoded symbol rate is $(10/3) \times (4/3) =$ 4.44 Gsymbols/s, requiring a bandwidth of 4.44 GHz.

(c) With (i) a requirement to transmit with an instantaneous bandwidth of 4.44 GHz and (ii) a maximum bandwidth of 10% of the carrier frequency, the lowest carrier frequency possible is 44.44 GHz

(d) No; not in this case.

(e) Data are being accumulated at the rate of 10 Gbit/s and so an outage of 5 minutes per day will require $5 \times 60 \times 10$ Gbit of storage capacity = 3 Tbit of storage capacity, as a minimum.

(f) A redundancy of 0.5% is equivalent to $[(10 \times 10^9) \times 0.5]/100 = 0.5 \times 10^8 = 50$ Mbit/s. If the 5 minutes of outage (corresponding to 3 Tbit of data) has to be sent over 24 hours, we need to find the time 3 Tbit of data may be sent at an equivalent rate of 50 Mbit/s. The time required to send 3 Tbit of data at a rate of 50 Mbit/s = $(3 \times 10^{12})/(50 \times 10^6)$ s = $6 \times 10^4$ s = 1000 minutes = 16.67 hours. Since this time is < 24 hours, the redundancy allocated can handle the anticipated 5 minutes of outage a day.

4. The broadcast of digital radio signals from satellites has a number of advantages and disadvantages. Some of the advantages are: crisp, clear sound; uninterrupted reception regardless of distance from your home location; and the ability to listen, commercial free, to your favorite radio station wherever you travel within the coverage region. The main disadvantage is the possible blockage by tall structures. This last issue may prove to be critical in the success, or otherwise, of two different approaches to the provisioning of such services.

   In the US, two companies (XM Radio and Sirius Satellite Radio) have approached the system design of a broadcast satellite radio system in two very different ways. One has adopted a GEO approach (www.xmradio.com) and the other a Tundra approach (www.siriusradio.com). The Tundra orbit is similar in concept to the Molniya orbit – a very high apogee and a relatively low perigee – but the orbital period of a Tundra orbit is one sidereal day rather than about half that amount. Three satellites are each in a different Tundra orbit, with the planes of the three Tundra orbits $120^o$ apart. By phasing the position of each of the three satellites in its Tundra orbit, there are always two satellites visible at high elevation over the coverage region. The Tundra approach therefore needs three satellites as opposed to the single satellite needed in the GEO approach, but the Tundra approach requires much fewer terrestrial repeaters to achieve full coverage in built up areas that cause blockage to GEO satellite signals. In the system level example below, the costs given are for example only and do not represent in any way the true costs of either of the proponent broadcast radio systems noted earlier.

   If programming and content support, and the unit cost of the car radio system, are assumed to be the same for both system approaches; the cost of a GEO satellite is \$200 M each, including launch; the cost of the Tundra satellites is \$125 M each, including launch; and the average cost of a terrestrial repeater is \$25 k each; find:

   (a) If no terrestrial repeaters are needed for either the GEO or Tundra approach, which system is the cheaper to bring to operational status – a single GEO satellite or three Tundra satellites?
   (b) If it is necessary to have an in-orbit spare for the GEO approach (i.e. two GEO satellites need to be launched prior to broadcast radio services being offered) but no in-orbit spare is required for the three Tundra satellites (since they provide a strong measure of in-orbit redundancy by virtue of there orbit placement), which system is now the cheapest to bring to operational status? [Still no terrestrial

repeaters required]

(c) If the operational control center costs for a GEO satellite are $6 M per year for one satellite and $1 M per year for each additional satellite; the operational control center costs for a Tundra satellite are $12 M per year for one satellite and $4 M per year for each additional satellite; and an operational lifetime of 10 years is assumed in this part of the question, which system is now the cheaper to bring into service and operate over a ten year lifetime (assuming 2 GEO and 3 Tundra satellites)? [Still no terrestrial repeaters required]

(d) Using the assumptions in part (c), if the Tundra approach requires ten times **fewer** terrestrial repeaters than the GEO approach, at what number of terrestrial repeaters for the Tundra approach are the total costs of the two satellite broadcast radio systems approximately equal to each other?

(e) Given the answer to (d), which approach might be the better to pursue? State your reasons, giving as many parameters that might influence your choice between the two system approaches.

(f) Using all of the assumptions in (d), which of the two approaches would you select – giving your reasons – for a digital satellite radio broadcast service to be offered in:

(i) Indonesia
(ii) Europe
(iii) The Pacific Islands
(iv) South America

**Solution to question 4**

(a) One GEO satellite = $200 M, while three Tundra satellites cost $125 × 3 = $375 M. In this case, the single GEO satellite is cheaper than three Tundra satellites that are required to get both systems to operational status.

(b) Two GEO satellites = $400 M, while three Tundra satellites cost $375 M. In this case, where immediate redundancy is required (i.e., in orbit spare satellites), three Tundra satellites are cheaper to implement than two GEOI satellites.

It could be argued that, if two Tundra satellites offer an adequate degree of in orbit redundancy, then perhaps only two Tundra satellites would suffice for the introduction of operational services. If that is the case, the answer to part (a) would be $200 M vs. $125 × 2 = $250M, and so the original answer would still hold.

(c) For the GEO case, we have two GEO satellites ($400 M) plus $(6 +1) × 10 total operational charges = $400 + $70 M = $470 M. For the Tundra case, we have three Tundra satellites ($375 M) plus $(12 +4 +4) × 10 total operational charges = $375 + $200 M = $575 M. The GEO alternative is cheaper with these numbers.

(d) In part (c), the GEO case was apparently \$575 - \$470 M = \$105 M cheaper over 10 years. If N is the number of repeaters the Tundra system requires, we need to know when $(N \times 0.025) + 105 = 10 \times (N \times 0.025)$ giving $0.225N = 105$ and $N = 466.67$. Rounding up to a whole number, we have N = 467 repeaters. Thus, as soon as 467 repeaters (for Tundra) and 4667 repeaters (for GEO) are implemented, then the costs of the two systems are nominally the same.

(e) In this part of the question, we are asked to judge which of the two systems might be better to implement in specific cases. The specific cases are as follows:
(i) Indonesia; for Indonesia, a huge archipelago stretched out close to the equator, a GEO architecture would probably be simpler to implement as there are no high latitudes to reach. In addition, there are few cities with very tall buildings that lead to major signal losses due to shadowing.
(ii) Europe; for Europe, a relatively compact landmass that is spread over more than $50^o$ in latitude, it is likely that a Tundra approach would confer operational advantages due to both the high northern latitudes of some of the countries and the density and height of the many conurbations.
(iii) The Pacific Islands; for the Pacific Islands, spread out over a very large ocean, but generally relatively close to the equator, a GEO architecture would appear to be preferred as there are no high latitude problems and very few cities with tall buildings to block signals.
(iv) South America; for South America, the same arguments that applied to Europe can be used here. We have a continent with a relatively large range of latitudes (in this case, southern latitudes) that would benefit from a satellite architecture that located satellites away from GEO. While the major cities (Buenos Aires, Santiago, Sao Paulo, etc.) tend not to be too far from the equator, on balance, the relatively high southern latitudes would seem to give an advantage to a Tundra approach.

5. This question concerns the effect of radiation on electronic equipment on a spacecraft.
   (a) What are the two principal effects that radiation has on electronic equipment?
   (b) What particles principally cause these effects?
   (c) What is the prime generator of these particles?
   (d) What causes the generation of these particles to fluctuate with time?
   (e) Is there a periodicity in these fluctuations? If yes, what is the approximate period of these fluctuations?
   (f) Is there a particular region, or are there particular regions, in near-earth space where concentrations of these particles are to be found? If yes, what is it (or are they)?
   (g) What orbital inclination of a satellite circling the earth will cause that satellite to receive the highest radiation dosage compared with other inclinations?
   (h) Which of the radiation particles is the strongest and therefore the hardest to protect against?
   (i) What are two ways to reduce unexpected performance or decreased lifetime in

electronic equipment exposed to radiation?

**Solution to question 5**

(a) The two main categories of radiation effects are generally classified into those that provide a cumulative effect over time (referred to as *total dose*) and those that occur randomly due to a single, highly energetic particle striking a vulnerable area of the electronic equipment (referred to as *single event upsets*)

(b) The two principle particles responsible for these radiation effects are electrons and protons, although other ionized particles can cause harmful effects.

(c) The prime generator of these particles is the sun.

(d) Enhanced radiation levels from the sun occur whenever there is sunspot activity, thus there is a periodicity in the radiation emitted from the sun. Occasional major solar upsets (sunspots, flares, corona discharges) can lead to a major influx of energetic particles into the near-earth environment.

(e) The periodicity will tend to track the sunspot cycle (approximately 11 years between minima and maxima).

(f) The magnetic field of the earth will trap many of the charged particles emitted from the sun that arrive in near-earth space. The characteristics of the magnetic field around the earth lead to the formation of two main areas of concentration, which are called the Van Allen belts. These belts have their maximum radiation levels at about 1,500 km and 15,000 km above the surface of the earth, although the radiation levels and the position of the maxima do fluctuate, and there is some evidence that there might be three belts, not two. The radiation never drops to zero; the Van Allen belts are just places around the earth with radiation levels that peak in those belts.

(g) Radiation is generally concentrated around the equator due to the magnetic lines of force and the way the radiation moves back and forth within these lines of force, and so circular, equatorial orbits that are located within one of the Van Allen belts would be the place where the maximum dosage would be reached. There are enhanced radiation zones over both of the poles (the auroral zones) but a satellite in a polar orbit would not constantly be in these zones, as an equatorial satellite would if orbiting at the height of a Van Allen belt peak radiation point.

(h) An energetic proton is generally the most likely to harm electronic equipment the most, and potentially cause single-event upsets, and even a latch up.

(i) Yes; the two main ways to protect electronic equipment from harmful radiation effects are to select solid state compounds that are resistant to radiation (called radiation hardened devices, or just ‘rad hard’) and/or to provide metallic shielding around the electronic equipment to limit the number of particles that get through. Shielding can cause problems, though, as the impact of ionized particles on the shield can lead to the production of secondary emissions that may be more harmful than the original particles.

6. The design of a communications satellite's antenna is fundamental to that satellite's ability to perform its assigned task. The portion of the earth's surface illuminated via that antenna is called the coverage. This question considers the definitions of coverage, frequency re-use, and capacity issues.

   (a) What is the difference between total coverage and instantaneous coverage for a satellite antenna illuminating the surface of the earth?
   (b) What is the fundamental difference between a "hopping" beam and a "scanning" beam that might be employed on advanced communications satellites?
   (c) A satellite needs to provide communications capability over a given coverage region. It has been predicted that the average number of users in the coverage region that access the satellite is 1% at any given time of those who have signed up with that satellite service. That is, for every 10,000 customers signed up, 100 are using the satellite at any given time.
      (i) How many instantaneous communications channels must the satellite be able to provide if the coverage region contains a potential user population of 120 million?
      (ii) If the bandwidth allocation for the satellite communications service permits only 6,000 channels to be provided without frequency re-use, how many separate beams will be required for this coverage region, ignoring interference between individual beams?
      (iii) If interference requirements between individual beams dictate that no frequency may be re-used by adjacent beams, what is the approximate new minimum number of separate beams that are required, ignoring for the moment the exact geometry of the beams? (NOTE: for this part of the question, assume that the bandwidth allocation has been divided into three separate bands to develop the frequency re-use pattern)
      (iv) If the physical size of the total coverage region remains the same between cases (ii) and (iii) above, what can you say about the size of the new individual instantaneous beams in part (iii) within the overall coverage?
      (v) In addition, what can you say about the effect this change in size has on the communications capability and satellite connectivity complexity issues?

**Solution to question 6**

(a) Instantaneous coverage provided by a satellite antenna is that portion of the earth's surface that is covered all at the same time, i.e. in that instant of time. The antenna may be able to redirect the beam from place to place on the earth, and so build up a total coverage area, but it will not be able to cover all of that area *at the same instant in time.* The difference between instantaneous coverage and total coverage becomes somewhat blurred when scanning and/or hopping beams are used to generate the total coverage. If the rapidity of the scanning or hopping is really fast, it may appear to a user that they have a continuous communications link, when they are only being afforded a small fraction of the time the antenna beam communicates with the total coverage. For example, referring to figure 4.8, this satellite is providing *CONUS* coverage. The beam remains fixed over the continental United States and so it would appear that the beam has a continuous coverage. However, such a beam might be formed of 100 smaller beamlets and the satellite's on board processor might be able to switch rapidly between the elemental beams so that the appearance of continuous coverage is given to any user in *CONUS* coverage.

(b) A *scanning* beam is generally one that moves the individual elemental beams (the beamlets referred to in the answer to the last part of the above question) in a continuous fashion, rather like a raster scan, across the total coverage. If there are 100 beamlets in a 10 by 10 pattern, the scan will move from beamlet position 1 through 10 in row 1, it will then move down to row 2 and scan from beamlet position 1 through 10 again, and so on. This form of active beam pointing is relatively simple to generate, but it may be inefficient if many of the beamlet positions do not have either a large population within the small, instantaneous coverage of that beamlet, or little traffic to carry. For example, in a regular scan over *CONUS*, such a scanning beam approach would allocate as much time to a beamlet over one of the great lakes as it would to, say, Columbus, Ohio. A *Hopping* beam overcomes this deficiency by moving the beamlets only to where there is traffic to be picked up and the dwell times over those locations can be made to match the volume of traffic. The approach is much more complex than a "simple" scanning beam approach as a high degree of control is required both in the antenna implementation and in the network architecture that would permit such random traffic allocations in time, space (pointing), and bandwidth (total dwell time).

(c) In this part of the question we have been told that the overall, average usage rate for a particular service is 1% of the customer base signed up. So
(i) For 120 million potential users we will need to provide $(120 \times 10^6 \times 0.01)$ instantaneous communications channels = $120 \times 10^4$ = 1.2 million channels.
(ii) The satellite can only provide 6,000 channels within any instantaneous coverage, and so there will have to be (assuming average population densities) $(1.2 \times 10^6)/6000$ separate elemental beams……….

7. (a) Define the terms "Zenith" from the point of view of an earth station on the surface of the earth and "Nadir" from the point of view of an earth orbiting satellite.
(b) A geostationary satellite is required to provide communications coverage over the whole of the earth (i.e. a global horn antenna). What is the maximum off-axis

angle that needs to be contained within the antenna coverage, measured from nadir?

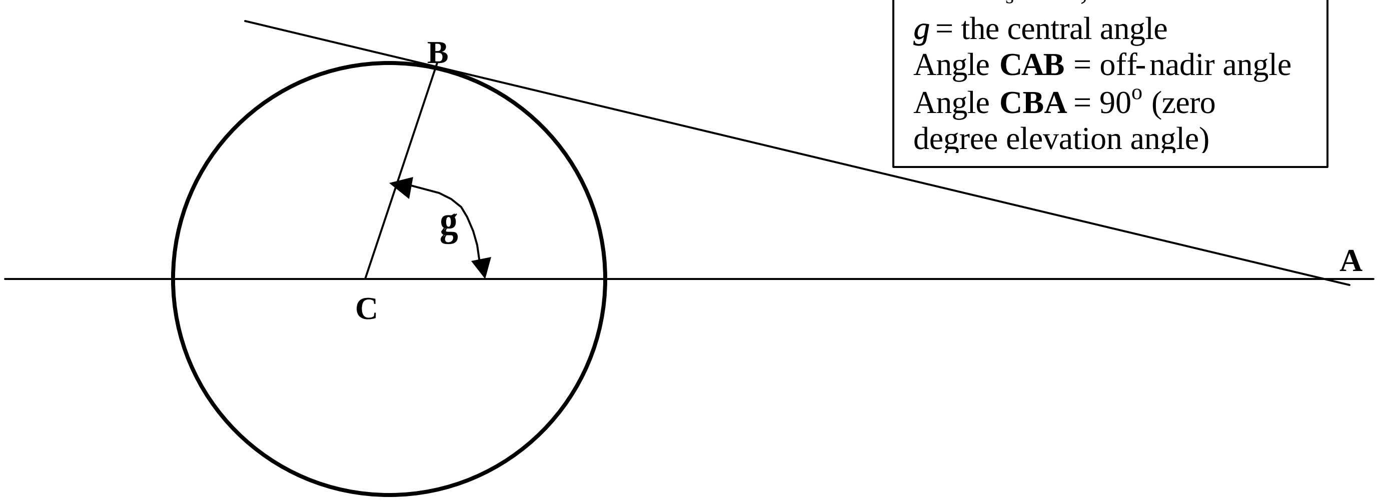
(c) A LEO satellite constellation is being designed to provide telecommunications service on a global basis at a downlink frequency of 12 GHz. An evaluation is being carried out to determine what would be the best altitude for the system (which will use circular orbits), what is the best mix of number of beams vs. complexity, etc. An initial determination of the optimum orbital height is 1,400 km. If the minimum operational elevation angle to any part of the coverage on the surface of the earth is $20^{o}$, find the following:

(i) What is the scan angle from nadir to edge of coverage?
(ii) What is the difference in path loss from nadir to edge of coverage?
(iii) What is the scan loss at edge of coverage if the parameter k (see equation (10.11)) is 1.4?
(iv) If the requirement is to provide the same power density at the edge of coverage as exists at nadir, what additional amount of power (in dB) is needed at the edge of coverage?
(v) In what ways do you think you could cope with this large difference in power requirements between nadir and edge of coverage power?

**Solution to question 7**

(a) "Zenith" from the point of view of an earth station on the surface of the earth is an elevation angle of $90^{o}$, or straight up, directly opposite to the direction from the earth station to the center of the earth. "Nadir" from the point of view of an earth orbiting satellite is the direction straight down towards the center of the earth from the satellite.

(b) A schematic of this part of the question is given below.

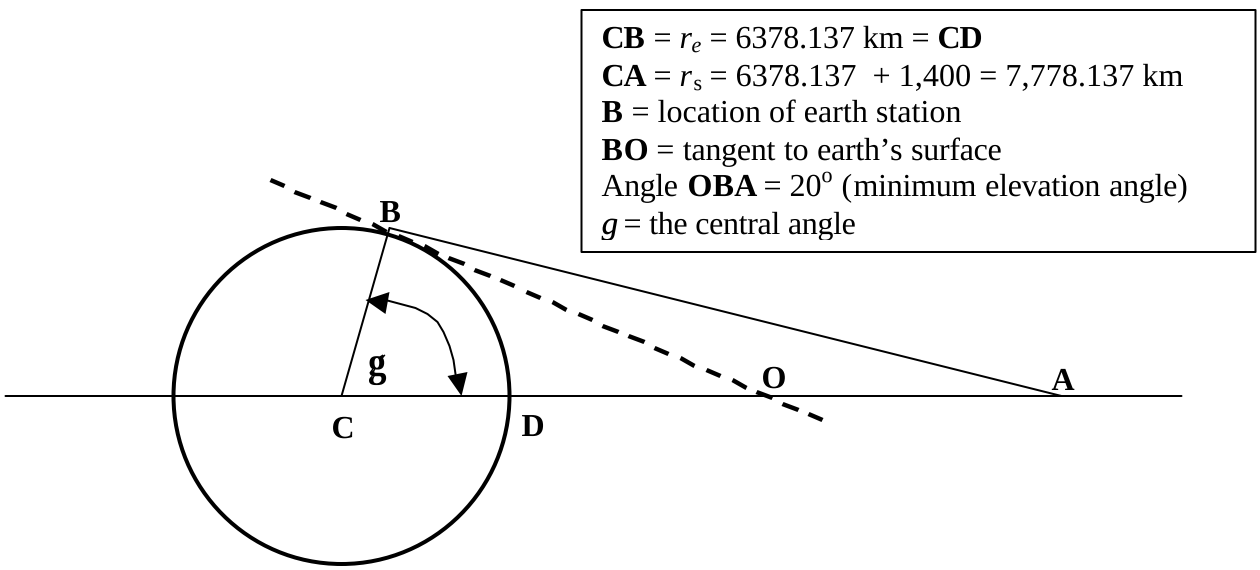



**CB** = $r_e$ = 6378.137 km
**CA** = $r_s$ = 42,164.17 km
*g* = the central angle
Angle **CAB** = off-nadir angle
Angle **CBA** = $90^{o}$ (zero degree elevation angle)

From equation (2.43), $\gamma \le \cos^{-1}(r_e/r_s)$. But, for an elevation angle of $0^o$, $\gamma = \cos^{-1}(r_e/r_s) = \cos^{-1}(6378.137/42164.17) = 81.2995^o$. Angle BAC = 180 – angle CBA – angle BCA = 180 – 90 – 81.2995 = 8.7005$^o$ = 8.7$^o$.

The maximum off-axis angle measured from nadir that needs to be contained within the antenna coverage = 8.7$^o$, giving a total beamwidth from the satellite of the horn antenna of twice this value = 17.4$^o$.

(c) A schematic for this LEO coverage is given below.




(a) By the law of sines, sin(ABC)/CA = sin(BAC)/CB, which gives sin (BAC) = CB $\times$ sin(ABC)/CA = 6378.137 $\times$ sin(110)/7,778.137 = 0.7706, and so the off-Nadir angle is 50.4038$^o$ = 50.4$^o$, making the total scan angle required for a symmetrical coverage of the earth = 100.8$^o$.

(ii) The path loss at Nadir (path length $d_1$ = 1,400 km) and the path loss at the edge of coverage (path length $d_2$ = **BA**) are $(4\pi d_2/\lambda)^2$ and $(4\pi d_1/\lambda)^2$, respectively Distance **BA** can be found as follows.

First: find the central angle *g* = 180 – 110 – 50.4 = 19.6$^o$.

Next: using the law of sines, **BA**/sin*g* = **BC**/sin50.4, and so
**BA** = 6378.137 $\times$ sin19.6/sin50.4 = 2,776.7934 km

Finally, find the wavelength for a frequency of 12 GHz = $2.997 \times 10^8/ 12 \times 10^9$ = 0.025 m.

The two path losses are $(4\pi 2{,}776{,}793.4/0.025)^2$ and $(4\pi 1{,}400{,}000/0.025)^2$ = $1.9482 \times 10^{18}$ and $4.9522 \times 10^{17} \Rightarrow$ 170.8551 dB and 164.9068 dB, which gives a difference in the path loss between a nadir link and an edge-of-coverage link of approximately 5.9 dB.

(iii) Equation (10.11) is –scan loss = (cosine $\phi$)$^k$ thus, for a value of $k$ = 1.4, the scan loss at the edge of coverage ($\phi$ = 50.4$^o$) = 0.6374 $\Rightarrow$ 1.9557 = 2.0 dB.

(iv) For the same power density to be received at the edge of coverage as that at nadir on the surface of the earth, an additional power of (5.9 + 2.0) = 7.9 dB must be provided to the edge of coverage compared with nadir.

(v) There are a number of ways the difference in power level between edge-of-coverage and nadir pointing could be handled. Additional power could be applied at the edge of coverage, additional coding could be applied, a reduced transmission bandwidth could be used, and so on. Virtually any technique for increasing the C/N by about 8 dB could be considered. In a practical application, though, it is almost certain that changes in bandwidth could not be tolerated, nor extreme changes in coding (which have the same consequence of reducing the information throughput for a constant channel bandwidth.). We are thus left with changing the raw power of the transmitted signal. This may be possible, if the amplifier characteristics permit it and the interference levels are not exceeded. Alternatively, if the performance of the edge of coverage user was satisfactory with the reduced C/N of about 8 dB dues to scan loss and differences in path loss, then the transmitted power levels could be progressively reduced from edge-of-coverage to nadir to keep the flux density relatively constant.

Another way that the differences in power level could be handled would be to keep the amplifier power levels constant, but reduce the beamwidth of the edge-of-coverage ‘beamlet’ so that the gain is effectively increased. If a gain of about 8 dB is required (from nadir to edge-of-coverage), this corresponds to an increase of a factor of 2.5 in the effective antenna diameter (8 dB $\Rightarrow$ 6.3096; $(6.3096)^{1/2}$ = 2.5119) , which leads to a beamwidth reduction of the same order. This type of gain loss compensation has an added benefit. As the beamlets point progressively away from nadir, the curvature of the earth makes the coverage area larger and larger for a given beamwidth. Increasing the gain of the beamlet’s antenna (and thus decreasing the beamwidth) will therefore compensate to some degree for the larger coverage area away from nadir.

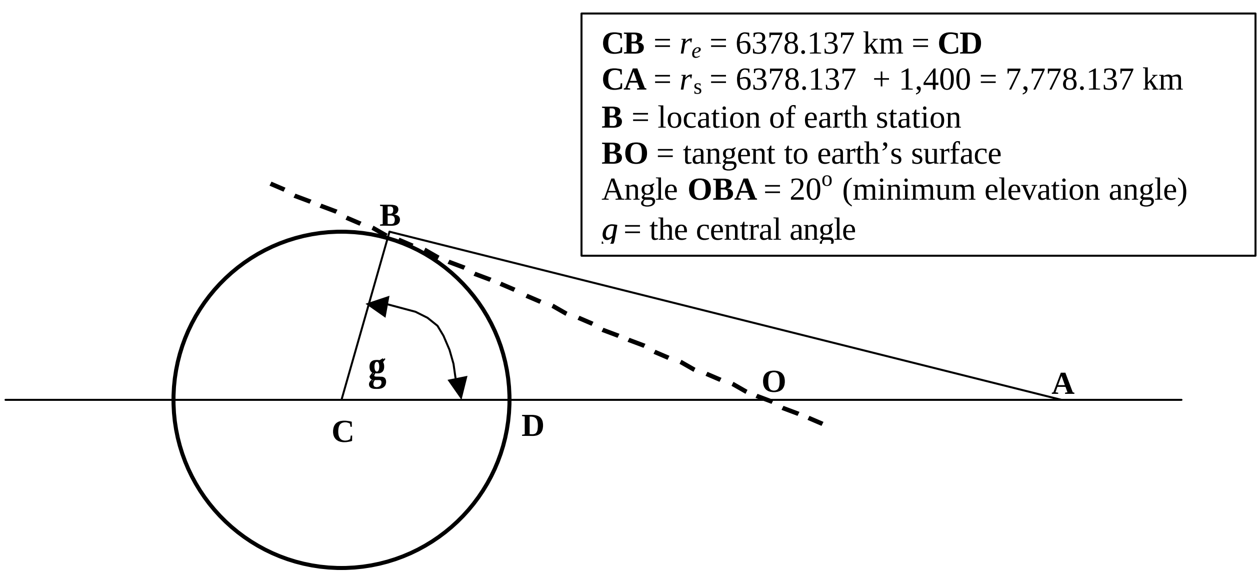
8. Historically, nearly every commercial service of whatever nature has been developed incrementally. Most major enterprises that exist today started as small operations in a single location. Global satellite systems do not have that luxury: they have to start global. The designers of the LEO satellite system in question 7 above want to examine how they could provide global coverage with different levels of service so that they can have a measure of incremental growth options. They have selected polar orbit for their system. Find the following:
(a) What is the minimum number of satellites that can be used to provide

continuous coverage around one polar orbit plane assuming operations can continue to down to an elevation angle of $0^o$?
(b) What is the minimum number of orbit planes that need to be used to provide continuous coverage over the entire globe?
(c) (i) Repeat (a) above for a minimum elevation angle of $20^o$
(ii) Repeat (a) above for a minimum elevation angle of $40^o$
(iii) Repeat (b) above for a minimum elevation angle of $20^o$
(iv) Repeat (b) above for a minimum elevation angle of $40^o$
(d) What are the advantages and disadvantages to the service provider if they adopt the constellation in part (b) above?
(e) What are the advantages and disadvantages to the service provider if they adopt the constellation in part (c) (iii) and part (c) (iv) above?

**Solution to question 8**

(a) Repeating the schematic from question 7 we have the diagram below.

We have already found the following parameters for an elevation angle of $20^o$:
Angle BAC = $50.4^o$
$g = 19.6^o$
**CB** = $r_e$ = 6378.137 km
**CA** = $r_s$ = 6378.137 + 1,400 = 7,778.137 km




In part (b) of this question, angle OBA is $0^o$. For this elevation angle, central angle $g$ = $\cos^{-1}(6{,}378.137/7{,}778.137) = \cos^{-1}(0.8200) = 34.9144^o$.

Arc **BD** = **CD** $\times$ (*g* in radians) = 6378.137 $\times$ (34.9144°) $\times$ $\pi$/180 = 3,886.6532 km

Arc **BD** is half the instantaneous coverage area from one satellite and so the total coverage is double this (assuming a symmetrical coverage) = 7,773.3065 km.

The circumference of the earth = $2\pi \times 6378.137$ = 40,075.0167 km

To have a complete coverage of one circumlocution of the earth will therefore require 40,075.0167/7,773.3065 satellites = 5.1555 satellites, which rounding up leads to a minimum of 6 satellites.

The minimum number of satellites to cover one polar orbit of the earth (one plane) with a minimum elevation angle of 0° = 6 satellites.

(b) Since the polar circumference of the earth is close to the equatorial circumference, there is therefore a requirement for 6/2 planes = 3 planes of polar orbiting satellites. (NOTE: you need to remember that a satellite in a polar orbit will cover ***both*** sides of the earth as it orbit and so 3 planes of satellites will give 6 *slices* of the earth.)

Thus, with 3 planes and 6 satellites per plane, the total number of satellites to cover the earth completely is 18 satellites.

(c)
(i) As above, but with a minimum elevation angle of 20°
Referring again to the diagram above, we have the following:

*g* = 19.6° (from the previous question)
Arc **BD** = **CD** $\times$ (*g* in radians) = 6378.137 $\times$ (19.6°) $\times$ $\pi$/180 = 2,181.8620 km

Arc **BD** is half the instantaneous coverage area from one satellite and so the total coverage is double this (assuming a symmetrical coverage) = 4,363.724 km.

The circumference of the earth = $2\pi \times 6378.137$ = 40,075.0167 km

To have a complete coverage of one circumlocution of the earth will therefore require 40,075.0167/4,363.724 satellites = 9.1837 satellites, which rounding up leads to a minimum of 10 satellites.

(c)
(iii) Since the polar circumference of the earth is close to the equatorial circumference, there is therefore a requirement for 10/2 planes = 5 planes of polar orbiting satellites. (NOTE: you need to remember that a satellite in a polar orbit will cover ***both*** sides of the earth as it orbit and so 5 planes of satellites will give 10 *slices* of the earth.)

(c)
(ii) As above, but with a minimum elevation angle of $40^o$
Referring to the figure, we have a new set of parameters.

| |
|---|
| **CB** = $r_e$ = 6378.137 km = **CD** |
| **CA** = $r_s$ = 6378.137 + 1,400 = 7,778.137 km |
| **B** = location of earth station |
| **BO** = tangent to earth's surface |
| Angle **OBA** = $40^o$ (minimum elevation angle) |
| ***g*** = the central angle |

By the law of sines, sin(ABC)/CA = sin(BAC)/CB, which gives sin (BAC) = CB × sin(ABC)/CA = 6378.137 × sin(130)/7,778.137 = 0.6282, and so the off-Nadir angle is $38.9147^o = 38.9^o$, making the total scan angle required for a symmetrical coverage of the earth = $77.8^o$.

The central angle ***g*** = 180 – 130 – 38.9 = $11.1^o$.

Arc **BD** = **CD** × (*g* in radians) = 6378.137 × ($11.1^o$) × π/180 = 1,235.6463 km

Arc **BD** is half the instantaneous coverage area from one satellite and so the total coverage is double this (assuming a symmetrical coverage) = 2,471.2927 km.

The circumference of the earth = 2π × 6378.137 = 40,075.0167 km

To have a complete coverage of one circumlocution of the earth will therefore require 40,075.0167/2,471.2927 satellites = 16.2162 satellites, which rounding up leads to a minimum of 17 satellites.

(c)
(iv) Since the polar circumference of the earth is close to the equatorial circumference, there is therefore a requirement for 17/2 planes = 8.5 planes of polar orbiting satellites, which again rounding up, <u>gives a requirement for 9 planes</u>. (NOTE: as before, you need to remember that a satellite in a polar orbit will cover ***both*** sides of the earth as it orbit and so 9 planes of satellites will give 18 *slices* of the earth.)

(d) The only advantage of the constellation for part (b) above (i.e. operating down to an elevation angle of $0^o$) is that fewer satellites are needed. The disadvantages are many: no significant overlap of coverages and so links are likely to be dropped before another satellite can effectively take over that link; scan loss will be large (due to the large off-nadir angle); path loss differences will be large, leading to power balancing problems; no redundancy possible if a satellite fails or many satellite beamlets fail; etc.

(e)As the elevation angle reduces, the propagation problems become less and the scan loss and path loss problems also reduce. The constellations in (c) (ii) and (iv) are therefore a better system design from the point of view of availability, and customer service and acceptance (due to easier hand-off and redundancy built in with a relatively large number of satellites per plane). The major disadvantage of these constellations is the large number of satellites required, which will lead to a potentially uneconomic systems.

# Chapter 12 Solution to Problems

**1.** Find the exact altitude of a GPS satellite that has an orbital period equal to precisely one half of a sidereal day. Use a value of mean earth radius $r_e$ = 6378.14 km and a sidereal day length of 23 hours 56 minutes 4.1 seconds.

**Answer:** The orbital period of the satellite is 11 hours 58 minutes 2.05 s = 43,082.05 s.
The orbital period is given by T where (Equation 2.6 squared on both sides)

$$T^2 = 4\,\pi^2\,a^3 / \mu$$

where $\mu$ = $3.986004418 \times 10^5$ km$^3$/s$^2$ and a is the radius of the orbit in km.

Hence

$$a^3 = T^2\mu / 4\,\pi^2 = 7.49602025 \times 10^{13}\ \text{km}^3$$

$$a = 26{,}561.764\ \text{km}$$

The orbital altitude above a mean earth radius of 6378.14 km is 20,183.62 km.

**2.** Find the maximum Doppler shift of the L1 signal frequency for a GPS satellite at an altitude of 20,200 km when the satellite has an elevation angle of $10^o$.
Hint: Maximum Doppler shift occurs when the observer is in the plane of the satellite orbit. Find the velocity of the satellite and the component of velocity towards the observer.

**Answer:** The velocity of the satellite in orbit is $2\,\pi\,a / T$ where T is the orbital period and a is the orbit radius. For a satellite with an orbit radius a = 20,200 + 6378.14 km = 26,578.14 km, the circumference of the orbit is $2\,\pi\,a$ = 166,995.38 km.

The orbital period is $T^2 = 4\,\pi^2\,a^3 / \mu$ where $\mu$ = $3.986004418 \times 10^5$ km$^3$/s$^2$.
Hence T = 43,121.90 s = 11 hrs 58 mins 41.8 s.
The velocity of the satellite is $v_s$ = 166995.38 / 43121.90 = 3.87264 km/s.

We must calculate the relative velocity of the satellite towards an observer who is in the plane of the satellite orbit when the satellite has an elevation angle of $10^o$. The geometry in the plane of the orbit is a triangle OGS, where O is the center of the earth, G is the observer at the earth's surface, and S is the satellite. When the satellite has an elevation angle of $10^o$, the angle

OGS = $90^o + 10^o = 100^o$. The known lengths of the sides of the triangle are OG = $r_e$, OS = a, and the angle between the satellite velocity vector and the line OS is $90^o$. We need to find the angle $\theta$, between the satellite velocity vector and the line SG.

Denoting the angle GSO as $\alpha$, we have

$$\sin\alpha / r_e = \sin 100^o / a$$

Hence

$$\sin\alpha = \sin 100^o \times r_e / a = 0.23633$$

$$\alpha = 13.670^o$$

The angle $\theta$ which defines the direction of the component of the satellite velocity towards the observer is given by

$$\theta = 90 - \alpha = 76.33$$

The component of velocity towards the observer is $v_r = v_s \cos\theta = 915.22$ m/s

The frequency of the L1 carrier signal for a GPS satellite is 1575.42 MHz, giving a wavelength of $\lambda = 0.190425$ m.

The maximum Doppler shift in the signal is

$$\Delta f = v_r / \lambda = 915.22 \text{ ms}^{-1} / 0.190425 \text{ m} = 4806.2 \text{ Hz}$$

A C/A code GPS receiver must be able to receive L1 signals that are shifted in frequency by up to 4.8 kHz. The shift will be an increase in frequency as the satellite approaches, falling to zero as the satellite passes overhead and then decreasing to a negative shift of – 4.8 kHz as the satellite reaches $10^o$ elevation before disappearing over the horizon.

**3.** An observer at the geographical north pole has a GPS receiver. At an instant in time, four GPS satellites all have the same range from the observer, and the GPS receiver records a measured delay time for the C/A signal of 0.17097528 s for each satellite. The four satellites' coordinates are calculated to be (0, -13280.5, 23002.5), (0, 13280.5, 23002.5), (-13280.5, 0, 23002.5), (13280.5, 0, 23002.5), where all distances are in km. Assuming an earth radius of 6378.0 km at the north pole, so that the observer's coordinates are (0,0, 6378), determine the clock offset error in the GPS receiver. (Use equations 12.1 and 12.3, and take the velocity of light in free space to be $2.99792458 \times 10^8$ m/s.)

**Answer:** From equation 12.1, the delay time of $T = 0.17097528$ s correspond to a psuedorange PR where

$$PR = T\,c = 0.17097528 \times 2.99792458 \times 10^8 = 51{,}257{,}099 \text{ m} = 51{,}257.099 \text{ km}$$

Equation 12.3 gives four simultaneous equations which give the psuedorange to the satellite

$$(X_i - U_x)^2 + (Y_i - U_y)^2 + (Z_i - U_z)^2 = (PR_i - \tau c)^2$$

where the receiver position is $(U_x, U_y, U_z)$ and the four satellites have positions $(X_i, Y_i, Z_i)$. The earth station location is known as (0, 0, 6378).

Hence the four simultaneous equations are (all distances in km)

$$(0 - 0)^2 + (13280.5 - 0)^2 + (23002.5 - 6378.0)^2 = (51{,}257.099 - \tau c)^2$$

$$(0 - 0)^2 + (-13280.5 - 0)^2 + (23002.5 - 6378.0)^2 = (51{,}257.099 - \tau c)^2$$

$$(13280.5 - 0)^2 + (0 - 0)^2 + (23002.5 - 6378.0)^2 = (51{,}257.099 - \tau c)^2$$

$$(-13280.5 - 0)^2 + (0 - 0)^2 + (23002.5 - 6378.0)^2 = (51{,}257.099 - \tau c)^2$$

Each of these equations gives the same result

$$13280.5^2 + (23002.5 - 6378.0)^2 = (51{,}257.099 - \tau c)^2$$

$$21{,}277.821 = (51{,}257.099 - \tau c)$$

Hence

$$\tau c = 29{,}979.278 \text{ km}$$

and the clock offset $\tau$ is

$$\tau = 0.10000010 = 100.000010 \text{ ms}$$

This simplified example illustrates how the four psuedorange equations can be solved to find clock offset error.

**4.** Accurate position location using GPS requires precise knowledge of the speed of light. In most applications, we use a velocity of light of $3.0 \times 10^8$ m/s. Solve Problem 3 above and then recalculate the clock offset using c = 3 x 10 m/s instead of the more precise value given in Problem 3. What is the error in the clock offset? What is the difference in the ranges to the satellites when the approximate value for $c = 3 \times 10^8$ m/s is used? Discuss the corresponding position error due to the approximation. Why is it essential to use the exact value of the velocity of EM waves?

**Answer:** From equation 12.1, the delay time of $T = 0.17097528$ s correspond to a psuedorange PR where

$$PR = T\,c = 0.17097528 \times 2.99792458 \times 10^8 = 51{,}257.099 \text{ km}$$

Using the approximate speed of light as $3.0 \times 10^8$ m/s, the corresponding psuedorange is

$$PR = T\,c = 0.17097528 \times 3.0 \times 10^8 = 51{,}292{,}584 \text{ m} = 51{,}292.584 \text{ km}$$

Following the solution to Problem 3, the four range equations reduce to

$$13280.5^2 + (23002.5 - 6378.0)^2 = (51{,}292.584 - \tau c)^2$$

$$21{,}277.821 = (51{,}292.584 - \tau c)$$

or

$$\tau c = 30{,}014.763 \text{ km}$$

Using the approximate velocity of EM waves of $3 \times 10^8$ m/s, the clock offset is

$$\tau = 1.0004921 \times 10^{-1} \text{ s} = 100.04921 \text{ ms}$$

The correct value from Problem 3 is $\tau = 100.000010$ ms

The range to the satellite using $c = 3 \times 10^8$ m/s is

$R_{approx} = PR - \tau c = 51{,}292.584 - 30{,}014.763 = 21{,}312.821$ km when the approximate speed of light is used in the calculations. The solution to Problem 3 using the exact speed of light gives

$$R_{exact} = PR - \tau c = 51{,}257.099 - 29{,}979.277 = 21{,}277.821 \text{ km}$$

The difference in range between the exact and the approximate calculation is 35.00 km.

In the general case, the position calculation requires four ranges, so when the error in the range values has a random distribution and many measurements are averaged to obtain the final result, the resulting error in the calculation of the position of the GPS receiver, for DOP = 1, is $\sqrt{4} \times \Delta R = 2\,\Delta R$. However, in this case, the error in the range is the same for each range value, and we cannot average the error. The position error would be similar in magnitude to $\Delta R$ – about 35 km. This is clearly not an acceptable error, so we must use the exact value for the velocity of EM waves in all GPS calculations.

**5.** A C/A code GPS receiver is located at the geographic south pole, coordinates $(0,0, z_p)$. Four GPS satellites are used to determine the radius of the earth at the south pole. At the instant of time that the measurement is made, the satellites have coordinates

#1: (0, -13280.500, -23002.500) #2 : (0, 13280.500, -23002.500),

#3: (13280.500, 0, -23002.500) #4: (0, 0, -26561.000).

The corresponding measured delay times for the C/A code sequences from the satellites are

#1: 0.12102731 s, #2: 0.12102731 s #3: 0.12102731 s #4: 0.11738995 s

Find the clock offset in the GPS receiver, and determine the radius of the earth at the south pole. Use a value for the velocity of light in free space $c = 2.99792458 \times 10^8$ m/s, and work your solution to a precision of 1 m. You will need to solve two simultaneous non-linear equations from the set in Equation 12.3 in which the unknowns are the clock offset and the value of $z_p$. Start with an estimated value $z_p = 6378$ km, and then solve the two simultaneous equations. This will give two unequal values for the clock offset. Use iteration of the value of $z_p$ to find the correct values for clock offset and earth radius at the south pole.

**Answer:** For satellites #1 through #3, the C/A code sequence delay time is T = 0.17097528 s correspond to a psuedorange PR where

$$PR_{1,2,3} = T_{1,2,3}\ c = 0.12102731 \times 2.99792458 \times 10^8 = 36{,}283.075 \text{ km}$$

For satellite #4, the delay time is T = 0.11738995 s and the psuedorange is

$$PR_4 = T_4\ c = 0.11738995 \times 2.99792458 \times 10^8 = 35{,}192.622 \text{ km}$$

Equation 12.3 gives four simultaneous equations which give the psuedorange to the satellite

$$(X_i - U_x)^2 + (Y_i - U_y)^2 + (Z_i - U_z)^2 = (PR_i - \tau c)^2$$

where the receiver position is $(U_x, U_y, U_z)$ and the four satellites have positions $(X_i, Y_i, Z_i)$. The earth station location at the south pole is known to be $(0, 0, z_p)$. Putting all distances in km and $c = 2.99792458 \times 10^5$ km/s:

The four simultaneous equations are

$$(0 - 0)^2 + (-13280.5 - 0)^2 + (-23002.5 - z_p)^2 = (36{,}283.075 - \tau c)^2$$

$$(-13280.5 - 0)^2 + (0 - 0)^2 + (-23002.5 - z_p)^2 = (36{,}283.075 - \tau c)^2$$

$$(+13280.5 - 0)^2 + (0 - 0)^2 + (-23002.5 - z_p)^2 = (36{,}283.075 - \tau c)^2$$

$$(0 - 0)^2 + (0 - 0)^2 + (-26561.000 - z_p)^2 = (35{,}192.622 - \tau c)^2$$

Each of first three equations gives the same result

$$13280.5^2 + (-23002.5 - z_p)^2 = (36{,}283.075 - \tau c)^2 \qquad (12.5.1)$$

The fourth equation gives

$$(-26561.0 - z_p)^2 = (35{,}192.622 - \tau c)^2 \qquad (12.5.2)$$

This is a pair of non-linear equations (because of the squares) with two unknowns, $z_p$ and $\tau$. There are several ways to solve such problems, but because we know the approximate value for $z_p$, iteration is one of the easiest, starting with an estimated value for $z_p$ of - 6378 km.

Computer or hand calculator equation solving routines can also be used solve this problem. (Note: $z_p$ is negative because the south pole is in the negative z direction for geocentric coordinates. The starting value of $z_p$ = - 6378 km is the mean radius of the earth at the equator. )

Substituting $z_p$ = - 6378 km in both equations and solving for $\tau$ c, noting that taking square roots leads to two possible answers (± root), only one of which is valid:

From 12.5.1 $\tau c = 15{,}005.253$ km

From 12.5.2 $\tau c = 15{,}009.622$ km

The values of $\tau$ c are not equal, with a difference of 4.369 km, so we must try another estimate.

Let's try $z_p$ = - 6368 km.

From 12.5.1 $\tau c = 14{,}999.002$ km

From 12.5.2 $\tau c = 15{,}001.622$ km

The values of $\tau$ c are now closer, at 2.62 km difference, and our revised estimate was in the correct direction, so we should try another estimate. Let's try $z_p$ = - 6360 km.

From 12.5.1 $\tau c = 14{,}991.187$ km

From 12.5.2 $\tau c = 14{,}911.621$ km

A final trial with $z_p$ = 6358 km gives

From 12.5.1 $\tau c = 14{,}989.624$ km

From 12.5.2 $\tau c = 14{,}989.622$ km

The difference is now 2 m, so we conclude that the radius of the earth at the south pole is 6358 km. Taking the mean of the two results above, the clock bias is

$$\tau = 14{,}989.623 \ / \ c = 0.050000 \text{ seconds} = 50.0000\text{ms}.$$

The values for the range error agree within 2 m, so we can find the clock offset error with considerable confidence. This example illustrates how a GPS receiver can calculate clock offset error and true range to the satellites within one meter, using the solution of simultaneous non-linear equations. This example is greatly simplified to make it possible to obtain a solution by hand calculation.

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Download Date: 25/01/2021

यशस्वी पुन्डीर

Yashasvi Pundeer

जन्म तिथि/DOB: 25/12/2001

पुरुष/ MALE

Mobile No: 7668711341

**6816 8878 1700**

VID : 9125 6163 6781 8326

Issue Date: 18/01/2021

मेरा आधार, मेरी पहचान

AADHAAR

भारतीय विशिष्ट पहचान प्राधिकरण

UNIQUE IDENTIFICATION AUTHORITY OF INDIA

**पता:**
S/O: संजीव कुमार, एन-23/24, लेन न 3, सिंह कॉलोनी,
रुद्रपुर, उधम सिंह नगर,
उत्तराखंड - 263153

**Address :**
S/O: Sanjeev Kumar, N-23/24, lane no 3,
singh colony, Rudrapur, Udham Singh
Nagar,
Uttarakhand - 263153

1947

help@uidai.gov.in

WWW

www.uidai.gov.in

P.O. Box No. 1947,
Bengaluru-560 001