

## **MAE 511 HW Set 1**

**Due Date: Sept 5, 2018 by 3pm**

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1. Use Mathematica™ to show that Equations 3.5-1 and Equations 3.5-2 from the Thomson book are equivalent. Hint: To accomplish this, use the matrix form of these equations as found in Equations 3.5-3 and 3.5-4.

Solution Procedure:

- RHS1 is the  $\omega$  matrix.
- rotmat is the rotation matrix in equation 3.5-3
- angvel is the matrix containing time derivatives of the angles as calculated from equation 3.5-3
- rotmat2 is the rotation matrix in equation 3.5-4
- angvel2 is the matrix containing time derivatives of the angles as calculated from equation 3.5-4

After defining the matrices as above, the inverse of rotmat is taken (In[55] in code). This matrix is same as rotmat2, as shown in line In[56].

Similarly, angvel and angvel2 are calculated from the equations 3.5-3 and 3.5-4 respectively and compared. These matrices are also equal as shown in line In[53].

Each step of the code is explained using comments.

2. Use matlab<sup>TM</sup> to simulate (using ode45) the equations of motion for a double pendulum found on page 78 of the Meirovitch book. Set  $m_1 = 2$  kg,  $m_2 = 3$  kg,  $L_1 = 0.8$  meters,  $L_2 = 0.6$  meters, and use initial conditions:  $\theta_1(0) = 0.2$  radians,  $\theta_2(0) = 0.3$  radians,  $d\theta_1/dt(0) = 0.4$  radians/second,  $d\theta_2/dt(0) = 0.5$  radians/second. Simulate the equations for 20 seconds and plot  $\theta_1(t)$  and  $\theta_2(t)$  versus time on separate plots.

#### Solution Procedure:

1. The equations are not in standard form, so the first step is to separate the expressions of  $\ddot{\theta}_1$  and  $\ddot{\theta}_2$ . For this purpose, the equations from the book are written in the  $\bar{A} \cdot \vec{x} = \bar{B}$  form.
2. The above equation is solved using the “linsolve” function of Matlab, with elements of  $\vec{x}$  being  $\ddot{\theta}_1$  and  $\ddot{\theta}_2$ ,  $\bar{A}$  containing their coefficients and  $\bar{B}$  containing the rest of the expression. The solution to this is a pair of 2<sup>nd</sup> order non-linear differential equations.
3. To convert this 2<sup>nd</sup> order differential equation into a 1<sup>st</sup> order (to use “ode45” function), state space method is used.

$$x1 = \theta_1$$

$$x2 = \dot{\theta}_1$$

$$y1 = \theta_2$$

$$y2 = \dot{\theta}_2$$

4. So, 4 equations are obtained of the following form:

$$\bar{x} = [x1 \quad x2 \quad y1 \quad y2]^T$$

5. A function is called that simultaneously assigns values to the  $\bar{x}$  (as written above) and LHS expressions that would be input into the “ode45” function.
6. The time span put in is [0 20] and the initial conditions are [0.2 0.3 0.4 0.5]. This completes all the inputs into the ode45 function and the output is  $\bar{x}$  at different values of t.
7. The values of  $\theta_1$  and  $\theta_2$  corresponds to x1 and y2 respectively, and a plot is of their values v/s time is made.