

```
In[ ]:= R1 = {{Cos[ψ] , Sin[ψ] , 0} , {-Sin[ψ] , Cos[ψ] , 0} , {0 , 0 , 1}};
R2 = {{1 , 0 , 0} , {0 , Cos[θ] , Sin[θ]} , {0 , -Sin[θ] , Cos[θ]}};
R3 = {{Cos[φ] , Sin[φ] , 0} , {-Sin[φ] , Cos[φ] , 0} , {0 , 0 , 1}};
```

```
R1 // MatrixForm
```

```
R2 // MatrixForm
```

```
R3 // MatrixForm
```

```
Out[ ]//MatrixForm=
```

$$\begin{pmatrix} \cos[\psi] & \sin[\psi] & 0 \\ -\sin[\psi] & \cos[\psi] & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

```
Out[ ]//MatrixForm=
```

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos[\theta] & \sin[\theta] \\ 0 & -\sin[\theta] & \cos[\theta] \end{pmatrix}$$

```
Out[ ]//MatrixForm=
```

$$\begin{pmatrix} \cos[\phi] & \sin[\phi] & 0 \\ -\sin[\phi] & \cos[\phi] & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

```
In[ ]:= A3 = R3.R2.R1;
```

```
Simplify[A3] // MatrixForm
```

```
Out[ ]//MatrixForm=
```

$$\begin{pmatrix} \cos[\phi] \cos[\psi] - \cos[\theta] \sin[\phi] \sin[\psi] & \cos[\theta] \cos[\psi] \sin[\phi] + \cos[\phi] \sin[\psi] & \sin[\theta] \sin[\phi] \\ -\cos[\psi] \sin[\phi] - \cos[\theta] \cos[\phi] \sin[\psi] & \cos[\theta] \cos[\phi] \cos[\psi] - \sin[\phi] \sin[\psi] & \cos[\phi] \sin[\theta] \\ \sin[\theta] \sin[\psi] & -\cos[\psi] \sin[\theta] & \cos[\theta] \end{pmatrix}$$

```
In[ ]:= B3 = Inverse[A3];
```

```
FullSimplify[B3] // MatrixForm
```

```
Out[ ]//MatrixForm=
```

$$\begin{pmatrix} \cos[\phi] \cos[\psi] - \cos[\theta] \sin[\phi] \sin[\psi] & -\cos[\psi] \sin[\phi] - \cos[\theta] \cos[\phi] \sin[\psi] & \sin[\theta] \sin[\psi] \\ \cos[\theta] \cos[\psi] \sin[\phi] + \cos[\phi] \sin[\psi] & \cos[\theta] \cos[\phi] \cos[\psi] - \sin[\phi] \sin[\psi] & -\cos[\psi] \sin[\theta] \\ \sin[\theta] \sin[\phi] & \cos[\phi] \sin[\theta] & \cos[\theta] \end{pmatrix}$$

```
In[ ]:= C3 = Dt[B3, t];
FullSimplify[C3] // TraditionalForm
```

Out[ ]//TraditionalForm=

$$\begin{pmatrix} \left( \frac{d\theta}{dt} \sin(\theta) \sin(\phi) - \cos(\phi) \left( \cos(\theta) \frac{d\phi}{dt} + \frac{d\psi}{dt} \right) \right) \sin(\psi) - \cos(\psi) \left( \frac{d\phi}{dt} + \cos(\theta) \frac{d\psi}{dt} \right) \sin(\phi) & \left( \cos(\phi) \frac{d\theta}{dt} \sin(\theta) + \left( \cos(\theta) \frac{d\phi}{dt} + \frac{d\psi}{dt} \right) \sin(\phi) \right) \sin(\psi) - \cos(\phi) \cos(\psi) \left( \frac{d\phi}{dt} + \cos(\theta) \frac{d\psi}{dt} \right) & \cos(\psi) \\ \cos(\psi) \left( \cos(\phi) \left( \cos(\theta) \frac{d\phi}{dt} + \frac{d\psi}{dt} \right) - \frac{d\theta}{dt} \sin(\theta) \sin(\phi) \right) - \left( \frac{d\phi}{dt} + \cos(\theta) \frac{d\psi}{dt} \right) \sin(\phi) \sin(\psi) & -\cos(\psi) \left( \cos(\phi) \frac{d\theta}{dt} \sin(\theta) + \left( \cos(\theta) \frac{d\phi}{dt} + \frac{d\psi}{dt} \right) \sin(\phi) \right) - \cos(\phi) \left( \frac{d\phi}{dt} + \cos(\theta) \frac{d\psi}{dt} \right) \sin(\psi) & \frac{d\psi}{dt} \sin(\theta) \\ \cos(\phi) \frac{d\phi}{dt} \sin(\theta) + \cos(\theta) \frac{d\theta}{dt} \sin(\phi) & \cos(\theta) \cos(\phi) \frac{d\theta}{dt} - \frac{d\phi}{dt} \sin(\theta) \sin(\phi) & 0 \end{pmatrix}$$

```
In[ ]:= FullSimplify[C3[[All, 1]]] // MatrixForm // TraditionalForm
```

Out[ ]//TraditionalForm=

$$\begin{pmatrix} \sin(\psi) \left( \sin(\theta) \sin(\phi) \frac{d\theta}{dt} - \cos(\phi) \left( \cos(\theta) \frac{d\phi}{dt} + \frac{d\psi}{dt} \right) \right) - \cos(\psi) \sin(\phi) \left( \cos(\theta) \frac{d\psi}{dt} + \frac{d\phi}{dt} \right) \\ \cos(\psi) \left( \cos(\phi) \left( \cos(\theta) \frac{d\phi}{dt} + \frac{d\psi}{dt} \right) - \sin(\theta) \sin(\phi) \frac{d\theta}{dt} \right) - \sin(\psi) \sin(\phi) \left( \cos(\theta) \frac{d\psi}{dt} + \frac{d\phi}{dt} \right) \\ \sin(\theta) \cos(\phi) \frac{d\phi}{dt} + \cos(\theta) \sin(\phi) \frac{d\theta}{dt} \end{pmatrix}$$

```
In[ ]:= FullSimplify[C3[[All, 2]]] // MatrixForm // TraditionalForm
```

Out[ ]//TraditionalForm=

$$\begin{pmatrix} \sin(\psi) \left( \sin(\phi) \left( \cos(\theta) \frac{d\phi}{dt} + \frac{d\psi}{dt} \right) + \sin(\theta) \cos(\phi) \frac{d\theta}{dt} \right) - \cos(\psi) \cos(\phi) \left( \cos(\theta) \frac{d\psi}{dt} + \frac{d\phi}{dt} \right) \\ -\cos(\psi) \left( \sin(\phi) \left( \cos(\theta) \frac{d\phi}{dt} + \frac{d\psi}{dt} \right) + \sin(\theta) \cos(\phi) \frac{d\theta}{dt} \right) - \sin(\psi) \cos(\phi) \left( \cos(\theta) \frac{d\psi}{dt} + \frac{d\phi}{dt} \right) \\ \cos(\theta) \cos(\phi) \frac{d\theta}{dt} - \sin(\theta) \sin(\phi) \frac{d\phi}{dt} \end{pmatrix}$$

```
In[ ]:= FullSimplify[C3[[All, 3]]] // MatrixForm // TraditionalForm
```

Out[ ]//TraditionalForm=

$$\begin{pmatrix} \sin(\theta) \cos(\psi) \frac{d\psi}{dt} + \cos(\theta) \sin(\psi) \frac{d\theta}{dt} \\ \sin(\theta) \sin(\psi) \frac{d\psi}{dt} - \cos(\theta) \cos(\psi) \frac{d\theta}{dt} \\ \sin(\theta) \left( -\frac{d\theta}{dt} \right) \end{pmatrix}$$

```
In[ ]:= D3 = A3.C3;
FullSimplify[D3] // TraditionalForm
```

Out[ ]//TraditionalForm=

$$\begin{pmatrix} 0 & -\frac{d\phi}{dt} - \cos(\theta) \frac{d\psi}{dt} & \cos(\phi) \frac{d\psi}{dt} \sin(\theta) - \frac{d\theta}{dt} \sin(\phi) \\ \frac{d\phi}{dt} + \cos(\theta) \frac{d\psi}{dt} & 0 & -\cos(\phi) \frac{d\theta}{dt} - \frac{d\psi}{dt} \sin(\theta) \sin(\phi) \\ \frac{d\theta}{dt} \sin(\phi) - \cos(\phi) \frac{d\psi}{dt} \sin(\theta) & \cos(\phi) \frac{d\theta}{dt} + \frac{d\psi}{dt} \sin(\theta) \sin(\phi) & 0 \end{pmatrix}$$