In[•]:=
```
R1 = {{1, 0, 0} , {0, Cos[θ1], Sin[θ1]}, {0 , -Sin[θ1] , Cos[θ1]}};
R2 = {{Cos[θ2] , 0 , -Sin[θ2]} , {0 , 1 , 0} , {Sin[θ2] , 0 , Cos[θ2]}};
R3 = {{Cos[θ3] , Sin[θ3] , 0} , {-Sin[θ3] , Cos[θ3] , 0} , {0, 0, 1}};
R1 // MatrixForm
R2 // MatrixForm
R3 // MatrixForm
```

Out[•]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \text{Cos}[\theta 1] & \text{Sin}[\theta 1] \\ 0 & -\text{Sin}[\theta 1] & \text{Cos}[\theta 1] \end{pmatrix}$$

Out[•]//MatrixForm=

$$\begin{pmatrix} \text{Cos}[\theta 2] & 0 & -\text{Sin}[\theta 2] \\ 0 & 1 & 0 \\ \text{Sin}[\theta 2] & 0 & \text{Cos}[\theta 2] \end{pmatrix}$$

Out[•]//MatrixForm=

$$\begin{pmatrix} \text{Cos}[\theta 3] & \text{Sin}[\theta 3] & 0 \\ -\text{Sin}[\theta 3] & \text{Cos}[\theta 3] & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

In[•]:=
```
A1 = R3.R2.R1;
Simplify[A1] // MatrixForm
```

Out[•]//MatrixForm=

$$\begin{pmatrix} \text{Cos}[\theta 2]\,\text{Cos}[\theta 3] & \text{Cos}[\theta 3]\,\text{Sin}[\theta 1]\,\text{Sin}[\theta 2] + \text{Cos}[\theta 1]\,\text{Sin}[\theta 3] & -\text{Cos}[\theta 1]\,\text{Cos}[\theta 3]\,\text{Sin}[\theta 2] + \text{Sin}[\theta 1]\,\text{Sin}[\theta 3] \\ -\text{Cos}[\theta 2]\,\text{Sin}[\theta 3] & \text{Cos}[\theta 1]\,\text{Cos}[\theta 3] - \text{Sin}[\theta 1]\,\text{Sin}[\theta 2]\,\text{Sin}[\theta 3] & \text{Cos}[\theta 3]\,\text{Sin}[\theta 1] + \text{Cos}[\theta 1]\,\text{Sin}[\theta 2]\,\text{Sin}[\theta 3] \\ \text{Sin}[\theta 2] & -\text{Cos}[\theta 2]\,\text{Sin}[\theta 1] & \text{Cos}[\theta 1]\,\text{Cos}[\theta 2] \end{pmatrix}$$

In[•]:=
```
B1 = Inverse[A1];
Simplify[B1] // MatrixForm
```

Out[•]//MatrixForm=

$$\begin{pmatrix} \text{Cos}[\theta 2]\,\text{Cos}[\theta 3] & -\text{Cos}[\theta 2]\,\text{Sin}[\theta 3] & \text{Sin}[\theta 2] \\ \text{Cos}[\theta 3]\,\text{Sin}[\theta 1]\,\text{Sin}[\theta 2] + \text{Cos}[\theta 1]\,\text{Sin}[\theta 3] & \text{Cos}[\theta 1]\,\text{Cos}[\theta 3] - \text{Sin}[\theta 1]\,\text{Sin}[\theta 2]\,\text{Sin}[\theta 3] & -\text{Cos}[\theta 2]\,\text{Sin}[\theta 1] \\ -\text{Cos}[\theta 1]\,\text{Cos}[\theta 3]\,\text{Sin}[\theta 2] + \text{Sin}[\theta 1]\,\text{Sin}[\theta 3] & \text{Cos}[\theta 3]\,\text{Sin}[\theta 1] + \text{Cos}[\theta 1]\,\text{Sin}[\theta 2]\,\text{Sin}[\theta 3] & \text{Cos}[\theta 1]\,\text{Cos}[\theta 2] \end{pmatrix}$$

In[•]:=
```
C1 = Dt[B1, t];
Simplify[C1] // TraditionalForm
```

Out[•]//TraditionalForm=

$$\begin{pmatrix} -\cos(\theta 3)\,\frac{d\theta 2}{dt}\,\sin(\theta 2) - \cos(\theta 2)\,\frac{d\theta 3}{dt}\,\sin(\theta 3) & & \\ \cos(\theta 2)\cos(\theta 3)\,\frac{d\theta 2}{dt}\,\sin(\theta 1) + \frac{d\theta 1}{dt}\,(\cos(\theta 1)\cos(\theta 3)\sin(\theta 2) - \sin(\theta 1)\sin(\theta 3)) + \frac{d\theta 3}{dt}\,(\cos(\theta 1)\cos(\theta 3) - \sin(\theta 1)\sin(\theta 2)\sin(\theta 3)) & -\cos(\theta 2)\,\frac{d\theta 2}{dt}\,\sin(\theta 1)\sin(\theta 3) - \frac{d\theta 3}{dt}\,(\text{co} & \\ -\cos(\theta 1)\cos(\theta 2)\cos(\theta 3)\,\frac{d\theta 2}{dt} + \frac{d\theta 1}{dt}\,(\cos(\theta 3)\sin(\theta 1)\sin(\theta 2) + \cos(\theta 1)\sin(\theta 3)) + \frac{d\theta 3}{dt}\,(\cos(\theta 3)\sin(\theta 1) + \cos(\theta 1)\sin(\theta 2)\sin(\theta 3)) & \cos(\theta 1)\cos(\theta 2)\,\frac{d\theta 2}{dt}\,\sin(\theta 3) + \frac{d\theta 3}{dt}\,(\text{cos} & \end{pmatrix}$$

In[•]:=

In[•]:=
```
FullSimplify[C1[[All, 1]]] // MatrixForm // TraditionalForm
```

Out[•]//TraditionalForm=

$$\begin{pmatrix} \sin(\theta 2)\,(-\cos(\theta 3))\,\frac{d\theta 2}{dt} - \cos(\theta 2)\sin(\theta 3)\,\frac{d\theta 3}{dt} \\ \cos(\theta 3)\left(\cos(\theta 1)\left(\sin(\theta 2)\,\frac{d\theta 1}{dt} + \frac{d\theta 3}{dt}\right) + \sin(\theta 1)\cos(\theta 2)\,\frac{d\theta 2}{dt}\right) - \sin(\theta 1)\sin(\theta 3)\left(\frac{d\theta 1}{dt} + \sin(\theta 2)\,\frac{d\theta 3}{dt}\right) \\ -\cos(\theta 1)\cos(\theta 2)\cos(\theta 3)\,\frac{d\theta 2}{dt} + \sin(\theta 1)\cos(\theta 3)\left(\sin(\theta 2)\,\frac{d\theta 1}{dt} + \frac{d\theta 3}{dt}\right) + \cos(\theta 1)\sin(\theta 3)\left(\frac{d\theta 1}{dt} + \sin(\theta 2)\,\frac{d\theta 3}{dt}\right) \end{pmatrix}$$

In[•]:=
```
FullSimplify[C1[[All, 2]]] // MatrixForm // TraditionalForm
```

Out[•]//TraditionalForm=

$$\begin{pmatrix} \sin(\theta 2)\sin(\theta 3)\,\frac{d\theta 2}{dt} - \cos(\theta 2)\cos(\theta 3)\,\frac{d\theta 3}{dt} \\ \sin(\theta 1)\,(-\cos(\theta 3))\left(\frac{d\theta 1}{dt} + \sin(\theta 2)\,\frac{d\theta 3}{dt}\right) - \sin(\theta 3)\left(\cos(\theta 1)\left(\sin(\theta 2)\,\frac{d\theta 1}{dt} + \frac{d\theta 3}{dt}\right) + \sin(\theta 1)\cos(\theta 2)\,\frac{d\theta 2}{dt}\right) \\ \cos(\theta 1)\cos(\theta 3)\left(\frac{d\theta 1}{dt} + \sin(\theta 2)\,\frac{d\theta 3}{dt}\right) + \sin(\theta 3)\left(\cos(\theta 1)\cos(\theta 2)\,\frac{d\theta 2}{dt} - \sin(\theta 1)\left(\sin(\theta 2)\,\frac{d\theta 1}{dt} + \frac{d\theta 3}{dt}\right)\right) \end{pmatrix}$$

In[•]:=
```
FullSimplify[C1[[All, 3]]] // MatrixForm // TraditionalForm
```

Out[•]//TraditionalForm=

$$\begin{pmatrix} \cos(\theta 2)\,\frac{d\theta 2}{dt} \\ \sin(\theta 1)\sin(\theta 2)\,\frac{d\theta 2}{dt} - \cos(\theta 1)\cos(\theta 2)\,\frac{d\theta 1}{dt} \\ \sin(\theta 1)\,(-\cos(\theta 2))\,\frac{d\theta 1}{dt} - \cos(\theta 1)\sin(\theta 2)\,\frac{d\theta 2}{dt} \end{pmatrix}$$

In[•]:=
```
D1 = A1.C1;
FullSimplify[D1] // TraditionalForm
```

Out[•]//TraditionalForm=

$$\begin{pmatrix} 0 & -\frac{d\theta 3}{dt} - \frac{d\theta 1}{dt}\sin(\theta 2) & \cos(\theta 3)\,\frac{d\theta 2}{dt} - \cos(\theta 2)\,\frac{d\theta 1}{dt}\sin(\theta 3) \\ \frac{d\theta 3}{dt} + \frac{d\theta 1}{dt}\sin(\theta 2) & 0 & -\cos(\theta 2)\cos(\theta 3)\,\frac{d\theta 1}{dt} - \frac{d\theta 2}{dt}\sin(\theta 3) \\ \cos(\theta 2)\,\frac{d\theta 1}{dt}\sin(\theta 3) - \cos(\theta 3)\,\frac{d\theta 2}{dt} & \cos(\theta 2)\cos(\theta 3)\,\frac{d\theta 1}{dt} + \frac{d\theta 2}{dt}\sin(\theta 3) & 0 \end{pmatrix}$$