<u>Project:</u> Solving optimization problems using analytical and heuristic techniques
DUE DATE: 12/11/18

The objective of this project is solve two optimization problems using analytical and heuristic techniques so that you can compare their effectiveness. The first problem is an unconstrained problem where you will not be able to open the objective function – it is a "black box". The second problem is a constrained optimization problem commonly used in engineering design optimization literature. For both problems, you will use multiple techniques and compare their effectiveness.

<u>Problem 1 – Unconstrained optimization using a "black-box" objective function</u>

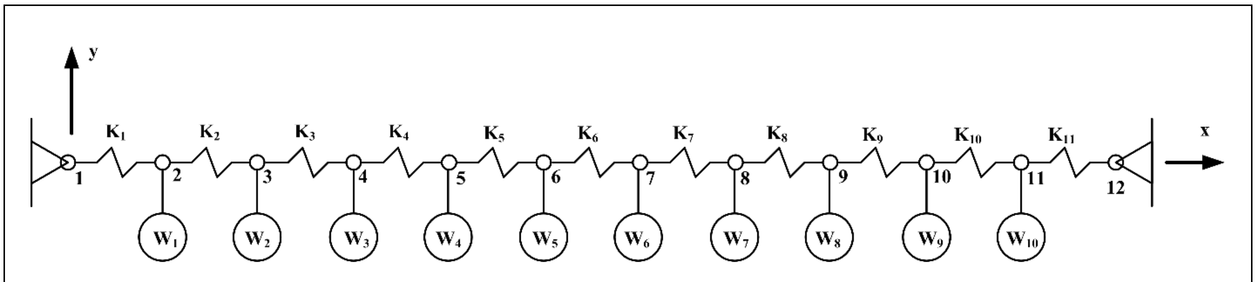
An eleven (11) spring system supporting ten (10) weights at the connections between the springs can be analyzed to determine the equilibrium position by minimizing potential energy. The deformation of spring *i* is given by:

$$\Delta L_i = \left[(X_{i+1} - X_i)^2 + (Y_{i+1} - Y_i)^2\right]^{1/2} - L_i^0$$

where the length L^0 is taken to be 10 m for each spring. Accounting for the stiffness of each spring and the weight of each mass, the potential energy of the system is given by:

$$PE = \sum_{i=1}^{N+1} \frac{1}{2} K_i \Delta L_i^2 + \sum_{j=1}^{N} W_j Y_{j+1}$$

PE has units of N-m and the coordinates are positive as shown in the figure below.



Mass-spring system before it deforms

The objective function for this problem has been coded in Matlab and uploaded to Moodle as "*projp1F.p*". You can call this function the same as any .m Matlab file, but you will not be able to open it. You will pass in the current x and y locations of the masses to get the objective function value in response. The input and output relationship of this file is given by:

```
[PE] = projp1F([x2;x3;x4;x5;x6;x7;x8;x9;x10;x11;y2;y3;y4;y5;y6;
                y7;y8;y9;y10;y11])
```

You will create your own code and use three approaches to solve this problem:

- BFGS
- Simulated annealing
- Genetic algorithm

For this problem, you are to report:

- Major algorithm parameters:
    - The starting location (or starting population) used for each approach
    - For BFGS – the 1D algorithm used for finding alpha_star
    - For simulated annealing:
        - Initial temperature
        - Value of n
        - Value of temperature reduction parameter, c
        - "Move limit" strategy for each move
    - For the genetic algorithm:
        - Type of encoding (binary or real)
        - Selection strategy
        - Crossover type and rate
        - Mutation type and rate
    - The convergence criteria used for each algorithm
- The best final solution from each algorithm and corresponding objective function value
- Results from an average of 5 runs for SA and GA
- A plot showing the value of the objective function as a function of the number of objective function evaluations.
- Show at least 5 plots (1 being the initial configuration, 1 the final configuration) of the masses (x-y location of all 10 masses) that depicts how each algorithm progresses
- Include a small write-up comparing the performance of the three algorithms

<u>Problem 2 – Constrained optimization problem</u>

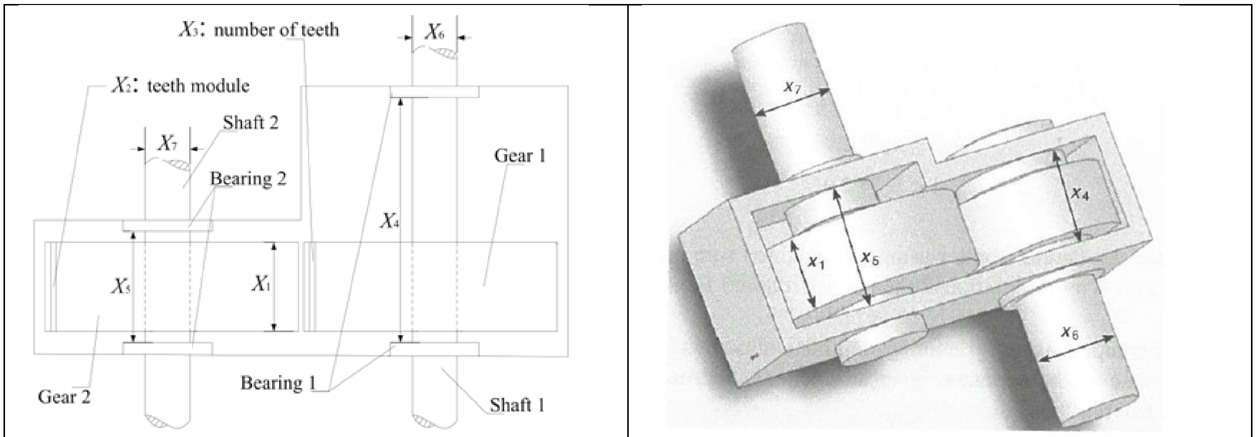
The Golinski speed reducer is depicted in the illustration below. The design of the speed reducer involves the design of a simple gear box. This mechanism can be used in a light airplane between the engine and the propeller to allow optimum rotating speed for each.

There are seven design variables in this problem:

- gear face width, x_1 (cm)
- teeth module, x_2 (cm)
- number of teeth of the pinion, x_3 (we will treat this as a continuous variable)
- distance between the bearing set 1, x_4 (cm)
- distance between the bearing set 2, x_5 (cm)
- diameter of shaft 1, x_6 (cm)
- diameter of shaft 2, x_7 (cm)

We are interested in minimizing the volume of the speed reducer since this translates to minimizing the weight of the device. The objective can be expressed as:

Minimize: $$F(\vec{x}) = 0.7854x_1x_2^2\left(\frac{10x_3^2}{3} + 14.933x_3 - 43.0934\right) - 1.508x_1(x_6^2 + x_7^2) + 7.477(x_6^3 + x_7^3) + 0.7854(x_4x_6^2 + x_5x_7^2)$$



Graphical representation of the speed reducer problem [1,2]

[1] Ren, X., Yadav, V., and Rahman, S., 2016, "Reliability-based design optimization by adaptive-sparse polynomial dimensional decomposition," *Structural and Multidisciplinary Optimization*, 53(3): 425-452.
[2] Messac, A., 2015, Optimization in Practice with Matlab®: For Engineering Students and Professionals. Cambridge University Press, 9781107109186.

Eleven inequality constraints are imposed by considering gear and shaft design practices. An upper limit on the bending stress of the gear tooth is imposed by g_1. The contact stress of the gear tooth is constrained using g_2. Constraints g_3 and g_4 are upper limits on the transverse deflections of the shafts. Dimensional constraints due to space limitations are imposed using g_5 through g_7. The constraints g_8 and g_9 are design requirements on the shaft based on past experience. The stress in the gear shaft is constrained by g_{10} and the stress in one of the gears is constrained by g_{11}. The constraints are given as follows:

$g_1(\vec{x}) : \dfrac{27}{x_1 x_2^2 x_3} \leq 1$	$g_2(\vec{x}) : \dfrac{397.5}{x_1 x_2^2 x_3^2} \leq 1$
$g_3(\vec{x}) : \dfrac{1.93 x_4^3}{x_2 x_3 x_6^4} \leq 1$	$g_4(\vec{x}) : \dfrac{1.93 x_5^3}{x_2 x_3 x_7^4} \leq 1$
$g_5(\vec{x}) : \dfrac{x_2 x_3}{40} \leq 1$	$g_6(\vec{x}) : \dfrac{x_1}{12 x_2} \leq 1$
$g_7(\vec{x}) : \dfrac{5 x_2}{x_1} \leq 1$	$g_8(\vec{x}) : \dfrac{1.5 x_6 + 1.9}{x_4} \leq 1$
$g_9(\vec{x}) : \dfrac{1.1 x_7 + 1.9}{x_5} \leq 1$	
$g_{10}(\vec{x}) : \dfrac{\sqrt{(745 x_5 / x_2 x_3)^2 + 157.5 \times 10^6}}{85 x_7^3} \leq 1$	
$g_{11}(\vec{x}) : \dfrac{\sqrt{(745 x_4 / x_2 x_3)^2 + 16.9 \times 10^6}}{110 x_6^3} \leq 1$	

All seven design variables have upper and lower bounds as given below:

$2.6 \leq x_1 \leq 3.6$	$0.7 \leq x_2 \leq 0.8$
$17 \leq x_3 \leq 28$	$7.3 \leq x_4 \leq 8.3$
$7.3 \leq x_5 \leq 8.3$	$2.9 \leq x_6 \leq 3.9$
$5.0 \leq x_7 \leq 5.5$	

You will solve this problem using three approaches:

- ALM
- Simulated annealing
- Genetic algorithm

For this problem, you are to report:

- Major algorithm parameters:
    - The starting location or starting population that you used for each approach
    - For ALM:
        - Initial values of the Lagrange multipliers
    - For simulated annealing:
        - Initial temperature
        - Value of n
        - Value of temperature reduction parameter, c
        - "Move limit" strategy for each move
        - Discussion of how your incorporated constraint violations
    - For the genetic algorithm:
        - Type of encoding (binary or real)
        - Selection strategy
        - Crossover type and rate
        - Mutation type and rate
        - Discussion of how your incorporated constraint violations
    - The convergence criteria used for each algorithm
- The best final solution from each algorithm and corresponding objective function value
- Results from an average of 5 runs for SA and GA
- A plot showing the value of the objective function as a function of the number of objective function evaluations.
- Include a small write-up comparing the performance of the three algorithms