Homework 4: Solving constrained optimization problems
DUE DATE: 10/23/18

1. Convert the following nonlinear optimization problem into a linear programming problem given the starting point of [1 ; -2]. Apply move limits of +/- ½ to each design variable. Set up the problem in canonical form. **Solve only the first move from your starting location using the Simplex method**. Show the tableau associated with each pivot.

$$\textit{Minimize}: \quad F = 2x_1^3 + 15x_2^2 - 8x_1x_2 + 15$$

$$\textit{Subject to}: \quad \begin{aligned} & x_1^2 + x_1x_2 + 1 = 0 \\ & 4x_1 - x_2^2 \le 4 \end{aligned}$$

2. Consider the following problem:

$$\textit{Minimize}: \quad F = (x_1 - 5)^2 + (x_2 - 5)^2$$

$$\textit{Subject to}: \quad \begin{aligned} & x_1 + 2x_2 \le 15 \\ & 1 \le x_i \le 10, \qquad i = 1, 2 \end{aligned}$$

Derive the conditions to be satisfied at the point [1; 7] by the search direction [S_1 ; S_2] if it is to be a useable feasible direction.

3. Consider the problem:

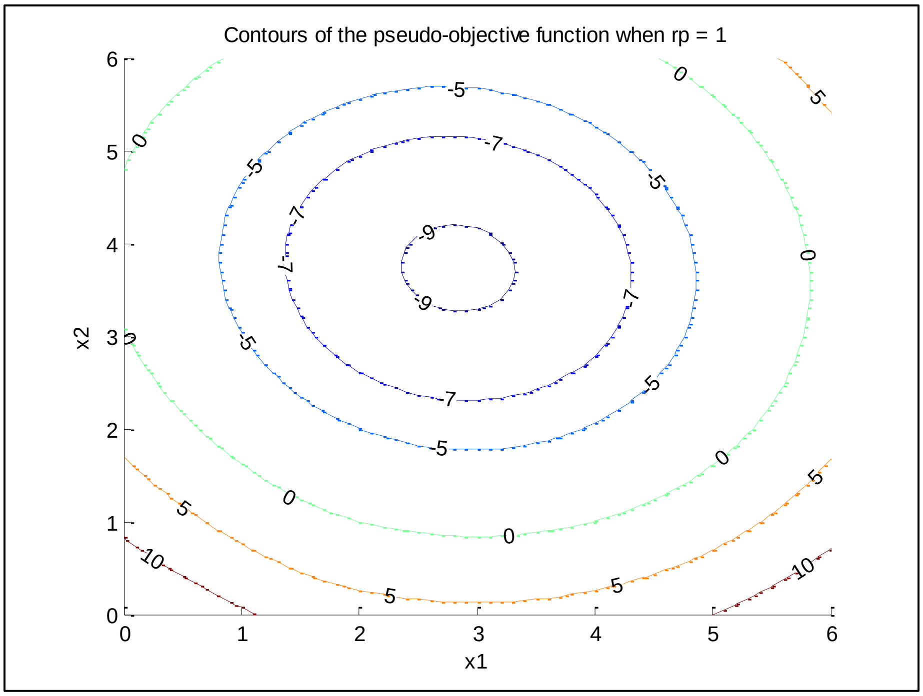
$$\textit{Minimize:} \quad F = x_1^2 + x_2^2 - 6x_1 - 8x_2 + 15$$

$$\textit{Subject to:} \quad \begin{aligned} & g_1 = 16 - 4x_1^2 - x_2^2 \le 0 \\ & g_2 = 3x_1 + 5x_2 - 15 \le 0 \\ & g_3 = -x_1 \le 0 \\ & g_4 = -x_2 \le 0 \end{aligned}$$

Using the exterior penalty function approach (you can use the excel method shown in class):

- Normalize the constraints; Start at the design location [1;1] with an initial r_p of 1
- Optimize the pseudo-objective function six times, increasing the value of r_p by a factor of 10 each time
- Report the value of the design variables, constraints, rp, F, and the pseudo-objective function at each iteration
- Create a series of contour plots in Matlab. The range for x_1 and x_2 should go from 0 to 6 in intervals of 0.1.
    - Create a contour plot for the original objective function (do not include constraints)

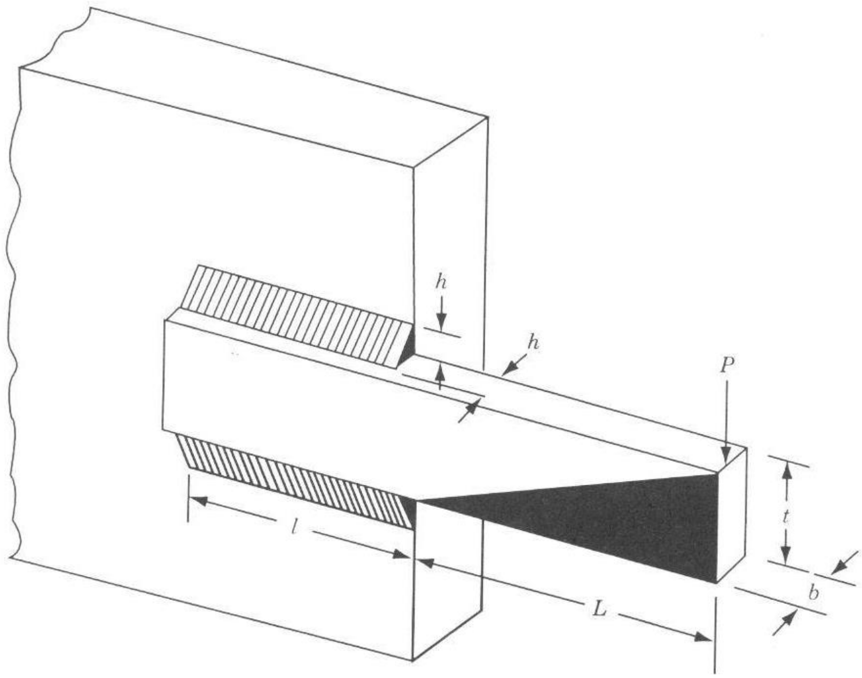
- Create a contour plot of the pseudo-objective function when rp = 1. Plot the contours of [-9 -7 -5 0 5 10]. You should get:



- Create a contour plot of the pseudo-objective function when rp = 10. Plot the contours of [-6 -5 -3 -1 0 5 10 15 20 30 40].
- Create a contour plot of the pseudo-objective function when rp = 100. Plot the contours of [-4 -2 -1 0 10 50 100 200 300 400].
- Create a contour plot of the pseudo-objective function when rp = 1000. Plot the contours of [-4 3 -2 -1 0 100 500 1000 2000 3000 4000]

4. The welded beam shown in the figure is designed for minimum cost subject to constraints on shear stress in the weld (τ), bending stress in the beam (σ), buckling load on the bar (P_c), end deflection of the beam (δ), and side constraints. The design variables are given by:

$$\begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{Bmatrix} = \begin{Bmatrix} h \\ l \\ t \\ b \end{Bmatrix}$$



The objective function of this problem is given by:

$$f(x) = 1.10471x_1^2 x_2 + 0.04811x_3 x_4 (14.0 + x_2)$$

with the following constraints:

$$g_1(x): \tau(x) - \tau_{\max} \le 0$$
$$g_2(x): \sigma(x) - \sigma_{\max} \le 0$$
$$g_3(x): x_1 - x_4 \le 0$$
$$g_4(x): 0.10471x_1^2 + 0.04811x_3 x_4 (14.0 + x_2) - 5.0 \le 0$$
$$g_5(x): 0.125 - x_1 \le 0$$
$$g_6(x): \delta(x) - \delta_{\max} \le 0$$
$$g_7(x): P - P_c(x) \le 0$$

Where:

$$\tau(\boldsymbol{X}) = \sqrt{(\tau')^2 + 2\tau'\tau''\frac{x_2}{2R} + (\tau'')^2}$$

$$\tau' = \frac{P}{\sqrt{2}x_1 x_2} \qquad \tau'' = \frac{MR}{J}$$

$$M = P\left(L + \frac{x_2}{2}\right) \qquad R = \sqrt{\frac{x_2^2}{4} + \left(\frac{x_1 + x_3}{2}\right)^2}$$

$$J = 2\left\{\sqrt{2}x_1x_2\left[\frac{x_2^2}{12} + \left(\frac{x_1 + x_3}{2}\right)^2\right]\right\}$$

$$\sigma(\boldsymbol{X}) = \frac{6PL}{x_4x_3^2} \qquad \delta(\boldsymbol{X}) = \frac{4PL^3}{Ex_3^3x_4}$$

$$P_c(\boldsymbol{X}) = \frac{4.013E\sqrt{\frac{x_3^2x_4^6}{36}}}{L^2}\left(1 - \frac{x_3}{2L}\sqrt{\frac{E}{4G}}\right)$$

and the following side bounds on the variables:

$$0.1 \le x_1 \le 2.0;\;\; 0.1 \le x_2 \le 10.0;\;\; 0.1 \le x_3 \le 10.0;\;\; 0.1 \le x_4 \le 2.0$$

Using the following inputs:

$$P = 6000\,lb$$
$$L = 14\,in$$
$$E = 30\times10^6\ psi$$
$$G = 12\times10^6\ psi$$
$$\tau_{\max} = 13{,}600\,psi$$
$$\sigma_{\max} = 30{,}000\,psi$$
$$\delta_{\max} = 0.25\,in$$

Also, use a starting point of:

$$\begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{Bmatrix} = \begin{Bmatrix} 0.4 \\ 6.0 \\ 9.0 \\ 0.5 \end{Bmatrix}$$

Optimize the problem by normalizing the constraints and use the Augmented Lagrangian Method (you can use Excel as shown in class). Report the values of the design variables at each iteration. At your optimum, report the design variables and the Lagrange multipliers.