Homework 1: Problem formulation, graphical optimization, and optimality conditions
DUE DATE: 9/13/18

1. Given the following constrained optimization problem:

Minimize: $F = x_1^2 + x_2^2 + x_3^2$

Subject to:

$g_1 : x_1 + x_2 + x_3 \geq 5$

$g_2 : 2 - x_2 x_3 \leq 0$

$x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 2$

For the two design space locations below, solve for the values of the Lagrange multipliers. Then, determine whether each point satisfies the KKT conditions for an optimum design. *Hint: Remember to put the problem in standard form – this includes normalizing constraints.*

Candidate design 1 = $\{{}^3\!/_2 \quad {}^3\!/_2 \quad 2\}$

Candidate design 2 = $\{2 \quad 1 \quad 2\}$

2. Given the following constrained optimization problem:

Minimize $\quad F = 2x_1 + 3x_2 - x_1^3 - 2x_2^2$

Subject to: $\quad x_1 + 3x_2 \leq 8$
$5x_1 + 2x_2 \leq 12$
$x_1 \geq 0, x_2 \geq 0$

a) normalize the constraints and put the problem into standard form
b) using the Matlab file 'hw1problem2contour.m' provided on Moodle, print out the contour plot for this problem, then draw the constraints and identify the optimal point graphically
c) demonstrate analytically that the point you identified satisfies the optimality conditions

3. A manufacturer produces two types of machine parts, P_1 and P_2, using lathes and milling machines. The machining times required by each part on the lathe and the milling machine and the profit per unit of each part are given by:

	Machine time (hr) required by each unit on:		
Machine part	**Lathe**	**Milling machine**	**Profit per unit**
P_1	5	2	$200
P_2	4	4	$300

The total machining times available in a week are 500 hours on lathes and 400 hours on milling machines.

a) formulate a formal optimization problem statement if the goal is to maximize profit by producing units of P_1 and P_2
b) create a contour plot for this problem using Matlab
c) plot the constraints and identify the optimal point graphically on your contour plot
d) demonstrate analytically that the point you identified satisfies the optimality conditions

4. Consider the slider-crank mechanism shown below with the crank rotating at a constant angular velocity ω. The mechanism has to satisfy Groshof's criterion $l \geq 2.5r$ to ensure 360º rotation of the crank. Additional constraints on the mechanism are given by $0.5 \leq r \leq 10$, $2.5 \leq l \leq 25$, and $10 \leq x \leq 20$. When the system is at $\theta = 40$ for $\omega = 70$ rad/s:

a) derive an expression for the velocity of the slider

hint 1: derive an expression for a and b in terms of the design variables r and l, and the constant θ.

hint 2: use geometry to relate position (x) and the expressions that you have derived for a and b

hint 3: think about how velocity (v) can be determined from position (x) with respect to changes in time

b) formulate an optimization problem in standard form to maximize the speed of the slider; your goal is to find the lengths of the crank and connecting rod that correspond to this velocity (*hint: speed can be assumed to be the absolute value of velocity)*
b) plot, using Matlab, the two-variable design space, showing performance contours and constraint boundaries (you can draw the constraints by hand)
c) identify the constrained minimum on the figure
d) at the minimum you identified, what are the Lagrange multipliers? Demonstrate that optimality conditions hold.