MAE 531
Project

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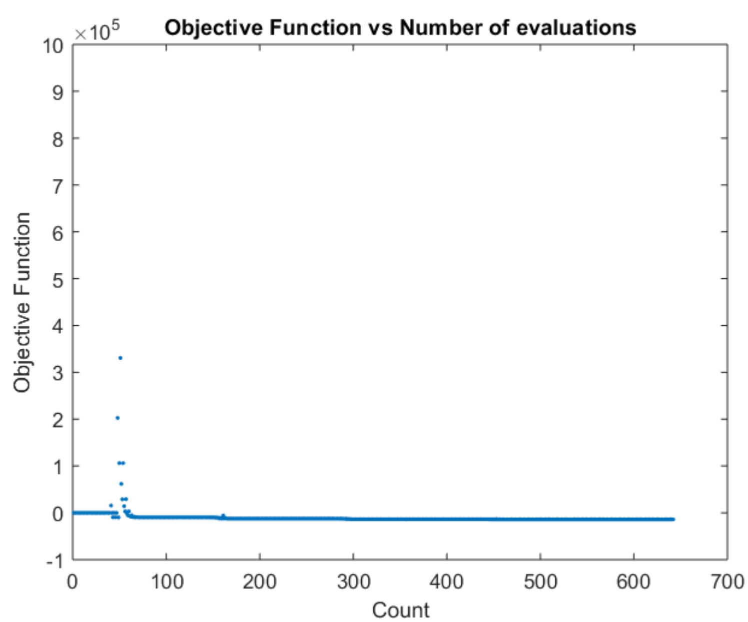
Problem 1:

- BFGS
  Initial x - position of the masses are taken as given in the problem: at intervals of 10m.
  All the y – position are taken same and are varied for each run as shown in the table below.
  Convergence criteria: **norm(grad(F)) <= 0.001**

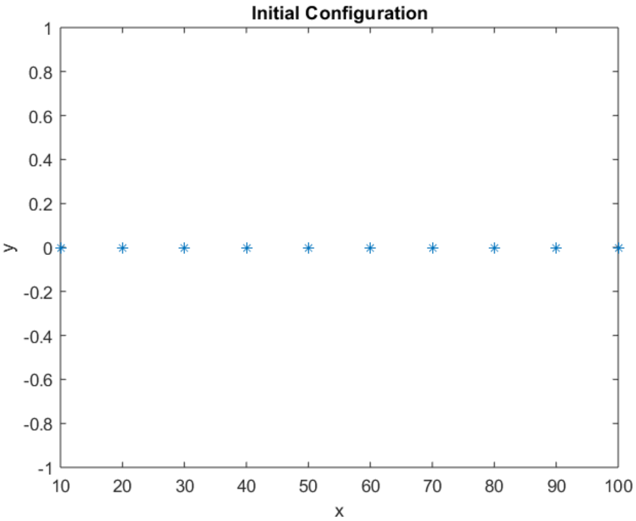
1D Algo	Golden Section

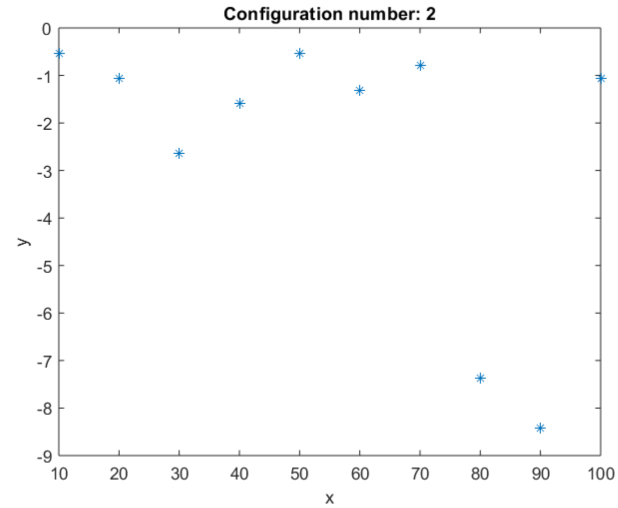
y init	0		-5		-10		-20	
	X_min	F_min	X_min	F_min	X_min	F_min	X_min	F_min
	10.01358		8.980585		8.181606		7.560437	
	20.05313		19.19848		18.25239		16.11279	
	29.98169		29.8742		28.97875		27.76328	
	40.0118		39.94038		39.55373		39.23416	
	50.20992		49.97346		49.89742		49.83796	
	60.23966		60.04395		60.10144		60.17503	
	70.57668		70.10048		70.55916		70.91265	
	79.71003		80.12532		81.09529		82.27505	
	90.68657		90.89985		92.28275		94.34324	
	100.1118	-14011.5	101.0079	-15080.4	101.5582	-25621.1	102.4229	-34231.9
	-0.79852		-4.18353		-6.01869		-8.34669	
	-1.61697		-5.25196		-9.72738		-15.9312	
	-3.94531		-5.74782		-10.9385		-20.1324	
	-2.39049		-5.45796		-10.6076		-20.5313	
	-0.84948		-5.15409		-10.2599		-20.226	
	-1.93888		-5.37905		-10.5009		-20.4971	
	-2.32573		-5.25815		-10.5128		-20.4282	
	-10.1396		-7.10517		-12.8044		-21.6063	
	-11.0057		-7.25665		-12.1751		-17.5198	
	-2.92025		-4.37698		-6.28664		-8.80823	

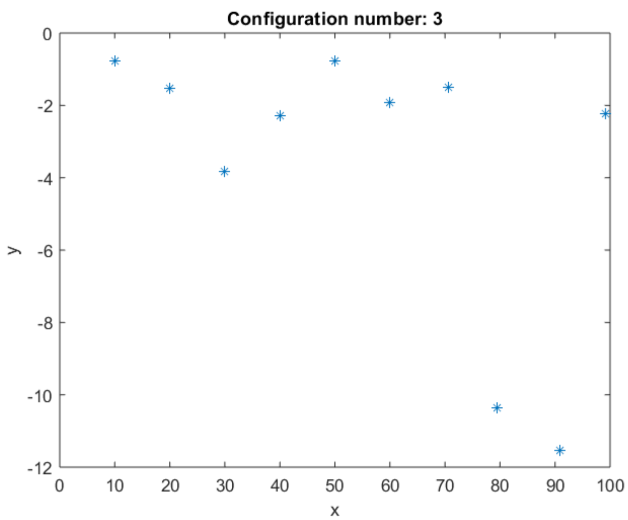
For y_init = 0:

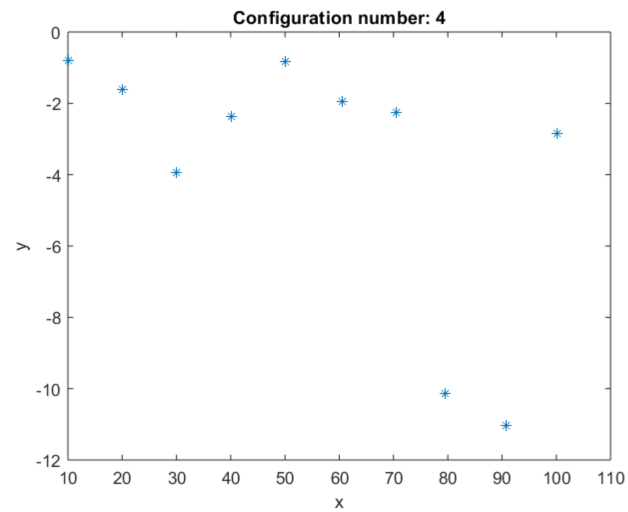


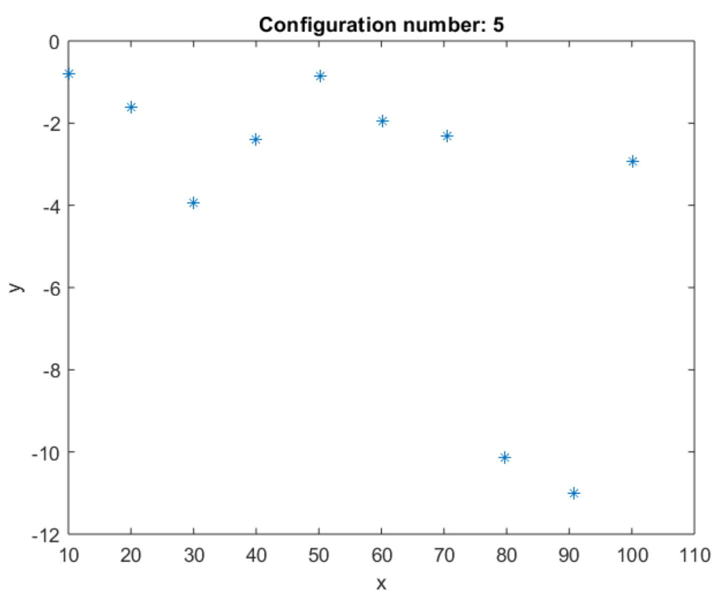
Location of the masses:











- Simulated annealing

  Starting location:
  x0 = [10;20;30;40;50;60;70;80;90;100]
  y0 = [0;0;0;0;0;0;0;0;0;0]

Init Temp	100000
n	500
c	0.4
Move limit	0.5

xmin	Fmin
8.2203622	-31299.3
18.836992	
29.32589	
40.95031	
52.004794	
62.691398	
73.401477	
86.087085	
93.893406	
102.42345	
-6.425394	
-13.26809	
-15.85485	
-16.8102	
-16.77121	
-20.78731	
-23.8404	
-23.96339	
-17.00079	
-7.424331	

xmin	Fmin
9.61636	-31201.8
18.96346	
30.79383	
43.45242	
53.50408	
63.63377	
73.13144	
85.28058	
95.72251	
102.8748	
-3.32205	
-10.6603	
-12.5876	
-11.4301	
-15.1323	
-17.5107	
-19.2239	
-23.0154	
-18.422	
-8.70819	

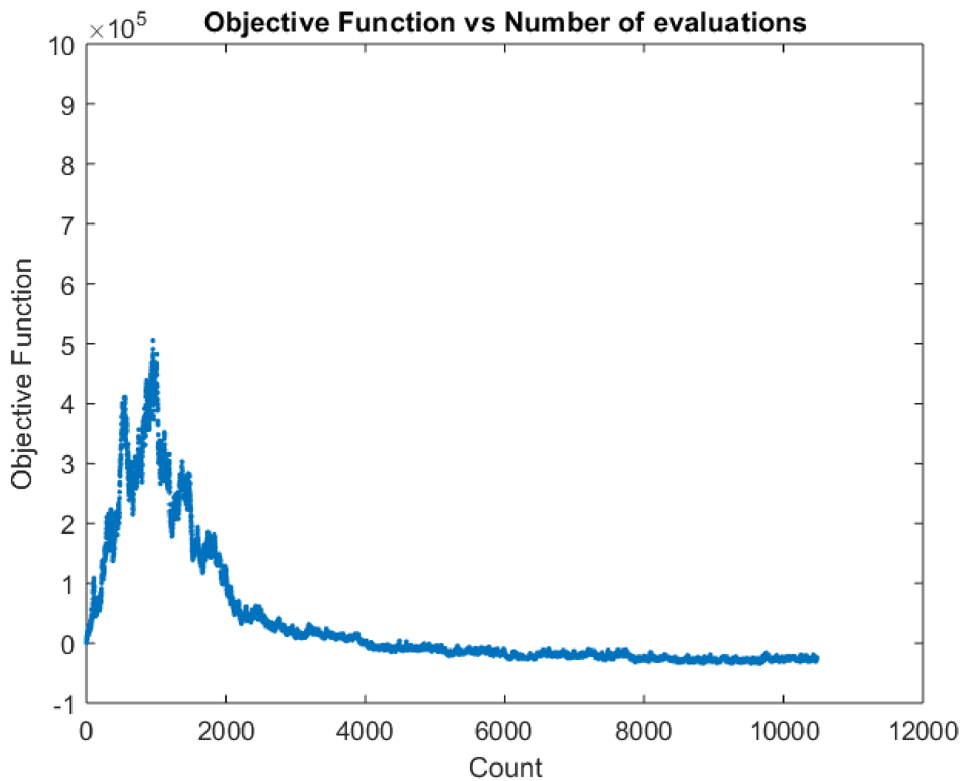
xmin	Fmin
10.22713	-30207.4
20.94831	
31.36427	
41.44388	
52.03584	
61.88273	
74.0164	
85.40964	
95.2755	
102.3024	
-0.57194	
-4.50141	
-7.1928	
-12.4724	
-13.9536	
-18.5482	
-20.4206	
-23.5808	
-18.7963	
-8.73649	

xmin	Fmin
5.47768	-30716.5
15.37664	
28.84076	
39.43354	
49.85366	
59.89269	
70.07879	
81.47494	
91.5879	
100.7151	
-9.15157	
-14.5254	
-20.229	
-22.8925	
-25.3663	
-24.6778	
-25.5221	
-21.7695	
-16.3228	
-7.12236	

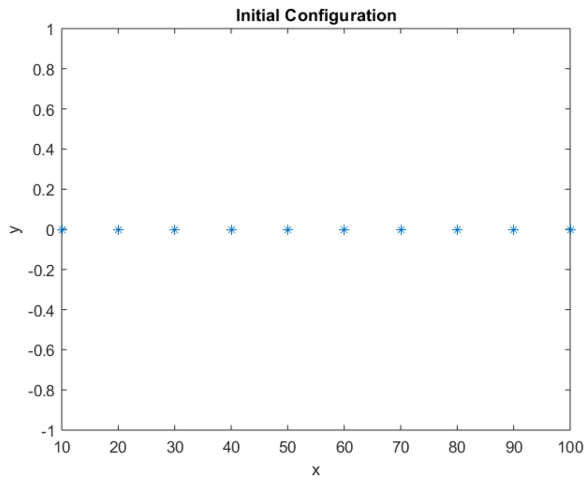
Avg F = -30786.9

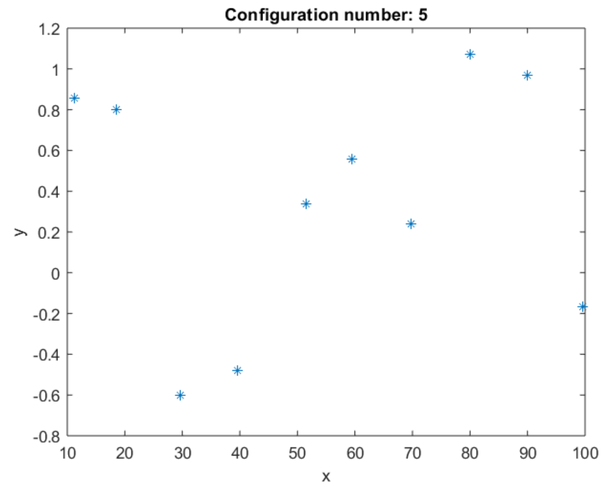
Convergence criteria: Temperature < 0.001 AND Objective function value between Temp changes < 0.001

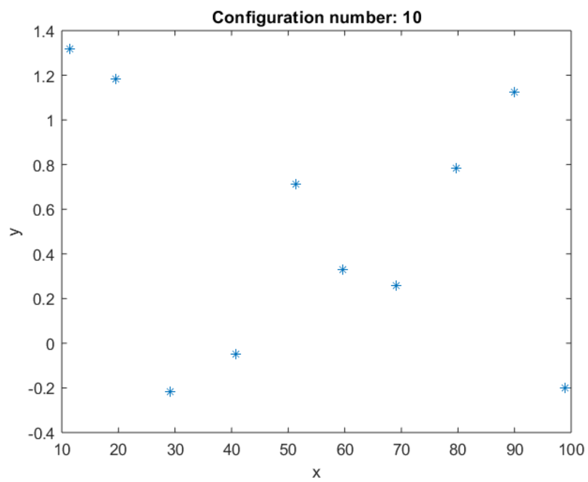
xmin	Fmin
9.204086	
19.79984	
30.32285	
41.59583	
50.96417	
60.77882	
70.47308	
83.67346	
94.72144	
101.8905	-30509.3
-5.38347	
-12.4111	
-14.9741	
-16.0782	
-18.6694	
-18.165	
-22.2685	
-24.5603	
-16.562	
-7.76473	

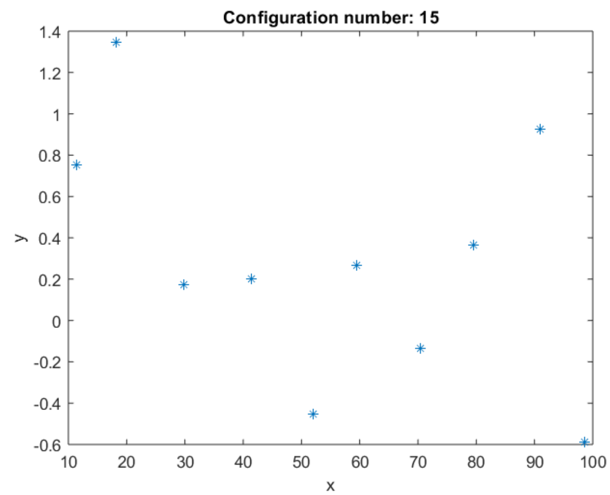


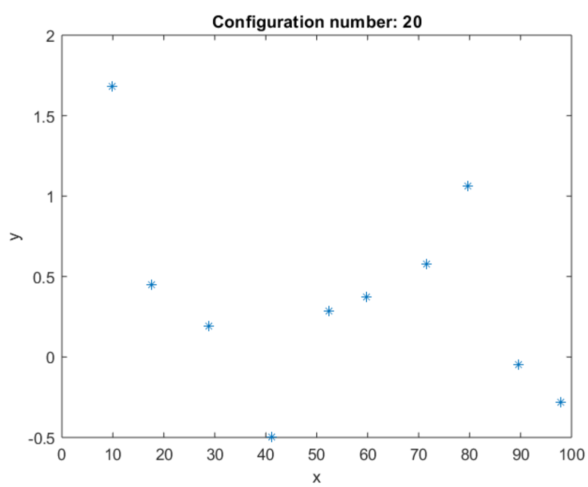
X-Y location of the masses:

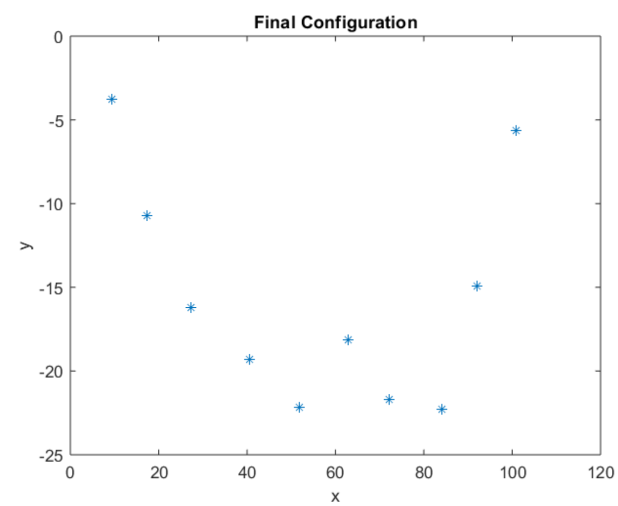












- Genetic algorithm

Starting Population:
x – Patterned numbers (Latin Hyper Cube) between 0 and 100
y – Patterned numbers (Latin Hyper Cube) between -30 and 30

xmin	Fmin
10.37004431	
21.43083024	
33.50830313	
44.72428875	
55.28547477	
61.43414132	
72.08701708	
81.4741801	
92.97082658	
100.9207174	-15460.8732
0.336905393	
-0.85515557	
1.589402305	
-1.03001268	
0.792496574	
-7.17457725	
-7.23645704	
-14.5161122	
-11.2126007	
-5.01381827	

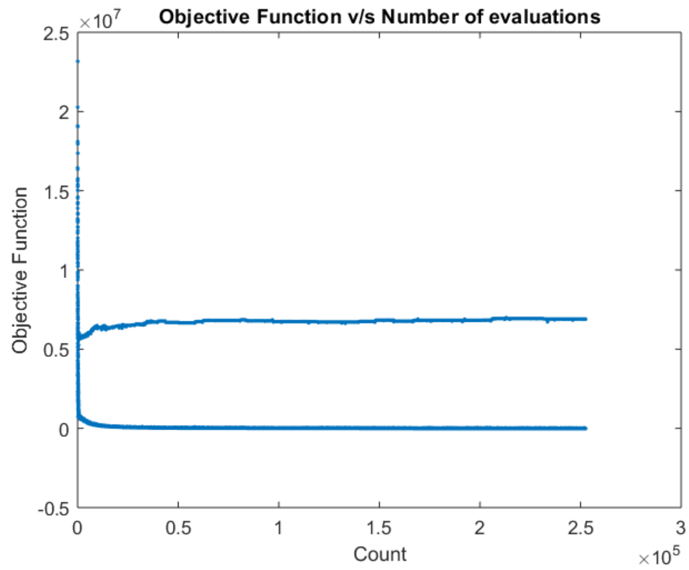
xmin	Fmin
9.580836	
19.74027	
30.66244	
41.45691	
50.95471	
54.49834	
64.79253	
75.47605	
88.20103	
99.72969	-11251.9
-2.2449	
-2.25344	
-5.6863	
-4.42556	
-1.37603	
-10.8031	
-7.61482	
-9.04501	
-7.89583	
-3.72047	

xmin	Fmin
9.987563	
20.65464	
31.67193	
42.11186	
52.6021	
57.57094	
67.18099	
76.92741	
89.37615	
100.0118	-9460.98
-1.94709	
-1.97058	
-1.69735	
-0.58891	
0.89902	
-7.39219	
-3.76302	
-6.48765	
-9.34181	
-3.96614	

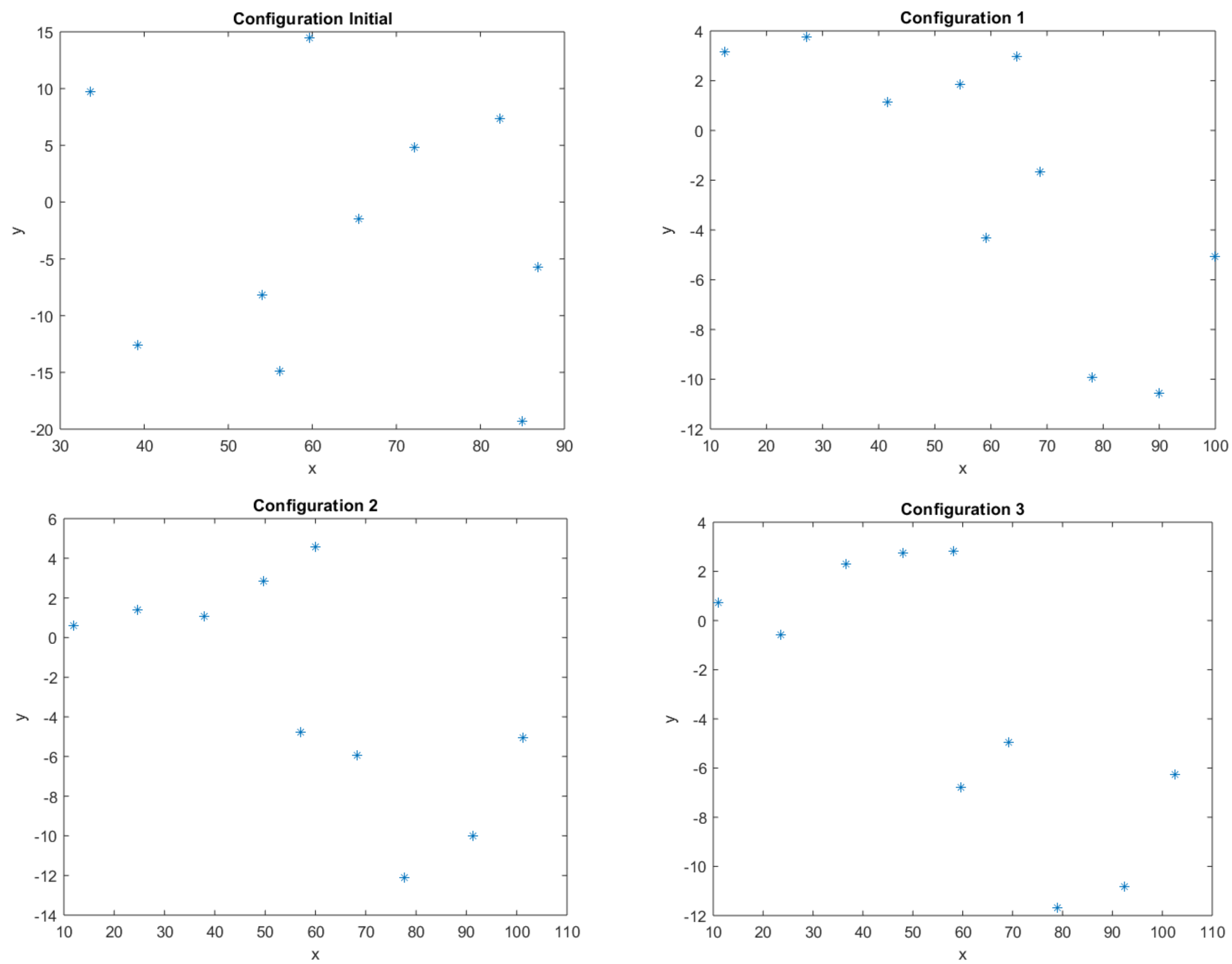
xmin	Fmin
10.21253	
21.99417	
33.75084	
45.22594	
55.23185	
57.02682	
66.42781	
77.07231	
89.42416	
99.27438	-13241.8
-4.35418	
-6.42159	
-8.13958	
-9.06498	
-4.89288	
-14.4262	
-10.5842	
-11.4544	
-8.24802	
-1.96835	

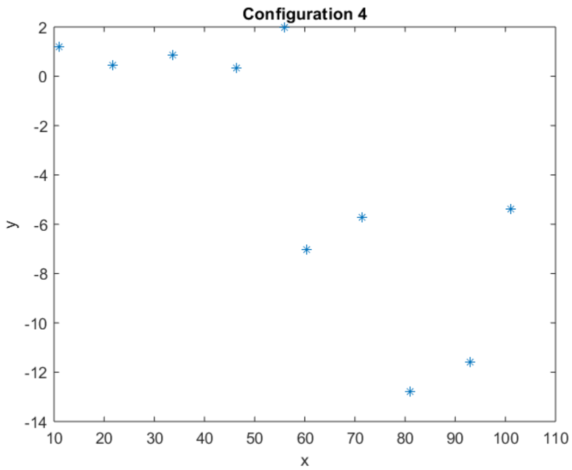
xmin	Fmin
10.22545	
21.41667	
34.1876	
44.68309	
52.49158	
56.98535	
67.48367	
78.08217	
89.77583	
99.87639	-10420.9
-3.9315	
-4.34687	
-4.2923	
-2.07977	
4.58754	
-4.95298	
-5.31352	
-9.16855	
-9.47763	
-4.70432	

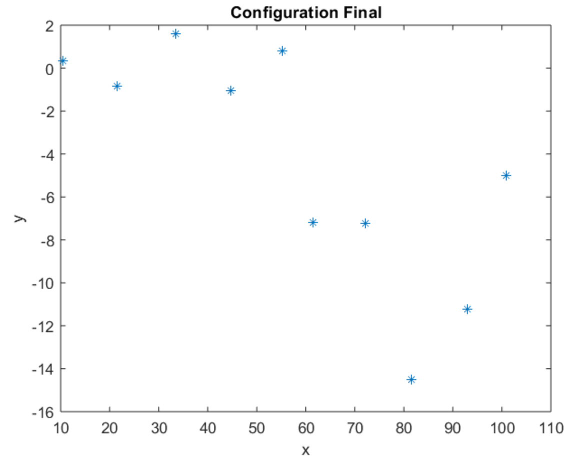
Avg F = -11967.3



X-Y location of the masses:







Encoding	Real
Selection	Tournament
Crossover	Arithmetic
Crossover rate	0.8
Mutation	Random value reassignment within bounds
Mutation rate	0.05

Convergence Criteria:
- 2500 Generations
- Average fitness of the entire population over 2 generations

Population size = 60
Parents were selected using tournament selection and children were formed by arithmetic crossover using $\lambda = 0.618$.
Number of children generated = pop_size * cross_rate
The bounds for mutation was set as +/- 2.5. So after mutation, the resulting variable would be within +/- 2.5 of the current value of the bound.

BFGS calculates the local minima, hence the minima value changes with different initial conditions as demonstrated above. Whereas, with SA and GA different starting locations were tried and optimum was found to be close in each case.
In GA, the number of function calls is the highest. The reason is that for each generation, the objective function is evaluated 100 times (population size). The parameters used for GA in this case doesn't result in the most efficient algorithm. The average F doesn't converge after 2500 generations, hence the convergence to global minima will take very high number of generations. For better performance, the parameters like selection, crossover and mutation can be changed like:

- Truncation selection was tried instead of tournament.
- The initial population was set as random between (-1,1) and the mutation was set as 2*current value

Problem 2:

- ALM

rp	100000
ε	0.0001

Start Location
3
0.75
20
8
8
3
5.2

xmin	fmin
3.506213	3503.546
0.700074	
19.64462	
7.982289	
8.236154	
3.3495	
5.286463	

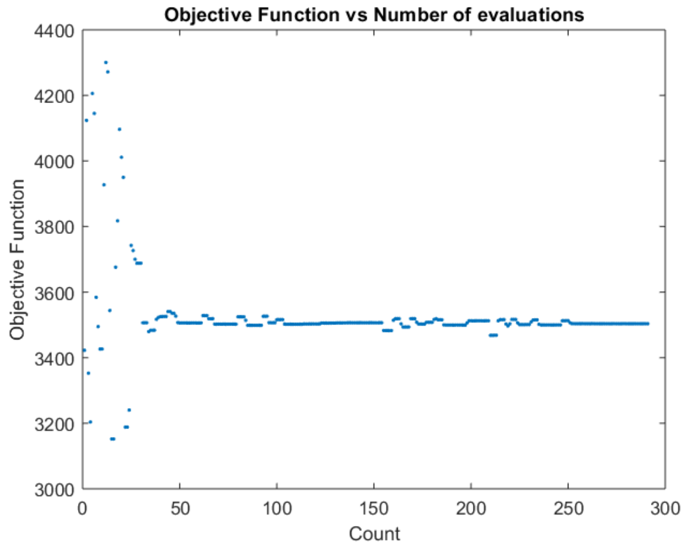
	λ init
g1	1
g2	1
g3	1
g4	1
g5	-1
g6	1
g7	-1
g8	-1
g9	-1
g10	1
g11	1
g12	1
g13	-1
g14	1
g15	-1
g16	1
g17	-1
g18	1
g19	-1
g20	1
g21	-1
g22	1
g23	-1
g24	1
g25	-1

Convergence Criteria:

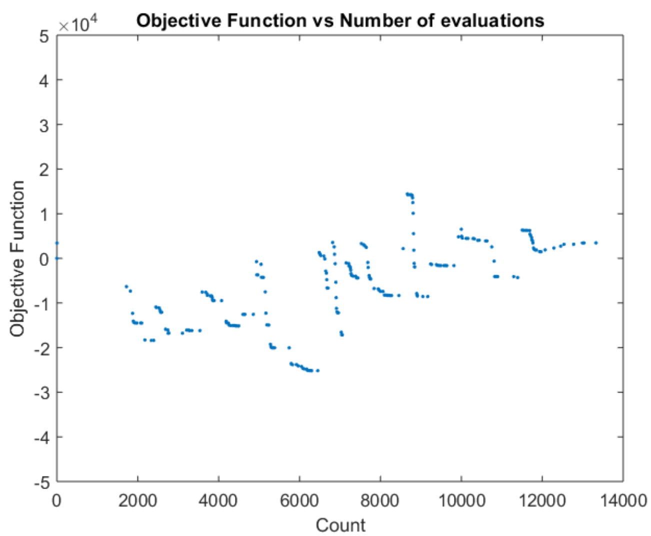
- Norm (x(i+1) – x(i)) < ε
- augF(x(i+1) – x(i)) < ε
- All the constraints are satisfied

fminunc was used to find the x in each iteration

Plot without considering funcCount of fminunc:



Plot after considering funcCount of fminunc: (The initial values were extremely large (+/- 10^9) , so not shown)



- Simulated annealing

Init Temp	100000
n	100
c	0.4
Move limit	0.001

x0	2.5
	0.65
	15
	7
	7
	2.5
	5

xmin	fmin
3.560404	
0.70006	
17.00596	
7.38496	3037.133
7.770553	
3.374216	
5.301769	

xmin	fmin
3.581022	
0.700773	
17.00433	
7.333398	3049.244
7.737336	
3.416073	
5.288052	

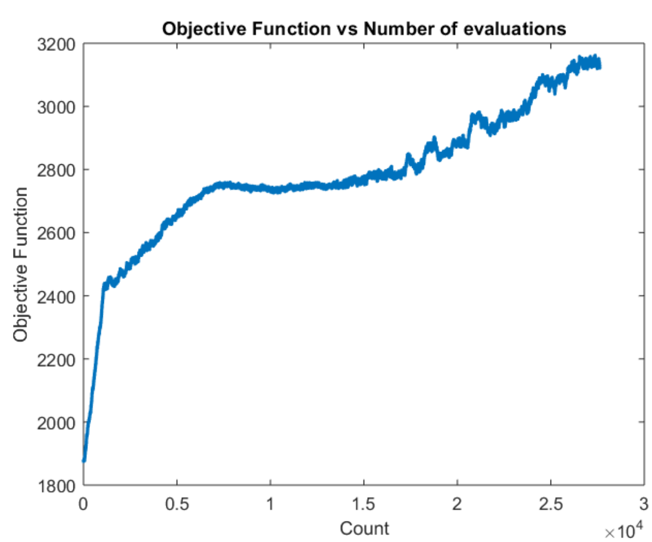
xmin	fmin
3.573229	
0.700905	
17.00122	
7.364103	3053.195
7.74801	
3.372561	
5.315958	

xmin	fmin
3.554931	
0.700006	
17.00105	
7.320409	3034.082
7.748703	
3.362698	
5.308391	

xmin	fmin
3.574812	
0.705931	
17.0008	
7.325117	3121.548
7.740063	
3.583451	
5.295993	

Exterior Penalty function was used. Hence, all the variables starting location had to be outside bounds.

Avg F min = 3059.04



- Genetic algorithm

Starting location of the population: random values within upper and lower bounds of each variable as given in the problem.

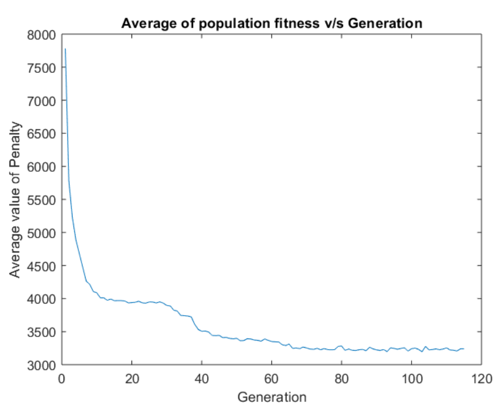
Encoding	Real
Selection	Tournament
Crossover	Arithmetic
Crossover rate	0.8
Mutation	Random value reassignment within bounds
Mutation rate	0.03

Population size = 100
Parents were selected using tournament selection and children were formed by arithmetic crossover using $\lambda = 0.618$.
Number of children generated = pop_size * cross_rate
The bounds for mutation was set as +/- 0.1. So after mutation, the resulting variable would be within +/- 0.1 of the current value of the bound.

To handle constraint violation, a penalty function similar to exterior penalty was used (instead of psi^2, just psi is used).

Reference to the following journal paper:
A COMPARISON OF SEQUENTIAL QUADRATIC PROGRAMMING, GENETIC ALGORITHM, SIMULATED ANNEALING, PARTICLE SWARM OPTIMIZATION AND HYBRID ALGORITHM FOR THE DESIGN AND OPTIMIZATION OF GOLINSKI'S SPEED REDUCER
By, Cenker Aktemur, Islam Gusseinov

Convergence Criteria:
- 500 Generations
- Average fitness value of the population of 2 consecutive generations <= 0.01
- All constraints <= 0



xmin	fmin
3.56678842	3184.910189
0.7	
17	
7.5656515	
8.3	
3.37859024	
5.5	

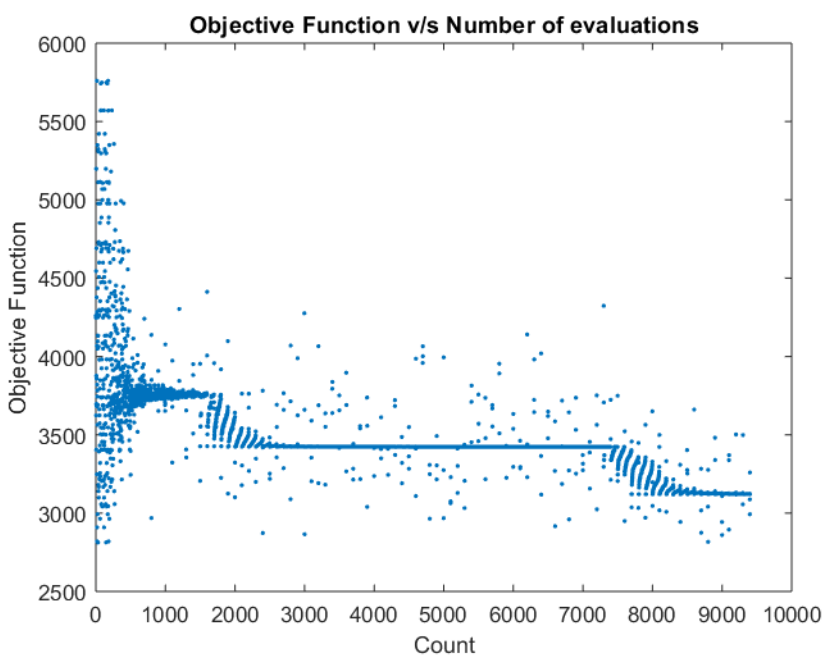
xmin	fmin
3.6	3160.283
0.7	
17.00001	
8.299998	
8.247306	
3.458755	
5.404919	

xmin	fmin
3.565763	3080.819
0.7	
17	
8.118489	
8.3	
3.363655	
5.344105	

xmin	fmin
3.6	3083.793
0.7	
17	
7.996323	
8.3	
3.397483	
5.316029	

xmin	fmin
3.545923	3121.16
0.7	
17	
8.3	
8.3	
3.596456	
5.315377	

Average F minimum = 3126.193



In this problem the GA parameters resulted in a very efficient performance of the code as compared to ALM and SA.
Apart from exterior penalty, dynamic penalty function was also attempted for SA with similar results.
In ALM, BFGS was attempted instead of fminunc. Different values of the initial Lagrange multipliers may result in a better performance. On application of different convergence criteria on constraints <= ε (varying the value of ε), it was seen that the performance decreased considerably as ε was decreased. It took more iterations and time for the function to converge.
After enforcing the constraints on convergence, SA showed good repeatability. To improve performance for different starting positions, initial temperature and n values can be increased.

In general, there is lot of flexibility while using the Heuristic based approaches over what all parameters can be changed to obtain the most efficient solution, which is lacking in the analytical approaches. Additionally, the analytical approaches usually do only local search for minima, whereas GA and SA do global search. However, this comes with a caveat: the number of objective function evaluations is generally higher in the Heuristic approaches.