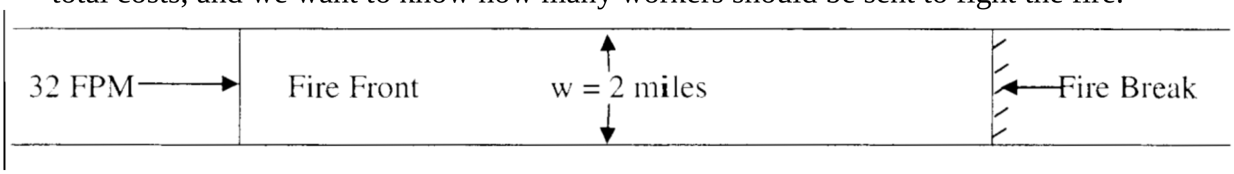
MAE 531 – Engineering Design Optimization | Fall 2018
Department of Mechanical and Aerospace Engineering | North Carolina State University

<u>Homework 2:</u> Optimization problems involving a single variable
DUE DATE: 9/25/18

1) A forest fire is burning down a 2 mile wide valley. The fire is moving to the right at a velocity of 32 feet per minute. The fire must be contained by cutting a fire break across the width of the valley (firefighters will cut trees perpendicular to the path of the fire so that when the fire reaches the break there is nothing to burn). A single worker can cut 2 ft of the fire break per minute. We are simplifying this problem by saying that we just need to create a line across the valley (the 2 ft cut by a single worker will be at the needed "width" to make the fire break).

   It costs $20 to transport each worker to the scene of the fire and back. Each worker is paid $6 per hour. The value of timber burned is $2000 per square mile. Our objective is minimizing total costs, and we want to know how many workers should be sent to fight the fire.



   a) Identify the single design variable that can be used to solve this problem
   b) Formulate the optimization problem statement to minimize cost
   c) Plot your objective as a function of the design variable you have chosen
   d) Use conditions of optimality to identify and validate your optimal solution

2) Write a code for the bounding phase algorithm discussed in class. Your code must be written in Matlab and:
   - Accept the following input variables (using the variable names in parenthesis):
     - starting location ('x0')
     - step size ('step_size')
   - Pass a design variable value ('x') to an objective function called ('objF')
   - Return the bounds to the user ('xleft') ('xright')

   Hint:
   ```
   function [xleft xright] = name_of_code_here(x0, step_size)
   ```

   Upload your code to Moodle.

3) Create a Matlab code for the 3-point quadratic approximation with refinement algorithm.

4) Create a Matlab code for the golden section algorithm.

5) Given the function:

$$f(x) = 2(x-3)^2 + e^{(0.5x^2)}$$

a) Plot the objective function over the range of [-1,4]
b) Using an initial point of 0 and a step size of 1.25, bound the optimum using the bounding phase algorithm we introduced in class. Report the results of your bounding.
c) Run your 3-point quadratic approximation with refinement algorithm code with the results of the bounding phase algorithm found in part (b). Use a convergence criteria value ε_x equal to 0.0001. At each iteration, store the values associated with the table below and report the results from each iteration of your code.

Recall that:

$$\frac{\tilde{x}^* - x_{min}}{x_{min}} \le \varepsilon_x$$

Iteration	**a_0**	**a_1**	**a_2**
1			
2			
.			
.			
Last			

x_1	**F_1**	**x_2**	**F_2**	**x_3**	**F_3**	$\widehat{x^*}$	$\widehat{F^*}$

d) Using the results that you tabulated for a_0, a_1, and a_2, make separate plots that show the parabola associated with your estimation function for every 5th iteration (1, 5, 10, 15, …). Also include the original objective function plot from part (a) on this graph.
e) Run your golden section algorithm code with the results of the bounding phase algorithm found in part (b). Use a convergence criteria value ε_x equal to 0.0001. At the beginning of each iteration, store the values associated with the table below and report the results from each iteration of your code. Calculate x_{opt} as the mid-point of x_L and x_R.

Recall that:

$$\frac{Current\ range}{Initial\ range} \le \varepsilon_x$$

Iteration	**x_L, F_L**	**x_1, F_1**	**x_2, F_2**	**x_R, F_R**	**x_{opt}, F_{opt}**
1					
2					
.					
.					
Last					