Integration

In this lesson, we will learn about definite and indefinite integrals of single and multiple integrations.

WE'LL COVER THE FOLLOWING

Integration is one of the two main operations of calculus, with its inverse operation, differentiation, being the other. Given a function f of a real variable x and an interval $[a, b]$ of the real line, the definite integral is denoted as:

$$\int_a^b f(x)\, dx$$

Integrals are calculated with the `integrate()` function. SymPy implements a combination of the Risch algorithm and an algorithm for computing integrals based on Meijer G-functions. These allow SymPy to compute a wide variety of indefinite and definite integrals.

Indefinite integrals

Integration uses syntax similar to differentiation. For the indefinite integral, we specify the function and the variable with respect to which the integration is performed.

```
integrate(y, x)
```

💡 The `integrate()` function does not add the constant of integration in the indefinite integral.

Let's see an example of integrating a polynomial below:

```
from sympy import *

def f(x):
    return (x**2 + x)

x = Symbol('x')

print(integrate(f(x), x))
```

SymPy allows for a range of integrals. Let's see them one by one

Rational

$$\int \frac{x^2}{x+1} dx$$

```
from sympy import *

def f(x):
    return (x**2 / x+1)

x = Symbol('x')
print(integrate(f(x), x))
```

Trigonometric

$$\int sin^2 x + cos2x \; dx$$

```
from sympy import *

def f(x):
    return (sin(x)**2 + cos(2*x))

x = Symbol('x')
print(integrate(f(x), x))
```

Logarithmic and Exponential

$$\int (x^2 e^x + \frac{1}{x})\, dx$$

```
from sympy import *

def f(x):
    return (x**2 * exp(x) + (1 / x))

x = Symbol('x')
print(integrate(f(x), x))
```

Definite integral

Definite integrals can be computed by providing a tuple having the variable of integration, and the limits of integration:

```
integrate(y, (x, lowerBound, upperBound))
```

$$\int_0^2 x^2 e^x dx$$

> 💡 For improper integrals, we use the `oo` symbol for infinite.

Proper Integral

Let's look at its implementation in SymPy:

```
from sympy import *

def f(x):
    return (x**2 * exp(x))

x = Symbol('x')
print(integrate(f(x), x))       # definite integral
print(integrate(f(x), (x, 0, 2)).evalf(5)) # limits are from 0 to 2
```

Improper Integral

$$\int_0^\infty (1+2x)e^{-x}dx$$

Let's see its implementation in SymPy:

```
from sympy import *

def f(x):
    return (1 + 2*x) * exp(-x)

x = Symbol('x')
print(integrate(f(x), x))       # definite integral
print(integrate(f(x), (x, 0, oo))) # limits are from 0 to inifinite
```

Multiple integrals

Multiple integrals are essential when dealing with vector calculus and are a stepping stone for solving many 2-D and 3-D problems in science.

$$\int\int\int x^2yz^3\ dxdydz$$

Multiple integrals can be performed by specifying several variables when the integration is performed.

Indefinite Integrals

For indefinite integrals, we use the following syntax:

```
integrate(f(x, y, z), x, y, z)
```

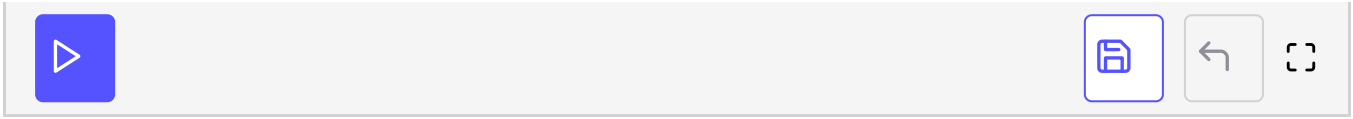
In the given code, the function would be integrated over `x`, first, then over `y`, then over `z`.

```
from sympy import *

def f(x, y, z):
    return x**2 + y + z**3
```

```
x, y, z = symbols('x y z')


indef_integ = integrate(f(x, y, z), x, y, z)
print(indef_integ)       # indefinite integral
```



Definite Integrals

$$\int_{-2}^{2}\int_{-3}^{3}\int_{0}^{4} x^2yz^3\,dxdydz$$

For definite integrals, we usually use tuples (we could also use lists) for providing limits of integration:

```
integrate(f(x, y, z), (x, lower_x, upper_x), (y, lower_y, upper_y), (z, lo
wer_z, upper_z))
```

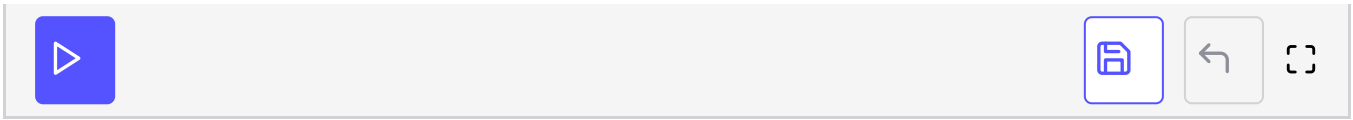
Let's see it's implementation below:

```
from sympy import *

def f(x, y, z):
    return x**2 + y + z**3

x, y, z = symbols('x y z')

def_integ = integrate(f(x, y, z), (x, 0, 4), (y, -3, 3), (z, -2, 2)) # Definite integral
print(def_integ)
```



Let's learn about limits in the next lesson.