Probability Distribution

This chapter discusses the concept of probability distribution.

Probability Distribution

When working with random variables, we can ask questions like "what is the probability that X equals 2 heads in three coin flips, or that Y equals a value greater than 2 on a die roll?". We'll work these example scenarios below.

Example 1

If we flip a coin three times, the sample space will look something like below:

$$S = HHH, HHT, HTH, HTT, THH, TTH, THT, TTT$$

Now what is P(X=2)? In other words, what is the probability that X (the number of heads in three coin flips) is exactly equal to two? The outcomes that satisfy exactly two heads include: HHT, HTH and THH. Therefore we can say

$$P(X=2) = \frac{number\ of\ occurrences\ of\ desired\ event}{total\ occurrences}$$

$$P(X=2) = \frac{3}{8}$$

Example 2

Now let's try to work out the dice example. If we roll a die, it can come face up with the following values:

$$S = 1, 2, 3, 4, 5, 6$$

We defined the random variable Y as the value that shows face-up on the die. The outcomes satisfying Y > 2 include 3, 4, 5, and 6.

For a fair die, the probability of P(Y= any value in S) is $1/6$ since any of the six values are equally likely to show face-up. However, if we ask what the probability is of Y > 2 happening, it would be equivalent of asking,

$$P(Y = 3 \; or \; 4 \; or \; 5 \; or \; 6) =? = \frac{4}{6} = \frac{2}{3}$$

We saw how to calculate the probabilities of random variables taking on different values. When we examine probabilities for the possible outcomes of a given random variable, we are working with *probability distribution* for that random variable. Let's go back to our examples.

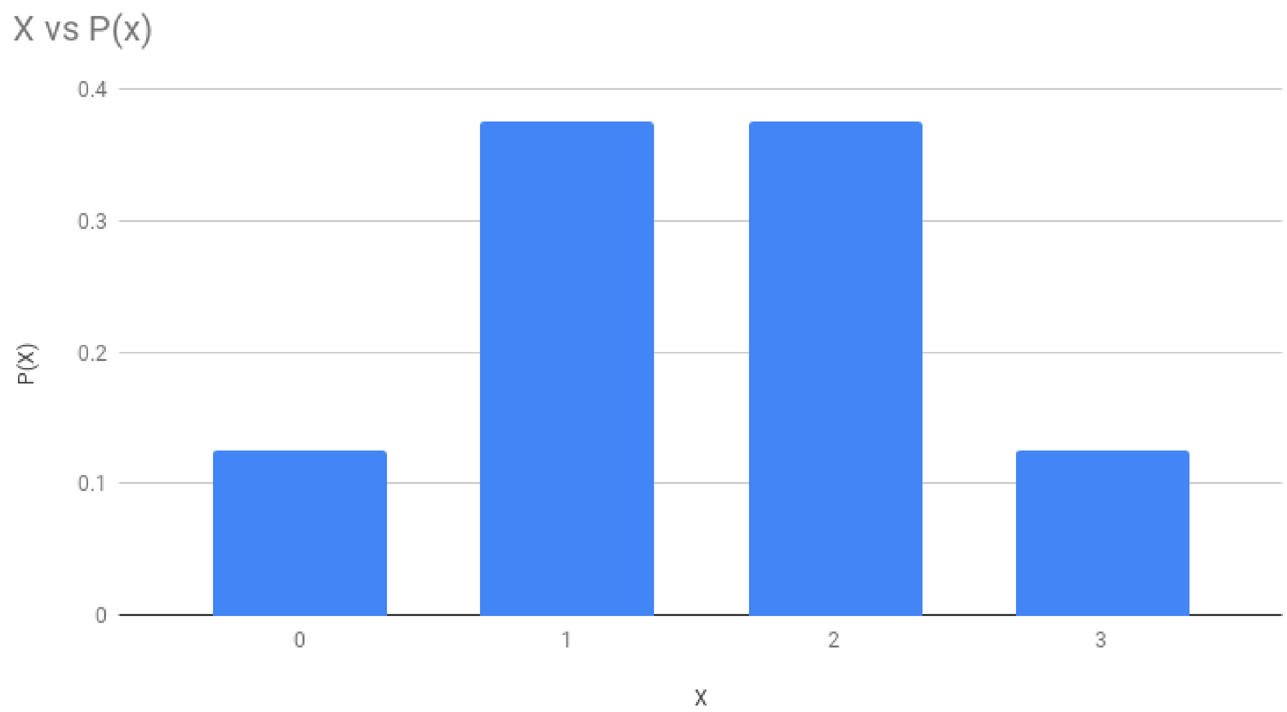
In case of the coin flip, we introduced the random variable X, which represented the number of times head shows up in 3 flips of a coin. What are various values that X can take on? Heads can show up 0 times, 1 time, 2 times, or 3 times in a 3-flip coin experiment. The distribution of probabilities for these various values that X can take on will constitute the *probability distribution* for X.

Values X can take on	Probability of X taking on value in first column
0	{ TTT }/8 = 1/8
1	{ HTT THT TTH }/8 = 3/8
2	{ HHT HTH THH }/8 = 3/8
3	{ HHH }/8 = 1/8

You can see from the table that X takes on various values with different

You can see from the table that X takes on various values with different probabilities. The probability varies depending on how often the desired outcome occurs in the entire sample space. In our example, head occurring exactly once or twice happens more often than head not occurring at all, thus the corresponding probabilities are also higher.

If we plot the above table as below, it'll represent the probability distribution for the random variable X.



Now we'll contrast the coin flip example with the die example. The random variable Y was defined as the die showing a value greater than 2 on a roll.

Values Y can take on	Probability of Y taking on value in first column
1	0
2	0
3	{ 3 }/6 = 1/6
4	{ 4 }/6 = 1/6

5	{ 5 }⁄6 = 1⁄6
6	{ 6 }⁄6 = 1⁄6

Note that we have listed probabilities for 1 and 2 as 0 since, by definition, Y can't take on those values. In contrast to X, one can see that Y has the same probability for the different values it can legally take on. This is an example of a *uniform probability distribution*.