

Q.1

a) Aggregate Method

The aggregate method calculates the total cost of series of operations & divides by no. of operations to find average.

- Suppose we have dynamic array that starts with a capacity of 2^0
- Each time we insert an element & array is full, the array size doubles.

To analyze cost,

- Single insertions: Inserting an element cost $O(1)$ unless resizing is required
- Resizing: Each time resizing happens, all elements need to be copied to new array so the cost is $O(K)$.

\therefore The total cost of inserting n elements

is $\log n$

$$T(n) \sum_{i=0}^{\log n} O(2^i) = O(n)$$

Thus the amortized cost per insertion is $O(1)$

b) Accounting method.

Here we assign credit to each operation, which covers not only immediate cost but also my future costs associated with operation.

for each operation, we charge cost of
3 units,

so units perform actual insertion
units to 'save-up' for future
resizing operations

When a resize operation occurs, the
saved units cover the copying cost
- Doubling from 1C to 2K requires copying
1C elements, which will be covered
by previously saved credits

Since each insertion is charged
a constant cost of 3 units,
the amortized cost per insertion
remains $O(1)$

$$(1) \alpha = (1) \alpha + 3 \quad (1) T$$

(1) α

bottom picture

and 2nd statement was for several direct
after bottom 2nd reading from odd