Do Sparsity Promoting Hierarchical Prior Models Plausibly Model Empirical Image Data?

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The Prior Model

- Conditionally Gaussian prior for x: $x|\theta \sim \mathcal{N}(0, D_\theta),\ D_\theta = \mathrm{diag}(\theta_1, \dots, \theta_n)$
- Where each θ_j is drawn i.i.d. from $\mathrm{GenGamma}(r_j, \beta_j, \vartheta_j)$
- With $\eta = r\beta - \frac{3}{2}$, r and η are shape parameters that control tail decay rate and peak behavior respectively

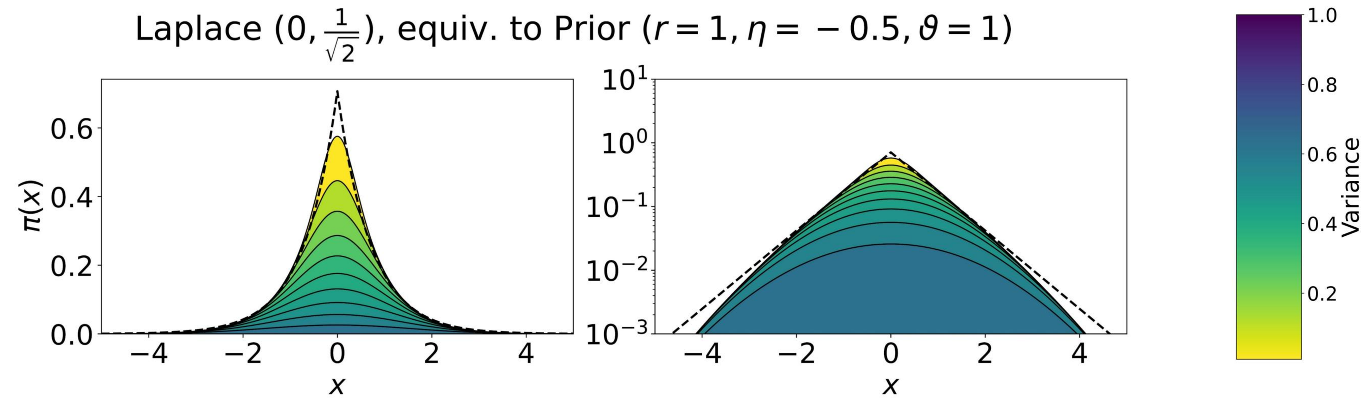


Fig. 1. Stylized diagram of constructing Laplace prior by stacking normal distributions with variances drawn from GenGamma hyperprior

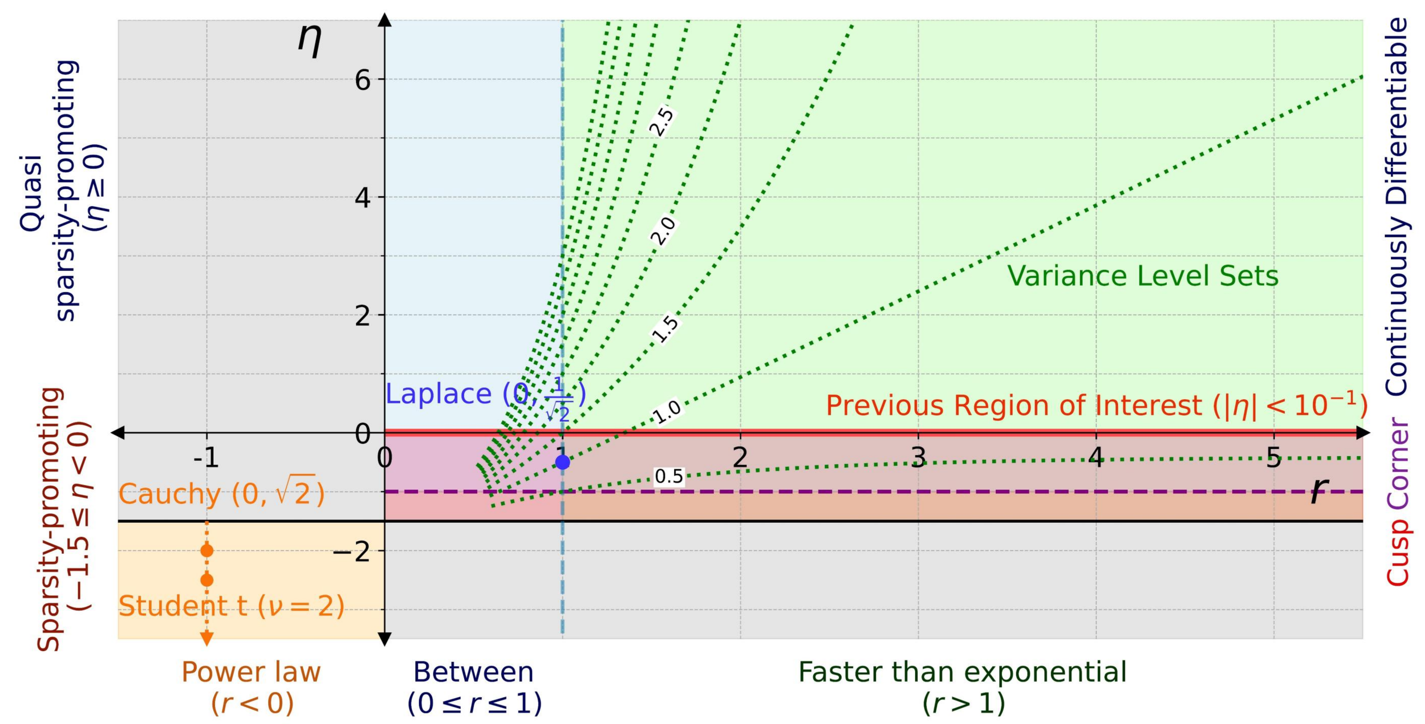


Fig. 2. Parameter space map depicting how the prior model relates to common sparsity-promoting priors and how the shape parameters r, η influence the regularizing effects of the prior. The previous region of interest (ROI) represents where the regularizing effect of the prior is equivalent to ℓ^p regularization, for appropriately picked p between 0 and 2.

Research Question

Ill-conditioned inverse problems occur frequently in imaging, wherein estimation based on likelihood alone is unstable and imprecise. While generalized gamma scale mixtures of normals have attracted interest in the computational imaging community for their generalization of standard approaches and scalable inferential algorithms (MAP estimation, uncertainty quantification), there is little work demonstrating their fit to real data. We study the following:

1. **Is the prior model realistic?**
   - To what extent does this prior model empirical datasets for any choice of parameters (r, η, ϑ)
2. **If so, where do the parameters lie?**
   - Are previously studied parameter ranges ($|\eta| < 10^{-1}$) realistic?
3. **And, under what representations do these results hold?**
   - Fourier, Wavelet, Learned Filters

Data

Name	# Images	Description
pastis	1590	Panoptic Agricultural Satellite Time Series, farm fields in France
agriVision	4500	Agricultural Vision, farmlands in US
spaceNet	3401	Multi-Sensor All Weather Mapping Dataset, centered in Rotterdam
coco	4050	**C**ommon **O**bjects in **Co**ntext, split into indoor / outdoor
segmentAnything	7072	segmentAnything dataset by Meta
syntheticMRI2D	3000	2D synthetic MRI brain images (axial / coronal / sagittal slices)
syntheticMRI3D	100	3D synthetic MRI brain images

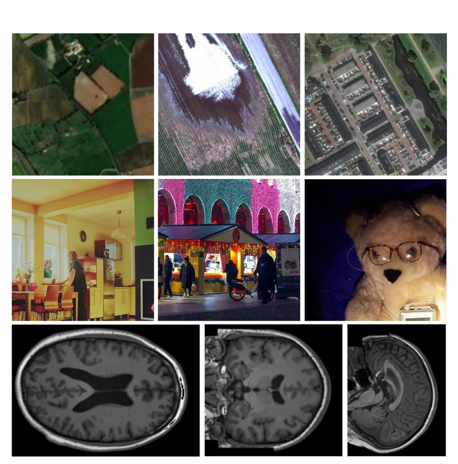


Fig. 3. Sample images

Normalized images before transform, number of coefficients ranged from $10^3 - 10^9$.

Data Augmentation

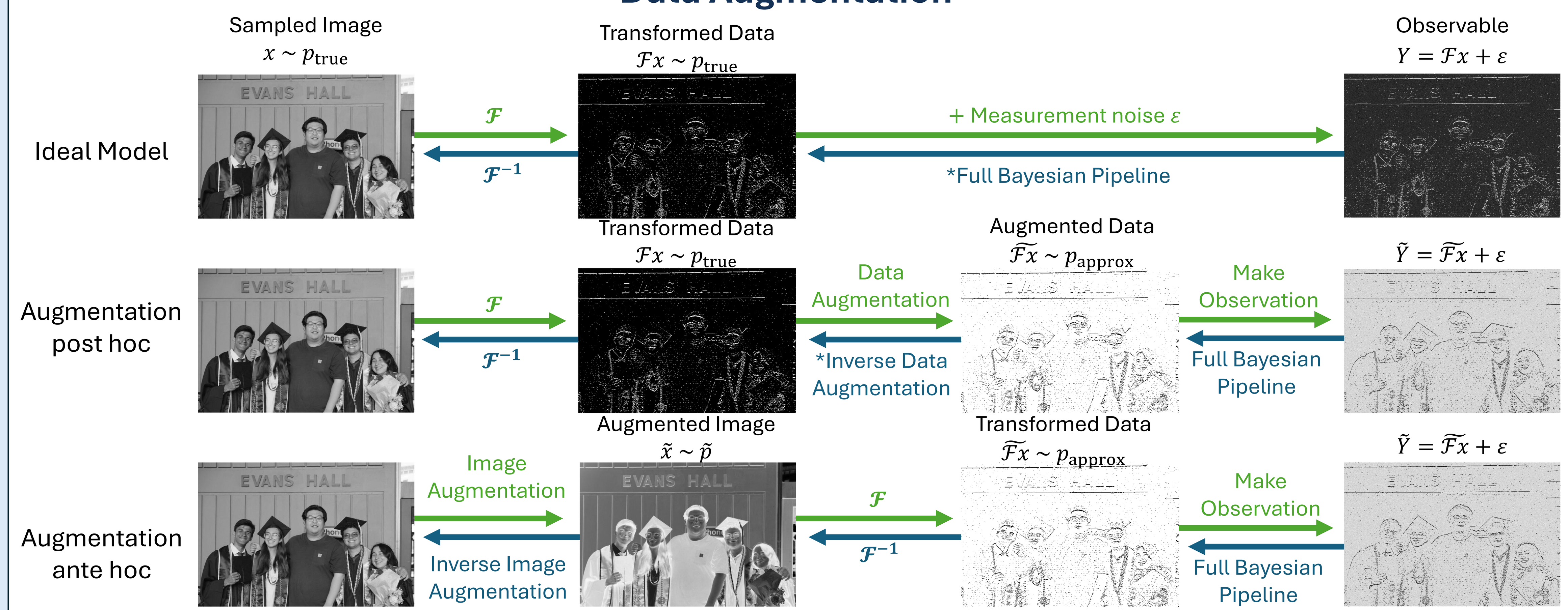


Fig. 4. Data augmentation model justifying groupings of coefficients into blocks (layers/bands)

Testing Procedure

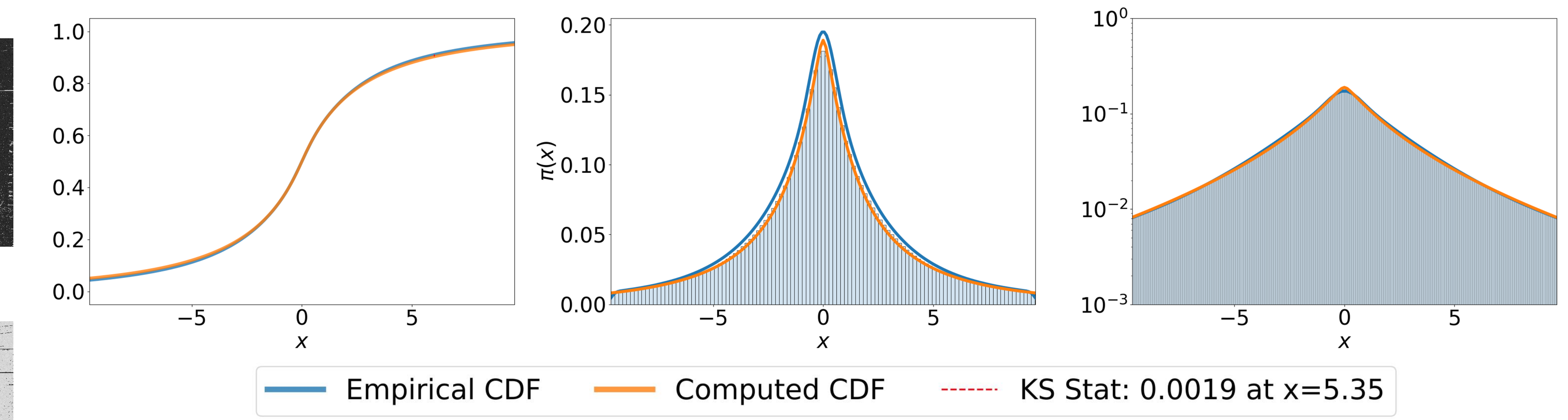
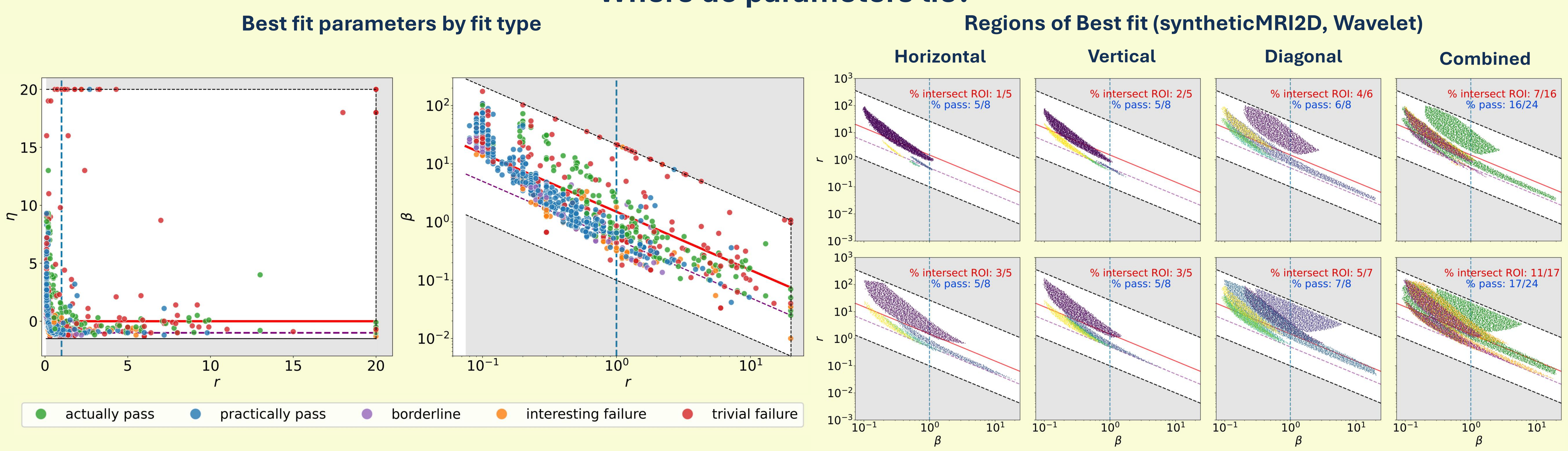


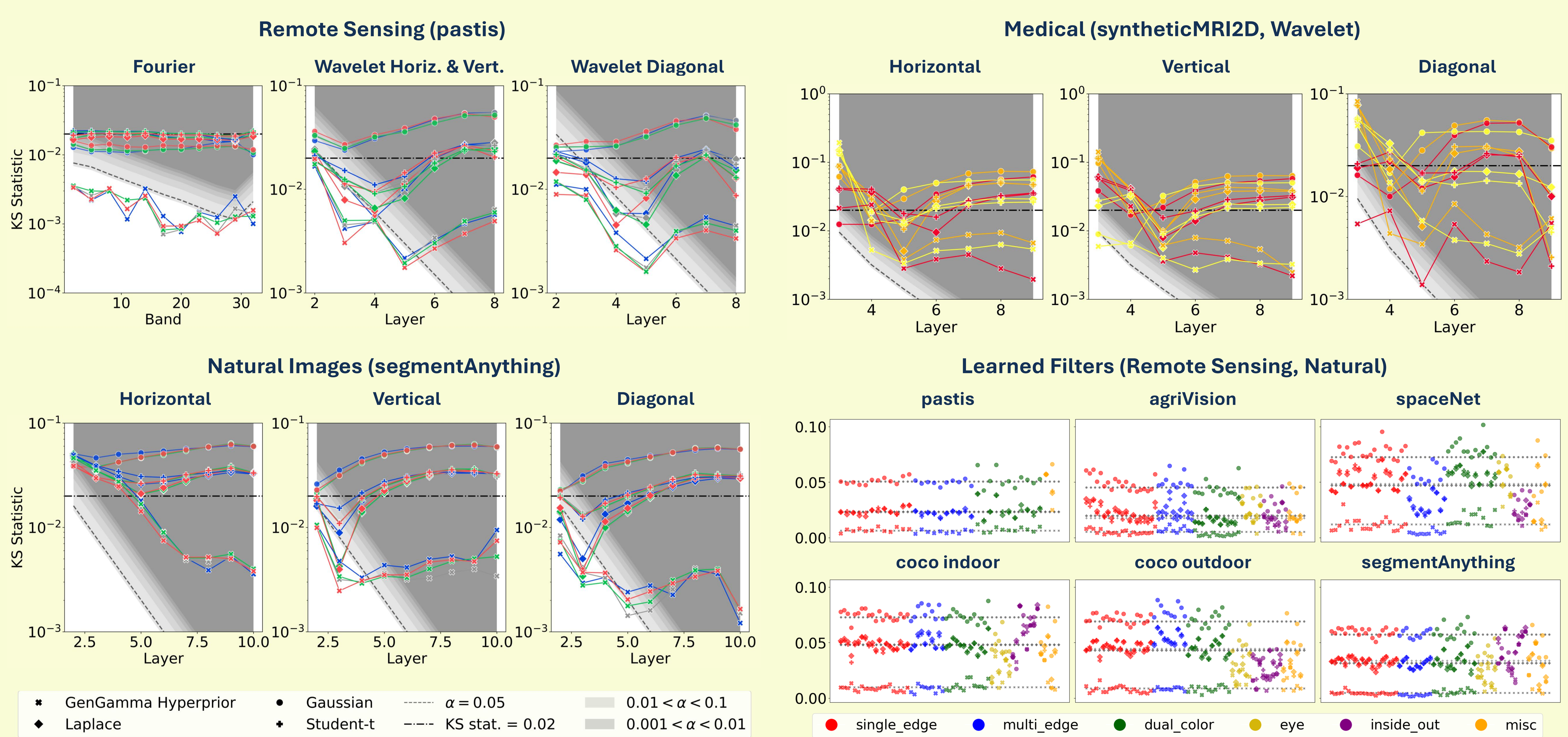
Fig. 5. Example fit categorized as practically pass

- Perform KS Test using the empirical CDF and $f(x; r, \eta, \vartheta)$
- Best-fit $r^*, \eta^* := \underset{r,\eta}{\mathrm{argmin}} \sup_x |F_{\text{empirical}}(x) - F(x; r, \eta)|$
- Perform grid search over (r, η) and local optimization to identify best-fit parameters
- Analyze fits individually to categorize fits as:
  1. Statistically significant (pass)
  2. Practically significant (pass)
  3. Borderline fits
  4. Nontrivial failure
  5. Trivial failures

Where do parameters lie?



Is the model realistic? And under what representations?



Conclusions

Transform (median # samples)	Dataset	% Actually Pass	% Practically pass	% Intersect Region Of Interest
Wavelet (5.8e5)	pastis	57.1	42.9	96.4
	agriVision	13.9	86.1	73.6
	spaceNet	4.7	48.4	46.9
	Remote sensing	**23.4**	**60.9**	**71.4**
	coco indoor	25	25	33.3
	coco outdoor	25	28.1	20.8
	segmentAnything	25	60.2	36.1
	Natural	**25.0**	**38.7**	**30.3**
	syntheticMRI2D	2.8	66.7	47.2
	syntheticMRI3D	6.1	53.1	57.1
	Medical	**4.1**	**61.2**	**51.2**
Fourier (5.5e5)	pastis	90.9	9.1	100
	agriVision	2.3	93.2	70.5
	spaceNet	0	100	50
	Remote sensing	**34.2**	**64.2**	**75.8**
Learned (4.8e8)	pastis	0	85.7	74.3
	agriVision	0	78.3	53.3
	spaceNet	0	42.9	46.4
	Remote sensing	**0.0**	**66.9**	**55.6**
	coco indoor	0	56.5	40.3
	coco outdoor	0	59.4	26.6
	segmentAnything	0	73.4	50
	Natural	**0.0**	**63.2**	**38.9**
Overall		**15.5**	**56.3**	**50.2**

- The prior model fits **71.8%** (76.6% incl. borderline fits) of the 1074 coefficient blocks tested, across all datasets and representations.
- The **median KS statistic per dataset-transform** ranges from **0.004 to 0.012** indicative of high-quality fits.
- Best-fit regions include both the top left quadrant (smooth tops, sub-exponential tail decay) and bottom right quadrant (corner or cusps with faster than exponential tail decay).
- Dataset type is the primary driver of performance, with remote sensing > medical > natural image sets

Next Steps

- Extending the established empirical framework to different parameter regimes (e.g. power law tails), different representations (Gabor).
- Studying the stability of posterior inference to model misspecification (within and outside the family of priors).

References