Module 3 : Satistical Inference 1

Samples : with replacement → N^n ways
without replacement → ${}^N C_n$ ways

$\{1,2,3,4,5\}$ → $N=5$ $n=2$ (in example)
↳ population   ↳ taking 2 entries from population

* without replacement : (1,2), (1,3), (1,4), (1,5), (2,3), (2,4), (2,5), (3,4), (3,5), (4,5)

$\mu_{\bar{x}}$ → mean of samples
$\sigma^2_{\bar{x}}$ → variance of ~~popula~~ samples
$\sigma_{\bar{x}}$ → SD of samples
μ → mean of population σ^2 → variance of population
σ → SD of population

$\{1,2,3,4,5\}$: $\mu = 3$ $\sigma^2 = 2$ $\sigma = \sqrt{2}$

* ~~μ~~ 1.5, 2, 2.5, 3, 2.5, 3, 3.5, 3.5, 4, 4.5 → mean of each entries
(1,2) (1,3) (1,4) (1,5) (2,3) (2,4) (2,5) (3,4) (3,5) (4,5)

$$\mu_{\bar{x}} = \frac{1.5+2+2.5(2)+3(2)+3.5(2)+4(1)+4.5(1)}{10}$$

(1) → no. of times it occurs.
10 → no. of entries

→ $\mu_{\bar{x}} = 3$

$$\sigma^2_{\bar{x}} = \frac{(-1.5)^2+(0.5)^2+(1.5)^2+(-1)^2+0^2+(-0.5)^2+(0.5)^2+0^2+(0.5)^2+(1)^2}{10}$$

$\sigma^2_{\bar{x}} = 0.75$ → $\sigma_{\bar{x}} = \sqrt{0.75}$

$\bar{x}_i$	f_i	$\bar{x}_i f_i$	$(\bar{x}_i - \mu_{\bar{x}})^2$	$(\bar{x}_i - \mu_{\bar{x}})^2 f_i$
1.5	1	1.5	2.25	2.25
2	1	2	1	1
2.5	2	5	0.25	0.5
3	2	6	0	0
3.5	2	7	0.25	0.5
4	1	4	1	1
4.5	1	4.5	2.25	2.25
	10	30		7.5

$$\mu_{\bar{x}} = \frac{\Sigma \bar{x}_i f_i}{\Sigma f_i} = \frac{30}{10} = 3$$

$$\sigma_{\bar{x}}^2 = \frac{(\bar{x}_i - \mu_{\bar{x}})^2 f_i}{\Sigma f_i} = \frac{7.5}{10} = 0.75$$

	With replacement	Without replacement
1	N^n ways	${}^N C_n$ ways
2	$\mu = \mu_{\bar{x}}$	$\mu = \mu_{\bar{x}}$
3	$\frac{\sigma^2}{n} = \sigma_{\bar{x}}^2$	$\left(\frac{N-n}{N-1}\right)\frac{\sigma^2}{n} = \sigma_{\bar{x}}^2$

3-1-24 Q. A population consists of 5 no.s {2,3,6,8,11}. Consider all possible samples of size 2 that can be drawn with replacement from this population.

i. Find μ & σ (population)

ii. Find the $\mu_{\bar{x}}$ & $\sigma_{\bar{x}}$ (sample means)

iii. Considering samples without replacement, find $\mu_{\bar{x}}$ & $\sigma_{\bar{x}}$

iv. Verify $\mu = \mu_{\bar{x}}$ & $\sigma_{\bar{x}}^2 = \sigma^2/n$ (with replacement)

v. Verify $\mu = \mu_{\bar{x}}$ & $\sigma_{\bar{x}}^2 = \left(\frac{N-n}{N-1}\right)\frac{\sigma^2}{n}$ (without replacement)

Ans. population : {2, 3, 6, 8, 11} , N = 5 & n = 2

i. μ & σ

$$\mu = \frac{2+3+6+8+11}{5} = \frac{30}{5} = 6$$

$$\sigma^2 = \frac{16+9+0+4+25}{5} = \frac{54}{5} = 10.8 \rightarrow \sigma = \sqrt{10.8} = 3.28$$

ii. With replacement : N^n ways $= 5^2 = 25$ samples

$$\mu_{\bar{x}} = \mu = 6$$

$$\sigma_{\bar{x}}^2 = \frac{\sigma^2}{n} = \frac{10.8}{2} = 5.4 \rightarrow \sigma_{\bar{x}} = 2.32$$

iii. Without replacement : ${}^NC_n$ ways $= {}^5C_2 = 10$ samples

$$\frac{N!}{(N-n)!\,n!}$$

$$\mu_{\bar{x}} = \mu = 6$$

$$\sigma_{\bar{x}}^2 = \left(\frac{N-n}{N-1}\right)\frac{\sigma^2}{n} = \left(\frac{5-2}{5-1}\right)\frac{10.8}{2} = \frac{3}{4}(5.4) = 4.05$$

$$\sigma_{\bar{x}} = \sqrt{4.05} = 2.012$$

iv. 10 samples : {2,3}, {2,6}, {2,8}, {2,11}, {3,6}, {3,8}, {3,11}, {6,8}, {6,11}, {8,11}

$$\mu_{\bar{x}} = \frac{2.5+4+5+6.5+4.5+5.5+7+7+8.5+9.5}{10} = \frac{60}{10} = 6$$

$$\sigma_{\bar{x}}^2 = \frac{12.25+4+1+0.25+2.25+0.25+1+1+6.25+12.25}{10} = \frac{40.5}{10} = 4.05$$

→ from iii. $\mu_{\bar{x}} = 6 = \mu$

$$\sigma_{\bar{x}}^2 = 4.05 \rightarrow \frac{\sigma^2}{n} \cancel{\frac{10.8}{2}}\left(\frac{N-n}{N-1}\right) = \frac{10.8}{2}\left(\frac{3}{4}\right) = 4.05$$

iv. Table banao aur karo ü

Q. Certain cubes manu. by company have mean life of 800 hrs & SD of 60 hrs. Find probability that a random sample with replacement of 16 cubes will have a mean lifetime

i. b/w 790 & 810 hrs

ii. less than 785 hrs

iii. more than 820 hrs

iv. b/w 770 & 880 hrs

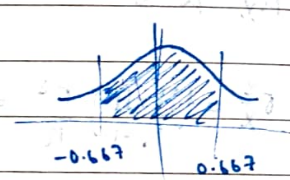
Ans $\mu = 800$ hrs $\quad \sigma = 60$ hrs $\quad n = 16$

$\mu_{\bar{x}} = 800$. $\quad \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{60}{4} = 15$ $\quad z = \frac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}} = \cancel{16}$ -

i. $P(790 < \bar{x} < 810)$

$\bar{x} = 790 \Rightarrow z_1 = \frac{790-800}{15} = \frac{-10}{15} = -0.667$

$\bar{x} = 810 \Rightarrow z_2 = \frac{810-800}{15} = \frac{10}{15} = 0.667$

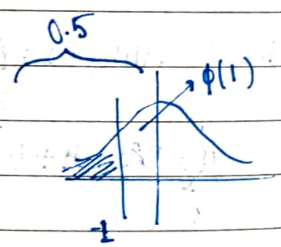


$\therefore P(-0.667 < z < 0.667) = 2\phi(0.667)$

$= 2(0.2486) = \underline{\underline{0.4972}}$

ii. $P(\bar{x} < 785)$

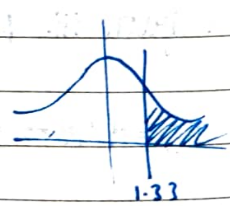
$\bar{x} = 785 \rightarrow z_1 = \frac{785-800}{15} = \frac{-15}{15} = -1$



$P(z < -1) = 0.5 - \phi(1) = 0.5 - 0.3413 = \underline{\underline{0.1587}}$

iii. $P(\bar{x} > 820)$

$\bar{x} = 820 \Rightarrow z = \frac{820-800}{15} = \frac{20}{15} = 1.334$



$P(z > 1.334) = 0.5 - \phi(1.33) = 0.4082$

$= \underline{\underline{0.0918}}$

iv. $P(770 < \bar{x} < 830)$

$\bar{x} = 770 \Rightarrow z_1 = \frac{770-800}{15} = -2$

$\bar{x} = 830 \Rightarrow z_2 = \frac{830-800}{15} = 2$

$P(-2<z<2) = 2\phi(2) = 2(0.4772) = \underline{0.9544}$

Q. The weights of 1500 ball bearings are distributed with $\mu = 635\,g$ & $\sigma = 1.36\,g$. If 300 samples of size 36 are drawn from this ~~If~~ ~~8~~ population.

i. Find $\mu_{\bar{x}}$ & $\sigma_{\bar{x}}$ if sampling is done WR & WoR.

ii. If sampling is done WR, find how many random samples would have their mean:

ⓐ b/w 634.76 & 635.24 g

ⓑ greater than 635.5 g

ⓒ less than 634.2 g.

Ans. $N = 1500 \quad \mu = 635\,g \quad \sigma = 1.36\,g, \quad n = 36$

i. WR: $\mu_{\bar{x}} = \mu = 635\,g$

$$\sigma^2_{\bar{x}} = \frac{\sigma^2}{n} = \frac{1.36^2}{36} = \frac{1.8496}{36} = 0.05137$$

$$\sigma_{\bar{x}} = 0.226$$

WoR: $\mu_{\bar{x}} = \mu = 635$

$$\sigma^2_{\bar{x}} = \left(\frac{N-n}{N-1}\right)\frac{\sigma^2}{n} = \left(\frac{1500-36}{1500-1}\right)\frac{1.8496}{36} = \underline{\underline{0.224}}$$

ii. ⓐ $P(634.76 < \bar{x} < 635.24)$

$$\bar{x} = 634.76 \Rightarrow z_1 = \frac{634.76-635}{\cancel{1.36}\ 0.226} = \frac{-0.24}{\cancel{1.36}\ 0.226} = \cancel{-0.176}\ \cancel{-}-1.06$$

$$\bar{x} = 635.24 \Rightarrow z_2 = \frac{635.24-635}{\cancel{1.36}\ 0.226} = \cancel{0.176}\ 1.06$$

$$\therefore P(-0.176 < z < 0.176) = 2\phi(\overset{1.06}{\cancel{0.176}}) = 2 \times 0.3554$$
$$= 0.7108$$

$\therefore$ No. of samples $= 300 \times 0.7108 = \underline{\underline{213}}$

ⓑ $P(\bar{x} > 635.5)$

$$\bar{x} = 635.5 \Rightarrow z = \frac{635.5-635}{\cancel{1.36}\ 0.226} = \cancel{0.367}\ 2.212$$

$\therefore P(z>2.212)=0.5-\phi(2.212)$

$\therefore$ No. of samples $=300x$

(c) $P(\bar{x}<634.2)$

$\bar{x}=634.2 \Rightarrow z_1=\frac{634.2-635}{0.226}=-3.54$

$P(z<-3.54)=0.5-\phi(3.54)$

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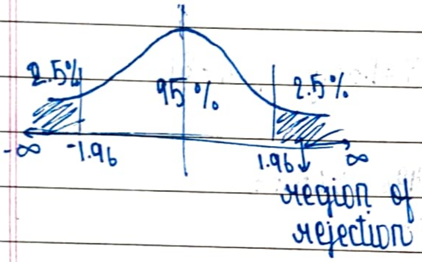
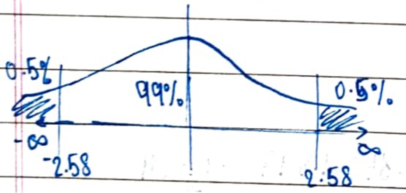
Level of Significance



95% of area under bell curve

5% : level of significance

$z_{0.05}$



99%

1% level of significance

$z_{0.01}$

$|z|\le 1.96 \Rightarrow -1.96\le z\le 1.96$

$\Rightarrow -1.96\le \frac{\bar{x}-\mu}{\sigma_{\bar{x}}}\le 1.96 \Rightarrow -1.96\le \frac{\bar{x}-\mu}{\sigma/\sqrt{n}}\le 1.96$

$\Rightarrow (-1.96)\frac{\sigma}{\sqrt{n}}\le \bar{x}-\mu\le (1.96)\frac{\sigma}{\sqrt{n}}$

$\Rightarrow (-1.96)\frac{\sigma}{\sqrt{n}}-\bar{x}\le -\mu\le \frac{\sigma}{\sqrt{n}}(1.96)-\bar{x}$

$\Rightarrow (1.96)\frac{\sigma}{\sqrt{n}}+\bar{x}\ge \mu\ge \frac{\sigma}{\sqrt{n}}(-1.96)+\bar{x}$

$-1.96\frac{\sigma}{\sqrt{n}}+\bar{x}\le \mu\le 1.96\frac{\sigma}{\sqrt{n}}+\bar{x} \Rightarrow \boxed{\mu=\pm 1.96\frac{\sigma}{\sqrt{n}}+\bar{x}}$

$\hookrightarrow$ confidence limits.

Q. A random sample of 400 items chosen from an infinite pop. is found to have $\bar{x} = 82$ & $\sigma_{\bar{x}} = 18$. Find 95% confidence limits of mean of population. Also find 99% confidence limits

$\bar{x}$ = sample mean (mean of one of the samples)

Ans. $\cancel{\mu_{\bar{x}} = \mu = 82}$ $\quad \sigma_{\bar{x}} = 18 = \sigma \quad n = 400 \quad \bar{x} = 82$

sample of mean = $\mu_{\bar{x}}$ (mean of all the means of samples)

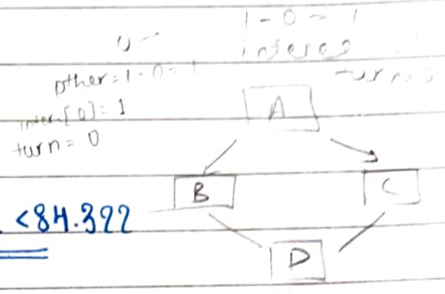
$\mu = \pm 1.96\left(\frac{18}{\sqrt{400}}\right) + 82$

$\Rightarrow \mu = 80.236$ or $83.764 \Rightarrow 80.236 < \mu \leq 83.764$

99% :

$\mu = \pm 2.58\left(\frac{18}{\sqrt{400}}\right) + 82$

$\mu = 79.678$ or $84.322 \Rightarrow 79.678 < \mu < 84.322$



Q. The life of a certain computer is approx. normally dist. with mean 800 hrs & SD of 40 hrs. If a random sample of 30 computers has an avg life 788 hrs. Test the hypothesis that avg life is 800 hrs at 5% LOS & 1% LOS.

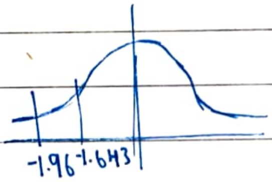
Ans. $\mu = 800 \quad \sigma = 40 = \sigma_{\bar{x}} \quad \bar{x} = 788 \quad n = 30$

5% LOS : $\qquad$ 1%

$\mu = \pm 1.96\left(\frac{40}{\sqrt{30}}\right) + 788$ $\qquad$ $\mu = \pm 2.58\left(\frac{40}{\sqrt{30}}\right) + 788$

$769.15 < \mu < 806.84$

$\Rightarrow \mu = 802.313$ or 773.686



$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{788 - 800}{40/\sqrt{30}} = -1.643$

95% accept and 5% reject

99% accept and 1% reject