HW3: Energy Transport and Storage

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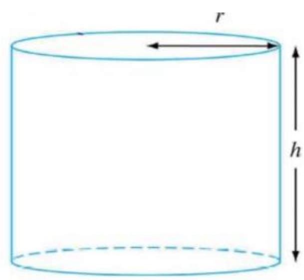
Q1]



Assuming that the cylinder is a solid cylinder

Outer area of cylinder, $A = 2\pi rh + 2\pi r^2$

Volume of cylinder, $V = \pi r^2 h$

Volume of the cylinder is fixed and the area is to be maximized. Formulating the optimization problem:

$$max_{r,h} 2\pi rh + 2\pi r^2$$

$$s.t. \pi r^2 h = V$$

$$r > 0\,; h > 0$$

Q2]

Defined variables:

x_1, x_2, x_3 : Time spent on course 1, 2 and 3 respectively (in hours)

g_1, g_2, g_3: Grades obtained in course 1, 2 and 3 respectively

It is also observed that after a particular time spent for each course, the grades are maximum.

From the given plots for Grade vs Time spent, equations of the straight lines can be found out as:

Course Number	Maximum time to be spent (hrs)	Points on the line	Equation of the line
1	6	(0,20) (6,100)	$g_1 = \frac{40}{3} x_1 + 20$
2	7	(0,40) (7,100)	$g_2 = \frac{60}{7} x_2 + 40$
3	8	(0,15) (8,100)	$g_3 = \frac{85}{8} x_3 + 15$

Based on these constraints, the point is to maximize the total grade, $g_1 + g_2 + g_3$

The optimization model has been set up as follows:

$$\max_{e_1,e_2,e_3,g_1,g_2,g_3 \leftarrow R} g_1 + g_2 + g_3$$

$$s.t.\ \ x_1 + x_2 + x_3 \leq 15$$

$$x_1 \leq 6; x_2 \leq 7\ ; x_3 \leq 8; x_1 \geq 0; x_2 \geq 0; x_3 \geq 0$$

$$g_1 = \frac{40}{3} x_1 + 20$$

$$g_2 = \frac{60}{7} x_2 + 40$$

$$g_3 = \frac{85}{8} x_3 + 15$$

Q3]

Given:

N: Number of buildings

P: Rebate per kWh, C: Incentive to building manager per kWh

Y: Minimum of demand response in total

Z_n: Reduction in energy consumption per minute for a building (kWh/min)

T: Maximum time the lights can be dimmed (min)

Defined variables:

$d_1, d_2, \ldots d_N$: Demand response from each building per day from (1-N) in kWh

$t_1, t_2, \ldots t_N$: Time the lights are dimmed for each building per day (in Mins)

Based on this, there are 4 constraints:

1) Maximum time light can be dimmed per building
2) Demand response per building as a function of time
3) Positivity of time and demand response
4) Demand storage minimum threshold

$$\max_{d_1,d_2 \ldots d_N,t_1,t_2 \ldots t_N \leftarrow R} \sum_{i=1}^{N} (P - C) d_i$$

$$s.t.\, t_i \leq T\ \forall\ i \leftarrow (1{,}2 \ldots, N)$$

$$d_i = Z_i * t_i \forall\ i \leftarrow (1{,}2 \ldots, N)$$

$$t_i \geq 0; d_i \geq 0\ \forall\ i \leftarrow (1{,}2 \ldots, N)$$

$$\sum_{i=1}^{N} d_i \geq Y$$

Q4]

Variables:

n_T: Number of tuna sandwiches made per day

n_C: Number of chicken sandwiches made per day

Constraints:

Profit, $P = 3.5n_T + 3n_C$

Time taken to make sandwiches: $t_T = 8n_T$; $t_C = 6n_C$

Total time taken, $8n_T + 6n_C \leq 180$ (Maximum time he can spend is 3 hours)

Budget: $4n_T + 6n_C \leq 120$

He has to make minimum of 5 sandwiches of each

The developed optimization model:

$$\max_{n_T, n_C \leftarrow R} 3.5n_T + 3n_C$$

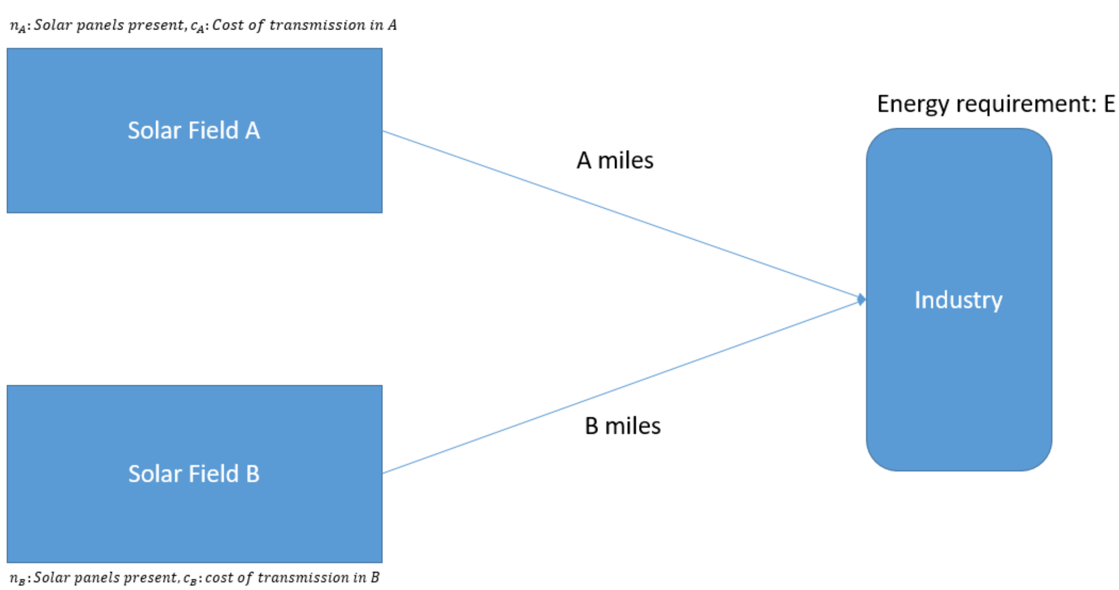
$$s.t. \, 4n_T + 3n_C \leq 90$$

$$2n_T + 3n_C \leq 60$$

$$n_T \geq 5 \, ; n_C \geq 5$$

Q5]

a. Energy source chosen here is **Solar**. It is assumed that there are two solar fields in an area, both of which can provide power to a small scale industry nearby.



b. The optimization problem at hand is to minimize the total cost of electricity, while satisfying the demand for the industry.

Variables: e_A, e_B: Amount of energy sourced from A and B respectively (kWh)

Objective function: $\min_{e_A,e_B} e_A c_A + e_B c_B$ $where$ c_A, c_B is the cost of transmission for A and B per kWh.

Transport limitations:

1. The total supply from A and B should satisfy the energy requirement of the industry

$e_A + e_B = E$

2. Suppliers from A and B will only supply energy to the plant if a minimum order is satisfied

$e_A \geq m_A$; $e_B \geq m_B$ where m_A, m_B are the minimum quantity to be placed.

3. Cost of transmitting electricity is also born by the industry, which incurs an additional cost per mile, which changes the objective function:

$\min_{e_A,e_B} e_A c_A + e_B c_B + A * e_A * x_A + B * e_B * x_B$ where A, B are the distances and , x_A, x_B are the additional cost per mile per kWh transmitted.

4. Suppliers have a maximum capacity (which might exceed total energy requirement of the industry as well).

$e_A \leq E_{A,\max}$, $e_B \leq E_{B,\max}$

c. Incorporating energy storage can further help with the cost minimization.

Assume that the energy which is transported to the industry has a pretty high average cost per kWh, which can be calculated as: $c_{avg} = \frac{e_A c_A + e_B c_B}{E}$. In order to further lower the cost of the electricity to the industry, the industry can plan to use the limited area available to it to store the incident solar energy, which would reduce the demand from the external solar farms. Assume that the number of storage devices (Batteries, Powerwalls, thermal storage, etc.) are in enough quantity to store the incident energy at the industry, and has an average payback period of 10 years due the saved cost of buying electricity externally.

$C_{storage} = 10 * 365 * c_{day}$

Let $E_{industry}$ be the energy generated by the industry itself. This reduces the demand from external sources by a factor. Due to this, the overall objective function reduces. Assume that $e_{A,1}, e_{B,1}$are the optimum solution without storage. Thus, the new objective function is:

$\min_{e_A,e_B} e_{A,new} c_A + e_{B,new} c_B + A * e_{A,new} * x_A + B * e_{B,new} * x_B$

Two additional constraints can be added: $e_{A,new} < e_{A,1}$; $e_{B,new} < e_{B,1}$

Thus, adding energy storage definitely leads to cost minimization in the longer run and would yield further savings post the payback period.

d.

Objective function is: $f(e_A, e_B) = (c_A + Ax_A)e_A + (c_B + Bx_B)e_B$

The function can be restated as: $f(e_A, e_B) = c_1 e_A + c_2 e_B$ **,** which is a linear function in each variable.

Calculating the hessian:

All the 4 second derivatives in the Hessian matrix will be zero, as the maximum power of the variables is one. Thus, $\nabla^2 f(e_1, e_2) = 0$

Thus, the objective function is a **convex function and has a global minimum.**