Process Systems Modeling

Homework II

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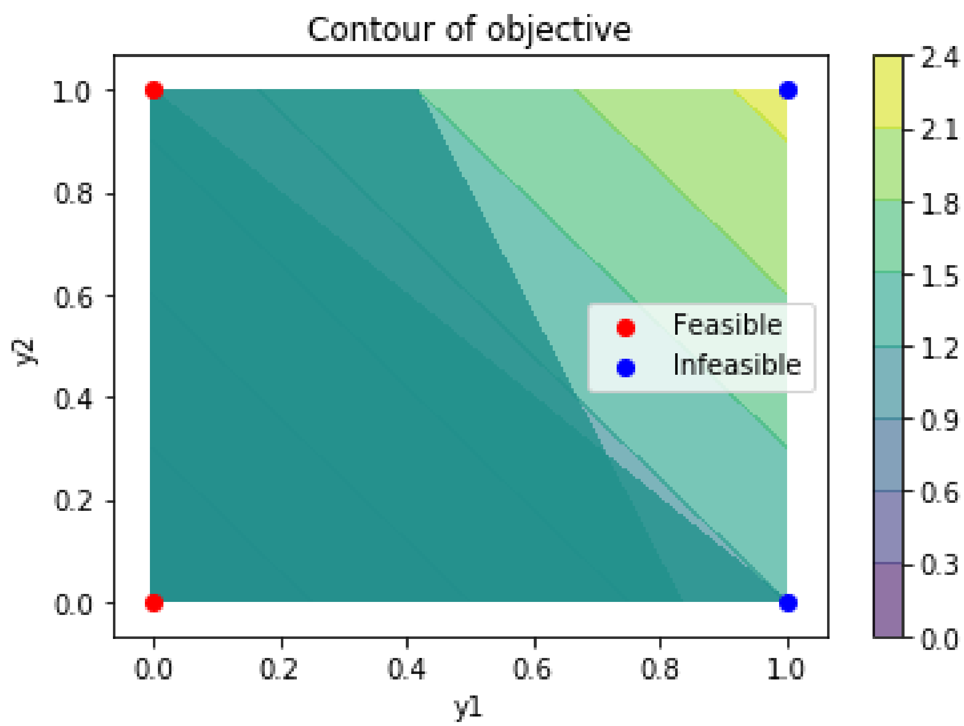
Answer 1:

a)

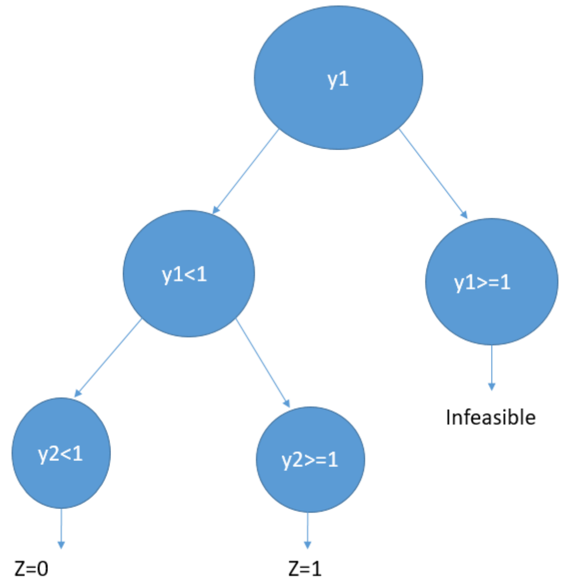


The contour plot for the objective and feasible region has been plotted. From the trend of contours, we can observe that there is a linear increase in the objective value as we move towards the right. The green shaded region indicates the intersection of the constraints with the objective. This is the feasible region of the relaxed binary variables to continuous variables.

b) From the relaxed LP, it can be observed that the point, y1=0 and y2=1 serves as the feasible solution based on the contour plot. The optimum however can be observed roughly around (5/7 , 2/7), which is the intersection of the two lines based on the contour plot. Thus, the optimum solution of the relaxed LP is 1.143 (however integer infeasible).

c) The 4 solutions to the integer program have been incorporated in the plot. There are 2 feasible solutions, (0, 1) and (0, 0), whereas two other options (1, 1) and (1, 0) violate the constraints. Thus, there are 2 feasible solutions in total.

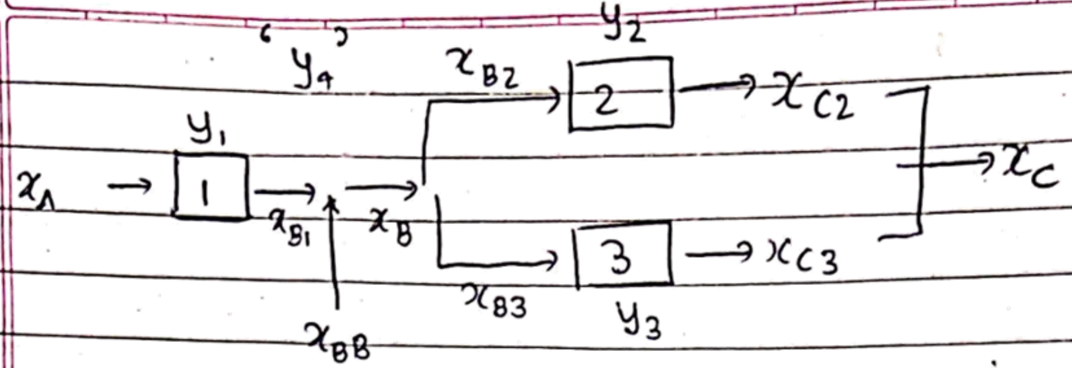
d)



The branch and bound algorithm was implemented for the two attributes, getting an optimum solution at y_2=1 and y_1=0. The optimum solution was obtained through the Gams software and thus reported.

(ysg_l.gms, ysg_r.gms, ysg_ll.gms, ysg_lr.gms)

Q2]



(a) Constraints are:

max $1800 \times x_C - (1000y_1 + 250x_A + 1500y_2 + 400x_{b2} + 2000y_3 + 550x_{b3} + 500x_A + 950x_{bb})$

s.t.

$x_{b1} + x_{bb} = x_b \quad \ldots (1)$

$x_{b2} + x_{b3} = x_b \quad \ldots (2)$

$x_{c2} + x_{c3} = x_c \quad \ldots (3)$

$x_{c2} = 0.82\,x_{b2} \quad \ldots (4)$

$x_{c3} = 0.95\,x_{b3} \quad \ldots (5)$

$y_2 + y_3 = 1 \quad \ldots (6)$

$y_1 + y_4 = 1 \quad \ldots (7)$

$x_c \le 10 \quad \ldots (8)$

$x_{b1} \le 0.9\,x_A \quad \ldots (9)$

$x_A \le 16y_1 \quad \ldots (10)$

$x_{b2} \le 16y_2 \quad \ldots (11)$

$x_{b3} \le 16y_3 \quad \ldots (12)$

Solving the model gave the following output:

$Z_{max} = 459.35$ \$/hr.

Process 2 is built

Chemical B was produced through process 1

Amount of C produced: 10 ton/hr.

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(b) There are a few additional constraints to be incorporated and a modification to objective function.

2 new variables are introduced.

$$x_c = x_{co} + x_s \text{, where } x_s \text{ is the excess.}$$

$$\max \; 1800x_{co} + 1500x_s - (1000y_1 + 250x_a + 1500y_2 + 400x_{b2} + 2000y_3 + 550x_{b3} + 500x_a + 450x_{bb})$$

st:

$x_c \leq 15$ ... Modifying (8) from (a)

$0 \leq x_s \leq 5y_5$

$x_c \leq 10 + 5y_5$

$x_c = x_{co} + x_s$

$x_{co} \leq 10$

$y_5 \in \{0, 1\}$.

The constraints in addition to those in (a) ensure:

$Z_{max} = 600$ \$/hr

Process 3 is built

Chemical B was produced through process 1

Amount of C produced: 13.680 ton/hr.

Answer 3:

a) Maximizing features

Formulated LP is:

Max $\sum_{i=1}^{6} y(i) * g(i)$

st. $\sum_{i=1}^{6} y(i) * c(i) \leq 35000$

Where y(i) are the binary variables for every feature, g(i) is the individual gain for every feature and c(i) is the cost for individual features.

Solution:

Optimal Objective: 25.00 mph

Feature 3 and 5 are to be used for maximizing the gain.

GAMS output:

```
----      37 VARIABLE y.L  decision variables

f3 1.000,    f5 1.000

MIP Solution:          25.000000    (3 iterations, 0 nodes)
Final Solve:           25.000000    (0 iterations)

Best possible:         25.000000
Absolute gap:           0.000000
Relative gap:           0.000000
```

b) Minimizing cost

Formulated LP is:

Min $\sum_{i=1}^{6} y(i) * c(i)$

st. $\sum_{i=1}^{6} y(i) * g(i) \leq 30$

Where y(i) are the binary variables for every feature, g(i) is the individual gain for every feature and c(i) is the cost for individual features.

Solution:

Features 1,3,5 are used.

Optimal Objective: 43k

GAMS Output:

```
----      37 VARIABLE y.L  decision variables

f1 1.000,    f3 1.000,    f5 1.000

MIP Solution:          43.000000    (2 iterations, 0 nodes)
Final Solve:           43.000000    (0 iterations)

Best possible:         39.175000
Absolute gap:           3.825000
Relative gap:           0.088953
```

The .gms files are uploaded for both the cases.

Answer 4:

y(i) refers to a mission selected by NASA. The objective is to maximize the value of the overall missions.

v(i) is the value of a single mission.

c(i,j) is the cost involved for every single mission for a time period

b(j) is the budget constraint for each of the time period

MIP is:

Max $\sum_{i=1}^{14} y(i) * v(i)$

s.t. $\sum_{i=1}^{14} y(i) * c(i,j) \leq b$(j) for all j

Two types of constraints are incorporated: Exclusive constraints, Dependent constraints

They are incorporated in the gams model.

GAMS Output:

```
Solution satisfies tolerances.

MIP Solution:          765.000000
Final Solve:           765.000000

Best possible:         826.250000
Absolute gap:           61.250000
Relative gap:            0.074130


----     81 VARIABLE y.L  Mission selection

m1  1.000,    m3  1.000,    m4  1.000,    m8  1.000,    m13 1.000,    m14 1.000
```