06665: Process Systems Modeling
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Group-5 Project Report

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Replication of:

Global Optimization of Gas Lifting Operations: A Comparative Study of Piecewise Linear Formulations

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Introduction:

The paper 'Global Optimization of Gas Lifting Operations: A Comparative Study of Piecewise Linear Formulations' by Misener et al. implements a classic formulation and three other novel piece wise linear methods to optimize the gas lifting problem.

The process of injecting compressed natural gas, called "lift gas", into the production tubing of an oil well is called continuous gas lifting. The weight of the column of fluid is reduced by the lift gas thereby increasing the pressure gradient between the reservoir and well fluid causing the fluid to be pushed to the surface. The fluid from the well, comprising of oil, gas, and water, is treated to separate water. The oil is sold, and the left over gas is either sold or recycled for use in another gas lifting operation. The optimization problem at hand involves allocation of gas to a field of oil wells using gas requirement curves. The reservoir dynamics are assumed to be constant over the period of a few days. Gas requirement curves are designed to relate gas injection ($q_{GAS,i}$) to oil production ($q_{OIL,i}$). These curves are piecewise linear functions and account for individual well factors such as reservoir pressure, composition, and depth. Our aim is to choose the gas injection level for each well so as to maximize oil production, thereby maximizing the profit.

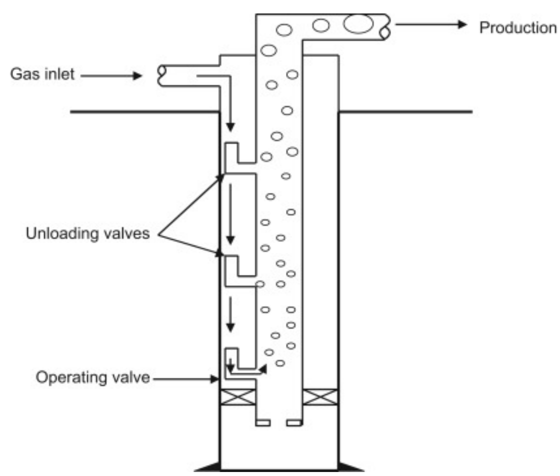

Figure 1: Schematic of gas lifting operation

Literature Review:

Scheduling and planning of oil and gas fields is critical and is termed as an important supervisory operation. The price of the available natural gas and the compression cost of injecting the oil in the well are important parameters to maximize the oil production, and thereby maximize the profit. The development of the models proposed in the paper is a gradual change. Initially, it was assumed that unlimited gas lift conditions were applicable to obtain the optimal profit for each well and after which, the gas requirement curve was computed (Kanu, et al., 1981). Later on, the condition of limited lift gas supply was introduced by Mason et al. (Mason, et al., 2007) and Buitrago et al. (Buitrago, et al., 1996) in order to find the optimal gas supply for a 6 well case, which requires continuously differentiable, concave gas requirement curves. The problem was transformed into a piecewise linear function by Fang and Lo (Fang & Lo, 1996). The referenced paper proposes four models for global optimality which are as follows, Classic Method (Nemhauser & Woolsey, 1988), Linear Segmentation Method (Floudas, 1995), Convex Hull Method (Sherali, 2001) and Special Structure Method (Keha, et al., 2004). All of the following models have been further explained and implemented to derive relevant results. The classic model in the paper was employed by Kosmidis et al.

(Kosmidis, et al., 2004), wherein he integrated the external factors such as pressure drop across tubing and merging of oil well lines into oil well scheduling optimization. The paper under consideration tries to computationally improve the classic model by implementing different piecewise linear algorithms.

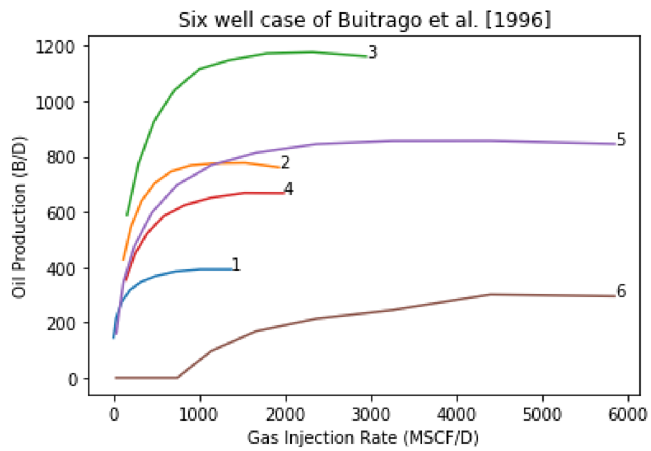

Figure 2: Test Case for Model Evaluation. Each vertex is a pair of (q_{Gas}, q_{OIL})

Problem Formulation Approaches

Terminologies

CM-Classic Method, **LSM**-Linear Segmentation Method, **CHM**-Convex Hull Method, **SSM**-Special Structure Method

Indices	i	Number of wells
	k	Number of vertex points
Data	$q_{GAS,i,k}$	Gas injection Rate (MSCF/Day)
	$q_{OIL,i,k}$	Oil Production rate (B/Day)
	$q_{GAS,LIMIT}$	Availability of Gas (MSCF/Day)
	$Q_{GAS,MIN}$	Lower bound for the gas rate (MSCF/Day)
Variables	$Q_{OIL,\ TOTAL}$	Objective to be maximized
	$\lambda \in [0, 1]^{N_{WELL} N_{VERTICES}}$	Convex combination weight used in CM and SSM
	$\lambda^L \in [0, 1]^{N_{WELL} N_{VERTICES-1}}$	Convex combination weight used in CHM
	$\lambda^R \in [0, 1]^{N_{WELL} N_{VERTICES-1}}$	Convex combination weight used in CHM
	$y \in \{0, 1\}]^{N_{WELL} N_{VERTICES-1}}$	Binary decision variable for CM, LSM, and CHM
	$Q_{GAS} \in R^{N_{WELL} N_{VERTICES-1}}$	Continuous variable used in LSM
	$m \in \{0, 1\}]^{N_{WELL}}$	Binary Variable to determine if well is activated
Parameters	$\alpha_{i,k}$	Intercept of a line segment used as in LSM
	$\beta_{i,k}$	Slope of a line segment used as in LSM
	$Q_{GAS,LIMIT}$	Availability of lift gas
	$Q_{GAS,MIN}$	Lower bound of the gas rate

1) Classic Method

The Classic Method (CM) serves the basis for designing the further 3 models. The basic idea of the classic method is to assign a convex combination weight, $\lambda \in [0,1]$ for every vertex, for each of the wells in consideration. Moreover, binary variable $y \in \{0,1\}$ has been taken into account to restrict the position of nonzero convex combination weights.

The formulated model is as follows:

$$\boldsymbol{max_{y,\lambda} Q_{OIL,TOTAL}} = \sum_{\boldsymbol{i}}^{\boldsymbol{N_{WELLS}}} \sum_{\boldsymbol{k}}^{\boldsymbol{N_{VERTICES}}} \boldsymbol{\lambda_{i,k} q_{OIL,i,k}} \quad (1)$$

$$\lambda_{i,1} \leq y_{i,1} \ \forall \ i = 1,2..N_{WELLS} \ (2)$$

$$\lambda_{i,k} \leq y_{i,k-1} + y_{i,k} \ \forall \ i = 1,2..N_{WELLS} \ and \ \forall \ k = 2..N_{VERTICES-1} \ (3)$$

$$\lambda_{i,N_{VERTICES}} \leq y_{i,N_{VERTICES-1}} \ \forall \ i = 1,2..N_{WELLS} \ (4)$$

$$\sum_{k=1}^{N_{VERTICES-1}} y_{i,k} = 1 \ \ \forall \ i = 1,2..N_{WELLS} \ (5)$$

$$\sum_{i}^{N_{WELLS}} \sum_{k}^{N_{VERTICES}} \lambda_{i,k} q_{GAS,i,k} \leq Q_{GAS,LIMIT} \ (6)$$

Equation (1) serves as the objective function, which we are looking to maximize by applying convex weights for every vertex point. Equations (2-4) limit the positive variables lambda to be less than the binary variable representing that line segment. This ensures that there is only a single entry and exit vertex is activated for every well. Equation (5) implies that for every well under consideration, there is a single segment activated to choose the optimal injection points. Lastly, for practical reasons, the supply of lift gas is limited, an additional constraint has been added through equation (6) for the upper bound of the injected gas.

2) Linear Segmentation Method

The Linear Segmentation Method (LSM) is an update to the classic model. LSM also uses a binary variable, y to restrict each well to a single entry and exit injection point, however, in this case, it introduces a new variable Q_{GAS} for every vertex. The striking difference between CM and LSM is that, in this case, the objective function is composed of linear functions with a slope, β and intercept, α.

Parameters α, β are evaluated for every well and vertex as follows:

$$\alpha_{i,k} = q_{OIL,i,k} - \beta_{i,k} q_{GAS,i,k} \ \forall \ i = 1,2..N_{WELLS} \ and \ \forall \ k = 1,2..N_{VERTICES-1} (7)$$

$$\beta_{i,k} = \frac{q_{OIL,i,k+1} - q_{OIL,i,k}}{q_{GAS,i,k+1} - q_{GAS,i,k}} \forall \ i = 1,2..N_{WELLS} \ and \ \forall \ k = 1,2..N_{VERTICES-1} (8)$$

The formulated model is:

$$\boldsymbol{max_{y,Q_{GAS}} Q_{OIL,TOTAL}} = \sum_{\boldsymbol{i}}^{\boldsymbol{N_{WELLS}}} \sum_{\boldsymbol{k}}^{\boldsymbol{N_{VERTICES}}} \boldsymbol{\alpha_{i,k} y_{i,k} + \beta_{i,k} Q_{GAS,i,k}} \ (9)$$

$$q_{GAS,i,k} y_{i,k} \leq Q_{GAS,i,k} \leq q_{GAS,i,k+1} y_{i,k} \ \forall \ i = 1,2..N_{WELLS} \ and \ \forall \ k = 1,2..N_{VERTICES-1} \ (10)$$

$$\sum_{k=1}^{N_{VERTICES-1}} y_{i,k} = 1 \ \ \forall \ i = 1,2..N_{WELLS} \ (11)$$

$$\sum_{i}^{N_{WELLS}} \sum_{k}^{N_{VERTICES}} Q_{GAS,i,k} \leq Q_{GAS,LIMIT} \ (12)$$

The two parameters, α and β are introduced, which represent the intercept and slope of the line segment respectively. Equation (9) is the objective function which we look to maximize. In comparison to the equations (5-6) in CM, equations (11-12) serve the same purpose. They look to restrict a single line segment for every well and the upper bound on the injection limit. However, equation (10) is an addition to the model. In this equation, the model is restricted to a linear model by using the Big M constraints. It ensures that, if $y_{i,k} = 0$, then $Q_{GAS,i,k} = 0$. On the contrary, if $y_{i,k} = 1$, then $Q_{GAS,i,k}$ is restricted within the required limits.

3) Convex Hull Method

The Convex Hull Method (CHM) like the LSM is a modification of the classic method. In this formulation, the combination weight is split into two parts, left and right. $\lambda^L \in [0, 1]^{N_{WELL} N_{VERTICES-1}}$ and $\lambda^R \in [0, 1]^{N_{WELL} N_{VERTICES-1}}$, are continuous variables and $y \in \{0, 1\}]^{N_{WELL} N_{VERTICES-1}}$ is a binary variable. The highlight of the convex hull method is that, for piecewise-linear functions, the convex hull of the constraint polytope (eqs 19-22) is equal to the linear programming relaxation of the constraint polytope, i.e,

$P_{LP} = \{(\lambda, y) | ((\lambda, y)$ satisfies (15) – (18) $\} = \text{conv}\{P_{LP} \cap \{(\lambda, y) | y \in \{0,1\}\}\} = P_c$ (13)

The formulated model is as follows:

$$\boldsymbol{max_{y,\lambda^L,\lambda^R}\, Q_{OIL,TOTAL} = \sum_i^{N_{WELLS}} \sum_k^{N_{VERTICES}} (\lambda^R{}_{i,k})(q_{OIL,i,k}) + (\lambda^L{}_{i,k})(q_{OIL,i,k})} \quad (14)$$

Where $\lambda_{i,k}^R$ and $\lambda_{i,k}^L$ can be described by:

$$\lambda_{i,k}^R + \lambda_{i,k}^L = y_{i,k} \ \forall\, i = 1,2..N_{WELLS} \ and \ \forall\, k = 1,2..N_{VERTICES-1} \quad (15)$$

$$\sum_{k=1}^{N_{VERTICES-1}} y_{i,k} = 1 \ \forall\, i = 1,2..N_{WELLS} \quad (16)$$

$$(\lambda^R, \lambda^L) \geq 0 \quad (17)$$

$$y_{i,k} \in \{0,1\} \quad (18)$$

$$\sum_i^{N_{WELLS}} \sum_k^{N_{VERTICES}} (\lambda^R{}_{i,k})(q_{GAS,i,k}) + (\lambda^L{}_{i,k})(q_{GAS,i,k}) \leq Q_{GAS,LIMIT} \quad (19)$$

4) Special Structure Method

The Special Structure Method (SSM) formulation, like the CHM and LSM is derived from the classic method. SSM implements the fact that any two $\lambda_{i,k}$ values should be adjacent without the introduction of a binary variable $y_{i,k}$. Thus the only variable used in SSM is the convex combination weight $\lambda \in [0, 1]^{N_{WELL} N_{VERTICES}}$ which is constrained using SOS2. The SOS2 special structure stipulates that $\forall\, i = 1,2..N_{WELLS}$ at most two $\lambda_{i,k}$ can be positive and, if two $\lambda_{i,k}$ are positive, they must be consecutive. GAMS allows for direct admissibility of SOS2 variables, therefore there's no need to explicitly implement the branch and cut algorithm.

The formulated model is as follows:

$$\boldsymbol{max_{y,\lambda} Q_{OIL,TOTAL} = \sum_i^{N_{WELLS}} \sum_i^{N_{VERTICES}} (\lambda_{i,k})(q_{OIL,i,k})} \quad \mathbf{(20)}$$

$$\sum_{k=1}^{N_{VERTICES}} \lambda_{i,k} = 1 \,, \forall\, i = 1,2..N_{WELLS} \quad (21)$$

$$\lambda \geq 0 \quad (22)$$

$$\lambda \in \text{SOS2} \quad (23)$$

$$\sum_i^{N_{WELLS}} \sum_{k=1}^{N_{VERTICES}} (\lambda_{i,k})(q_{GAS,i,k}) \leq Q_{GAS,LIMIT} \quad (24)$$

Addition of a Lower Bound to Injection

To generalize the model further, another aspect is considered, which is to add a lower bound on the gas flow rate. This constraint can be universally applied to all the four models described in the paper. Addition of the new constraint involves introduction of a new parameter: $Q_{GAS,\ MIN}$ and a set of binary variables m_i $\forall\ i = 1,2..N_{WELLS}$ which gets activated when $Q_{GAS,i}$ not equal to 0.

The additional constraint is as follows:

$$\sum_{k}^{N_{VERTICES}} (q_{GAS,i,k}) \geq Q_{GAS,min} m_i \forall\ i = 1,2..N_{WELLS} \quad (25)$$

With the introduction of the additional constraint and binary variable m_i the computational requirements increase which further increases with increase in supply limit.

Analysis of the Formulation

On analyzing the performance of the various model implementations we can conclude that the Classic model (CM) forms the basis for the formulation of the other three models. However, it is more primitive and computationally less efficient in some cases when compared to the other models. The Linear Segmentation Model (LSM) is a robust Big-M reformulation, however it involves several parameters such as the slopes and intercepts of the line segments and constraints which makes it computationally slow. The Convex Hull model (CHM) is also a big-M reformulation. Like all other big-M models the goal of CHM is to create a model whose relaxation is as close as possible to the convex hull of the original constraint. The big-M part ensures equivalence with the original model whereas the convex hull part ensures that the integer relaxations are of good quality, thus resulting in a robust model. However, the convex hull part involves the introduction of multiple extraneous variables and constraints which adversely affects the computational efficiency of the model in some cases. The fastest and most efficient model is the Special Structure Model (SSM) using the SOS2 variable. Its computational efficiency can be attributed to the usage of the SOS2 variable, which enables it to avoid using one extra binary variable. The SSM Model can be easily implemented in solvers such as GAMS and CPLEX, where the use of SOS2 variable is directly admissible and there's no need for an additional branch and cut implementation. This model out performs the other three models in most cases.

Figure: 2 represents the results obtained for the 6 well case. It can be observed from the figure that the sixth well does not start producing oil as soon as the lift gas is injected. This lag between the injection of lift gas and production of oil gives rise to non-convexity that forces the usage of mixed-integer linear programming instead of linear programming. If it can be assumed that pathological wells such as well 6 do not exist, the problem could be formulated using linear programming methods.

Results and Discussion

1) Test cases for 56 wells by Buitrago et al.

The four models were run for the two test cases presented in Buitrago et. al (Buitrago, et al., 1996) for the 6 well case and the 56 well case. The supply limit was set to 4600 MSCF/day for the 6 well test case and 22500 for the 56 well test case, in order to compare the results with literature.

Table 1: Results for 6 well test case with 4600 MSCF/Day Supply Limit

6 well case	**4600 MSCF/Day**			
	CM	**LSM**	**CHM**	**SSM**
Objective	3669.148	3662.6294	3655.1	3665.623
No of iterations	16	142	27	12
CPU (s)	0.122	0.202	0.148	0.155

Table 2: Results for 56 well test case with 22500 MSCF/Day

56 well case	**22500 MSCF**			
	CM	**LSM**	**CHM**	**SSM**
Objective	22648.399	22720.401	22719	22726.133
No of iterations	71	1244	380	90
CPU (s)	0.173	0.462	0.237	0.15

Based on the Table 1 and 2, the objective value obtained matches to a satisfactory degree with those mentioned in the literature. Although the number of iterations and CPU times vary with those in literature, the trend observed in these values is roughly the same.

2) Test cases for 200 wells

Further detailed study of the efficacy of the formulated models was done for the 200 well test cases, which have been cited as the supporting material in the paper. The detailed study of the effect of various parameters has been done in this section, by varying the gas supply limit. Five cases have been taken into account, tabulating its results for the three advanced models, namely, LSM, CHM and SSM. The test results are also visualized after close verification with the test results.

Table 3: Results for 200 well test case

200 well case	**3000 MSCF**		
	LSM	**CHM**	**SSM**
Objective	66188.509	66142.609	66188.509
No of iterations	31	132	709
CPU (s)	0.503	0.306	0.211
200 well case	**7000 MSCF**		
	LSM	**CHM**	**SSM**
Objective	82436.406	82430.709	82435.853
No of iterations	2029	106	644

CPU (s)	1.416	0.259	0.214
200 well case	**10000 MSCF**		
	LSM	**CHM**	**SSM**
Objective	92124.142	92073.699	92124.142
No of iterations	2153	122	653
CPU (s)	1.423	0.27	0.176
200 well case	**50000 MSCF**		
	LSM	**CHM**	**SSM**
Objective	165851.584	165845.499	165851.584
No of iterations	2277	153	494
CPU (s)	1.525	0.277	0.198
200 well case	**70000 MSCF**		
	LSM	**CHM**	**SSM**
Objective	186537.3157	186532.049	186537.316
No of iterations	3567	536	482
CPU (s)	1.77	0.37	0.242

By comparing the results in Table 3 to those in literature, the objective value was deemed to be satisfactory and was thus plotted in Figure 3, from which it can be observed that the objective value seems to increase linearly with the gas supply limit. Moreover, the objective values of the three models coincide indicating that all the models ultimately converge to the same objective value.

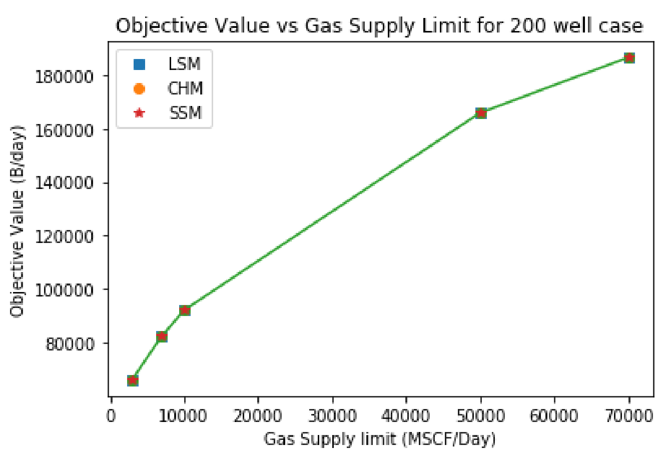

Figure 3: Trend of Objective Value with Gas Supply Limit for 200 wells

3) Analysis of imposition of Lower Bound Constraint

To make the models more inclusive, a lower bound constraint was introduced for the 200 well case and its objective was tabulated. In total, 3 different values of lower bounds were considered (100, 500, 1000) with 5 values of the gas supply limit (3000, 7000, 10000, 50000, 70000 MSCF).

Table 4: Results for 200 well case with a lower bound of 100 units

200 well case	**3000 MSCF**	**100 LB**	
	LSM	**CHM**	**SSM**
Objective	66188.509	66142.609	66188.509
No of iterations	31	132	709
CPU (s)	0.493	0.277	0.19
200 well case	**7000 MSCF**	**100 LB**	
	LSM	**CHM**	**SSM**
Objective	82436.406	82430.709	82435.853
No of iterations	2029	106	644
CPU (s)	1.051	0.241	0.204
200 well case	**10000 MSCF**	**100 LB**	
	LSM	**CHM**	**SSM**
Objective	92124.142	92073.699	92124.142
No of iterations	2153	122	653
CPU (s)	1.087	0.25	0.18
200 well case	**50000 MSCF**	**100 LB**	
	LSM	**CHM**	**SSM**
Objective	165851.584	165845.499	165851.584
No of iterations	2277	153	494
CPU (s)	1.169	0.257	0.192
200 well case	**70000 MSCF**	**100 LB**	
	LSM	**CHM**	**SSM**
Objective	186537.316	186532.049	186537.316
No of iterations	3567	536	482
CPU (s)	1.29	0.349	0.234

Table 5: Results for 200 well case with a lower bound of 500 units

200 well case	**3000 MSCF**	**500 LB**	

	LSM	**CHM**	**SSM**
Objective	66188.509	66142.609	66188.509
No of iterations	31	132	709
CPU (s)	0.547	0.291	0.19
200 well case	**7000 MSCF**	**500 LB**	
	LSM	**CHM**	**SSM**
Objective	82436.406	82430.709	82435.853
No of iterations	2029	106	644
CPU (s)	1.067	0.225	0.206
200 well case	**10000 MSCF**	**500 LB**	
	LSM	**CHM**	**SSM**
Objective	92124.142	92073.699	92124.142
No of iterations	2153	122	653
CPU (s)	1.106	0.242	0.184
200 well case	**50000 MSCF**	**500 LB**	
	LSM	**CHM**	**SSM**
Objective	165851.584	165845.499	165851.584
No of iterations	2277	153	494
CPU (s)	1.179	0.255	0.187
200 well case	**70000 MSCF**	**500 LB**	
	LSM	**CHM**	**SSM**
Objective	186537.316	186532.049	186537.316
No of iterations	3567	536	482
CPU (s)	1.346	0.337	0.218

Table 6: Results for 200 well case with a lower bound of 1000 units

200 well case	**3000 MSCF**	**1000 LB**	
	LSM	**CHM**	**SSM**
Objective	66188.509	66142.609	66188.509
No of iterations	31	132	709
CPU (s)	0.494	0.271	0.189
200 well case	**7000 MSCF**	**1000 LB**	
	LSM	**CHM**	**SSM**
Objective	82436.406	82430.709	82435.853

No of iterations	2029	106	644
CPU (s)	1.094	0.227	0.202
200 well case	**10000 MSCF**	**1000 LB**	
	LSM	**CHM**	**SSM**
Objective	92124.142	92073.699	92124.142
No of iterations	2153	122	653
CPU (s)	1.063	0.241	0.178
200 well case	**50000 MSCF**	**1000 LB**	
	LSM	**CHM**	**SSM**
Objective	165851.584	165845.499	165851.584
No of iterations	2277	153	494
CPU (s)	1.221	0.238	0.181
200 well case	**70000 MSCF**	**1000 LB**	
	LSM	**CHM**	**SSM**
Objective	186537.3157	186532.049	186537.3157
No of iterations	3567	536	482
CPU (s)	1.368	0.339	0.227

The objective value after adding the lower bound constraint were found to be in accordance with those in the paper. Although the change in the absolute value is insignificant, the addition of the lower bound ensures more inclusivity and generalization of the model to pertinent situations.

Conclusion

From our results and the referenced literature it can be observed that the Special Structure Model is the most efficient and simplest of all four models. It is convenient to implement in solvers such as CPLEX and GAMS. The oil production increases with increase in gas injection only up to a certain limit, thereafter it becomes constant and does not increase with increase in supply of lift gas. From Figure 1, the reduced returns of an unlimited gas supply is evident. The classic method is the least efficient of all methods described in the paper, therefore for larger and more challenging gas lifting problems, it's advisable to use one of the other 3 methods. This paper also highlights the importance of the piecewise linearization technique as a mathematical tool to solve higher order non linearity. It can be applied to a wide variety of problems such as separation synthesis, heat exchanger networks, reactor networks, design and scheduling.

Further Improvements

In our project work, we have majorly focused on optimizing the gas allocation using piecewise linear functions. However, there are other algorithms and optimization techniques which can be used to solve the problem at hand. The piecewise linear model can be efficiently used for optimization, however it is not satisfactory in simulating the yielded gas-life performance curves. To serve this dual purpose, the Newton-Reduction Method can be used, wherein, the formulation can be transformed to solve a composite residual function by restricting the availability of gas to an equality constraint (sensitivity of well flow rate to a change in gas injected should be the same for all wells). Thus, using this, an equal slope based solution can be obtained. As proposed by Mayo et. al (O. Mayo, 1999), a Lagrange multiplier approach can be used to model the gas-lift performance curves as a second order polynomial, with further impositions of a KKT (Karush-Kuhn-Tucker) conditions to ensure that the solution is globally optimal. This method is able to handle wells with unlimited and limited gas supply and the lift-gas deficit is shared equally amongst the wells. Applying this method to the constraints mentioned in the paper, using equations (1) and (6), the constrained problem can be converted to an unconstrained problem as follows:

$$\min \mathrm{Q}_{\mathrm{oil,total=}} -Q_i + \lambda(L_i - Q_{GAS,LIMIT}) \quad (26)$$

Where $L_i$the supply to each is well and Q_i is the objective function

The KKT conditions for optimality can be obtained by setting:

$$\frac{\partial \text{ Qoil,total}}{\partial Q_i} = \frac{\partial \text{ Qoil,total}}{\partial L_i} = 0 \ (27)$$

In addition to this, two additional optimization techniques, Sequential Linear Programming (SLP) and Sequential Quadratic Programming (SQP) can be used to find the optimal solution, through a Taylor Series Expansion and Hessian Update procedure for SLP and SQP respectively (Kashif Rashid, 2012). Further complicated algorithms to solve the Gas Lift Allocation problem at hand is through implementing genetic algorithms such as a Tabu Search Method (for higher complex cases) and a Neural-Network based production simulation method for product optimization.

References

Buitrago, S., Rodrıguez, E. & Espin, D., 1996. *Global Optimization Techniques in Gas Allocation for Continuous Flow Gas Lift Systems.* Calgary,Alberta, SPE Gas Technology Symposium.

Fang, W. Y. & Lo, K. K., 1996. A Generalized Well-Management Scheme for Reservoir Simulation. . *SPE ReserV. Eng,* Volume SPE Number 29124.

Floudas, 1995. *Nonlinear and Mixed-Integer Optimization: Fundamentals and Applications.* New York: Oxford University Press.

Kanu, E. P., Mach, J. & Brown, K. E., 1981. Economic Approach to Oil Production and Gas Allocation in Continuous Gas Lift.. *J. Pet. Technol..*

Kashif Rashid, W. B. a. B. C., 2012. A Survey of Methods for Gas-Lift Optimization. *Modelling and Simulation in Engineering.*

Keha, A. B., de Farias, I. R. J. & Nemhauser, 2004. G. L. Models for representing piecewise linear cost functions.. *Oper. Res. Lett..*

Kosmidis, V. D., Perkins, J. D. & Pistikopoulos, E. N., 2004. Optimization of Well Oil Rate Allocations in Petroleum Fields. *Ind. Eng. Chem. Res.*

Mason, T. L., Bagirov, A., Klunder, J. & van Berkel, J., 2007. *Development and Application of the Discrete Gradient Method for Non-Smooth Non-convex Production Optimization.* Veracruz, Mexico, International Oil Conference and Exhibition.

Nemhauser, G. L. & Woolsey, L. A., 1988. *Integer and Combinatorial Optimization.* New York: J. Wiley.

O. Mayo, F. B. a. J. A., 1999. Procedure optimizes lift gas allocation. *Oil & Gas Journal,* 97(13), pp. 38-41.

Sherali, 2001. On mixed-integer zero-one representations for separable lower-semicontinuous piecewise-linear functions. *Oper. Res. Lett..*