

Q2

(1) is true . For that note that all s1 sparse vectors are s2 sparse also (as s1 sparse have atmost s1 non zero elements, they don't have greater than s2 non zero elements ) , hence if the RIC constanr for s2 is  $\delta_{s_2}$  , then all the s2 sparse vectors  $\theta$  obey

$$(1 - \delta_{s_2})\|\theta\|^2 \leq \|A\theta\|^2 \leq (1 + \delta_{s_2})\|\theta\|^2$$

(we can always find a  $\delta_{s_2}$  for a matrix A(as we have seen the bounds on max value of  $\frac{\|A\theta\|^2}{\|\theta\|^2}$  in Q1 , though it may be large in which case A doesn't obey RIP of order s2)

Now let v be any s1 sparse vector , since it is also s2 sparse , hence all s1 sparse vectors v , satisfy the above inequality ,

$$(1 - \delta_{s_2})\|v\|^2 \leq \|Av\|^2 \leq (1 + \delta_{s_2})\|v\|^2$$

We need to find  $\delta_{s_1}$  for s1 sparse vectors , we already know that  $\delta_{s_2}$  satisfies the inequality for s1 sparse vectors. Since  $\delta_{s_1}$  is the smallest constant  $\alpha$  satisfying the inequality

$$(1 - \alpha)\|v\|^2 \leq \|Av\|^2 \leq (1 + \alpha)\|v\|^2$$

and  $\alpha = \delta_{s_2}$  satisfies the inequality , hence  $\delta_{s_1}$  is atleast as small as  $\delta_{s_2}$  (and can be even smaller) and hence we get

$$\delta_{s_1} \leq \delta_{s_2}$$