Q4)
Suppose measurement matrix is a m x n matrix where ith column of matrix is represented as vector v_i of length m. Then y is also a vector of length m.
We are given y and measurement matrix and wish to find x , thus we need to express y as a linear combination of some $v_i's$.

A) Suppose $y = a_i * v_i$, then we can find a_j for some non zero v_j such that $y = a_i * v_i = a_j * v_j$ as length of each vector is 1 (2 vectors of length 1 are always linearly dependent).Thus x can be a_j at jth position and zero elsewhere or a_i at ith position and zero elsewhere . Hence unique estimation is not possible for any measurement matrix (matrix that can uniquely estimate all signals). If index is known (let's say i) then we know that $y = a_i * v_i$ is the only possibility as x has non zero element at ith position only hence x can be uniquely estimated by $a_i = \frac{y}{v_i}$ (in cases where all v_i are non zero , then can estimate all signals uniquely).
B) Suppose we found a solution $y = a_i * v_i$,where length of vector is 2 then we should not be able to find another a_j for some non zero v_j such that $y = a_i * v_i = a_j * v_j$ ie $a_i * v_i - a_j * v_j \neq 0$ for any pair of (i,j) . This will ensure all signals x can be uniquely estimated. Hence any 2 columns of measurement matrix should be linearly independent . Such a matrix can be constructed , for eg $v_i^t = [1\ b_i]$ (ith column) and all b_i are different , then no 2 columns would be linearly dependent. To find x , let $y^t = [y_1\ y_2]$, then $\frac{y_2}{y_1}$ gives the value of b_i and hence index i of the non zero element obtained and then by setting $a_i = y_1$, we get $y = a_i * v_i$ hence x is non zero at ith position with value a_i
C) Here since 2 non zero elements are there in x , let a_i and a_j be its non zero elements at ith and jth position , then we need to express $y = a_i * v_i + a_j * v_j$ uniquely where each vector is of length 3, but unique estimation of all signals is not possible for any measurement matrix as any 4 vectors of length 3 will always be linearly dependent. Hence there exists a_i, a_j, a_k, a_l such that not all of them are 0 and $a_i * v_i + a_j * v_j - a_k * v_k - a_l * v_l = 0$, so $y = a_i * v_i + a_j * v_j = a_k * v_k + a_l * v_l$ (not unique) hence the signal (a_i , a_j) (non zero with these values at ith and jth position) and the signal (a_k , a_l) (non zero with these values at kth and lth position) will give the same y vector and hence unique estimation is not possible . No special instances of measurement matrix exists which can uniquely estimate all signals as we can't find any matrix having all set of 4 columns linearly independent as length of each column is just 3.
D) Here , the expression for y is same , just the length of column vectors is 4 here . Unique estimation is possible here , as here we can construct a matrix such that any 4 columns are linearly independent and then there exists no (a_i, a_j, a_k, a_l) such that not all of them are zero and $a_i * v_i + a_j * v_j - a_k * v_k - a_l * v_l = 0$ and we can find the signal by trying all possible pairs(i,j) and try to express $y = a_i * v_i + a_j * v_j$ by solving 3 linear equations for each value of vector y (gauss elimination method , easily implementable) and then when we find a solution to the linear system , that solution is unique since no 4 columns are linearly dependent and $y = a_i * v_i + a_j * v_j = a_k * v_k + a_l * v_l$ is not possible.

To construct such a measurement matrix where any set of 4 columns are linearly independent , as in B part set $v_i^t = [1\ b_i\ b_i^2\ b_i^3]$ (ith column) and all b_i are different. Hence no 4 columns will be linearly dependent . The proof is as follows , suppose there exists 4 columns that are linearly dependent then there exists a_i, a_j, a_k, a_l (not all zero) such that
$a_i * v_i + a_j * v_j + a_k * v_k + a_l * v_l = 0.$
Expanding this gives a system of 4 linear equations in 4 variables . Writing in matrix form VA=0 , where $A^t = [a_i\ a_j\ a_k\ a_l]$ and $V = [v_i\ v_j\ v_k\ v_l]$. Notice that columns of V are of the form $[1\ b_i\ b_i^2\ b_i^3]^t$, hence V is a Vandermonde's matrix having non zero determinant as all b_i are different. Hence V is a full rank matrix having rank 4 and hence by rank-nullity theorem it has a trivial null space. Since VA=0 , pre multiply by V^{-1} on both sides since V is invertible , hence we

get A=0 ie a_i, a_j, a_k, a_l all are 0 . This is a contradiction , hence any 4 columns of the constructed measurement matrix are linearly independent and hence we can uniquely estimate any signal from its measurement in such cases .
When some set of 4 columns are linearly dependent for measurement matrix , then we can not uniquely estimate all signals as we will be able to find a_i, a_j, a_k, a_l that are not all zero and hence the same y(formed using signal (a_i, a_j) or $(-a_k, -a_l)$) can be written in 2 ways (can give 2 signals) as $y = a_i * v_i + a_j * v_j = -a_k * v_k - a_l * v_l$

This very closely resembles the proof given in slides that if any set of 2S columns is linearly independent , then we can uniquely estimate S sparse signals.