Q1)

A) As S increases , δ_{2S} increases . Intuitively this can be thought like this . As m is staying the same , the chances of any 2S columns of measuring matrix being linearly independent decreases as the length of column is fixed , by increasing S , we are including more columns hence they can be linearly dependent. The RIC constant is related to this property even though 2S column being linearly independent doesn't imply RIP of order S , but vice versa is true. Now in the error bound C1 and C2 are increasing functions of δ_{2S} and hence they increase as S increase , and they increase in such a way that overall error term increases , hence as S increases and m is kept same , the reconstruction worsens which seems reasonable.

B) M is hidden in the error term in various places . ϵ is upper bound on magnitude of η and that depends on the number of measurements we take . $\epsilon = \sigma\sqrt{m}$ is generally taken and hence the error depends on m. Also Theorem 3 holds only when the matrix satisfies RIP of order 2S and $\delta_{2S} < 0.41$ and if we vary m then it is possible that the matrix doesn't obey RIP (mainly when decrease m) or the value of δ_{2S} could change and hence C1,C2 would change as they are functions of δ_{2S}. Hence the error bound strongly depends on m. Also as m increases error bound should decrease as looks intuitive (reconstruction should be better). This can be explained by the fact that as m increase it looks as if δ_{2S} should decrease (ie RIC constant decrease) as the matrix has better chance of satisfying RIP . Also any 2S columns would be linearly independent with more probability when m is increased (as more freedom and more length of vector) , hence C1 and C2 could decrease , thereby decreasing the error term even though ϵ increases. As m decreases, chances of matrix satisfying RIP decrease ($m \geq 2S$ is necessary) we have seen in the construction of random matrices that they satisfy RIP with overwhelmingly high probability when $m \geq CSlog(n/S)$ and when m decreases , there are chances that this inequality won't be satisfied .

C) Theorem 3A is more useful as it relaxes the condition on δ_{2S} , ie now it can have higher value also . There can be 2 cases during reconstruction , when δ_{2S} <0.41 , then the reconstruction would be similar due to both theorems . In the case when δ_{2S}>0.41 and δ_{2S}<0.6246 , then theorem 3 can't be applied and according to that we can't reconstruct(or get error bound) , but since we have a theorem 3A , it tells that we can still reconstruct , though the error may be higher as C1,C2 are higher as δ_{2S} is higher , but at least it gives the possibility of reconstruction . Hence theorem 3A covers more measurement matrices than theorem 3. The measurement matrices satisfying δ_{2S} <0.41 are a subset of matrices satisfying δ_{2S} <0.6246 and hence using theorem 3A we can do compressed reconstruction over larger set of matrices. As δ_{2S} can be larger here , so as explained in B , we can even select smaller values of m (decreasing m , can likely increase δ_{2S}) and hence we can compress the signal more and reconstruct from it also within an error bound.