$Q5$: Lensless Compressive Sensing Imaging

a) Architecture of Rice Single Pixel Camera(RSPC)

- Light from a scene passes through a lens (say lens1), and is focused on DMD array, an arrangement of as many mirrors as there are pixels in the image, each pointing either towards or away from a second lens (say lens 2).
- Light from lens2 is focused on a photodiode, which thus records a single scalar value per measurement.
- m such values are recorded, each for a different arrangement of DMD mirrors. Mirror facing the lens2 corresponds to multiplication of that pixel value by one and the one facing away is zero.
- Due to random binary matrix (0/1) not following RIP, two captures are taken for each measurement.

$$y_1 = \phi_1 x$$

$$y_2 = \phi_2 x$$

$$\acute{y}_1 - y_2 = (\phi_1 - \phi_2)x$$

$$y' = \phi' x$$

Here ϕ_1 *contains a* zero where ϕ_2 *contains a one and vice versa*. Hence, y' *is the new effective measurement with corresponding sensing matrix* ϕ' *with random* 1/-1.

- m such measurements are taken. Thus,

$$y = \phi x \,, \phi = [\,\varphi_1 \mid \varphi_2 \mid \varphi_3 \;\dots\; \varphi_m\,]$$
$$y = [\,y_1 \mid y_2 \mid y_3 \dots y_m\,]$$

Architecture of Lensless Camera (LC)

- Essentially there are two components: an aperture assembly and a sensor.
- Aperture Assembly: made up of a two-dimensional array of aperture elements. The transmittance of each aperture element can be individually controlled.
- Sensor is a single detection element. The integration of the ray intensities coming from one aperture element is defined as a pixel value of the image. An image is defined by the pixels which correspond to the array of aperture elements in the aperture assembly.
- An image can be captured by using the sensor to take as many measurements as the number of pixels, when one of the aperture elements is completely open and all others are completely closed, which corresponds to the binary transmittance 1 (open), or 0 (closed).
- m measurements are done.

$$\mathrm{y} = \phi \mathrm{f}$$
$$where\ f\ is\ the\ vectorized\ image.$$

$y = [\,y_1 \mid y_2 \mid y_3 \dots y_m\,]$ are the measurements.

$\phi = [\,\varphi_1 \mid \varphi_2 \mid \varphi_3 \;\dots\; \varphi_m\,]$ are matrices containing individual transmittances for each trial.

$$Individual\ \varphi_i's\ are\ vectorised\ to\ give\ m\ rows\ corresponding\ to\ each\ \varphi_i.$$

Differences

- In LC, no planar image is explicitly formed. The image is not formed by any physical mechanism, unlike RSPC where image is first formed on micrometer array. This may impact resolution due to the discretization being done. The quality of image from LC is only affected by the resolution of pixelization (the number of the aperture elements in the aperture assembly).
- In LC, there are no lenses. In RSPC, a lens is employed to form an image of the scene on the micromirror array. The micromirror array then performs the functions of both pixelization and projection. Therefore, RSPC creates an “analog” image of the scene on a plane. This may create artifacts due to the process of formation. The sharpness may depend on the focal point of the scene, so that an object may appear blurred because it is out of focus. LC is free from such artifacts because there are no lenses and analog image involved.

b) Cost function used in Reconstruction:

- L1 minimization of total variation is used.
- $$\min_{f} \quad \begin{matrix} TV(f)\ such\ that\ y = \phi f \\ where\ TV(f) = \sum_x \sum_y \sqrt{(f_x^2(x,y) + f_y^2(x,y))} \end{matrix}$$
- Here TV represents Total Variation.
  f is the vectorized image to be reconstructed.
  $\phi\ is\ the\ sensing\ matrix.$

  $f_x\,(x,y) = f(x,y) - f(x-1,y)$

  $f_y(x,y) = f(x,y) - f(x,y-1)$

  $f(x,y)\ is\ the\ value\ at\ pixel\ at\ (x,y)\ before\ vectorization.$

  Here, appropriate zero padding is done for evaluations at border values.