Q4

In the whole document wherever $|a|$ is written , it means $||a||_{l_2}$. Other norms are explicitly specified.

1) By triangle inequality $|a+b| \leq |a| + |b|. Take\ a = \phi x^* - y\ and\ b = y - \phi x.$ $y = \phi x + z\ and\ |z|_{l_2} \leq \epsilon\ hence |y - \phi x|_{l_2} \leq \epsilon\ and\ |y - \phi x^*|_{l_2} \leq \epsilon$ as that is our optimisation problem constraint, hence adding both , we get the desired inequality.

2) infinity norm means the max of the absolute values and 2 norm is the square root of sum of squares of values. Every square term is less than the square of the term with max absolute value , hence the l2 norm is less than square root of s times square of max absolute term (as atmost s values) hence l2 norm is less than $s^{\frac{1}{2}}$ times infinite norm. 1 norm means sum of absolute values , any term in T_j are less than all terms of T_{j-1} in absolute value hence s times inifinity norm of T_j is less than 1 norm of T_{j-1}. Divide by square root s on both sides.(as either Tj and T(j-1) both have s terms or Tj has less than s terms and T(j-1) has s terms , in both cases inequality holds)

3) For first inequality take the 1st and 3rd term of the previous inequality(2). since $j \geq 2$ hence $j-1 \geq 1$. For the second part , we'll sum up 1 norms of T1,T2,.. so on , till all but last , this is essentially sum of absolute values of all terms of h excluding T0 positions and the positions corresponding to last set . Hence these sum of absolute values will be less than sum of absolute values of h excluding T0.

4) As all the T_j are disjoint , if we denote the 2 norm of h_{T_j} by a_j then we need to prove $a_2^2 + a_3^2 + a_4^2 .. \leq (a_2 + a_3 + ..)^2$ (square both sides then take square root) which is true since when we open the square , the square terms cancel out and we are left with terms $2 * a_i * a_j$ which are ≥ 0 as all 2 norms are positive. The second part of inequality is same as the inequality established between 1st and 3rd term in previous inequality (3) .

5) triangle inequality . a) $|x_i + h_i| \geq |x_i| - |h_i|$ and b) $|x_i + h_i| \geq |h_i| - |x_i|$. Summing "a" over T_0 and "b" over T_0^c and adding both , we get the required inequality

6) take the leftmost term and the right most terms of the previous inequality (5) and rearrange to get $|h_{T_0^c}|_{l_1} \leq |h_{T_0}|_{l_1} + |x_{T_0^c}|_{l_1} + |x|_{l_1} - |x_{T_0}|_{l_1}$ and now $|x|_{l_1} - |x_{T_0}|_{l_1} = |x_{T_0^c}|_{l_1}$, which gives the required inequality .

7) a) proof of $|h_{T_0}|_{l_1} \leq s^{\frac{1}{2}} |h_{T_0}|_{l_2}$. Use cauchy shwarz in which the absolute terms of $|h_{T_0}|$ form a s length vector and the other vector of s length is all 1's vector . Take dot product , it gives the 1 norm of $|h_{T_0}|$ and the magnitude of

vector formed by absolute values is 2 norm of $|h_{T_0}|$ and magnitude of all 1's vector is $s^{\frac{1}{2}}$.
Actual proof of 7 . Using first and last term of inequality (4) (11 in the paper) and using inequality(6) (12 in paper) , and using $|h_{T_0}|_{l_1} \leq s^{\frac{1}{2}}|h_{T_0}|_{l_2}$, we get this inequality . Note that $|x_{T_0^c}|_{l_1} = |x - x_s|_{l_1}$

8) first part , cauchy schwarz , dot product is less than product of magnitudes . for the second part $|\phi h_{T0UT1}| \leq \sqrt{1+\delta_{2s}}|h_{T0UT1}|$ by RIP as h_{T0UT1} is 2S sparse . And $|\phi h| \leq 2\epsilon$ as proved in inequality(1) (9 in paper) as x*=x+h.

9) lemma 2.1 of this paper directly . T0 and Tj are s sparse hence s'=s and s+s'=2s.

10) Taking $|h_{T_0}|_{l_1} = a$ and $|h_{T_1}|_{l_1} = b$ this inequality is same as $a + b \leq \sqrt{2}\sqrt{a^2+b^2}$ as $a^2+b^2-2ab \geq 0$ hence $a^2+b^2 \geq 2ab$ hence $2(a^2+b^2) \geq a^2+b^2+2ab$ hence $2(a^2+b^2) \geq (a+b)^2$ which gives the desired inequality.

11) The left inequality is same as that of RIP left inequality h_{T0UT1} is 2S sparse as total 2S non zero here. For the second part , use inequality (9 as in this document) and replace T1 in place of T0 (inequality still holds , same proof). Now observe $h_{T0UT1} = h_{T0} + h_{T1}$ and add both the inequalities(after putting T0 and T1 in 9(as per this document)) to get $| < \phi h_{T0UT1}, \phi h_{Tj} > | \leq \delta_{2S}|h_{Tj}|(|h_{T0}| + |h_{T1}|)$ and using inequality (10 as in this document) , we get $| < \phi h_{T0UT1}, \phi h_{Tj} > | \leq \sqrt{2}\delta_{2S}|h_{Tj}||h_{T0UT1}|$. Now as mentioned in the paper , below equation 13 of paper and using triangle inequality get

$$|\Phi h_{T_0 \cup T_1}||_2^2 \leq |\langle \Phi h_{T_0 \cup T_1}, \Phi h \rangle| + |\langle \Phi h_{T_0 \cup T_1}, \sum_{j \geq 2} \Phi h_{T_j} \rangle|$$

(triangle inequality $a - b \leq |a| + |b|$)
using triangle inequality again , as $|\langle \Phi h_{T_0 \cup T_1}, \sum_{j \geq 2} \Phi h_{T_j} \rangle| \leq \sum_{j \geq 2} |\langle \Phi h_{T_0 \cup T_1}, \Phi h_{T_j} \rangle|$. Now using the inequality $| < \phi h_{T0UT1}, \phi h_{Tj} > | \leq \sqrt{2}\delta_{2S}|h_{Tj}||h_{T0UT1}|$ proved above and using inequality (8 as given in this document) and the above 2 triangle inequalities to get summation outside the mod , we get the desired inequality.

12)Use the previous inequality (11 in this document) and take the leftmost and rightmost term and divide by $||h_{T_0 \cup T_1}||_{l_2}$ on both sides . Then divide both sides by $1 - \delta_{2s}$. And then use leftmost and rightmost terms of inequality (3 here and 10 in paper) to get the desired inequality .

13) Use inequality (6 here and 12 in paper) on $|h_{T_0^c}|_{l_1}$ and then use $|h_{T_0}|_{l_1} \leq s^{\frac{1}{2}}|h_{T_0}|_{l_2}$ as proved in 7a here and then use

$$||h_{T_0}||_{l_2} \leq ||h_{T_0} + h_{T_1}||_{l_2} \leq ||h_{T_0 \cup T_1}||_{l_2}$$

(obvious as T0 and T1 are disjoint and hence rhs of this has greater support and has all elements of lhs as well as extra elements) (this gives second term of rhs in required inequality) and using the fact that $|x_{T_0^c}|_{l_1} = |x - x_s|_{l_1}$ and the definition of e_0 , we get the whole inequality.

14)For first inequality , take $a = ||h_{T_0 \cup T_1}||_{l_2}$ and $b = ||h_{T_0 \cup T_1{}^c}||_{l_2}$ and then (as $T_0 \cup T_1$ and $(T_0 \cup T_1)^c$ are disjoint) , it is same as $\sqrt{a^2 + b^2} \leq (a + b)$ (which is true as a,b positive and $2ab \geq 0$) For the second part , use inequality (7 here and 13 in paper) and then $||h_{T_0}||_{l_2} \leq ||h_{T_0} + h_{T_1}||_{l_2} \leq ||h_{T_0 \cup T_1}||_{l_2}$ and for third part use implication of inequality proven above (13 here and 14th second inequality in paper) , then add the two terms and take $(1 - \rho)^{-1}$ common.

15) Use inequality 15 of paper to get $||h_{T_0}||_{l_1} + ||h_{T_0^C}||_{l_1} \leq (1 + \rho)||h_{T_0^C}||_{l_1}$ and then use the implication derived from the inequality derived after 15 of paper $||h_{T_0^C}||_{l_1} \leq 2(1 - \rho)^{-1}||x_{T_0^C}||_{l_1}$ to get the required inequality.