Digital assignment - 2

Signals and systems

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17BEC0186

① (a) $w(t) = \cos t$

Apply Fourier transform:

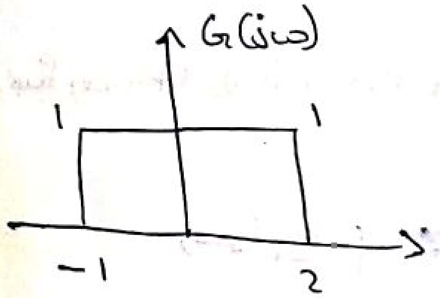
$$w(j\omega) = \pi[\delta(\omega-1)+\delta(\omega+1)]$$

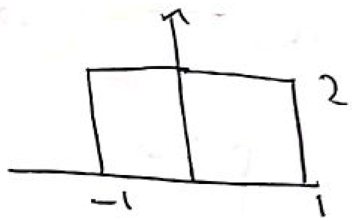
$$g(t) = x(t)\cos t.$$

Apply F.T

$$G(j\omega) = \frac{1}{2\pi}[x(j\omega) + w(j\omega)]$$

$$G(j\omega) = \frac{1}{2}\times[j(\omega-1)] + \frac{1}{2}\times[j(\omega+1)]$$





$$\therefore\ x(t) = \frac{2\sin t}{\pi t}$$

(b) $$\left(\frac{2\sin t}{\pi t}\right)\cos t = x_1(t)\cos(2/3\,t).$$

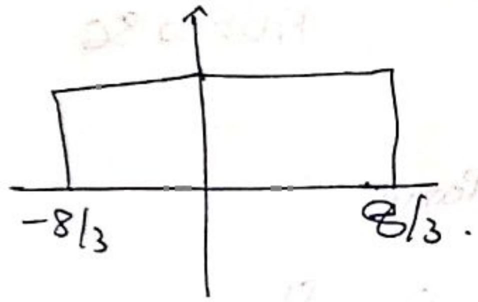
$$x_1(t) = \frac{2\sin t\cos t}{\pi t\cos(2/3\,t)} = \frac{\sin 2t}{\pi t\cos(2/3\,t)}.$$

$$= \frac{2\sin 2t}{\pi t}\left[e^{-2/3t} + e^{2/3t}\right]$$

$x_1(j\omega)$.

Apply fourier transform.

$$\left[\frac{2\sin 2t}{\pi t}\, e^{-2/3 t} + \frac{2\sin 2t}{\pi t}\, e^{2/3 t}\right]$$



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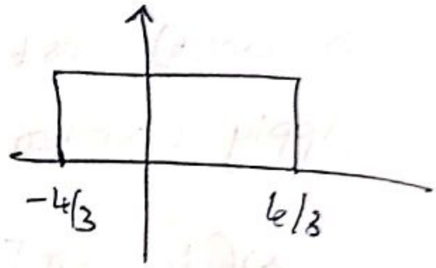
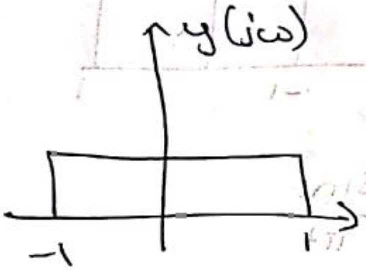
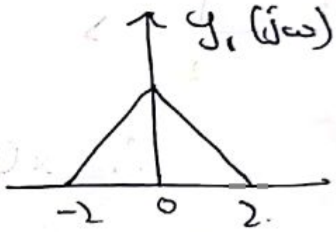


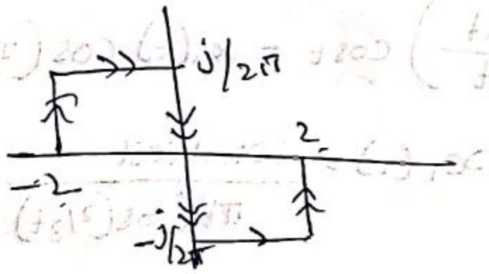
(2) (a) Fourier transform of $x(t) = t\left(\frac{\sin t}{\pi t}\right)^2$

$$\frac{\sin t}{\pi t} \xleftrightarrow{F.T} \text{Rectangular function } y(j\omega)$$

$$\left(\frac{\sin t}{\pi t}\right) \xleftrightarrow{F.T} \left(\tfrac{1}{2\pi}\right)\left[\text{Rect. fun } (y(j\omega)) + \text{Rect. fun } y(j\omega)\right]$$







$$t\left(\frac{\sin t}{\pi t}\right)^2 \xleftrightarrow{F.T} x(j\omega) = j\frac{d}{d\omega}\left[y_1(j\omega)\right]$$

$$x(j\omega) = \begin{cases} j/2\pi & -2 \le \omega < 0 \\ -j/2\pi & 0 \le \omega < 2 \\ 0 & \text{Otherwise.} \end{cases}$$

(b) Parseval's theorm.

$$\int_{-\infty}^{\infty} t^2 \left(\frac{\sin t}{\pi t}\right)^4 dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |x(j\omega)|^2 d\omega$$

$$= \left(\frac{1}{2\pi^3}\right).$$

(3) a) $[e^{-\alpha t} \cos\omega_0 t] u(t), \alpha > 0.$

$[e^{-\alpha t} \cos\omega_0 t] \; u(t).$

$$= \tfrac{1}{2} e^{-\alpha t} e^{j\omega_0 t} u(t) + \tfrac{1}{2} e^{-\alpha t} e^{-j\omega_0 t} u(t).$$

$$\therefore x(j\omega) = \frac{1}{2(a - j\omega_0 + j\omega)} - \frac{1}{2(a - j\omega_0 + j\omega)}$$

b) $e^{-3|t|} \cdot \sin 2t.$

$$x(t) = e^{-3t} \sin 2t \; u(t) + e^{3t} \sin 2t \; u(-t).$$

$$x_1(t) = e^{-3t} \sin 2t \, u(t).$$

Apply fourier transfom

$$x_1(j\omega) = (1/2j)/_{3-2j+j\omega} - \frac{(1/2j)}{3+2j+j\omega}$$

$$x_2(t) = e^{3t} \sin(2t) u(-t) = -x_1(-t).$$

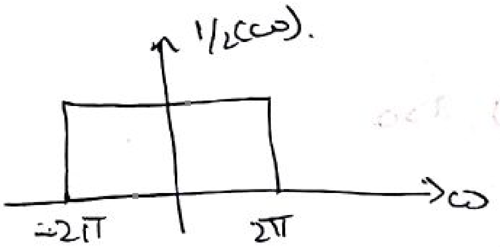
$$x_2(j\omega) = -x_1(-j\omega).$$

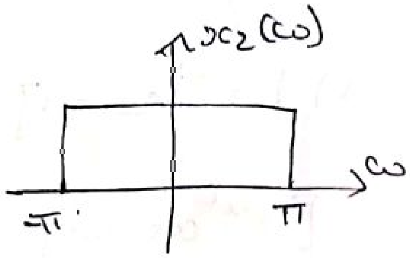
$$x_2(j\omega) = \frac{(1/2j)}{3-2j-j\omega} - \frac{(1/2j)}{3+2j-j\omega}$$

$$x(j\omega) = x_1(j\omega) + x_2(j\omega).$$

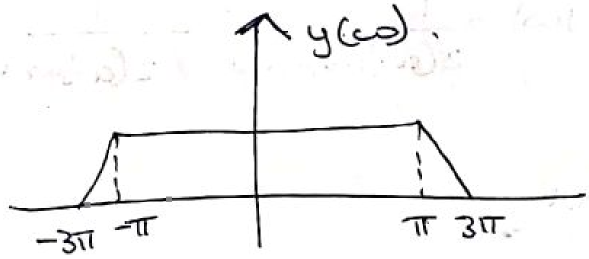
$$\therefore \boxed{x(j\omega) = \frac{3j}{9+(\omega+2)^2} - \frac{3j}{9+(\omega-2)^2}}$$

(c) $\left(\frac{\sin \pi t}{\pi t}\right)\left(\frac{\sin 2\pi t}{\pi t}\right)$.





$$y(\omega) = \frac{1}{2\pi}\left(x_1(t) * x_2(t)\right).$$



$$y(\omega) = \begin{cases} -\omega/\pi + 3. & \pi \le \omega \le 3\pi \\ 2 & |\omega| = \pi. \\ \omega/\pi + 3. & -3\pi \le \omega \le \pi \end{cases}$$

(4)

sol:- $H(j\omega) = \frac{(\sin^2(3\omega)).\cos\omega}{\omega^2}$

$$= \frac{1}{\omega^2}\left(\frac{1-\cos(6\omega)}{2}\right)(\cos\omega).$$

$$= \frac{1}{2\omega^2}\left(\cos\omega - \cos(\omega)\cos(6\omega)\right).$$

$$= \frac{1}{2\omega^2}\left[\cos(\omega) - \frac{1}{2}\cos(7\omega) - \frac{1}{2}\cos(5\omega)\right)$$

$$= \frac{1}{2\omega^2} \cos(\omega) - \frac{1}{4\omega^2}\cos(7\omega) - \frac{1}{4\omega^2}\cos(5\omega).$$

$$H(j\omega) = \frac{1}{2\omega^2}\left(\frac{e^{j\omega}+e^{-j\omega}}{2}\right) - \frac{1}{4\omega^2}\left(\frac{e^{j7\omega}+e^{-j7\omega}}{2}\right).$$

$$-\frac{1}{4\omega^2}\left(\frac{e^{j5\omega}+e^{-j5\omega}}{2}\right)$$

$$\operatorname{sgn}(t) \longleftrightarrow 2/j\omega$$

$$\operatorname{sgn}(t) * \operatorname{sgn}(t) \longleftrightarrow (2/j\omega)(2/j\omega).$$

$$\left[-\sqrt{\frac{\pi}{2}}\, t \;\; + \operatorname{sgn}(t) \longleftrightarrow \frac{1}{\omega^2} \right]$$

$$H(j\omega) = \frac{1}{4\omega^2}(e^{j\omega}+e^{-j\omega}) \cdot -\frac{1}{8\omega^2}(e^{j7\omega}+e^{-j7\omega}).$$

$$-\frac{1}{8\omega^2}(e^{j5\omega}+e^{-j5\omega}).$$

By using fourier transform.

h

$$h(t) = \tfrac{1}{4}\left(-\sqrt{\pi/2}\,(t-1)\operatorname{sgn}(t-1) - \sqrt{\pi/2}\,(t+1)\operatorname{sgn}(t+1)\right).$$

$$-\tfrac{1}{8}\left(-\sqrt{\pi/2}\,(t-7)\operatorname{sgn}(t-7) - \sqrt{\pi/2}\,(t+7)\operatorname{sgn}(t+7)\right)$$

$$-\tfrac{1}{8}\left(-\sqrt{\pi/2}\,(t-5)\operatorname{sgn}(t-5) - \sqrt{\pi/2}\,(t+5)\operatorname{sgn}(t+5)\right)]$$

$$h(t) = \tfrac{1}{4}\sqrt{\pi/2}\,[(t-1)\operatorname{sgn}(t-1) + (t+1)\operatorname{sgn}(t+1)] +$$

$$\tfrac{1}{4}\sqrt{\pi/2}\,[(t-7)\operatorname{sgn}(t-7) + (t+7)\operatorname{sgn}(t+7)] +.$$

$$\tfrac{1}{4}\sqrt{\pi/2}\,[(t-5)\operatorname{sgn}(t-5) + (t+5)\operatorname{sgn}(t+5)].$$

(5)

(a) we ~~may~~ write.

$$x(t) = \sum_{k=-\infty}^{\infty} \frac{\sin(k\pi/4)}{(k\pi/4)} \delta(t-k\pi/4).$$

$$= \frac{\sin t}{\pi t} \sum_{k=-\infty}^{\infty} x\delta(t-k\pi/4).$$

Therefore $g(t) = \sum_{k=-\infty}^{\infty} x\delta(t-k\pi/4)$.

(b) $G(j\omega) = \frac{\pi \delta x}{x/4} \sum_{k=-\infty}^{\infty} (\omega - 2\pi k/x/4).$

$$= 8x \sum_{k=\infty} \delta(\omega - 8t).$$

$$x(j\omega) = \frac{1}{2\pi} \left\{ f_t \left\{ \frac{\sin t}{xt} \right\} \cdot G(j\omega) \right\}.$$

if fourier transform $\left\{ \frac{\sin t}{xt} \right\}$ by $A(j\omega)$.

$$x(j\omega) = (1/2\pi) \left[A(j\omega) \delta x \sum_{k=-\infty}^{\infty} \delta(\omega - 8t) \right]$$

$\therefore$ $x(j\omega)$ may is be viewed as replication of $4A(j\omega)$ every 8 rad/sec.

$$A(j\omega) = \begin{cases} 1, & |\omega| \leq 1. \\ 0, & \text{otherwise} \end{cases}$$

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(6) The fourier transform of the signal $x[n]$ is $X(e^{j\omega})$. From conjugation and conjugate symmetry

if $x[n]$ is real valued function then it's transform $X(e^{j\omega})$ is conjugate symmetric.

$$\text{odd}\{x[n]\} \xleftrightarrow{F.T} j\,\text{Im}[X(e^{j\omega})]$$

consider the following fact about function $x[n]$.

$$\rightarrow \text{Im}[X(e^{j\omega})] = \sin\omega - \sin 2\omega$$

$$j\,\text{Im}[X(e^{j\omega})] = j\sin\omega - j\sin 2\omega$$

$$j\,\text{Im}[X(e^{j\omega})] = j\left(\frac{e^{j\omega}-e^{-j\omega}}{2j}\right) - j\left(\frac{e^{j2\omega}-e^{-j2\omega}}{2j}\right)$$

$$j\,\text{Im}[X(e^{j\omega})] = \tfrac{1}{2}\left(e^{j\omega} - e^{-j\omega} - e^{2j\omega} + e^{-2j\omega}\right).$$

odd part of the function $x[x]$

$$\text{odd}\{x[x]\} = F^{-1}\{j\,\text{Im}\{x(e^{j\omega})\}\}$$

$$= \tfrac{1}{2}\left(\delta(n+1) - \delta(n-1) - \delta(n+2) + \delta(n-2)\right).$$

$$\text{odd}\{x[n]\} = \frac{x[n] - x[-n]}{2}.$$

Given, $x[x] = 0$ for $n>0$.

$\therefore x[n] = 2\,\text{odd}\{x[n]\}$.

$x[n] = \delta[n+1] - \delta[n+2]$ for $n<0$.

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using parseval's theorme

$$x[0] = \frac{1}{2}\int_{-\pi}^{\pi} |X(e^{j\omega})|^2 d\omega.$$

$$= \sum_{n=-\infty}^{\infty} |x[n]|^2.$$

$$= \frac{1}{2}\int_{-\pi}^{\pi} |X(e^{j\omega})|^2 d\omega$$

$$= \sum_{n=-\infty}^{-1} \left[|x[n]|^2 + |x[0]|^2\right] + \sum_{n=0}^{\infty} |x[n]|^2.$$

using the condition $x[n]=0$ for $n>0$

$$\therefore \sum_{n=0}^{\infty} |x[n]|^2 = 0.$$

$$\frac{1}{2\pi}\int_{-\pi}^{\pi} |X(e^{j\omega})|^2 d\omega = \sum_{n=-\infty}^{-1} |x[n]|^2 + |x[0]|^2$$

$$\left(\because \frac{1}{2\pi}\int_{-\pi}^{\pi} |X(e^{j\omega})|^2 d\omega = 3.\right)$$

$$3 = |x[0]|^2 + \sum_{n=-\infty}^{-1} |x[n]|^2.$$

$$3 = |x[0]|^2 + 2$$

$$x[0] = \pm 1$$

$\therefore$ consider $x[0]=0$ ay $x[0]>0$.

$x[n] = \delta[n]$ for $n=0$.

Function for $x[n]$.

$x[n] = \delta[n+1] - \delta[n+2]$ for $n<0$

$= \delta[n]$ for $n=0$.

1. This function $x[n]$ is the sum of three impulsive functions

The signal $x[n]$ is ; $\boxed{\delta(n)+\delta(n+1)\cdot-\delta[n-2]}$

(7) ~~[illegible]~~

(a) $H(e^{j\omega}) = H_1(e^{j\omega})\, H_2(e^{j\omega})$

$$= \left(\frac{2-e^{-j\omega}}{1+1/2\, e^{-j\omega}}\right)\left(\frac{1}{1-1/2\, e^{-j\omega}+1/4\, e^{-j2\omega}}\right)$$

$$= \frac{(2-e^{-j\omega})}{(1+1/2\, e^{-j\omega})(1-1/2\, e^{-j\omega}+1/4\, e^{-j2\omega})}$$

$$H(e^{j\omega}) = \frac{(2-e^{-j\omega})}{(1-1/2\, e^{-j\omega}+1/4\, e^{-j2\omega}+1/2\, e^{-j\omega}-1/4\, e^{-j2\omega}+1/8\, e^{-j3\omega})}$$

$$H(e^{j\omega}) = \frac{(2-e^{-j\omega})}{(1+1/8\, e^{-j3\omega})}$$

Sub $\dfrac{4(e^{j\omega})}{X(e^{j\omega})}$ for $H(e^{j\omega})$

$$\frac{4(e^{j\omega})}{X(e^{j\omega})} = \frac{2-e^{-j\omega}}{1+1/8\, e^{-j3\omega}}$$

$$\left(4(e^{j\omega})+1/8\, e^{-j3\omega}4(e^{j\omega})\right) = (2X(e^{j\omega})-e^{-j\omega}X(e^{j\omega}))$$

Apply Inverse fourier transform

$y[n] = \frac{1}{8} y[n-3] = 2x[n] - x[n-1]$

Thus, the difference equation of overall system is

$$\boxed{y[n] + \frac{1}{8} y[n-3] = 2x[n] - x[n-1]}$$

(b). $H(e^{j\omega}) = \left(\frac{2 - e^{-j\omega}}{1 + \frac{1}{2} e^{-j\omega}}\right) \left(\frac{1}{1 - \frac{1}{2} e^{-j\omega} + \frac{1}{4} e^{-j2\omega}}\right)$

$$H(e^{j\omega}) = \left(\frac{2e^{j\omega} - 1}{e^{j\omega} + \frac{1}{2}}\right) \left(\frac{e^{j2\omega}}{e^{j2\omega} - \frac{1}{2} e^{j\omega} + \frac{1}{4}}\right)$$

$$\frac{H(e^{j\omega})}{e^{j\omega}} = \frac{(2e^{j\omega} - 1) e^{j\omega}}{(e^{j\omega} + \frac{1}{2})(e^{j\omega} - (\frac{1}{4} + j\frac{\sqrt{3}}{4}))(e^{j\omega} - (\frac{1}{4} - j\frac{\sqrt{3}}{4}))}$$

By using partial fractions.

$$H(e^{j\omega}) = \left(\frac{A}{e^{j\omega} + \frac{1}{2}}\right) + \left(\frac{B}{e^{j\omega} - \frac{1}{2}(\frac{1}{2} + j\frac{\sqrt{3}}{2})}\right) + \left(\frac{C}{e^{j\omega} - \frac{1}{2}(\frac{1}{2} - j\frac{\sqrt{3}}{4})}\right)$$

By simplify,

$A = 4/3 \quad B = 0.333 + 0.577j \quad C = 0.333 - 0.577j$

$$\frac{H(e^{j\omega})}{e^{j\omega}} = \frac{4/3}{e^{j\omega} + \frac{1}{2}} + \left[\frac{0.333 + 0.577j}{e^{j\omega} - \frac{1}{2}(\frac{1}{2} + j\frac{\sqrt{3}}{2})}\right] + \left(\frac{0.333 - 0.577j}{e^{j\omega} - \frac{1}{2}[-\frac{1}{2} - j\frac{\sqrt{3}}{4})}\right)$$

$\frac{1}{\sqrt{2}} + j\frac{\sqrt{3}}{2} = 60^\circ$

$$H(e^{j\omega}) = \left(\frac{4/3}{1+1/2\,e^{-j\omega}}\right) + \left(\frac{1+i\sqrt{3}}{3}\right)\left(\frac{1}{1-1/2\,e^{-j\omega}(e^{-60^\circ})}\right)$$

$$+\left(\frac{1-i\sqrt{3}}{3}\right)\left(\frac{1}{1-1/2\,e^{-j\omega}(e^{-j60})}\right)$$

Inverse fourier transform

$$h(n) = 4/3\,(-1/2)^n u[n] + \left(\frac{1+i\sqrt{3}}{3}\right)\left(1/2\,e^{j60^\circ}\right)^m u[n]$$

$$+\left(\frac{1-i\sqrt{3}}{3}\right)\left(1/2\,e^{-j60^\circ}\right)^m u[n]$$