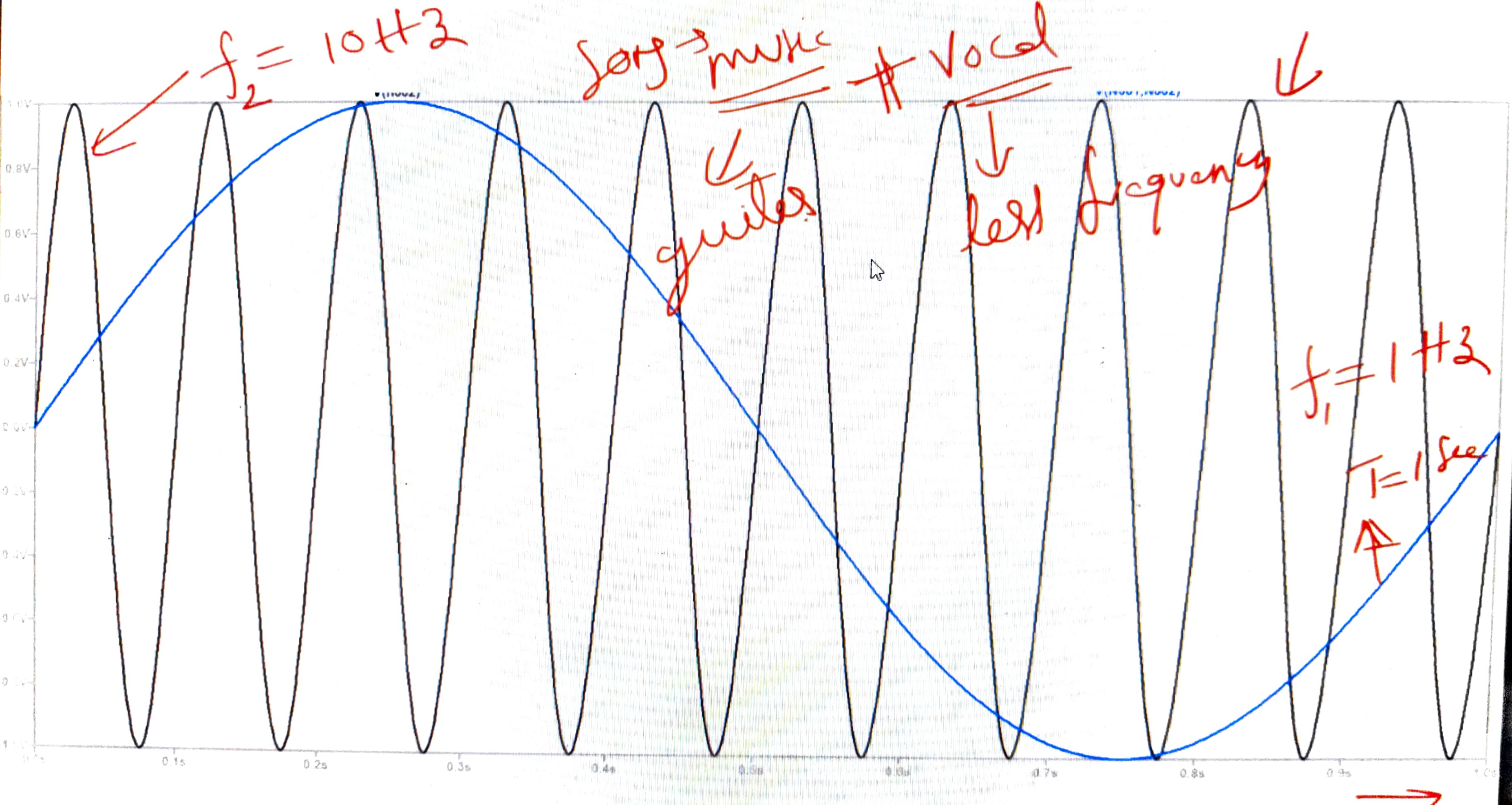
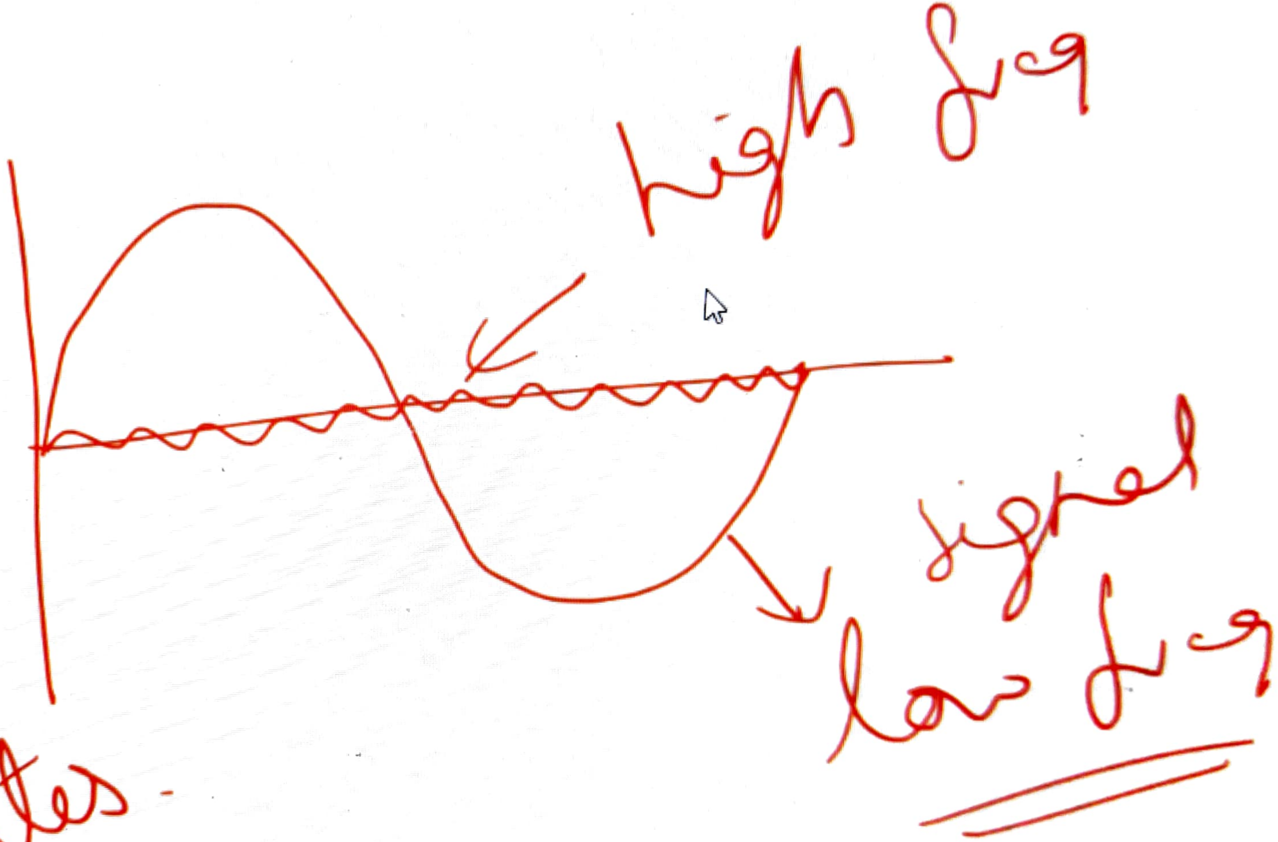
Active Filters



$f_2 = 10 Hz$
Song → music
vocal
guiter
less frequency
$f_1 = 1 Hz$
$T = 1 sec$
0.8V
0.6V
0.4V
0.2V
0.0V
0.1s
0.2s
0.3s
0.4s
0.5s
0.6s
0.7s
0.8s
0.9s

Separate signal from noise

↓ filter.



Electric filters

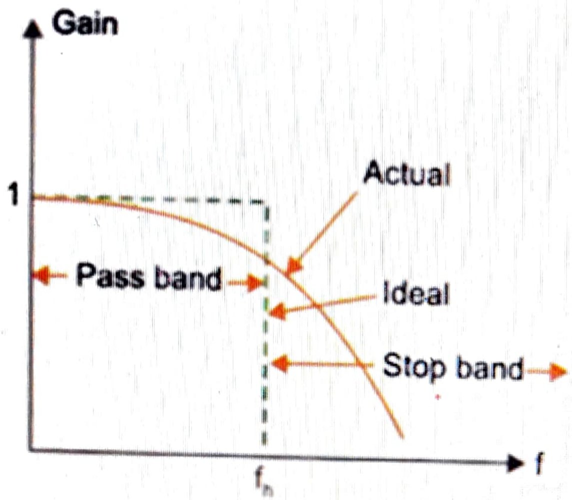
Filter: Which allows/pass the input signals to the output in desired range of frequencies

- Passive filters (R, L, C)
  - Input signal is not amplified
- Active filters (transistors, op-amps)
  - Input signal is amplified

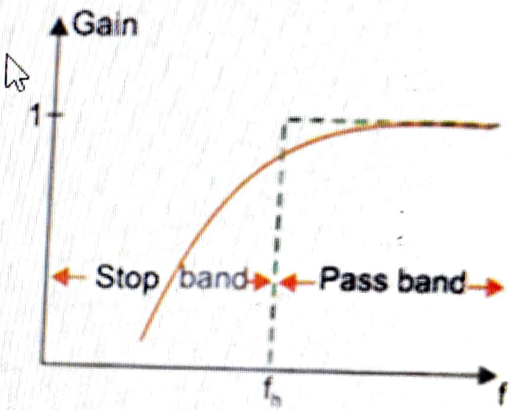
Filter classification

- Low pass filter (LPF)
- High pass filter (HPF)
- Band pass filter (BPF)
- Band reject filter/Band stop filter (BSF)

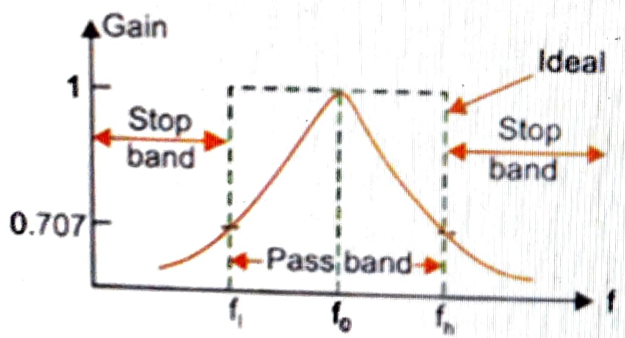
Frequency response of various filters



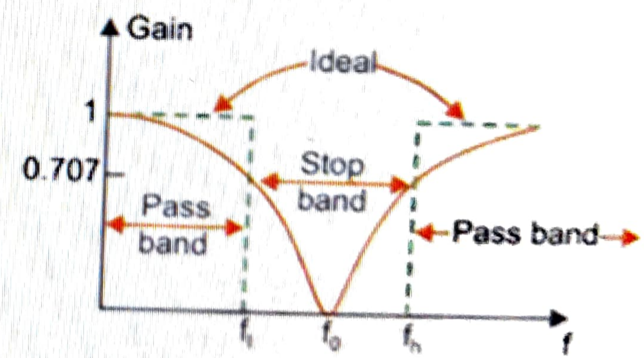
(a) **LPF**



(b) **HPF**



(c) **BPF**



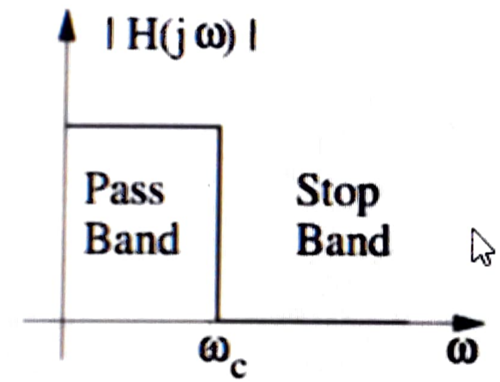
(d) **BSF**

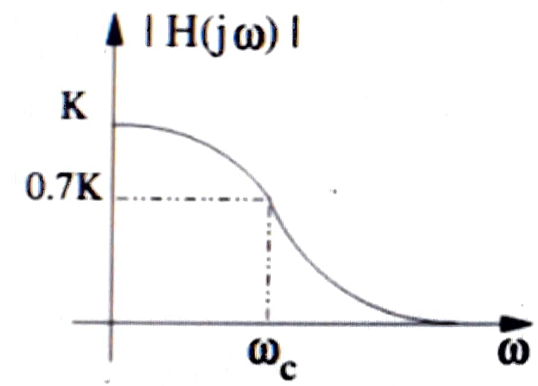
Passive Low-Pass RL filter

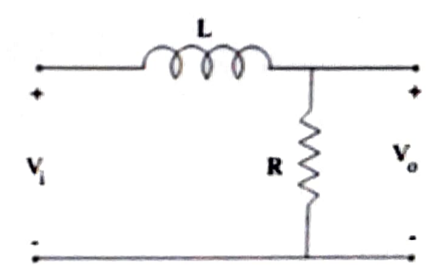
- Transfer function of RL LPF

$$V_o = \frac{R}{R + j\omega L} V_i \quad \rightarrow \quad H(j\omega) = \frac{V_o}{V_i} = \frac{R}{R + j\omega L} = \frac{1}{1 + j(\omega L/R)}$$

$$|H(j\omega)| = \frac{1}{\sqrt{1 + (\omega L/R)^2}}$$







Cut-off frequency

- Ideal filter: The frequency between the pass- and-stop bands is called the cut-off frequency (ω_c)
- Practical filter: The frequency at which the magnitude $|H(j\omega)|$ is reduced to $1/\sqrt{2}$ (=0.7) times the maximum magnitude

AC

R L C

Impedance — Z

$Z = R + jX_L - jX_C$

$= R + j(X_L - X_C)$

$X_L \rightarrow$ inductive

Passive Low-Pass RL filter

RC

DC ckt Resistance

- Transfer function of RL LPF

$$V_o = \frac{R}{R + j\omega L} V_i \quad \rightarrow \quad H(j\omega) = \frac{V_o}{V_i} = \frac{R}{R + j\omega L} = \frac{1}{1 + j(\omega L/R)}$$

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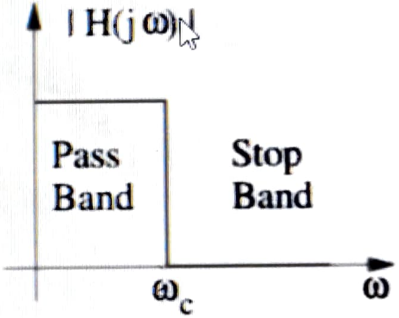
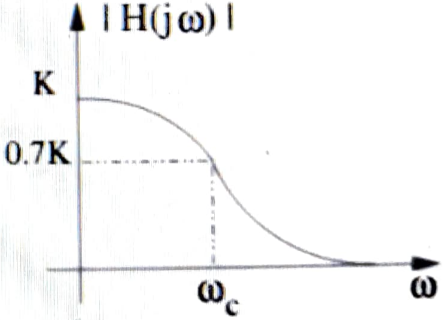
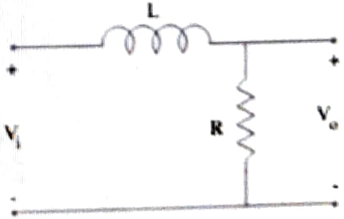






Cut-off frequency

- Ideal filter: The frequency between the pass- and-stop bands is called the cut-off frequency (ω_c)
- Practical filter: The frequency at which the magnitude $|H(j\omega)|$ is reduced to $1/\sqrt{2}$ (=0.7) times the maximum magnitude

$$Z = R + j(X_L - X_C)$$

real imaginary part

$$|Z| = \sqrt{R^2 + (X_L - X_C)^2}$$

$$\theta = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$

RL ckt

$$Z = R + jX_L$$

$$|Z| = \sqrt{R^2 + X_L^2}$$

RC

$$|Z| = \sqrt{R^2 + X_C^2}$$

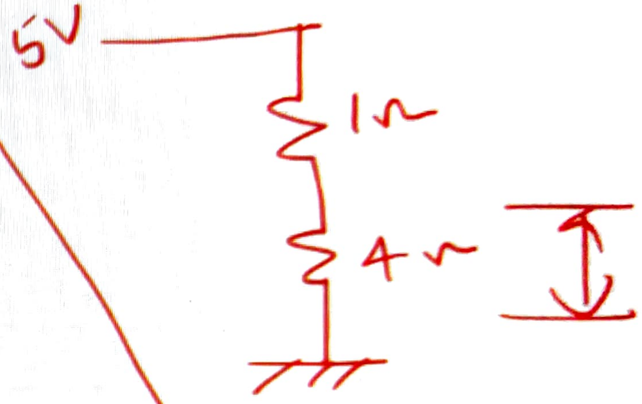
$$|H(j\omega)| = \frac{1}{\sqrt{1+\left(\frac{\omega L}{R}\right)^2}}$$

at $f = 0$ Hz

$$\rightarrow |H(j0)| = \frac{1}{0} = \infty$$

$f = \infty$ Hz

$$\rightarrow |H(j\infty)| = \frac{1}{\infty} = 0$$



$$\frac{4}{1+4} \times 5 = \underline{\underline{4V}}$$

Cut-off frequency of RL LPF

$$|H(j\omega)| = \frac{1}{\sqrt{1+\left(\frac{\omega L}{R}\right)^2}}$$

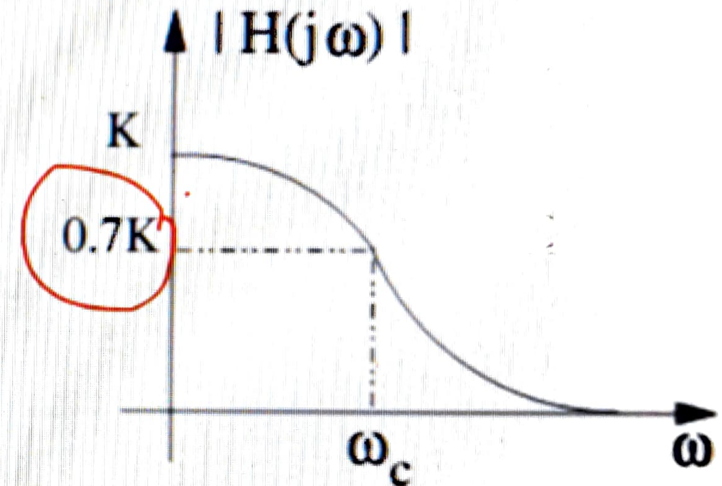
$$At\ f = 0\,Hz,$$

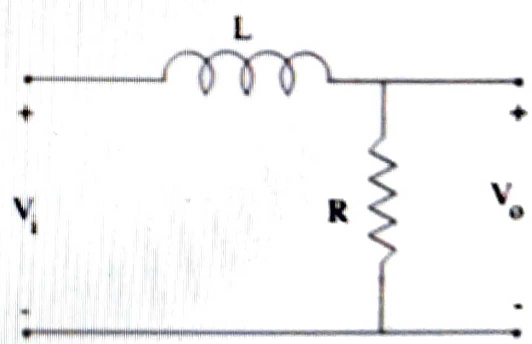
$$|H(j\omega)| = 1$$

$$At\ \omega = \omega_c, |H(j\omega)| = 1/\sqrt{2}$$

$$\frac{1}{\sqrt{1+\left(\frac{\omega_c L}{R}\right)^2}} = \frac{1}{\sqrt{2}}$$

$$\omega_c = \frac{R}{L}\ and\ H(j\omega) = \frac{1}{1+j\left(\frac{\omega}{\omega_c}\right)}$$





Cut-off frequency of RL LPF

$2\,KHz$

$f_c = 1\,KHz$

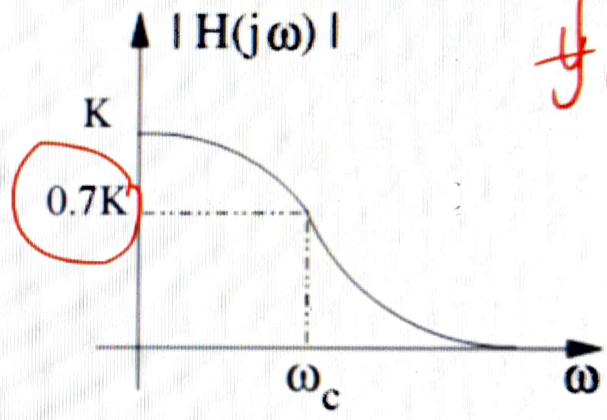
$$|H(j\omega)| = \frac{1}{\sqrt{1+\left(\frac{\omega L}{R}\right)^2}}$$

$At\ f = 0\,Hz,$

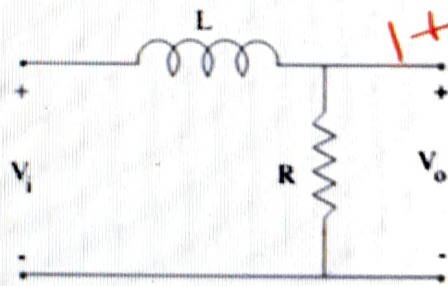
$$|H(j\omega)| = 1$$

$$At\ \omega = \omega_c, |H(j\omega)| = 1/\sqrt{2}$$

$$\frac{1}{\sqrt{1+\left(\frac{\omega_c L}{R}\right)^2}} = \frac{1}{\sqrt{2}} = 0.7$$



$$1+\left(\frac{\omega_c L}{R}\right)^2 = 2$$



$$\frac{\omega_c L}{R} = 1$$

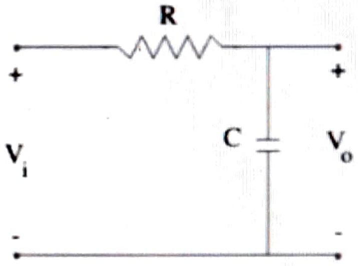
$$\omega_c = \frac{R}{L}\ and\ H(j\omega) = \frac{1}{1+j\left(\frac{\omega}{\omega_c}\right)}$$

$$\omega_c = \frac{R}{L} \Rightarrow \left(f_c = \frac{R}{2\pi L}\right)$$

RC Low Pass Filter Circuit

$$V_o = \frac{1/(j\omega C)}{R + 1/(j\omega C)} V_i = \frac{1}{1 + j(\omega RC)} V_i$$

$$H(j\omega) = \frac{1}{1 + j\omega RC}$$



$$= \frac{Z_C}{Z} V_i$$

$$X_C = (2\pi f) C$$

$$\omega$$

$$j^2 = -1$$

$$j = -\frac{1}{j}$$

$$Z_C = -jX_C = -j/\omega C$$

$$= \frac{1}{j\omega C}$$

$$v_o = \frac{Z_C}{R - jX_C}$$

$$= \frac{-jX_C}{R - jX_C}$$

$$|H| = \frac{1}{\sqrt{1 + (\omega RC)^2}}$$

Low Pass Filter Example

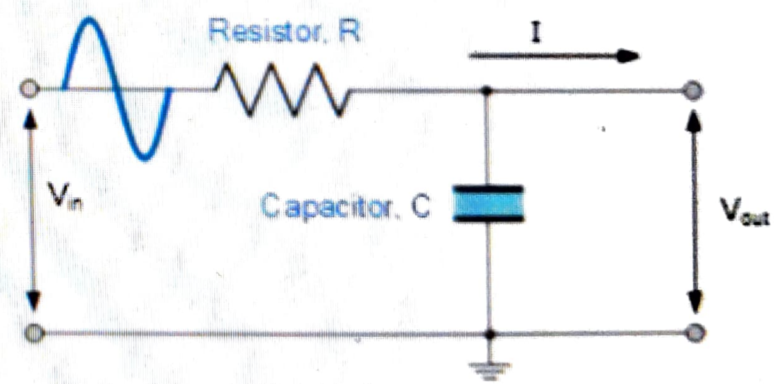
A Low Pass Filter circuit consisting of a resistor of 4.7 kΩ in series with a capacitor of 47 nF is connected across a 10 V sinusoidal supply. Calculate the output voltage (V_{out}) at a frequency of 100 Hz, 200 Hz and again at frequency of 10kHz, 11 kHz.

$$X_c = \frac{1}{2\pi fC}$$

$$Z = R - jX_c$$

$$|Z| = \sqrt{R^2 + X_c^2}$$

$$V_{out} = V_{in}\frac{X_c}{\sqrt{R^2 + X_c^2}} = V_{in}\frac{X_c}{Z}$$



Low Pass Filter Example

$X_c = 10\sqrt{47}$

A Low Pass Filter circuit consisting of a resistor of 4.7 kΩ in series with a capacitor of 47 nF is connected across a 10 V sinusoidal supply. Calculate the output voltage (V_{out}) at a frequency of 100 Hz, 200 Hz and again at frequency of 10kHz, 11 kHz.

$G \rightarrow 100 \rightarrow$

$200\,Hz \rightarrow$

$10\,KHz \rightarrow$

$11\,KHz \rightarrow$

$$X_c = \frac{1}{2\pi fC}$$

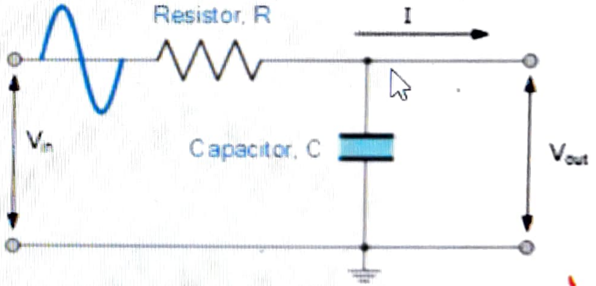
$$Z = R - jX_c$$

$$|Z| = \sqrt{R^2 + X_c^2}$$

$$V_{out} = V_{in}\frac{X_c}{\sqrt{R^2 + X_c^2}} = V_{in}\frac{X_c}{Z}$$

$X_c = \frac{1}{2\pi f_c}$ $2\pi f_c R = 1$ $\rightarrow f_c = \frac{1}{2\pi RC}$

$G = \frac{1}{\sqrt{2}}$



$\frac{X_c}{\sqrt{R^2 + X_c^2}} = \frac{1}{\sqrt{2}}$

$\frac{1}{\sqrt{\left(\frac{R}{X_c}\right)^2 + 1}} = \frac{1}{\sqrt{2}}$

$\frac{R}{X_c} = 1$

$X_c = R$