A* Search

- The most common informed search algorithm is A* search (pronounced “A-star search”), a best-first search that uses the evaluation function,

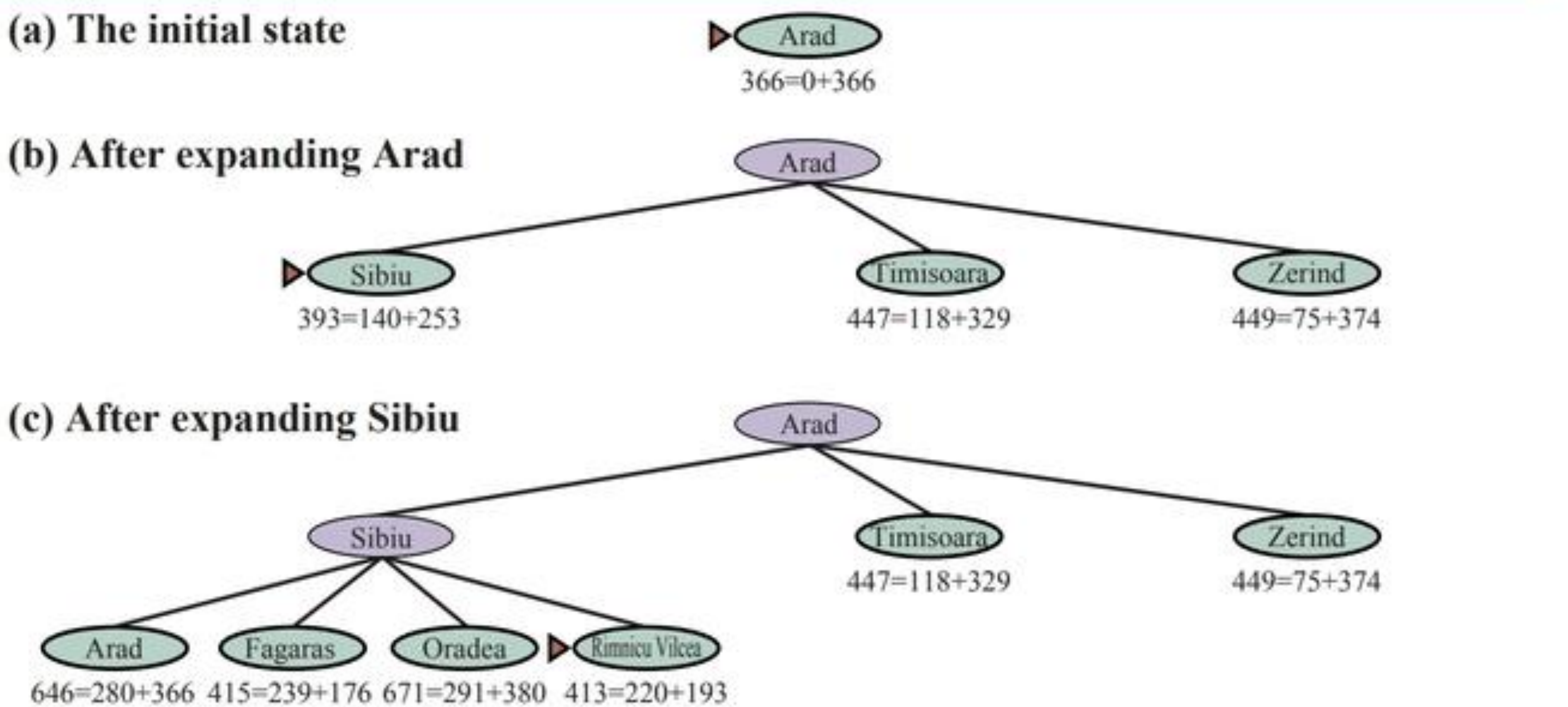
$f(n) = g(n) + h(n)$

where $g(n)$ is the path cost from the initial state to node n, and $h(n)$ is the estimated cost of the shortest path from n to a goal state, so we have,

$f(n)$ = estimated cost of the best path that continues from n to a goal.

Arad	366	**Mehadia**	241
Bucharest	0	**Neamt**	234
Craiova	160	**Oradea**	380
Drobeta	242	**Pitesti**	100
Eforie	161	**Rimnicu Vilcea**	193
Fagaras	176	**Sibiu**	253
Giurgiu	77	**Timisoara**	329
Hirsova	151	**Urziceni**	80
Iasi	226	**Vaslui**	199
Lugoj	244	**Zerind**	374

Figure 3.16 Values of h_{SLD}—straight-line distances to Bucharest.


(a) The initial state
Arad
366=0+366
(b) After expanding Arad
Arad
Sibiu
393=140+253
Timisoara
447=118+329
Zerind
449=75+374
(c) After expanding Sibiu
Arad
Sibiu
Timisoara
447=118+329
Zerind
449=75+374
Arad
646=280+366
Fagaras
415=239+176
Oradea
671=291+380
Rimnicu Vilcea
413=220+193

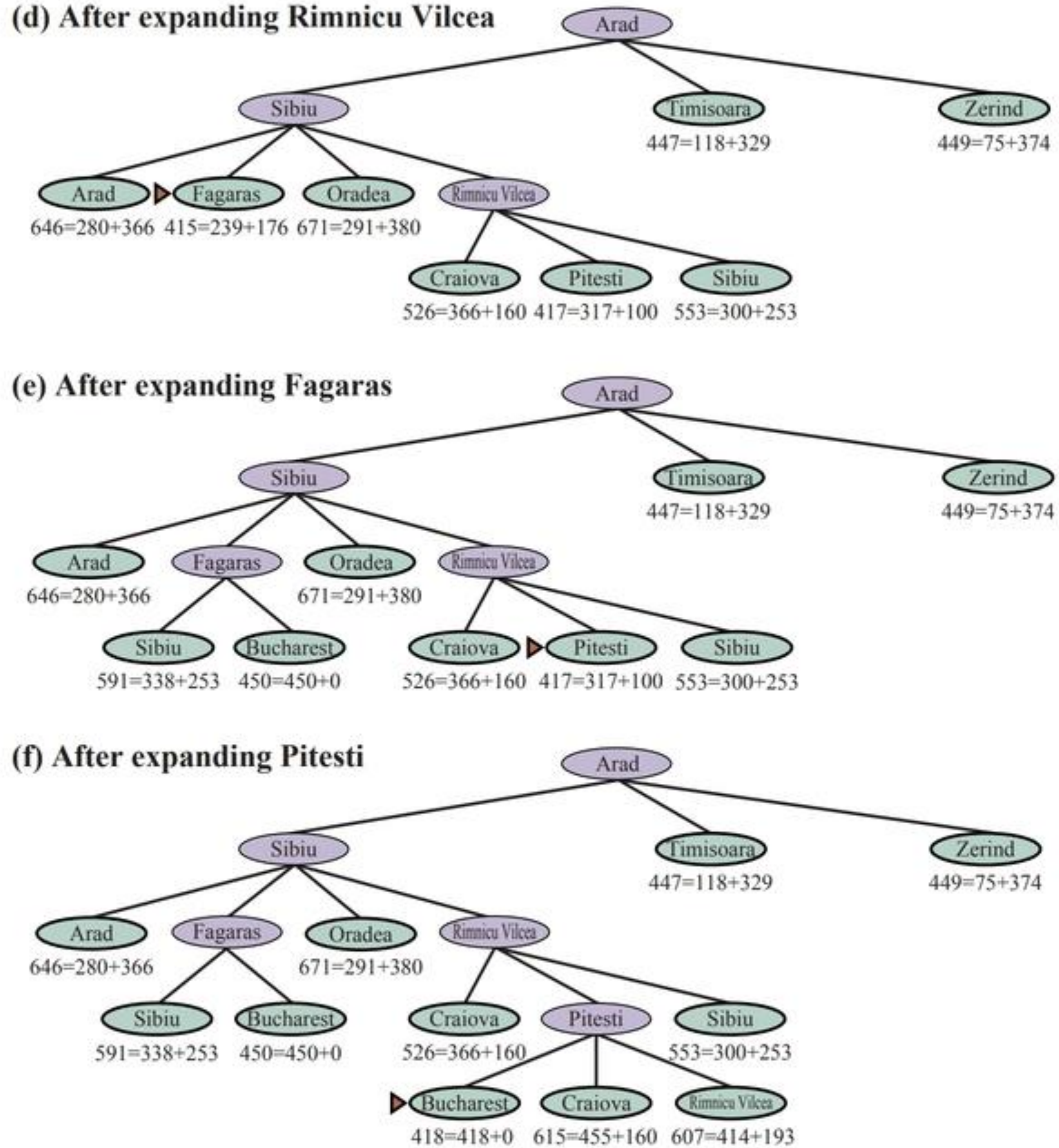


Figure 3.18 Stages in an A* search for Bucharest. Nodes are labeled with $f = g + h$. The h values are the straight-line distances to Bucharest taken from Figure 3.16.

A* search is complete.!!

- *Whether A* is cost-optimal depends on certain properties of the heuristic.*

- A key property is admissibility an admissible heuristic is one that never overestimates the cost to reach a goal. (An admissible heuristic is therefore optimistic.)

- With an admissible heuristic, A* is cost-optimal.

- Suppose the optimal path has cost C*, but the algorithm returns a path with cost C > C*. **Then there must be some node n which is on the optimal path and is unexpanded.**

- The notation g*(n) denote the cost of the optimal path from the start to n, and h*(n) denote the cost of the optimal path from n to the nearest goal, we have:

$$
\begin{aligned}
f(n) &> C^* \quad \text{(otherwise } n \text{ would have been expanded)} \\
f(n) &= g(n) + h(n) \quad \text{(by definition)} \\
f(n) &= g^*(n) + h(n) \quad \text{(because } n \text{ is on an optimal path)} \\
f(n) &\leq g^*(n) + h^*(n) \quad \text{(because of admissibility, } h(n) \leq h^*(n)\text{)} \\
f(n) &\leq C^* \quad \text{(by definition, } C^* = g^*(n) + h^*(n)\text{)}
\end{aligned}
$$

- a suboptimal path must be wrong—it must be that A* returns only cost-optimal paths.

- A heuristic i(n) is consistent if, for every node n and every successor n’ of n generated by an action a, we have:

$$h(n) \leq c(n, a, n') + h(n').$$

- This is a form of the triangle inequality, which stipulates that a side of a triangle cannot be longer than the sum of the other two sides.

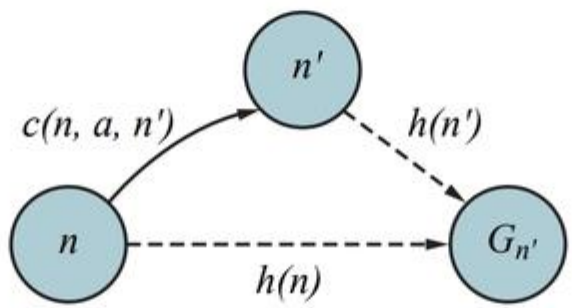


Figure 3.19 Triangle inequality: If the heuristic h is **consistent**, then the single number $h(n)$ will be less than the sum of the cost $c(n, a, a')$ of the action from n to n' plus the heuristic estimate $h(n')$.

Search contours

- A useful way to visualize a search is to draw contours in the state space, just like the contours in a topographic map.

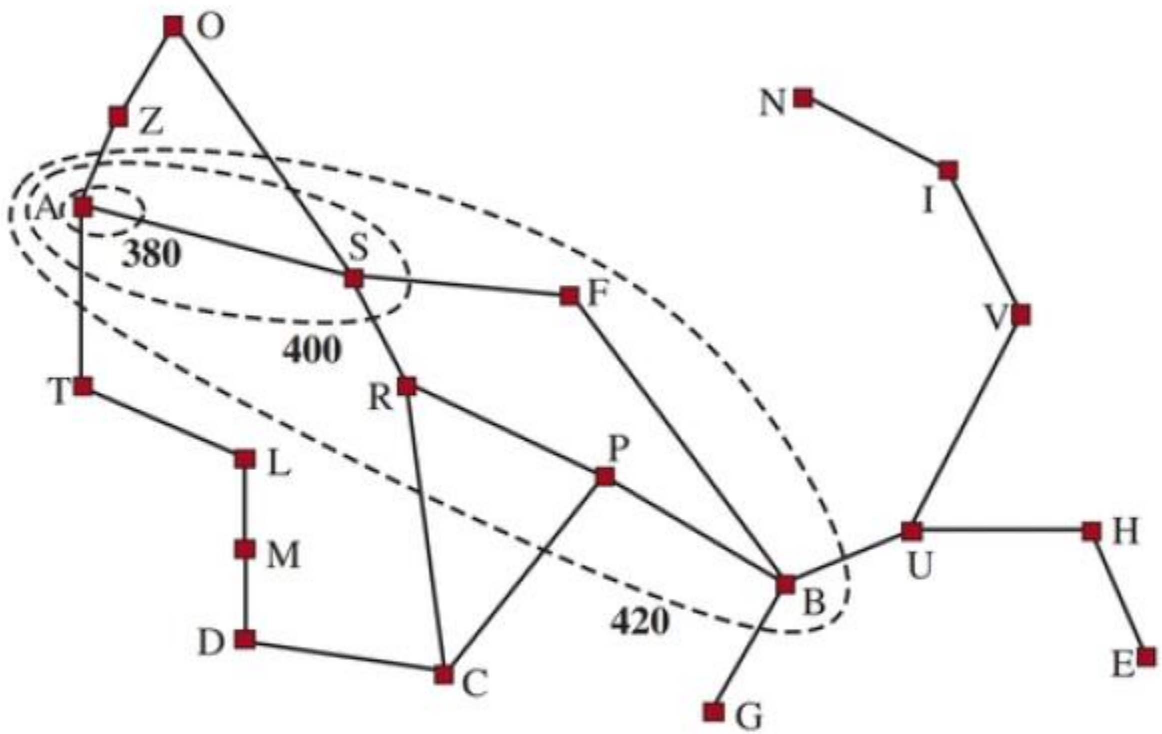
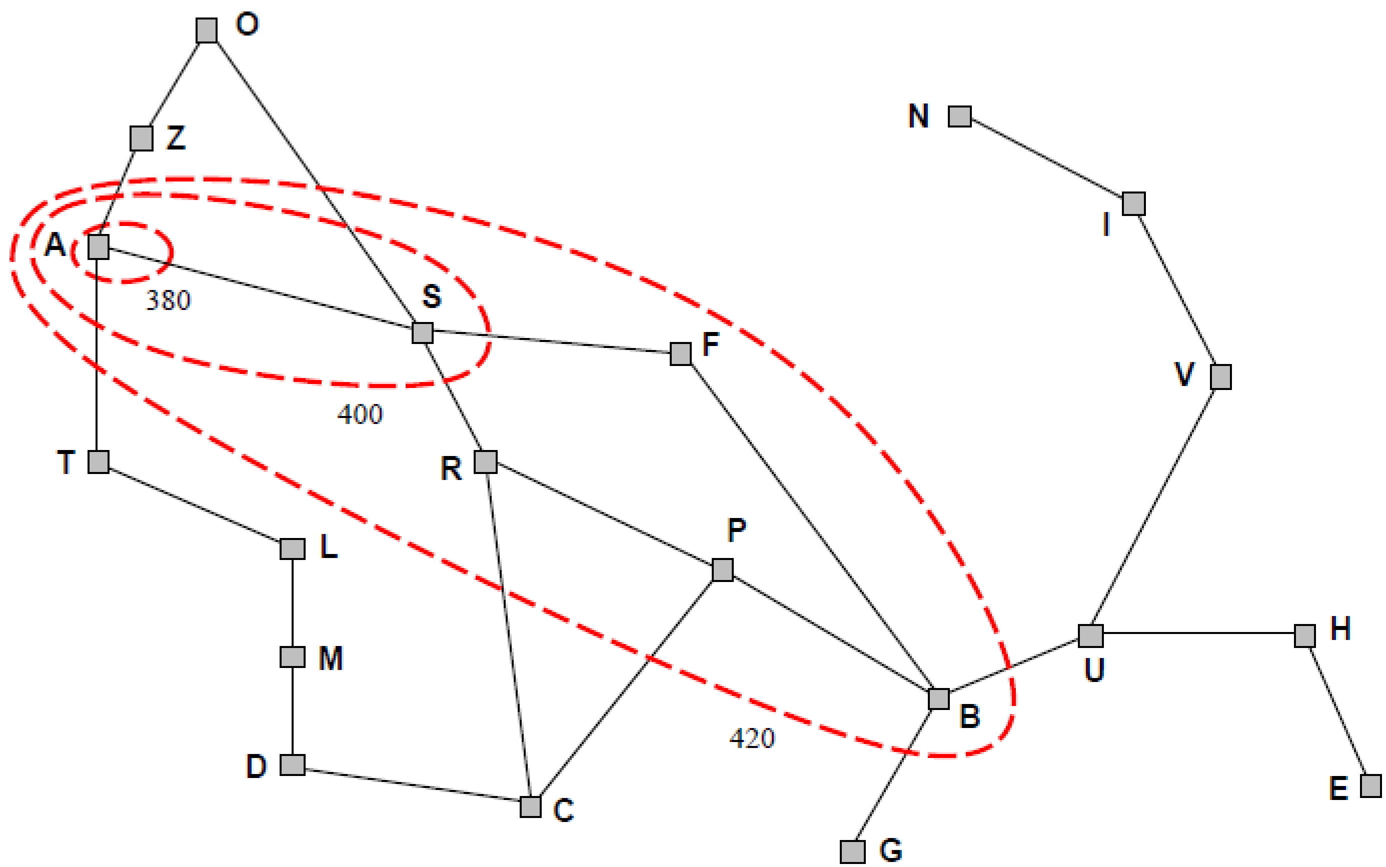


Figure 3.20 Map of Romania showing contours at $f = 380$, $f = 400$, and $f = 420$, with Arad as the start state. Nodes inside a given contour have $f = g + h$ costs less than or equal to the contour value.

Optimality of A* (more useful)

Lemma: A* expands nodes in order of increasing f value*

Gradually adds "f-contours" of nodes (cf. breadth-first adds layers)
Contour i has all nodes with $f = f_i$, where $f_i < f_{i+1}$



Properties of A*

Complete?? Yes, unless there are infinitely many nodes with $f \leq f(G)$

Time?? Exponential in [relative error in $h \times$ length of soln.]

Space?? Keeps all nodes in memory

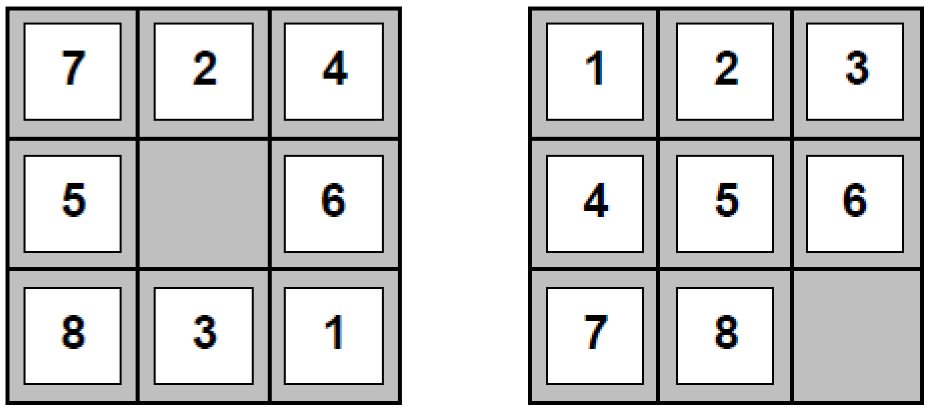
Optimal?? Yes—cannot expand f_{i+1} until f_i is finished

A* expands all nodes with $f(n) < C^*$
A* expands some nodes with $f(n) = C^*$
A* expands no nodes with $f(n) > C^*$

Admissible heuristics

E.g., for the 8-puzzle:

$h_1(n)$ = number of misplaced tiles
$h_2(n)$ = total Manhattan distance
(i.e., no. of squares from desired location of each tile)



Start State Goal State

$\underline{h_1(S)\ =}??$
$\underline{h_2(S)\ =}??$

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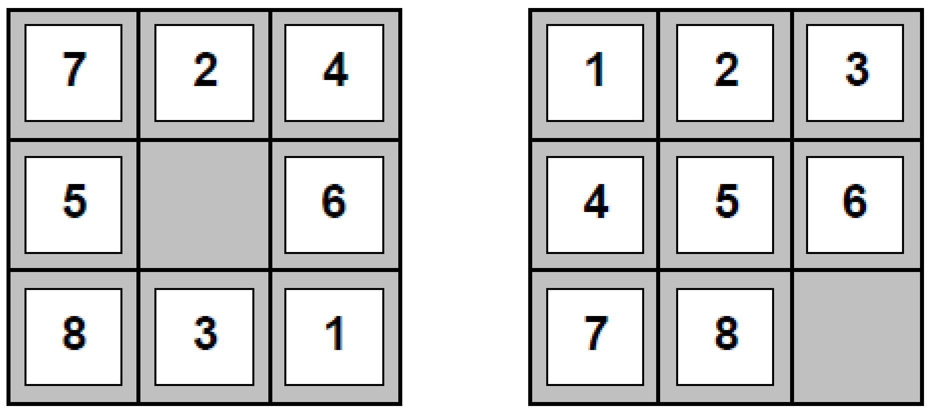


Start State   Goal State

$h_1(S) =??$ 6
$h_2(S) =??$ 4+0+3+3+1+0+2+1 = 14

Dominance

If $h_2(n) \geq h_1(n)$ for all n (both admissible)
then h_2 dominates h_1 and is better for search

Typical search costs:

$d = 14$ IDS = 3,473,941 nodes
A$^*(h_1)$ = 539 nodes
A$^*(h_2)$ = 113 nodes
$d = 24$ IDS $\approx$ 54,000,000,000 nodes
A$^*(h_1)$ = 39,135 nodes
A$^*(h_2)$ = 1,641 nodes

Given any admissible heuristics h_a, h_b,

$$h(n) = \max(h_a(n), h_b(n))$$

is also admissible and dominates h_a, h_b

Relaxed problems

Admissible heuristics can be derived from the **exact** solution cost of a **relaxed** version of the problem

If the rules of the 8-puzzle are relaxed so that a tile can move **anywhere**, then $h_1(n)$ gives the shortest solution

If the rules are relaxed so that a tile can move to **any adjacent square**, then $h_2(n)$ gives the shortest solution

Key point: the optimal solution cost of a relaxed problem is no greater than the optimal solution cost of the real problem

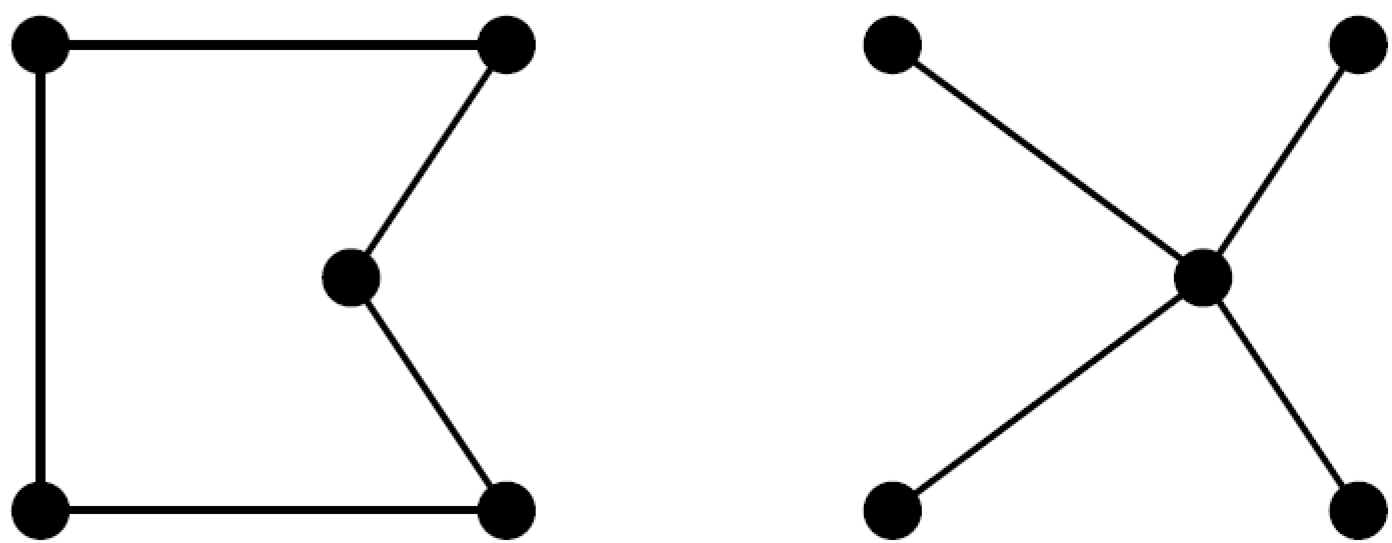
- A problem with fewer restrictions on the actions is called a relaxed problem.

- The state-space graph of the relaxed problem is a supergraph of the original state space because the removal of restrictions creates added edges in the graph.

- *Hence, the cost of an optimal solution to a relaxed problem is an admissible heuristic for the original problem.*

Relaxed problems contd.

Well-known example: travelling salesperson problem (TSP)
Find the shortest tour visiting all cities exactly once



Relaxed problems contd.

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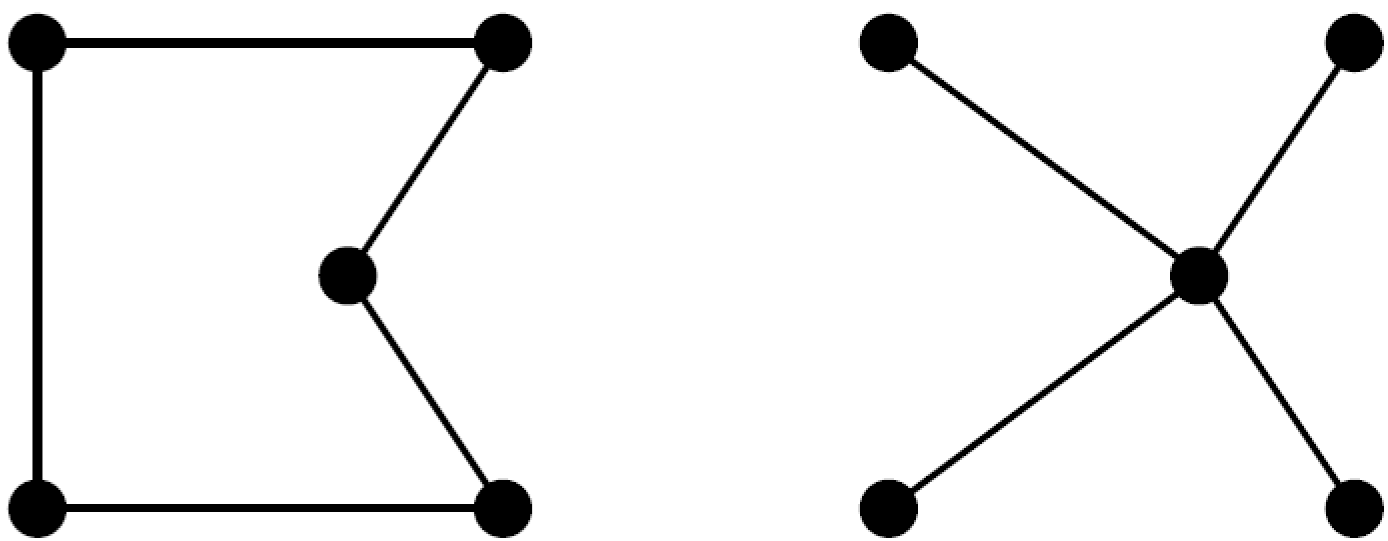

Minimum spanning tree can be computed in $O(n^2)$
and is a lower bound on the shortest (open) tour

Summary

Heuristic functions estimate costs of shortest paths

Good heuristics can dramatically reduce search cost

Greedy best-first search expands lowest h
– incomplete and not always optimal

A* search expands lowest $g + h$
– complete and optimal
– also optimally efficient (up to tie-breaks, for forward search)

Admissible heuristics can be derived from exact solution of relaxed problems