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①

M3 G-2

Q-1

$$a_n = \frac{\sin(n^2+1)}{n^2+1} - n^2 + \sqrt{n^4+n^2}$$

$$-1 \le \sin(n^2+1) \le 1$$

$$\frac{-1}{n^2+1} \le \frac{\sin(n^2+1)}{n^2+1} \le \frac{1}{n^2+1}$$

as we know $\frac{-1}{n^2+1}$, $\frac{1}{n^2+1}$ converge to 0

$\frac{\sin(n^2+1)}{n^2+1}$ converges to 0

$$a_n = \frac{\sin(n^2+1)}{n^2+1} + n^2\left(\sqrt{1+\frac{1}{n^2}} - 1\right)$$

$$v_n = \lim_{n\to\infty} \frac{\sqrt{1+\frac{1}{n^2}} - 1}{1/n^2}$$

$$v_n = \lim_{n\to\infty} \frac{\left(\sqrt{1+\frac{1}{n^2}} - 1\right)\left(\sqrt{1+\frac{1}{n^2}} + 1\right)}{1/n^2\left(\sqrt{1+\frac{1}{n^2}} + 1\right)}$$

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$$\lim_{n\to\infty} \frac{\left(\sqrt{1+\frac{1}{n^2}}-1\right)}{\frac{1}{n^2}\left(\sqrt{1+\frac{1}{n^2}}+1\right)} = u_n$$

$$= 1/2$$

Hence the ~~series~~ u_n converges and the limit at $n\to\infty$ is $1/2$.

~~a_n has 2 limit points 0 & 1/2~~
~~therefore it will oscillate b/w 0 & 1/2.~~

therefore a_n converges at $1/2$.

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Q-2

$$\sum_{n=1}^{\infty} \frac{5n^2-3n+4}{n^3+5n}$$

$$u_n = \sum_{n=1}^{\infty} \frac{5n^2-3n+4}{n(n^2+5)}$$

Applying limit form

let $v_n = \frac{1}{n}$

$$\lim_{n\to\infty} \frac{u_n}{v_n} = \lim_{n\to\infty} \frac{\frac{5n^2-3n+4}{n(n^2+5)}}{1/n}$$

$$= \lim_{n\to\infty} \frac{5n^2-3n+4}{n^2+5} = 2 \neq 0$$ and finite

Acc. to p test
if v_n diverges then U_n also diverges

Therefore, V_n diverges

Hence U_n also diverges.

Hence does not converges.

Q.3

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$$f(x) = \frac{x\cos x}{x^2} + 2x + \sin 1/x$$

given interval $[-1, 1]$

and we know that $\lim_{n \to \infty} f_n(x) = f(x)$

$$\lim_{n \to \infty} \frac{x\cos x}{n^2} + 2x + \sin 1/n \quad \text{on } -1, 1$$

$$\frac{\cos x}{n^2} \to 0$$

$$\sin 1/n \to \sin(0) = 0$$

then $f(x) = 2x \qquad -1 \le x \le 1$

Hence it will be a point wise convergence.

$\lim_{n \to \infty} f(x) =$

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Q-9 $x^4 + 4x^3 + 6x^2 - x - 3 = 0$

x	0	1	2
$f(x)$	-3	7	67

$f(0) \cdot f(1) < 0$

$\therefore$ a root lies b/w 0 & 1

$a = 0, \quad b = 1$

now,

n	a_n	b_n	$x_n = \frac{a_n + b_n}{2}$	$f(x_n)$
0	0	1	0.5	-1.4375
1	0.5	1	0.75	1.6289
2	0.5	0.75	0.625	-0.150099
3	0.625	0.75	0.6875	0.671676
4	0.625	0.6875	0.65625	0.243699

$\therefore$ $\boxed{\text{Solution} = 0.65625}$

Q-5 (c)

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[illegible]

(a) $f(x) = \sqrt{x}$ $f(5.75) = ?$

$x = 5.75$

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$
5	2.236			
		0.213		
			-0.016	
6	2.449			0.001
		0.197		
7	2.646		-0.015	
		0.182		
8	2.828			

$x = 5.75$, $x_0 = 5$, $h = 1$

$$f(x) = f(x_0) + \frac{(x-x_0)\Delta f(x_0)}{\underline{1}\,h} + \frac{(x-x_0)(x-x_1)(x-x_2)\Delta^3 f(x_0)}{\underline{2}\,h}$$

$$+ \frac{(x-x_0)\Delta^3 f(x_0)(x-x_1)(x-x_2)}{\underline{3}\,h^3}$$

$$= 2.236 + \frac{(5.75-5)(0.213)}{1} + \frac{(5.75-5)(5.75-6)(-0.016)}{\underline{2}(1)}$$

$$+ \frac{(5.75-5)(0.001)(5.75-6)(5.75-7)}{6\times 1}$$

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$= 2.236 + 0.15975 + 0.0015 + 0.000039$

$= 2.397$ (upto 3 decimals)

$$\boxed{f(5.75) = \sqrt{5.75} = 2.397}$$

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Q-5 (b)

$I = \int_0^1 2x - 3x^2 \, dx$

$f(x) = 2x - 3x^2$

$I = \left[2 \cdot \frac{x^2}{2} - \frac{3x^3}{3}\right]_0^1 = [x^3 - x^2]_0^1$

$= 0$

$N = 6, \quad h = \frac{1-0}{6}$ Nodes $= 0, \frac{1}{6}, \frac{2}{6}, \frac{3}{6}, \frac{4}{6}, \frac{5}{6}, 1$

$I' = \frac{h}{2}\Big(f(x_0) + 2\big(f(x_1) + f(x_2) + f(x_3) \dots + f(x_{n-1})\big)$
$+ f(x_n)\Big)$

$= \frac{1/6}{2}\Big(f(0) + 2\big(f(1/6) + f(2/6) + f(3/6) + f(4/6)$
$+ f(5/6)\big) + f(1)\Big)$

$= \frac{1}{12}\Big(0 + 2\Big[\left(\frac{2}{6} - \frac{3}{36}\right) + \left(\frac{4}{6} - \frac{12}{36}\right) + \left(\frac{6}{6} - \frac{27}{36}\right)$
$+ \left(\frac{8}{6} - \frac{48}{36}\right) + \left(\frac{10}{6} - \frac{75}{36}\right)\Big] + (2-3)\Big)$

$= \frac{1}{12}\left(2\left(\frac{9}{36} + \frac{12}{36} + \frac{9}{36} + 0 - \frac{15}{36}\right) - 1\right)$

$= \frac{1}{12}\left(2\left(\frac{1}{4} + \frac{1}{3} + \frac{1}{4} - \frac{5}{12}\right) - 1\right)$

$= \cancel{\frac{}{108}} = \cancel{0.1018}$ $= \cancel{\frac{-1}{72}} = \cancel{-0.0139}$

Error $=$ $\cancel{I' - I = 0.1018 - 0} = \cancel{\boxed{0.1018}}$

$= -1/72 = -0.0139$

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$$\text{Error} = |I' - I|$$

$$= |-0.0139 - 0|$$

$$= 0.0139$$