Gate-Level Minimization
Karnaugh map or K-map

Gate-Level Minimization

- Design task of finding an optimal gate-level implementation of the Boolean functions describing a digital circuit
- The complexity of implementing digital logic gates is directly related to the complexity of the algebraic expression
- Simple straightforward procedure for minimizing Boolean functions is
  - Karnaugh map or K-map

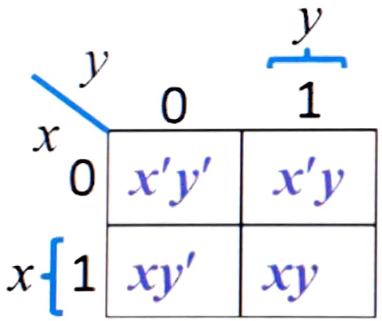
Karnaugh map

- Diagram of squares
- Each square represents one minterm of the function that is to be minimized

Two-variable K-map

- Two variables
- Four minterms
- The K-map consists of four squares, one for each minterm

m_0	m_1
m_2	m_3



00	01
10	11

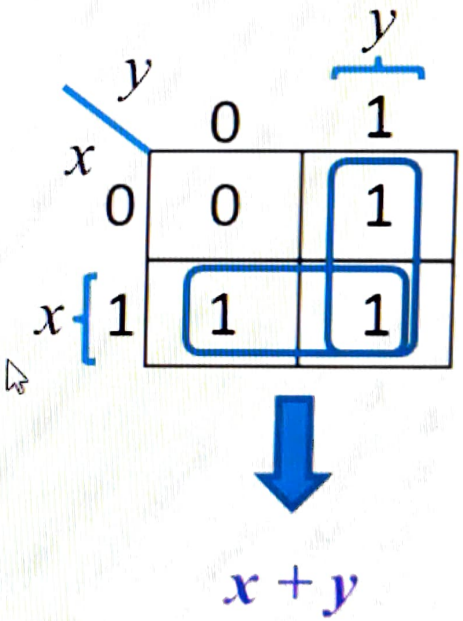
0	0
0	1
1	0
1	1

Two-variable K-map

- Minimize the function $m_1 + m_2 + m_3$
- $m_1 + m_2 + m_3 = x'y + xy' + xy = x + y$

m_0	m_1
m_2	m_3

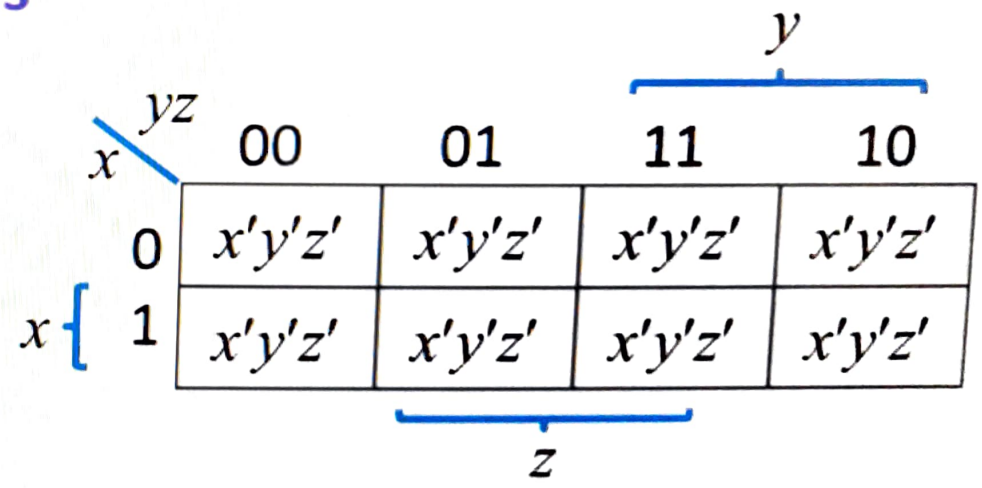
$x'y'$	$x'y$
xy'	xy



Three-variable K-map

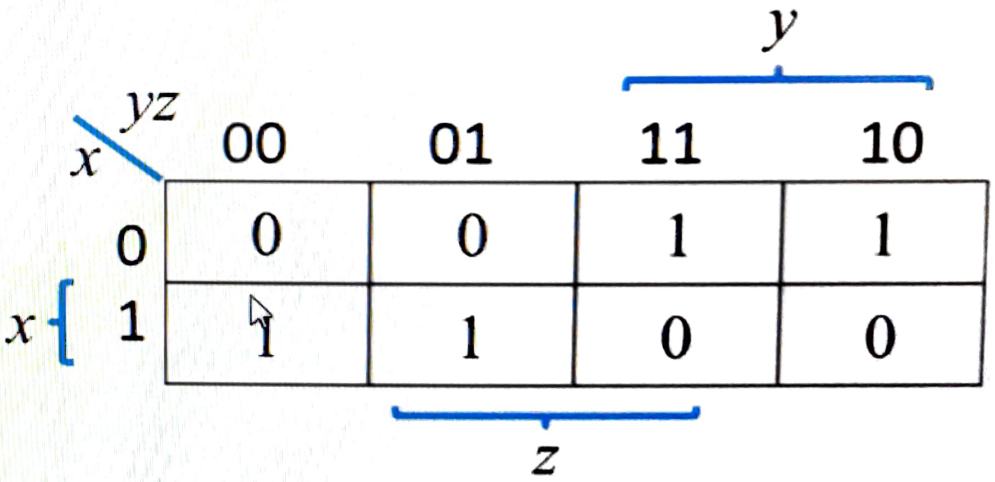
- Three variable K-map
- There are eight minterms
- ∴ Eight squares
- Characteristic: **Only one bit changes**

m_0	m_1	m_3	m_2
m_4	m_5	m_7	m_6



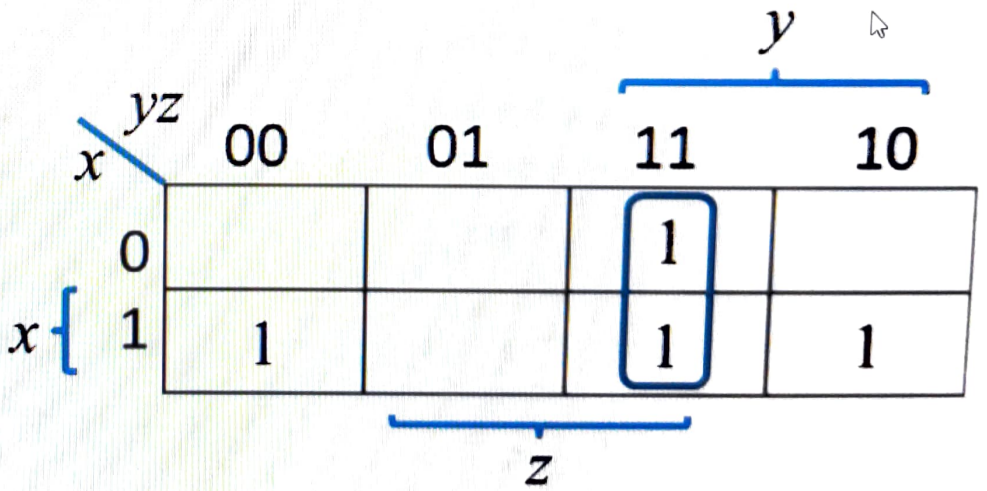
Three-variable K-map

- Simplify the Boolean function F(x, y, z) = Σ(2, 3, 4, 5)
- xy′ + x′y



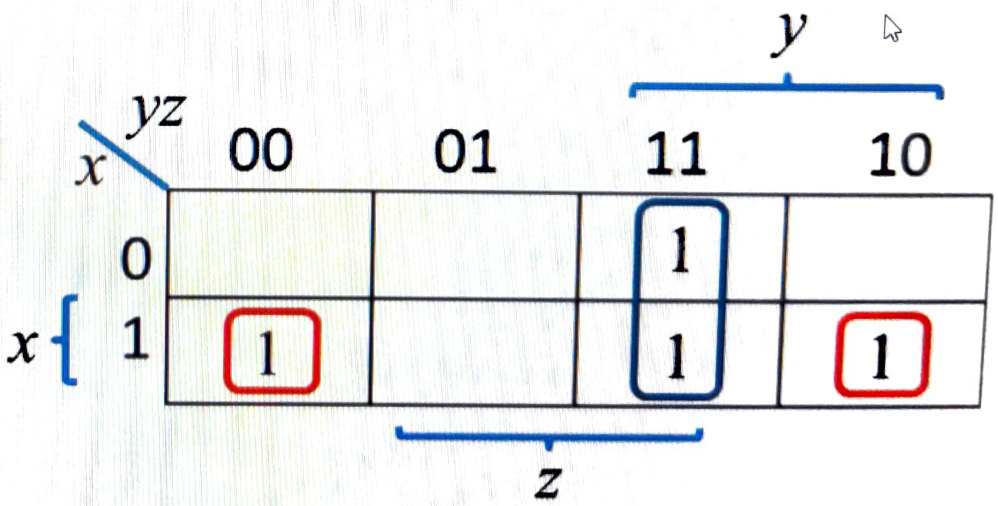
Three-variable K-map

- Simplify the Boolean function F(x, y, z) = Σ(3, 4, 6, 7)
- $xz' + yz$



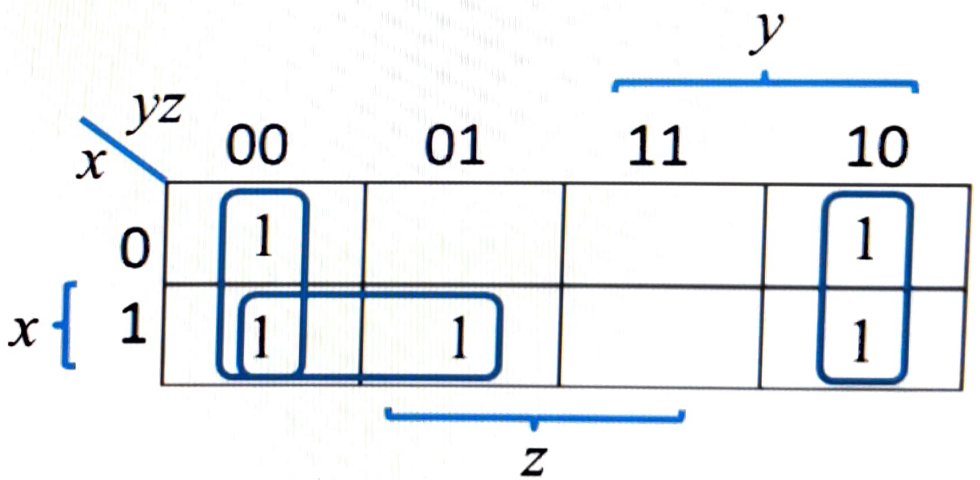
Three-variable K-map

- Simplify the Boolean function F(x, y, z) = Σ(3, 4, 6, 7)
- $xz' + yz$



Three-variable K-map

- Simplify the Boolean function F(x, y, z) = Σ(0, 2, 4, 5, 6)
- F = z′+xy′



Three-variable K-map

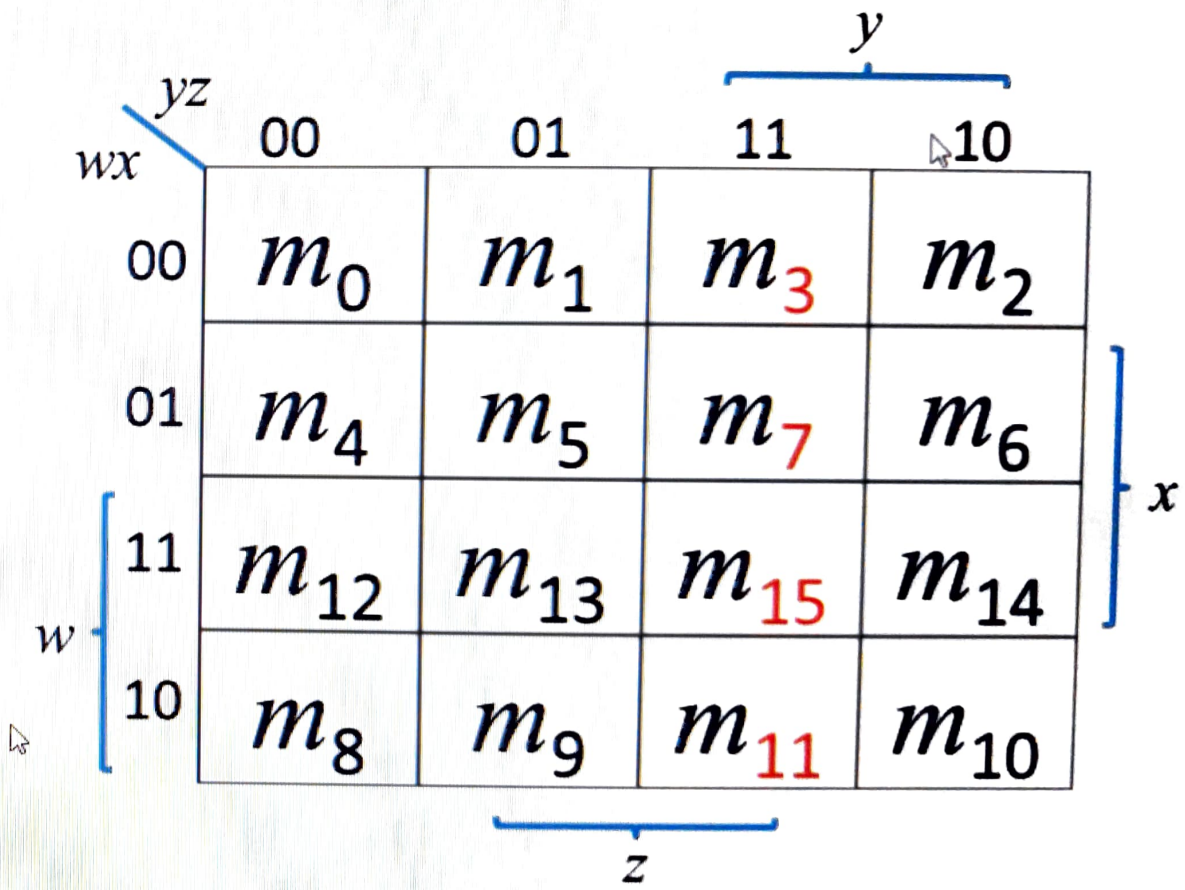
- For the Boolean function F = A′C+A′B+AB′C+BC
  - Express the function as a sum of minterms
  - Find the minimal Sum-of-products

Four-variable K-map

- One square represents one minterm with four literals
- Two adjacent squares represent a term with three literals
- Four adjacent squares represent a term with two literal
- Eight adjacent squares represents a function one literal
- Sixteen adjacent squares represents a function that is always equal to 1

Four-variable K-map

m_0	m_1	m_3	m_2
m_4	m_5	m_7	m_6
m_{12}	m_{13}	m_{15}	m_{14}
m_8	m_9	m_{11}	m_{10}



Four-variable K-map

Simplify the Boolean function F(x, y, z) = Σ(0, 1, 2, 4, 5, 6, 8, 9, 12, 13, 14)

wx \ yz	00	01	11	10
00	1	1		1
01	1	1		1
11	1	1		1
10	1	1		

m_0	m_1	m_3	m_2
m_4	m_5	m_7	m_6
m_{12}	m_{13}	m_{15}	m_{14}
m_8	m_9	m_{11}	m_{10}

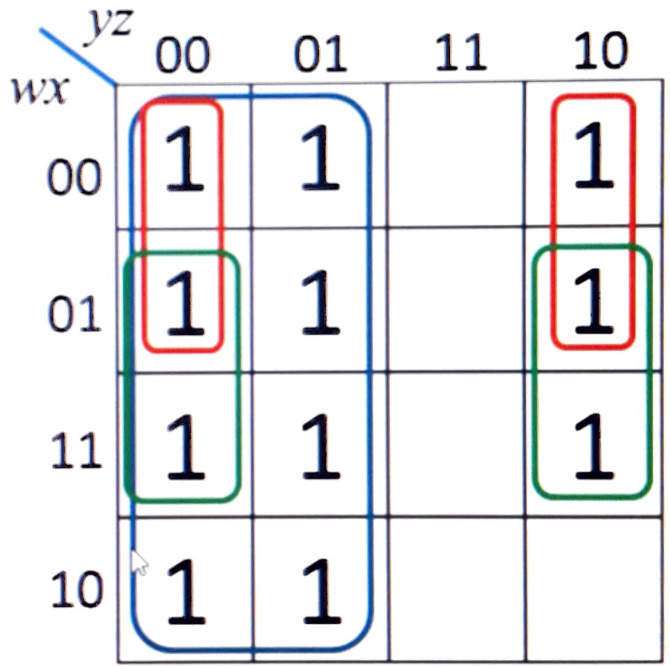
Four-variable K-map

Simplify the Boolean function F(x, y, z) = Σ(0, 1, 2, 4, 5, 6, 8, 9, 12, 13, 14)

y'

$w'z'$

xz'



m_0	m_1	m_3	m_2
m_4	m_5	m_7	m_6
m_{12}	m_{13}	m_{15}	m_{14}
m_8	m_9	m_{11}	m_{10}

Four-variable K-map

Simplify the Boolean function given in standard SOP form

F = A′B′C′ + B′CD′ + A′BCD′ + AB′C′

Add missing variables

F = A′B′C′ (D+D′)+(A+A′) B′CD′ + A′BCD′ + AB′C′(D+D′)

m_0	m_1	m_3	m_2
m_4	m_5	m_7	m_6
m_{12}	m_{13}	m_{15}	m_{14}
m_8	m_9	m_{11}	m_{10}

AB \ CD	00	01	11	10
00	1	1		1
01				1
11				
10	1	1		1

Four-variable K-map

m_0	m_1	m_3	m_2
m_4	m_5	m_7	m_6
m_{12}	m_{13}	m_{15}	m_{14}
m_8	m_9	m_{11}	m_{10}

Simplify the Boolean function given in standard SOP form

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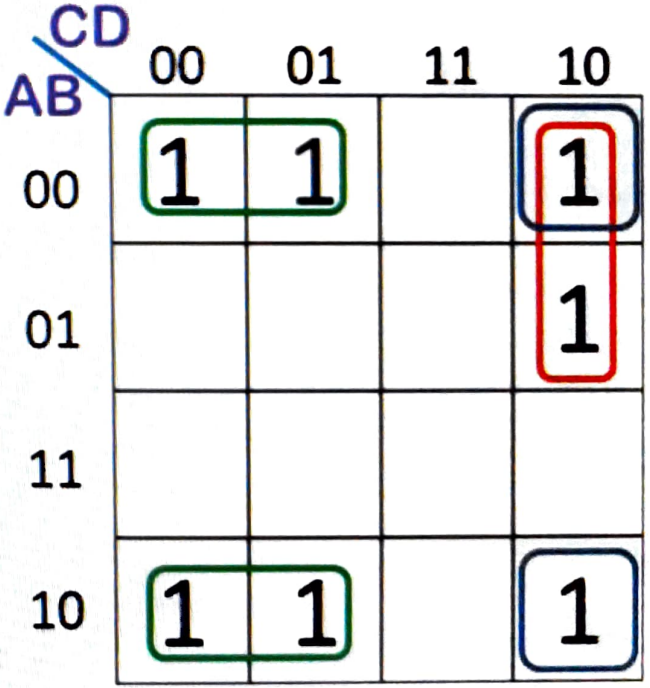
Add missing variables

F = A′B′C′ (D+D′)+(A+A′) B′CD′ + A′BCD′ + AB′C′(D+D′)

$$B'C'$$

$$A'CD'$$

$$B'CD'$$



Four-variable K-map

m_0	m_1	m_3	m_2
m_4	m_5	m_7	m_6
m_{12}	m_{13}	m_{15}	m_{14}
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Simplify the Boolean function given in standard SOP form

F = A′B′C′ + B′CD′ + A′BCD′ + AB′C′

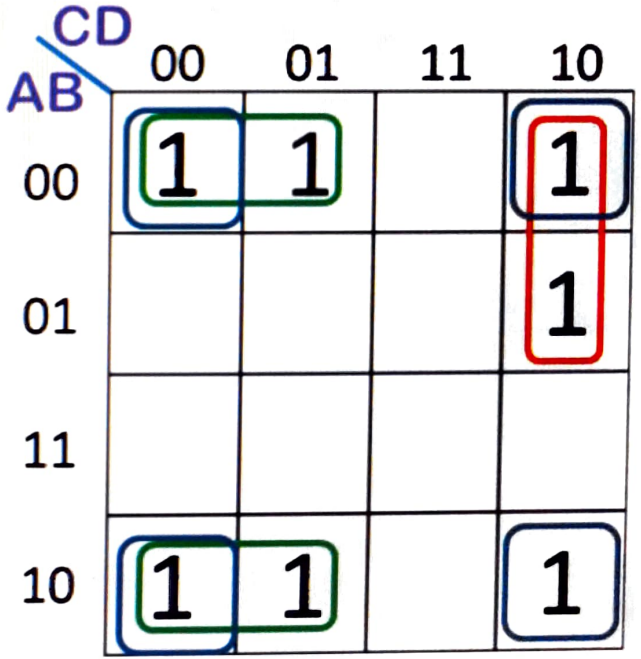
Add missing variables

F = A′B′C′ (D+D′)+(A+A′) B′CD′ + A′BCD′ + AB′C′(D+D′)

$$B'C'$$

$$A'CD'$$

$$B'D'$$

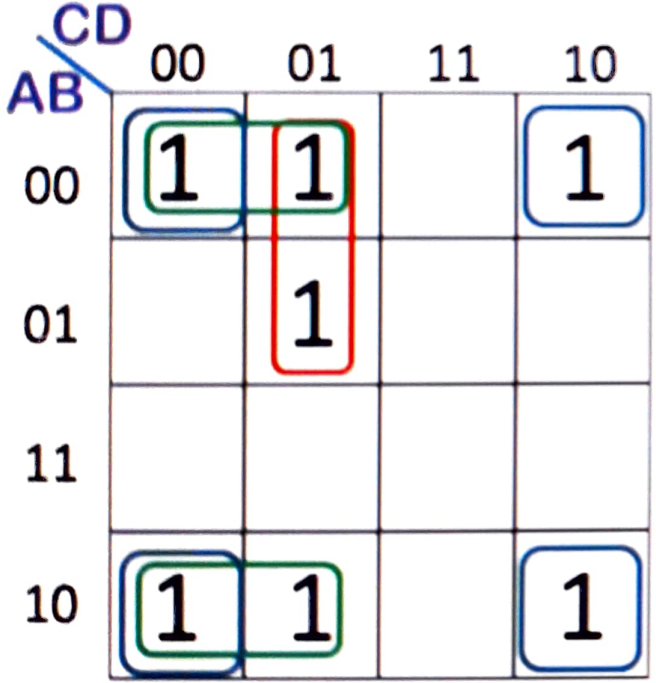


Four-variable K-map

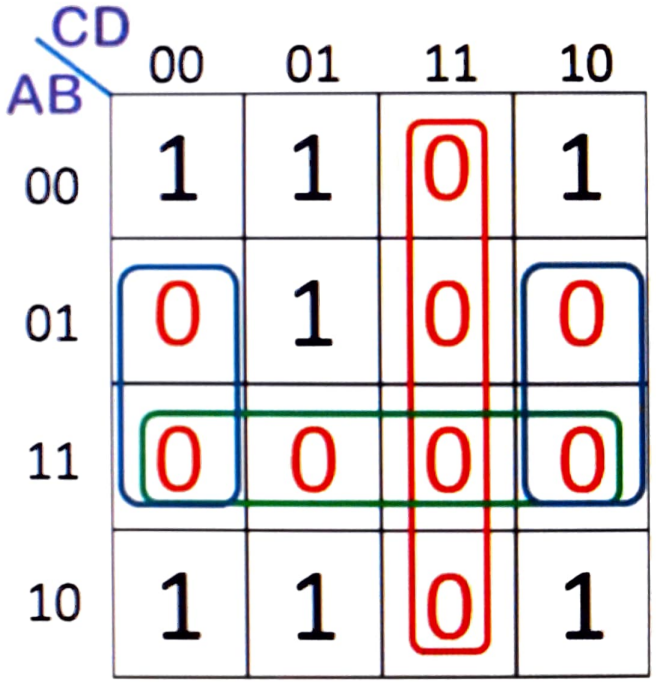
m_0	m_1	m_3	m_2
m_4	m_5	m_7	m_6
m_{12}	m_{13}	m_{15}	m_{14}
m_8	m_9	m_{11}	m_{10}

Simplify the Boolean function into SOP and POS forms

$F(A, B, C, D) = \Sigma(0, 1, 2, 5, 8, 9, 10)$



$B'C' + A'C'D + B'D'$



$(A' + B')(C' + D')(B' + D)$

Don’t care conditions

- The logical sum of the minterms of a Boolean function is equal to 1
  - For the sum of other terms the function is equal to 0
  - Ex: The four bit binary code for the decimal digits has six combinations that are not used, the unused are termed don’t care conditions