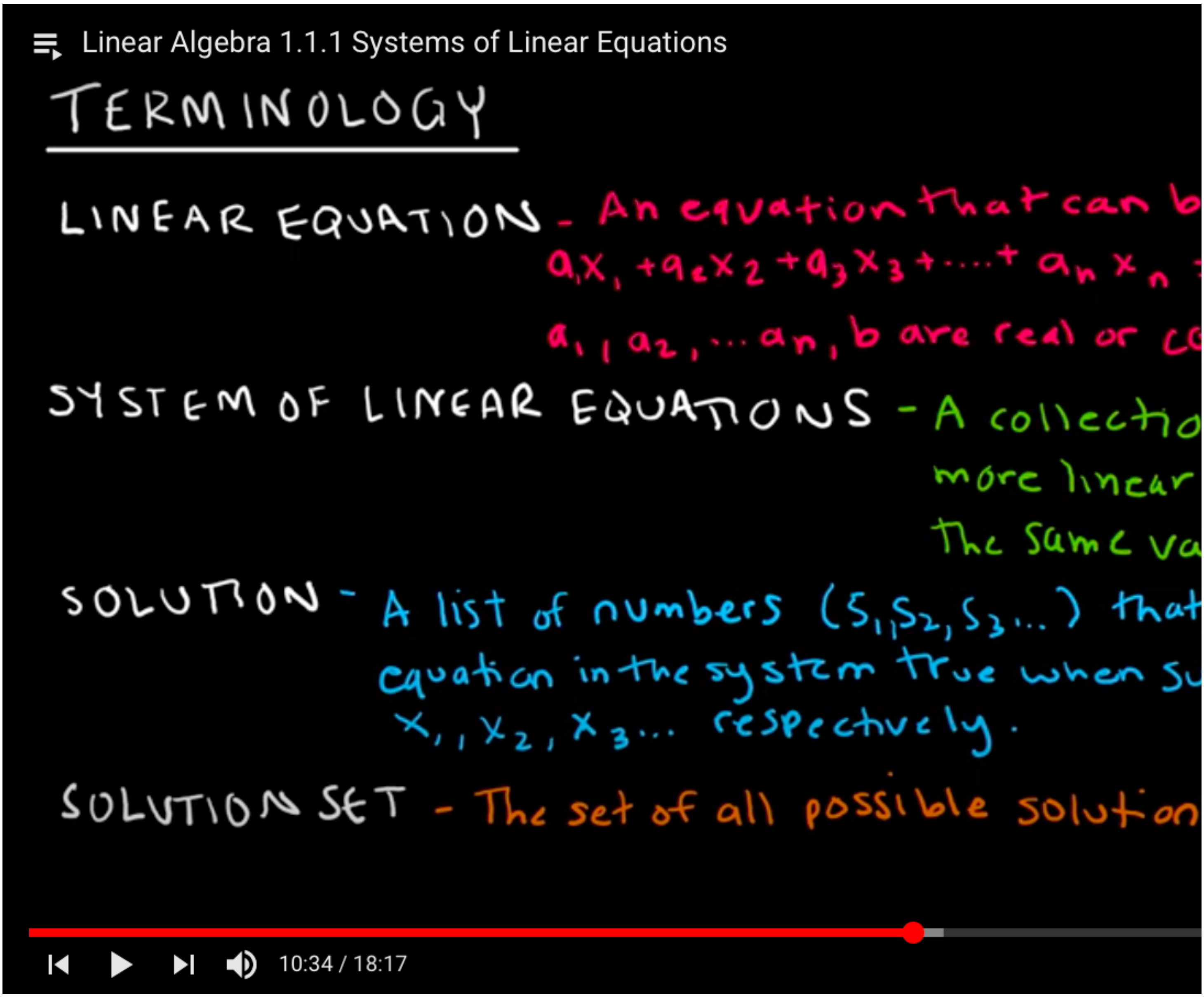
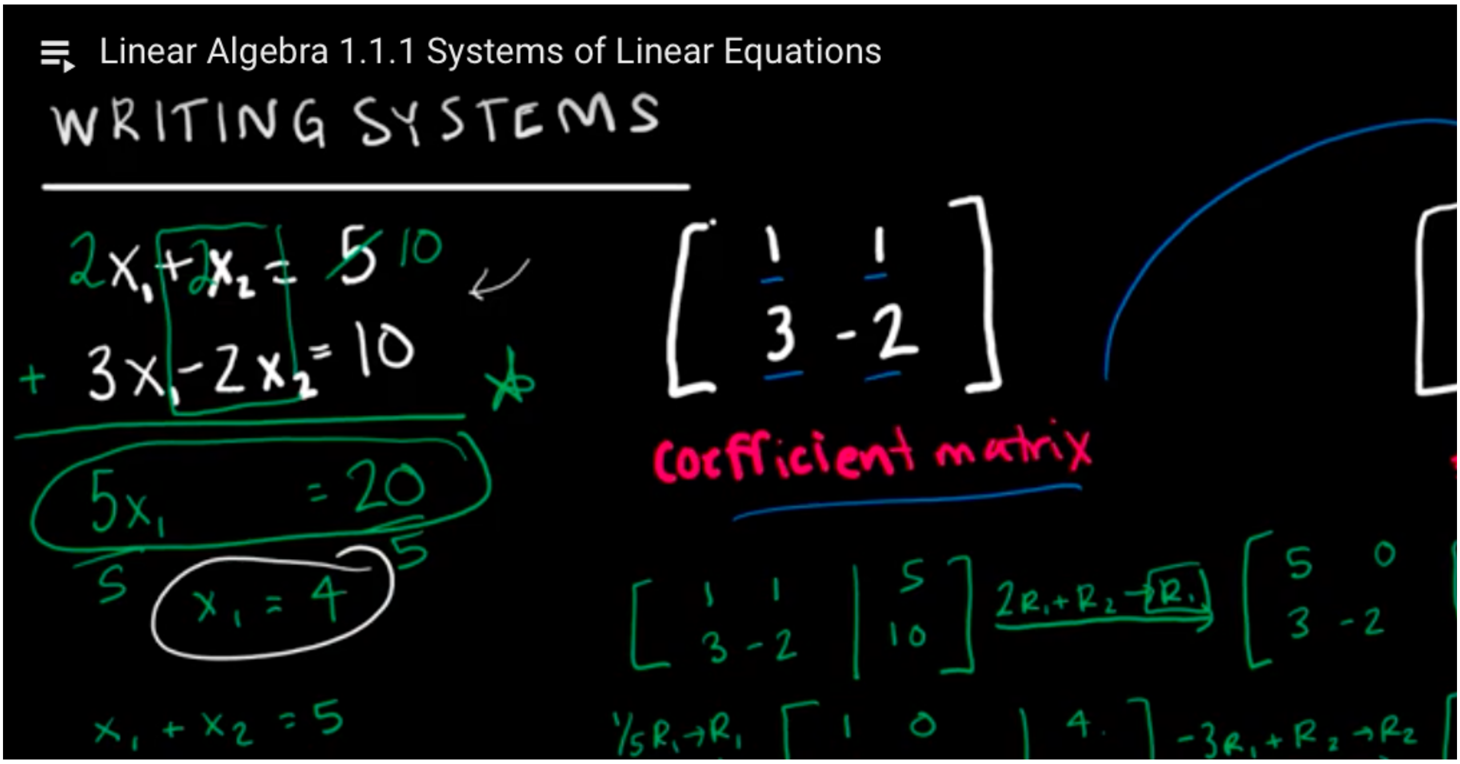
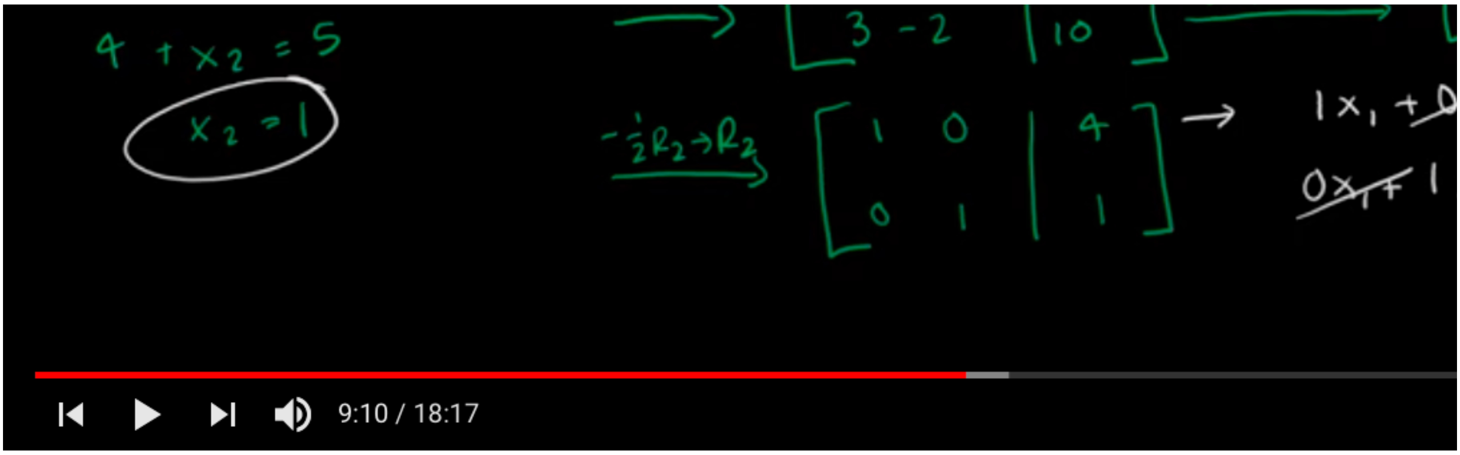


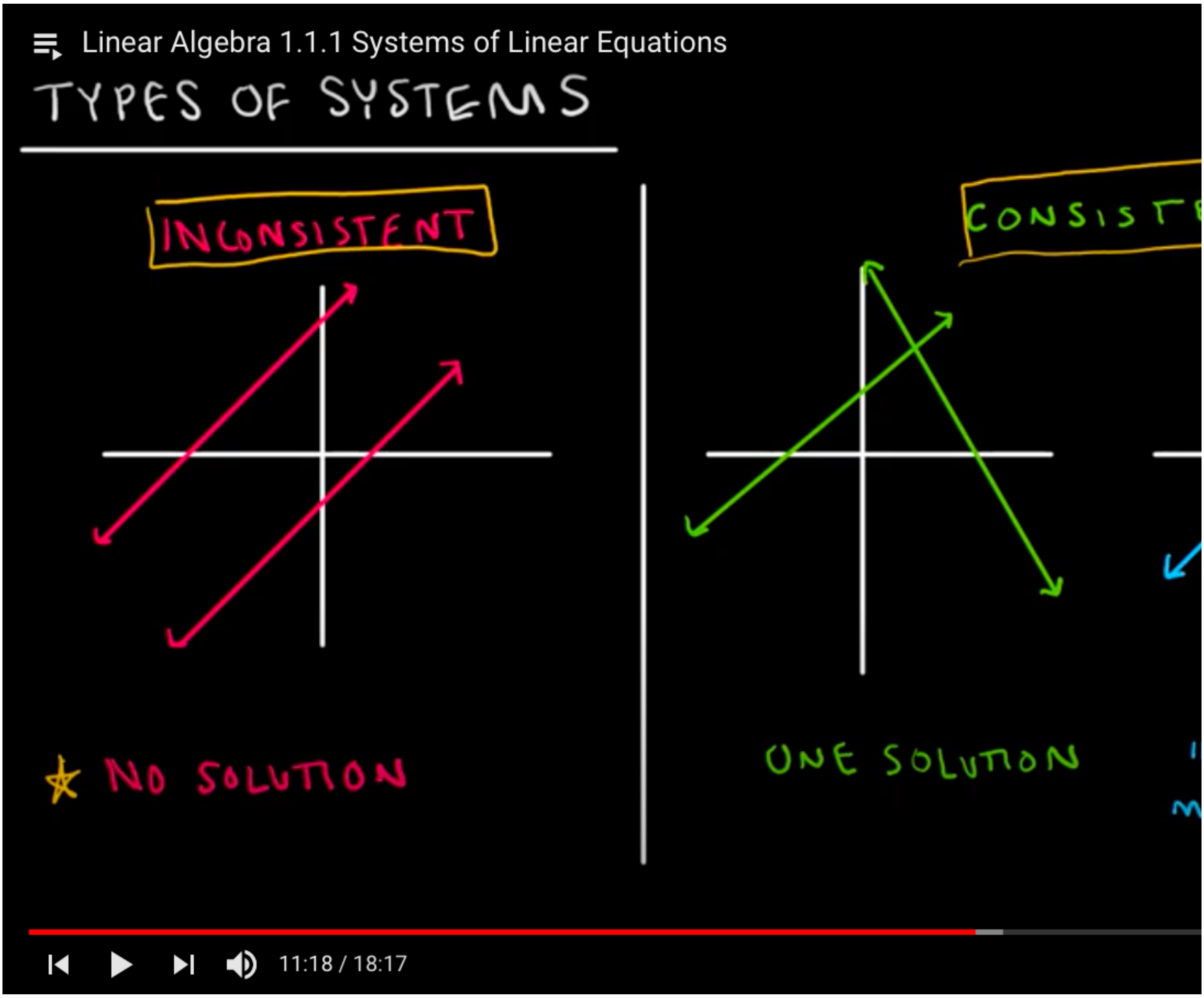
Linear Algebra 1.1.1 Systems of Linear Equations
TERMINOLOGY
LINEAR EQUATION - An equation that can b
$a_1x_1 + a_2x_2 + a_3x_3 + \dots + a_nx_n$
$a_1, a_2, \dots a_n, b$ are real or co
SYSTEM OF LINEAR EQUATIONS - A collectio
more linear
The same va
SOLUTION - A list of numbers $(s_1, s_2, s_3 \dots)$ that
equation in the system true when su
$x_1, x_2, x_3 \dots$ respectively.
SOLUTION SET - The set of all possible solution
10:34 / 18:17



Linear Algebra 1.1.1 Systems of Linear Equations
WRITING SYSTEMS
$2x_1 + 2x_2 = 5$ 10
$+\ 3x_1 - 2x_2 = 10$
$5x_1 = 20$
5 5
$x_1 = 4$
$x_1 + x_2 = 5$
coefficient matrix
$2R_1 + R_2 \to R_1$
$\frac{1}{5}R_1 \to R_1$
$-3R_1 + R_2 \to R_2$



4 + x2 = 5
x2 = 1
−½R2→R2
9:10 / 18:17



Linear Algebra 1.1.1 Systems of Linear Equations
TYPES OF SYSTEMS
INCONSISTENT
NO SOLUTION
ONE SOLUTION
11:18 / 18:17

SOLVE THE SYSTEM (AGAIN)

THIS TIME USE AN **AUGMENTED MATRIX** AND

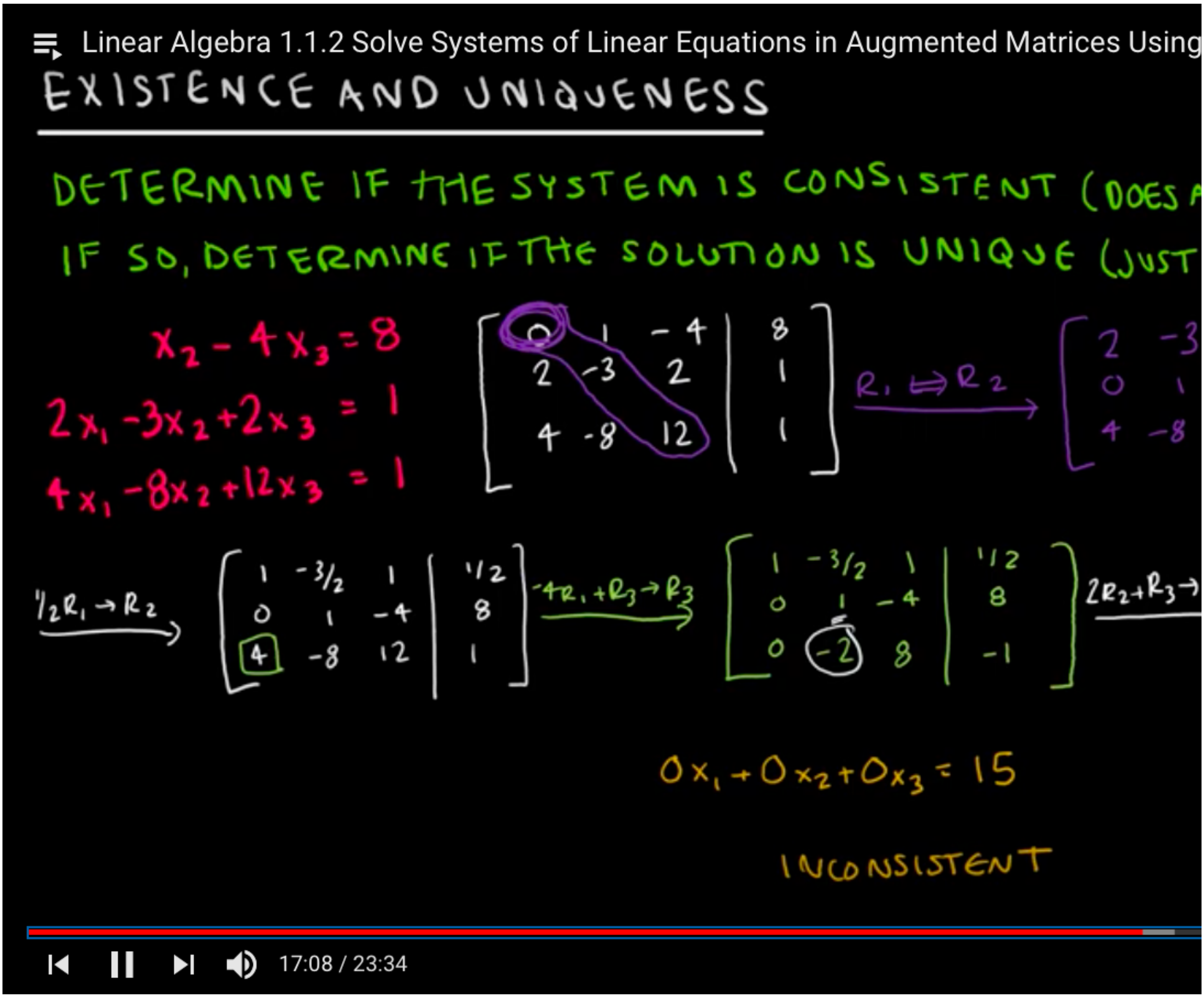
$$
\begin{aligned}
x_1 - 2x_2 + x_3 &= 0\\
2x_2 - 8x_3 &= 8\\
-4x_1 \quad x_2 + \quad x_3 &= -9
\end{aligned}
$$

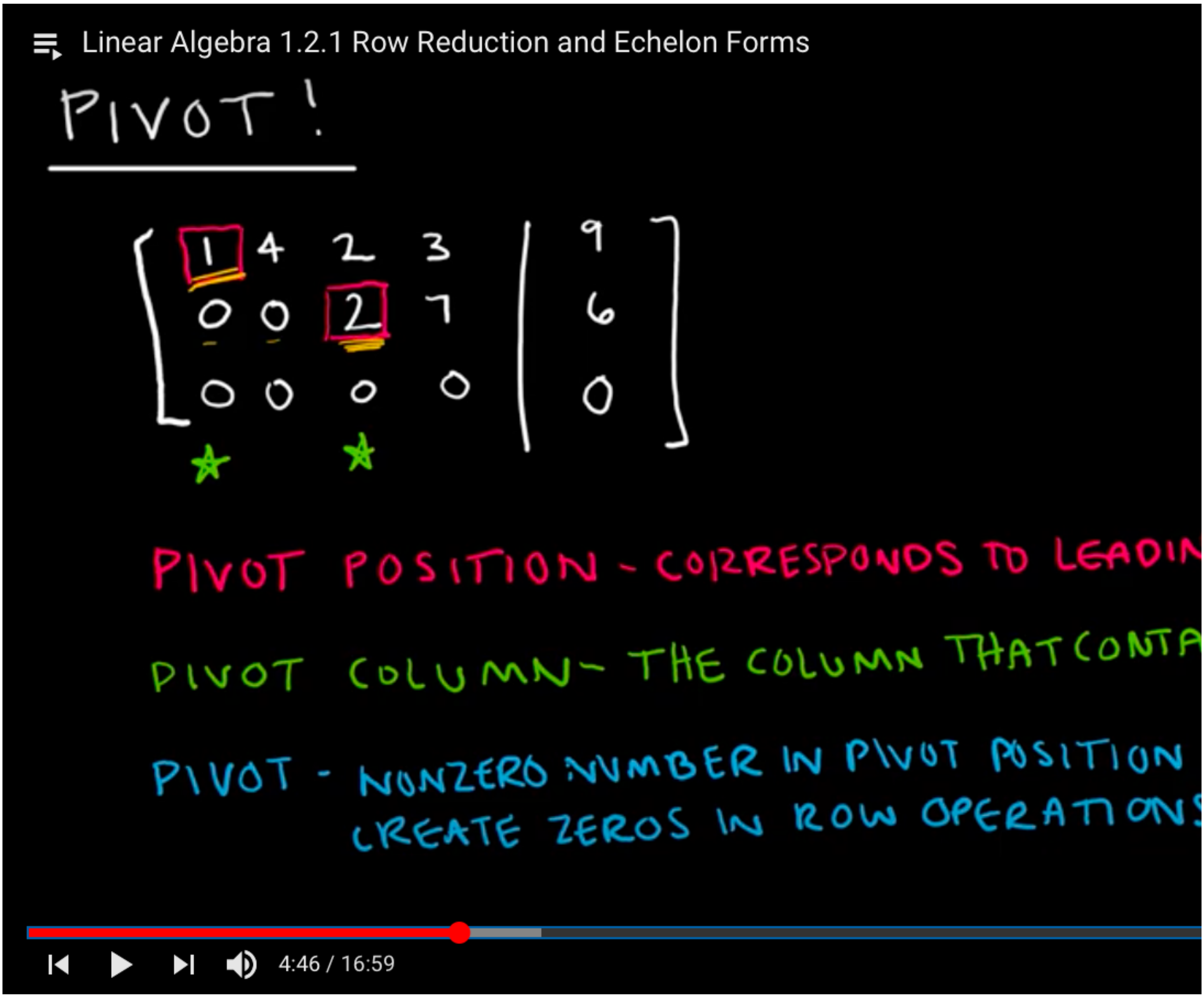
$$
\left[\begin{array}{ccc|c}
1 & -2 & 1 & 0\\
0 & \boxed{2} & -8 & 8\\
\boxed{-4} & \boxed{5} & 9 & -9
\end{array}\right]
\xrightarrow{4R_1 + R_3 \to R_3}
$$

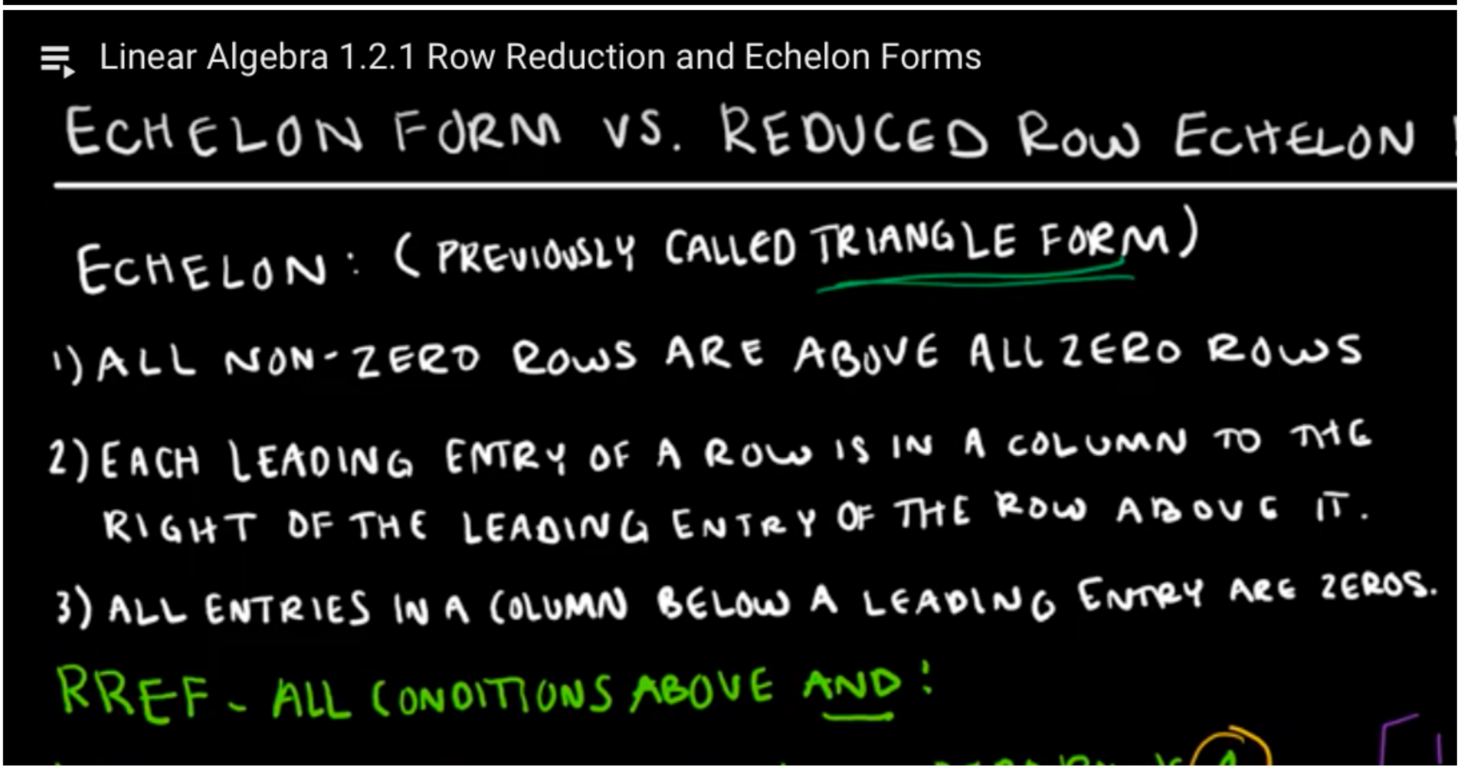
$$
\xrightarrow{\frac{1}{2}R_2 \to R_2}
\left[\begin{array}{ccc|c}
1 & -2 & 1 & 0\\
0 & 1 & -4 & 4\\
0 & -3 & 13 & -9
\end{array}\right]
\xrightarrow{3R_2 + R_3 \to R_3}
\left[\begin{array}{ccc|c}
1 & -2 & 1 & 0\\
0 & 1 & -4 & 4\\
0 & 0 & 1 & 3
\end{array}\right]
$$

Back Subst.

$$
\begin{aligned}
x_1 - 2x_2 + 1x_3 &= 0\\
1x_2 - 4x_3 &= 4\\
1x_3 &= 3
\end{aligned}
\quad\rightarrow\quad
\begin{aligned}
x_2 - 4(3) &= 4\\
x_2 - 12 &= 4\\
x_2 &= 16
\end{aligned}
\qquad
\begin{aligned}
x_1 - 2(16) + 3 &= 0\\
x_1 - 32 + 3 &= 0\\
x_1 - 29 &= 0\\
x_1 &= 29
\end{aligned}
$$


Linear Algebra 1.1.2 Solve Systems of Linear Equations in Augmented Matrices Using
EXISTENCE AND UNIQUENESS
DETERMINE IF THE SYSTEM IS CONSISTENT (DOES
IF SO, DETERMINE IF THE SOLUTION IS UNIQUE (JUST
$x_2 - 4x_3 = 8$
$2x_1 - 3x_2 + 2x_3 = 1$
$4x_1 - 8x_2 + 12x_3 = 1$
$\left[\begin{array}{ccc|c} 0 & 1 & -4 & 8 \\ 2 & -3 & 2 & 1 \\ 4 & -8 & 12 & 1 \end{array}\right]$
$R_1 \Leftrightarrow R_2$
$\frac{1}{2}R_1 \to R_2$
$\left[\begin{array}{ccc|c} 1 & -3/2 & 1 & 1/2 \\ 0 & 1 & -4 & 8 \\ \boxed{4} & -8 & 12 & 1 \end{array}\right]$
$-4R_1 + R_3 \to R_3$
$\left[\begin{array}{ccc|c} 1 & -3/2 & 1 & 1/2 \\ 0 & 1 & -4 & 8 \\ 0 & -2 & 8 & -1 \end{array}\right]$
$2R_2 + R_3 \to$
$0x_1 + 0x_2 + 0x_3 = 15$
INCONSISTENT
17:08 / 23:34


Linear Algebra 1.2.1 Row Reduction and Echelon Forms
PIVOT!
1 4 2 3 | 9
0 0 2 7 | 6
0 0 0 0 | 0
PIVOT POSITION - CORRESPONDS TO LEADIN
PIVOT COLUMN - THE COLUMN THAT CONTA
PIVOT - NONZERO NUMBER IN PIVOT POSITION
CREATE ZEROS IN ROW OPERATIONS
4:46 / 16:59


Linear Algebra 1.2.1 Row Reduction and Echelon Forms
ECHELON FORM VS. REDUCED ROW ECHELON
ECHELON: (PREVIOUSLY CALLED TRIANGLE FORM)
1) ALL NON-ZERO ROWS ARE ABOVE ALL ZERO ROWS
2) EACH LEADING ENTRY OF A ROW IS IN A COLUMN TO THE RIGHT OF THE LEADING ENTRY OF THE ROW ABOVE IT.
3) ALL ENTRIES IN A COLUMN BELOW A LEADING ENTRY ARE ZEROS.
RREF - ALL CONDITIONS ABOVE AND:

4) THE LEADING ENTRY IN EACH NON-ZERO ROW IS 1

5) EACH LEADING 1 IS THE ONLY NON-ZERO ENTRY IN THE COLUMN

3:40 / 16:59

Linear Algebra 1.1.2 Solve Systems of Linear Equations in Augmented Matrices Using

PRACTICE

SOLVE THE SYSTEM USING ROW OPERATIONS AN

$$\begin{aligned} x_1 - 2x_2 + x_3 &= 0 \\ 2x_2 - 8x_3 &= 8 \\ 5x_1 \qquad - 5x_3 &= 10 \end{aligned}$$

$$\left[\begin{array}{ccc|c} \boxed{1} & -2 & 1 & 0 \\ 0 & 2 & -8 & 8 \\ 5 & 0 & -5 & 10 \end{array}\right] \xrightarrow[\frac{1}{2}R_2 \to R_2]{-5R_1 + R_3 \to R_3}$$

$$\xrightarrow{\frac{1}{10}R_3 \to R_3} \left[\begin{array}{ccc|c} 1 & -2 & 1 & 0 \\ 0 & 1 & -4 & 4 \\ 0 & 1 & -1 & 1 \end{array}\right] \xrightarrow{R_2 - R_3 \to R_3} \left[\begin{array}{ccc|c} 1 & -2 & 1 & 0 \\ 0 & 1 & -4 & 4 \\ 0 & 0 & -3 & 3 \end{array}\right]$$

$$\xrightarrow{2R_2 + R_1 \to R_1} \left[\begin{array}{ccc|c} 1 & 0 & -7 & 8 \\ 0 & 1 & -4 & 4 \\ 0 & 0 & 1 & -1 \end{array}\right] \xrightarrow[7R_3 + R_1 \to R_1]{4R_3 + R_2 \to R_2} \left[\begin{array}{ccc|c} 1 & 0 & 0 & \\ 0 & 1 & 0 & \\ 0 & 0 & 1 & - \end{array}\right]$$

23:30 / 23:34

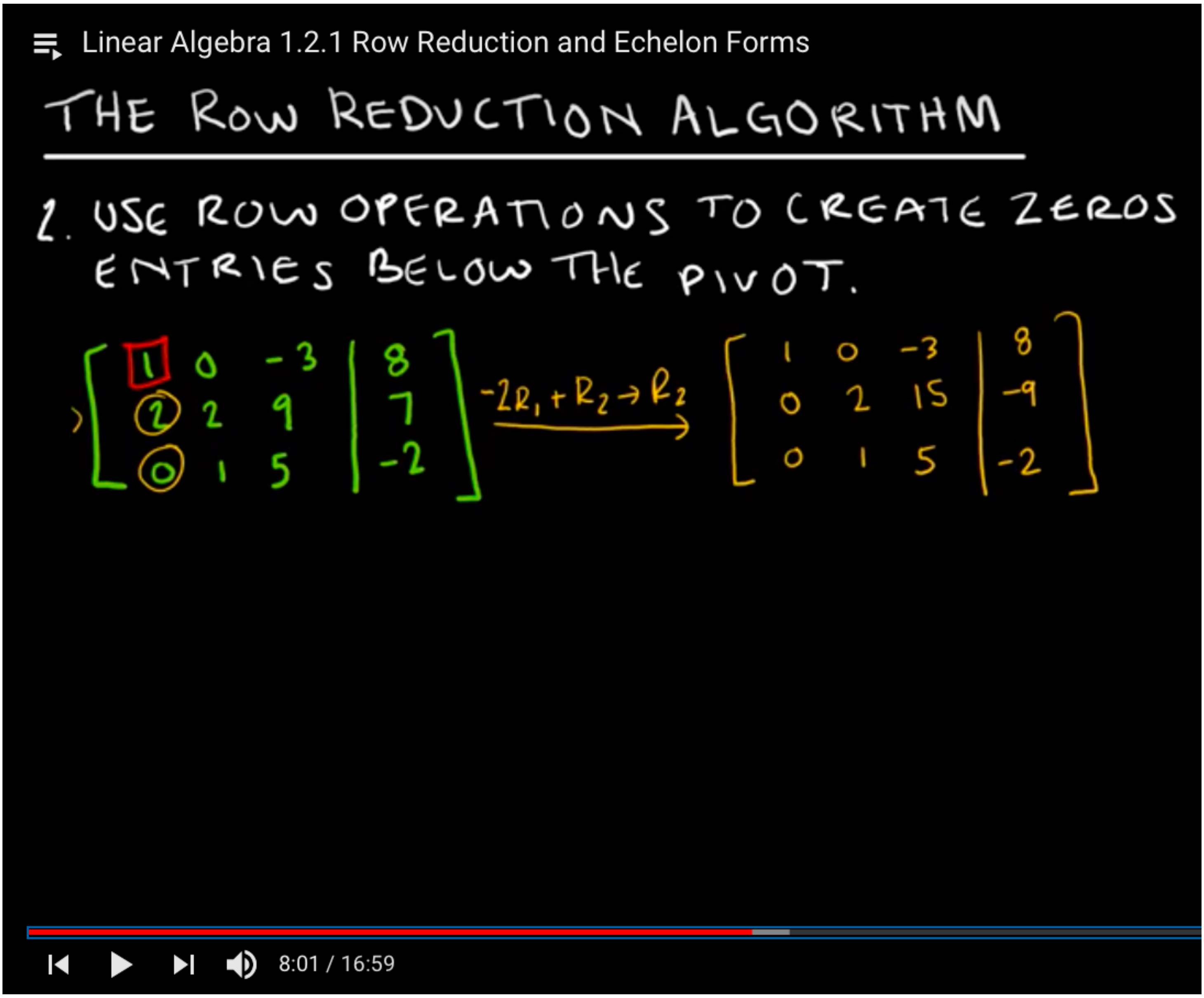
Linear Algebra 1.2.1 Row Reduction and Echelon Forms

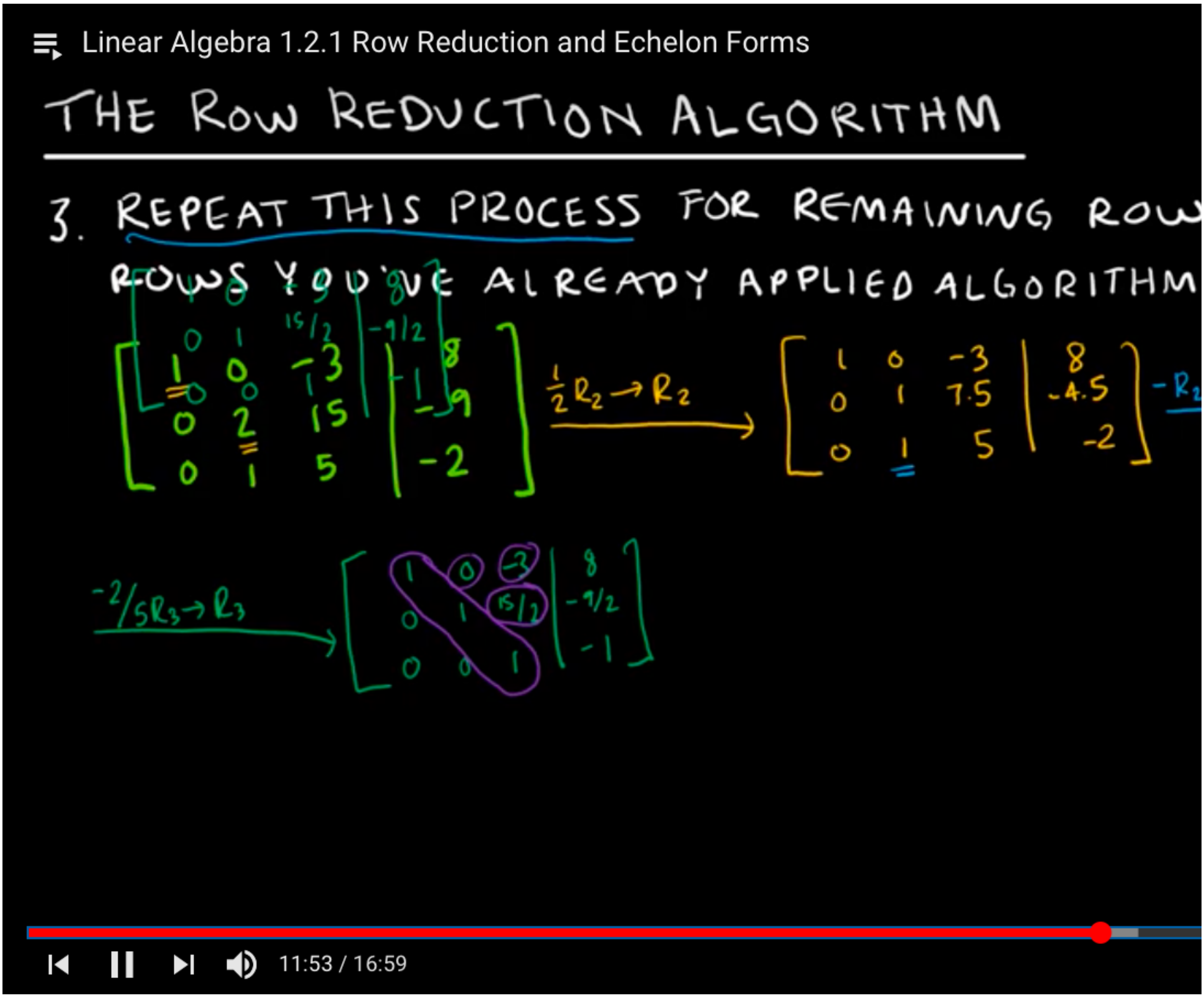
THE ROW REDUCTION ALGORITHM

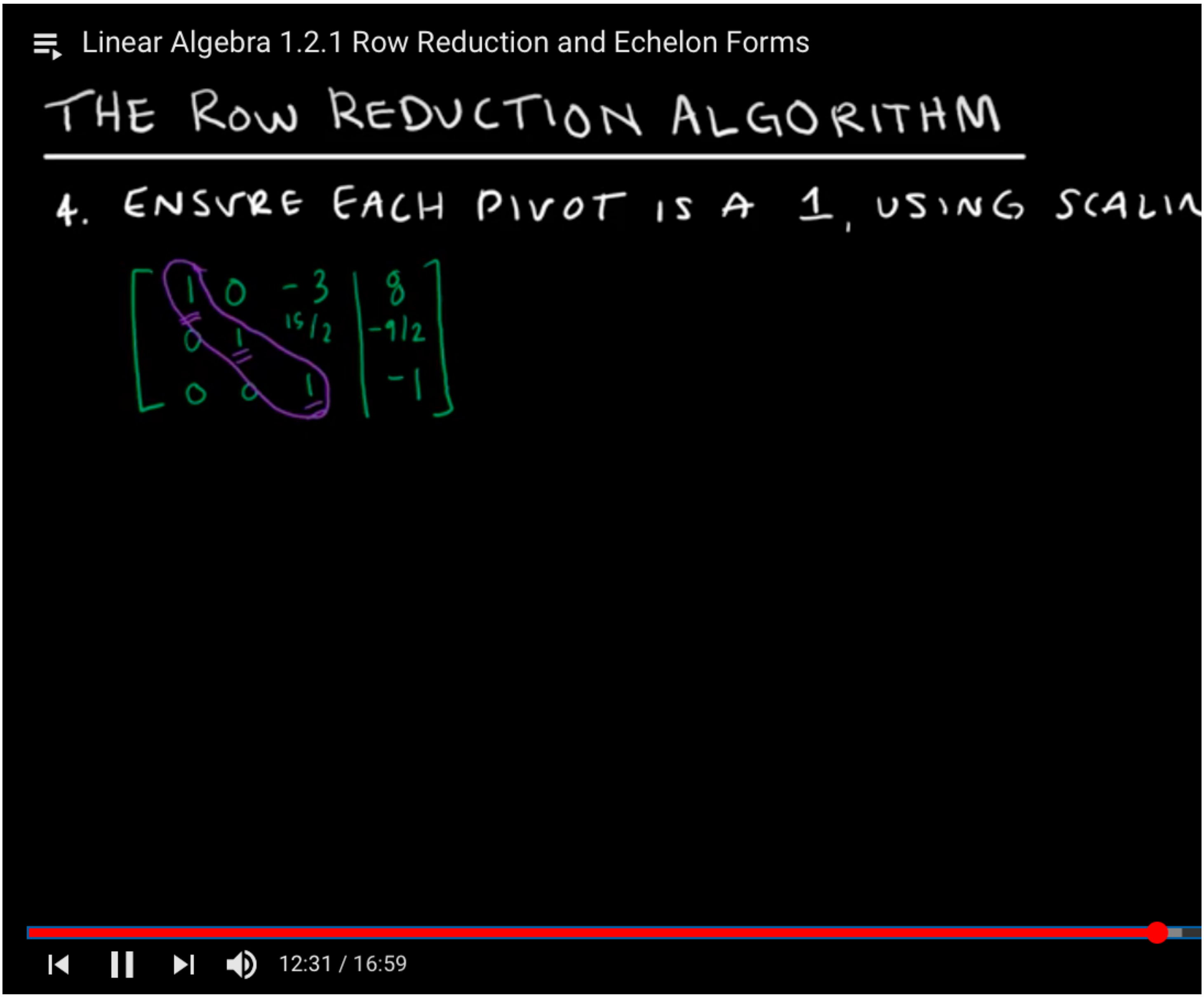
1. BEGIN AT LEFTMOST NONZERO COLUMN, WHICH COLUMN. SELECT A NONZERO ENTRY AS PIV INTERCHANGE, IF NECESSARY, TO MOVE THAT INTO THE PIVOT POSITION (ROW 1).

$$
\begin{aligned}
x_1 \qquad\quad - 3x_3 &= 8\\
2x_1 + 2x_2 + 9x_3 &= 7\\
x_2 + 5x_3 &= -2
\end{aligned}
\qquad
\left[\begin{array}{ccc|c}
1 & 0 & -3 & 8\\
2 & 2 & 9 & 7\\
0 & 1 & 5 & -2
\end{array}\right]
$$

6:21 / 16:59


Linear Algebra 1.2.1 Row Reduction and Echelon Forms
THE ROW REDUCTION ALGORITHM
2. USE ROW OPERATIONS TO CREATE ZEROS ENTRIES BELOW THE PIVOT.
$$\left[\begin{array}{ccc|c} \boxed{1} & 0 & -3 & 8 \\ 2 & 2 & 9 & 7 \\ 0 & 1 & 5 & -2 \end{array}\right] \xrightarrow{-2R_1+R_2 \to R_2} \left[\begin{array}{ccc|c} 1 & 0 & -3 & 8 \\ 0 & 2 & 15 & -9 \\ 0 & 1 & 5 & -2 \end{array}\right]$$
8:01 / 16:59


Linear Algebra 1.2.1 Row Reduction and Echelon Forms
THE ROW REDUCTION ALGORITHM
3. REPEAT THIS PROCESS FOR REMAINING ROW
ROWS YOU'VE ALREADY APPLIED ALGORITHM
½R2 → R2
-2/5R3 → R3
11:53 / 16:59


Linear Algebra 1.2.1 Row Reduction and Echelon Forms
THE ROW REDUCTION ALGORITHM
4. ENSURE EACH PIVOT IS A 1, USING SCALIN
1 0 -3 | 8
0 1 15/2 | -9/2
0 0 1 | -1
12:31 / 16:59

Linear Algebra 1.4.2 Computation of Ax

PRACTICE

GIVEN $A = \begin{bmatrix} 2 & 1 \\ -4 & 2 \end{bmatrix}$, $\vec{u} = \begin{bmatrix} -4 \\ 2 \end{bmatrix}$, $\vec{v} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$

1. $A(\vec{u}+\vec{v})$

$$\begin{bmatrix} 2 & 1 \\ -4 & 2 \end{bmatrix}\left(\begin{bmatrix} -4 \\ 2 \end{bmatrix} + \begin{bmatrix} 3 \\ -1 \end{bmatrix}\right) = \begin{bmatrix} 2 & \\ -4 & \end{bmatrix}$$

$$\begin{bmatrix} 2(-1)+(1)(1) \\ (-4)(-1)+(2)(1) \end{bmatrix} = \begin{bmatrix} -2+1 \\ 4+2 \end{bmatrix} =$$

2. $A\vec{u} + A\vec{v}$

$$\begin{bmatrix} 2 & 1 \\ -4 & 2 \end{bmatrix} \cdot \begin{bmatrix} -4 \\ 2 \end{bmatrix} + \begin{bmatrix} 2 & 1 \\ -4 & 2 \end{bmatrix}\begin{bmatrix} 3 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} 2(-4)+1(2) \\ -4(-4)+2(2) \end{bmatrix} + \begin{bmatrix} 2(3)+1(-1) \\ -4(3)+2(-1) \end{bmatrix}$$

$$\begin{bmatrix} -8+2 \\ 16+4 \end{bmatrix} + \begin{bmatrix} 6+-1 \\ -12+-2 \end{bmatrix} = \begin{bmatrix} -6 \\ 20 \end{bmatrix} + \begin{bmatrix} 5 \\ -14 \end{bmatrix}$$

6:54 / 9:41

Linear Algebra 1.4.2 Computation of Ax

EXAMPLE - REVISTED

$2 \times \boxed{3 \quad\quad 3} \times 1$

FIND Ax $\quad A = \begin{bmatrix} 2 & -1 & 0 \\ 3 & 4 & 1 \end{bmatrix}$ AND $\vec{x} = \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix}$.

PREVIOUSLY: $\underline{2}\begin{bmatrix} 2 \\ 3 \end{bmatrix} + \underline{3}\begin{bmatrix} -1 \\ 4 \end{bmatrix} + \underline{-1}\begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$$= \begin{bmatrix} 4 \\ 6 \end{bmatrix} + \begin{bmatrix} -3 \\ 12 \end{bmatrix} + \begin{bmatrix} 0 \\ -1 \end{bmatrix} =$$

ROW-VECTOR RULE FOR COMPUTING Ax

IF Ax IS DEFINED, THE i^{th} ENTRY IN Ax IS THE SUM OF

OF CORRESPONDING ENTRIES FROM ROW i OF A AND FRO

$$\begin{bmatrix} 2(2)+(-1)(3)+0(-1) \\ 3(2)+4(3)+(1)(-1) \end{bmatrix} = \begin{bmatrix} 4+-3+0 \\ 6+12+-1 \end{bmatrix} =$$

3:10 / 9:41

Linear Algebra 1.4.1 The Matrix Equation Ax=b

PRACTICE

LET $A = \begin{bmatrix} 1 & 3 & 4 \\ -4 & 2 & -6 \\ -3 & -2 & -7 \end{bmatrix}$ AND $b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$. IS THE

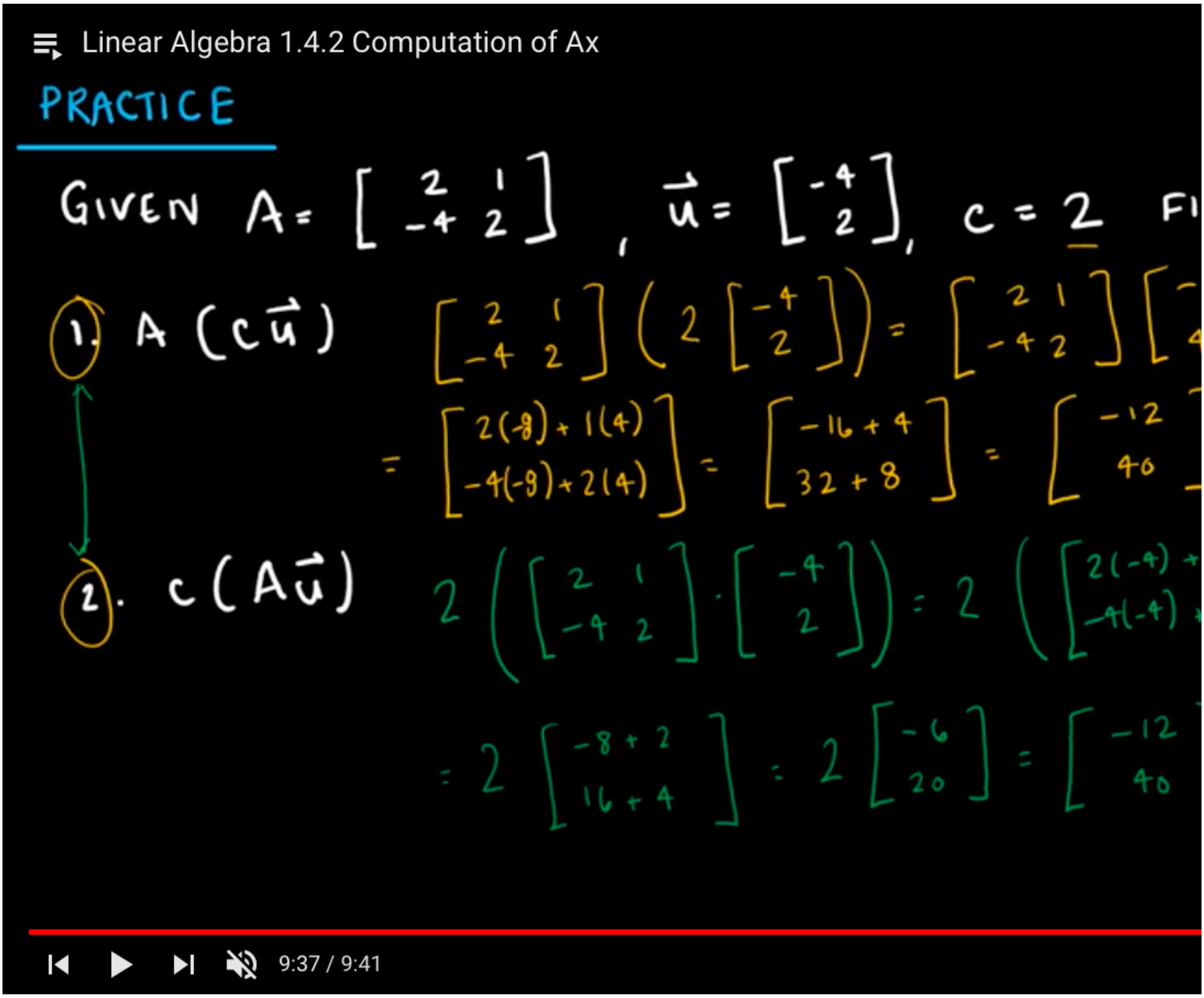
$Ax=b$ CONSISTENT FOR ALL POSSIBLE B ?

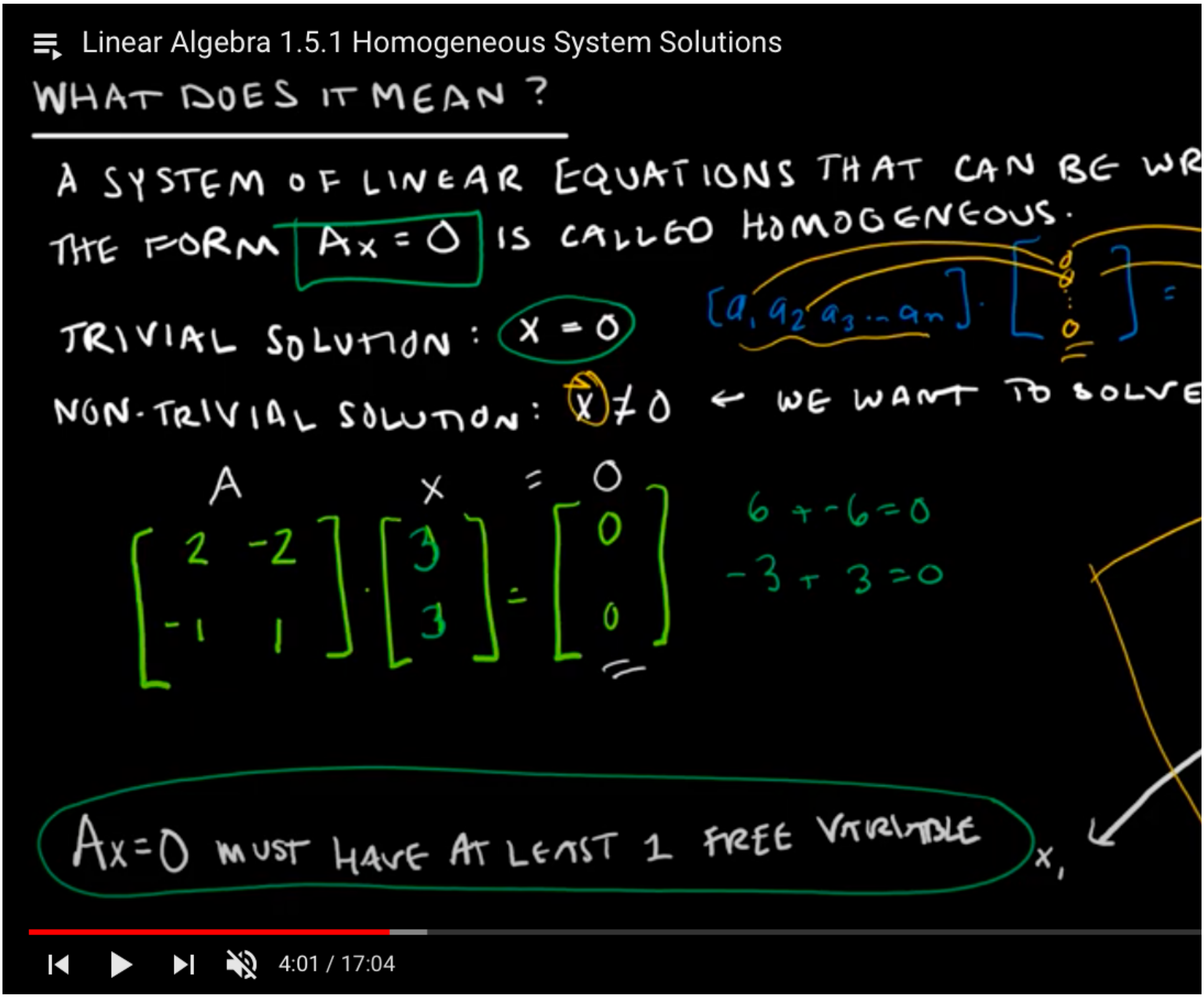
$$\left[\begin{array}{ccc|c} 1 & 3 & 4 & b_1 \\ -4 & 2 & -6 & b_2 \\ -3 & -2 & -7 & b_3 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 3 & 4 & b_1 \\ 0 & 14 & 10 & b_2+4b_1 \\ 0 & 7 & 5 & b_3+3b_1 \end{array}\right] \sim \begin{bmatrix} 1 & 3 \\ 0 & 14 \\ 0 & 0 \end{bmatrix}$$

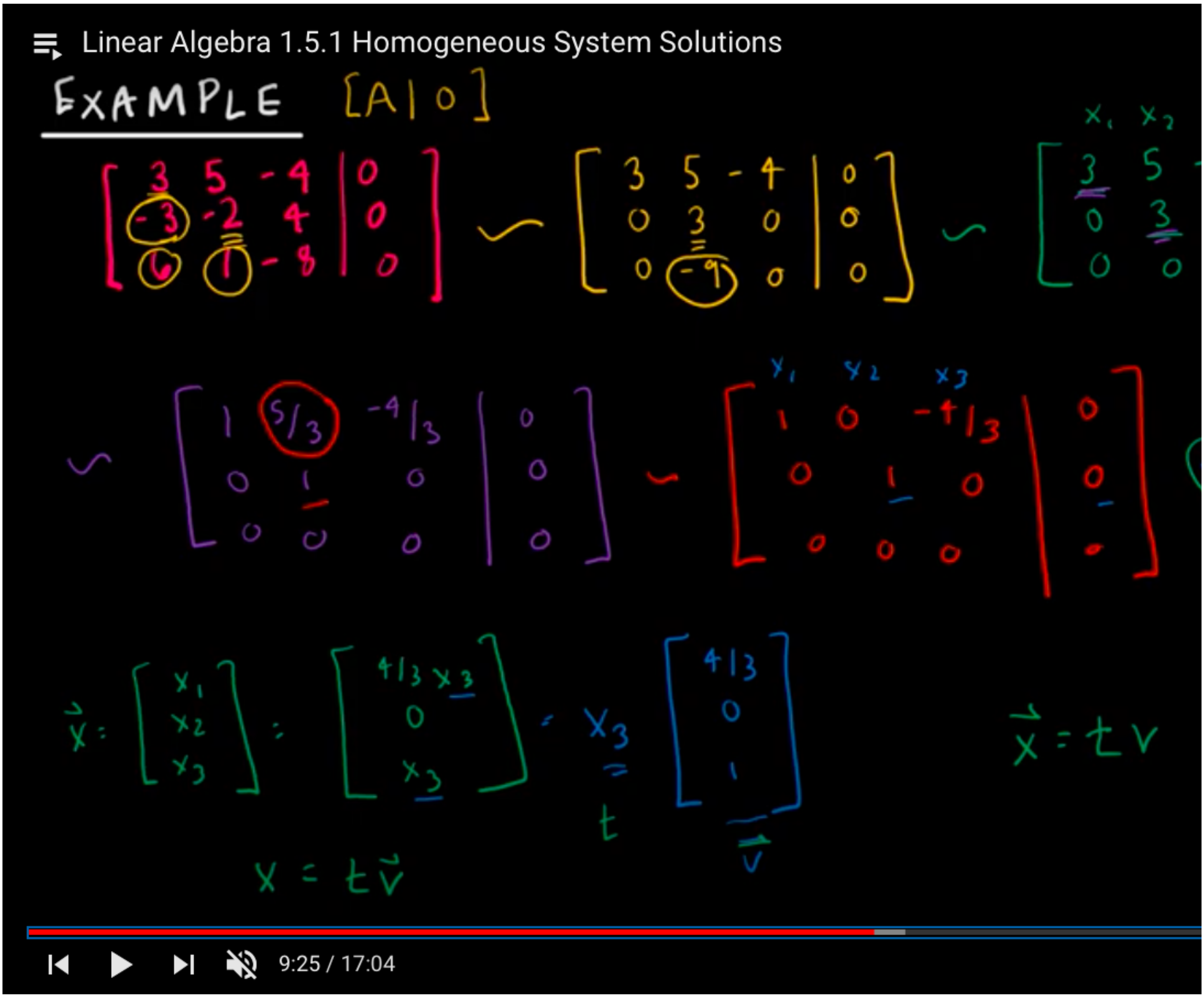
$$\left[\begin{array}{ccc|c} 1 & 3 & 4 & b_1 \\ 0 & 14 & 10 & b_2+4b_1 \\ 0 & 0 & 0 & \boxed{b_1-\frac{1}{2}b_2+b_3} \end{array}\right]$$

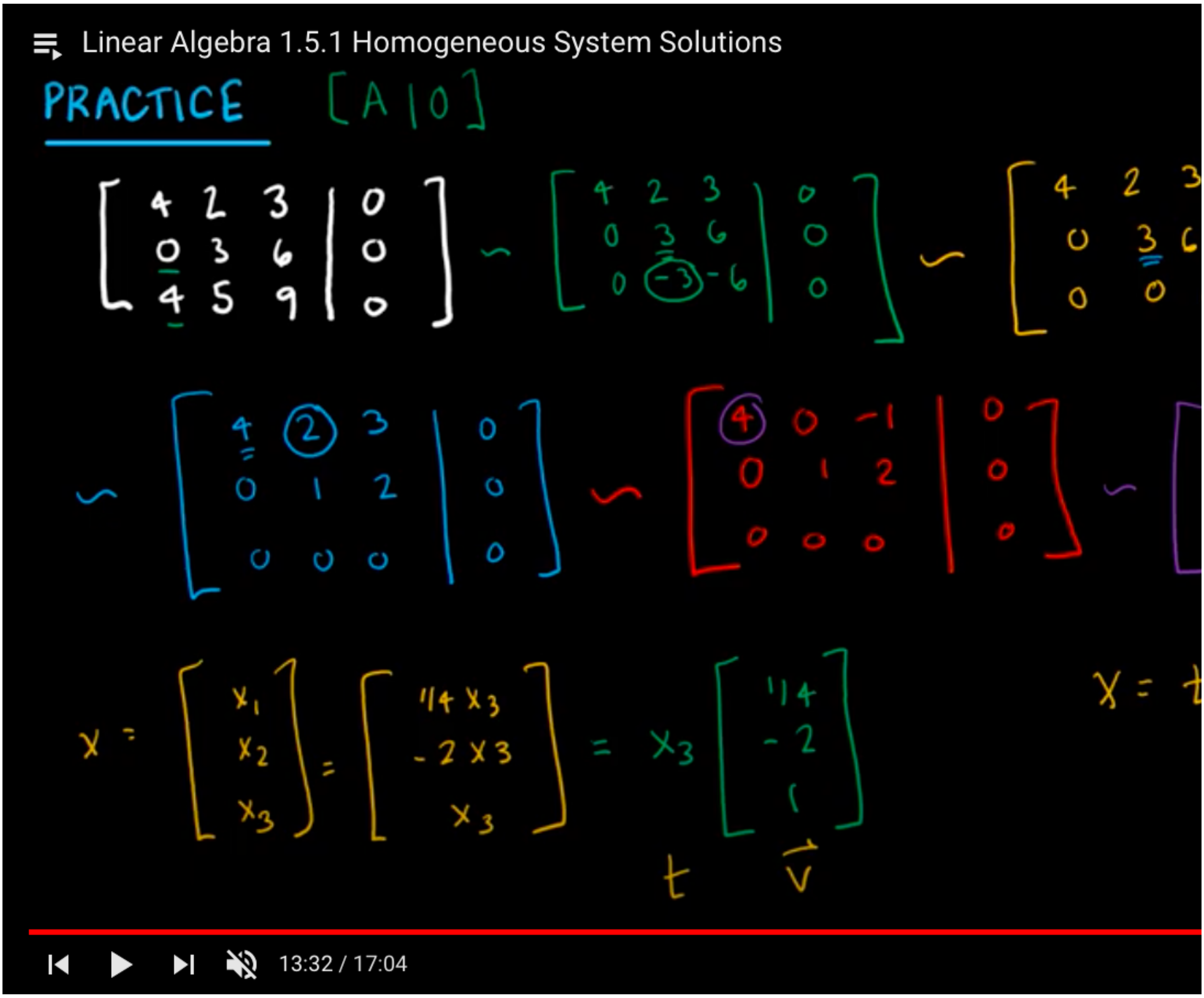
INCONSISTENT

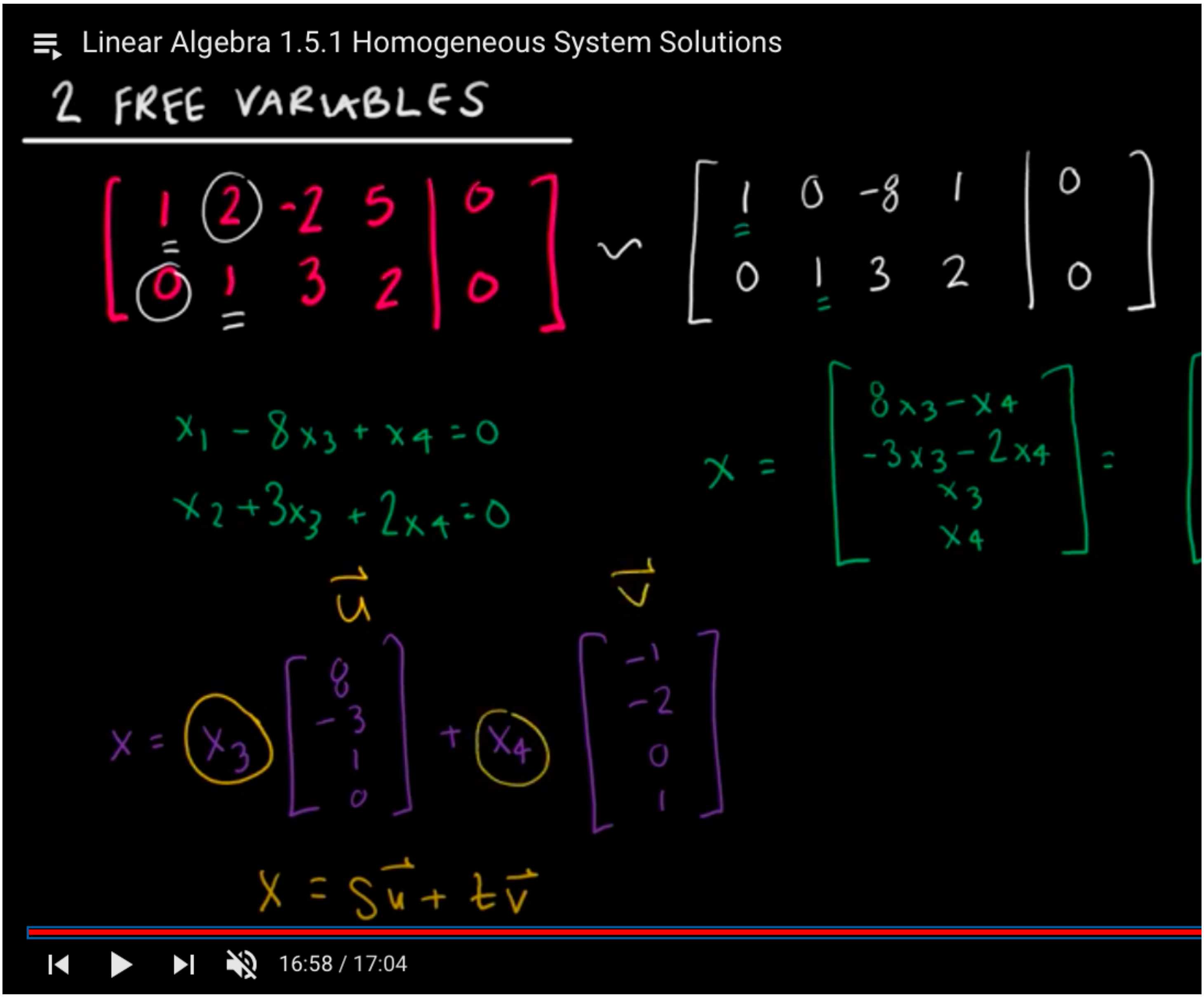
11:00 / 11:10

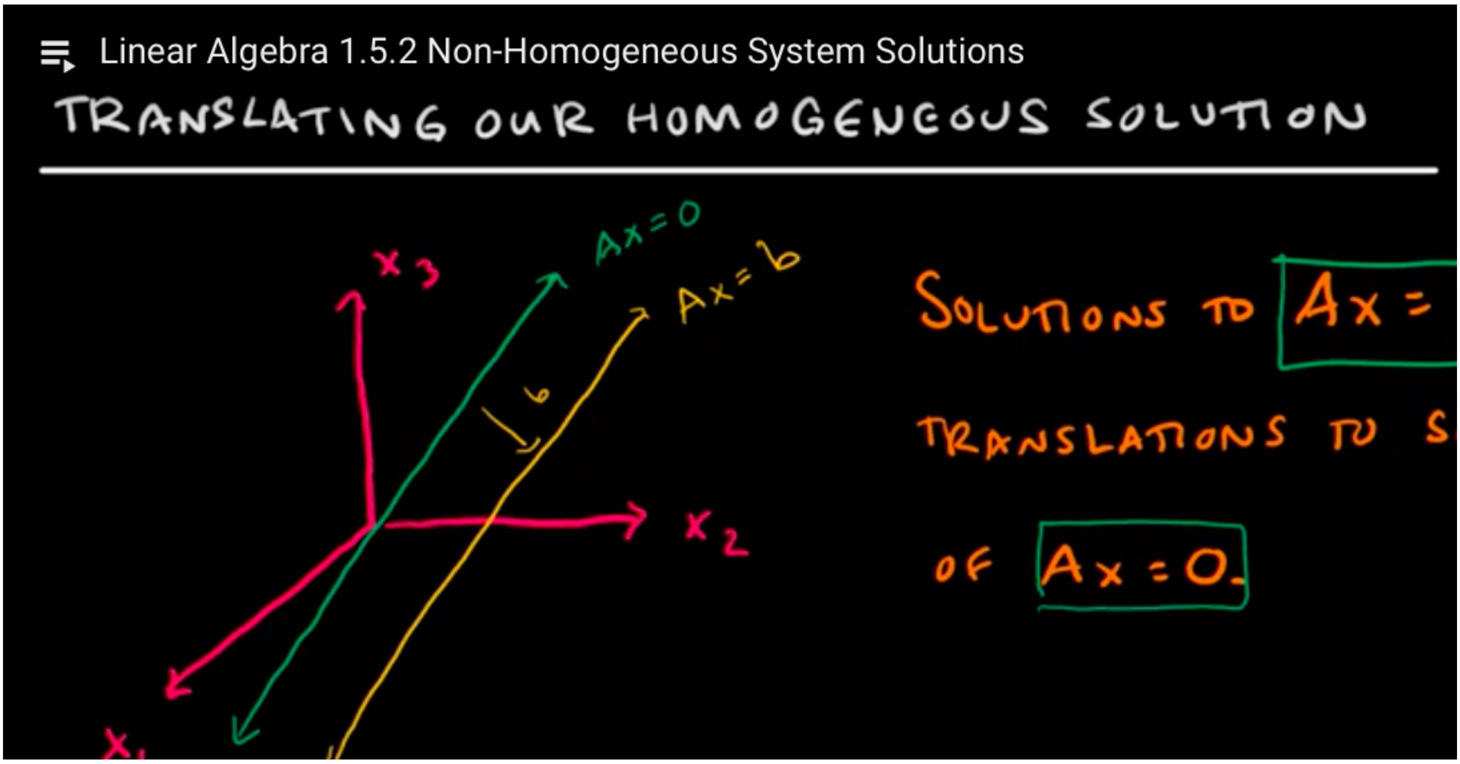

Linear Algebra 1.4.2 Computation of Ax
PRACTICE
GIVEN A= [2 1; -4 2], u = [-4; 2], c = 2 FI
1. A(cu)
[2 1; -4 2](2[-4; 2]) = [2 1; -4 2]
= [2(-8) + 1(4); -4(-8) + 2(4)] = [-16 + 4; 32 + 8] = [-12; 40]
2. c(Au)
2([2 1; -4 2] · [-4; 2]) = 2([2(-4) +; -4(-4)
= 2[-8 + 2; 16 + 4] = 2[-6; 20] = [-12; 40]
9:37 / 9:41

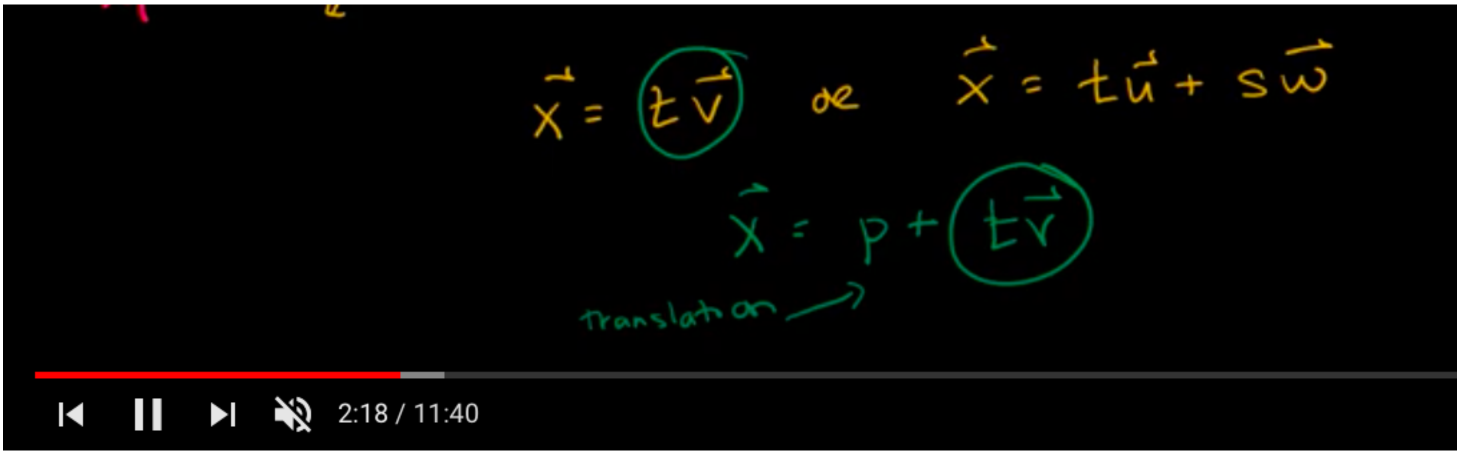

Linear Algebra 1.5.1 Homogeneous System Solutions
WHAT DOES IT MEAN ?
A SYSTEM OF LINEAR EQUATIONS THAT CAN BE WR
THE FORM Ax = 0 IS CALLED HOMOGENEOUS.
TRIVIAL SOLUTION : x = 0
NON-TRIVIAL SOLUTION : x ≠ 0 ← WE WANT TO SOLVE
[a1 a2 a3 ... an] · [0 ... 0] =
A x = 0
[2 -2; -1 1] · [3; 3] = [0; 0]
6 + -6 = 0
-3 + 3 = 0
Ax=0 MUST HAVE AT LEAST 1 FREE VARIABLE
x1
4:01 / 17:04

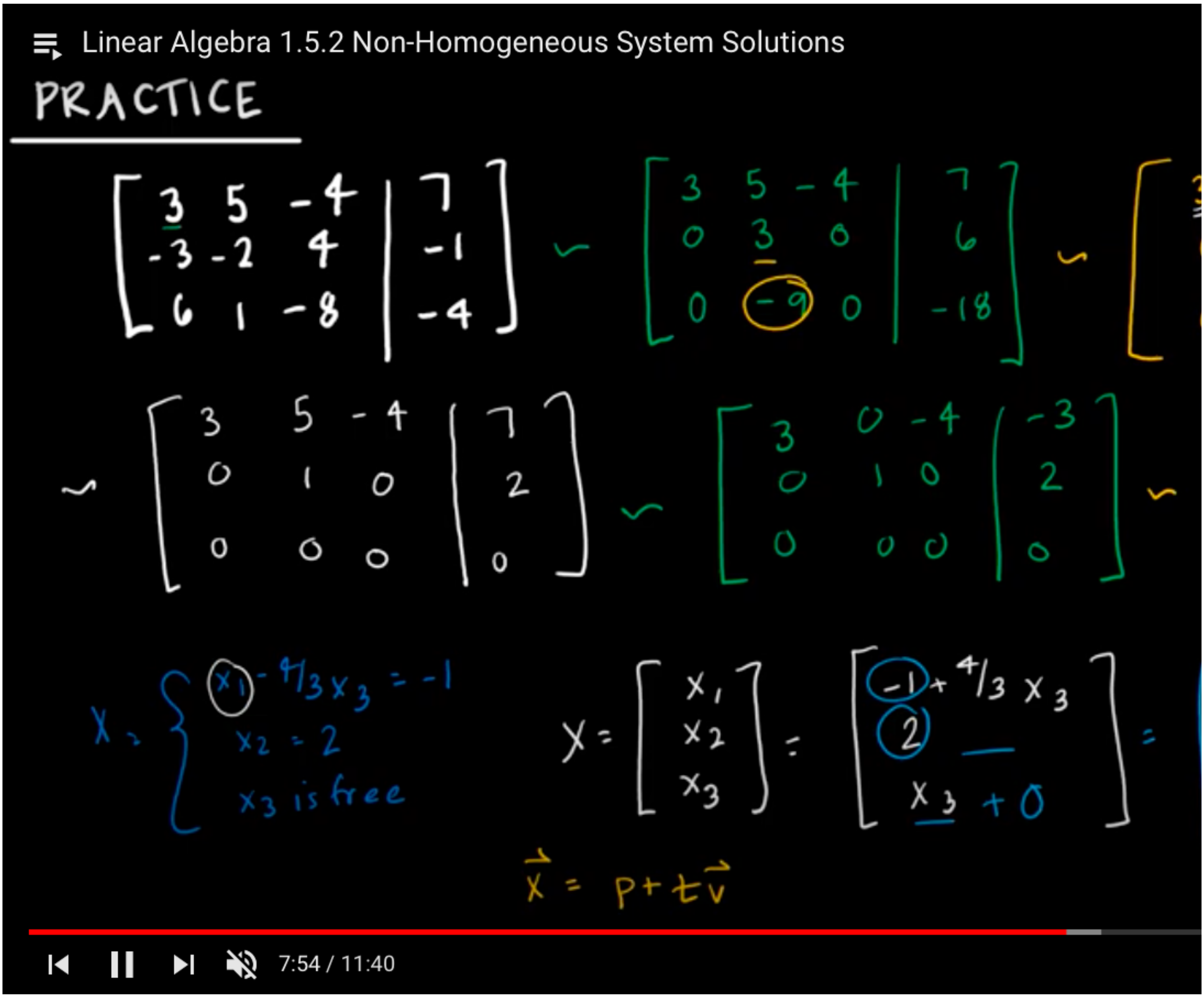

Linear Algebra 1.5.1 Homogeneous System Solutions
EXAMPLE [A | 0]
x = tv
9:25 / 17:04

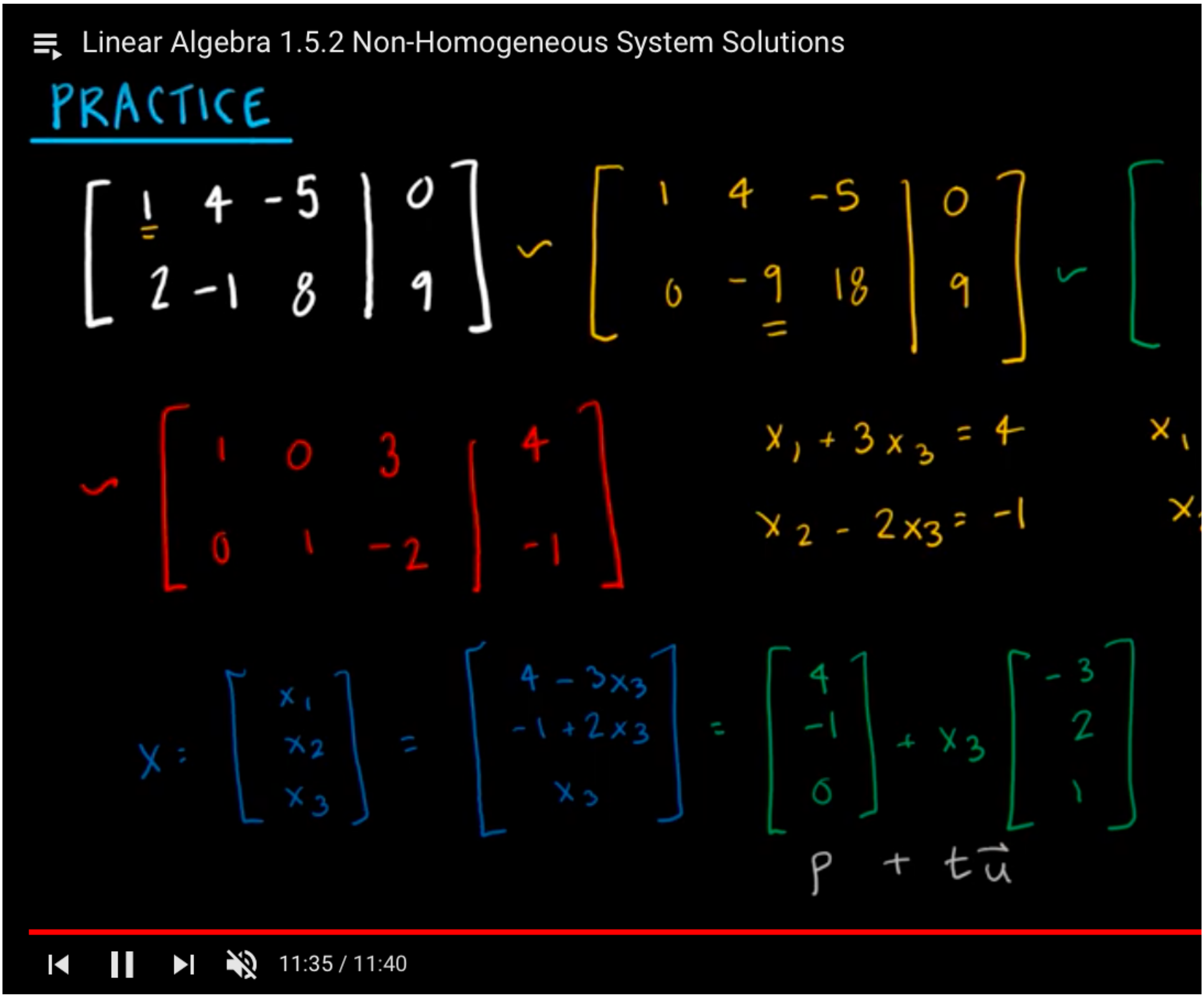

Linear Algebra 1.5.1 Homogeneous System Solutions
PRACTICE
[A | 0]
13:32 / 17:04


Linear Algebra 1.5.1 Homogeneous System Solutions
2 FREE VARIABLES
$\begin{bmatrix} 1 & 2 & -2 & 5 & 0 \\ 0 & 1 & 3 & 2 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -8 & 1 & 0 \\ 0 & 1 & 3 & 2 & 0 \end{bmatrix}$
$x_1 - 8x_3 + x_4 = 0$
$x_2 + 3x_3 + 2x_4 = 0$
$x = \begin{bmatrix} 8x_3 - x_4 \\ -3x_3 - 2x_4 \\ x_3 \\ x_4 \end{bmatrix} =$
$\vec{u}$
$\vec{v}$
$x = x_3 \begin{bmatrix} 8 \\ -3 \\ 1 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} -1 \\ -2 \\ 0 \\ 1 \end{bmatrix}$
$x = s\vec{u} + t\vec{v}$
16:58 / 17:04


Linear Algebra 1.5.2 Non-Homogeneous System Solutions
TRANSLATING OUR HOMOGENEOUS SOLUTION
x_3
$Ax = 0$
$Ax = b$
b
x_2
x_1
SOLUTIONS TO $Ax =$
TRANSLATIONS TO S
OF $Ax = 0$.


x = tv or x = tu + sw
x = p + tv
translation
2:18 / 11:40


Linear Algebra 1.5.2 Non-Homogeneous System Solutions
PRACTICE
x1 - 4/3 x3 = -1
x2 = 2
x3 is free
X = [x1, x2, x3] = [-1 + 4/3 x3, 2, x3 + 0] =
x = p + tv
7:54 / 11:40


Linear Algebra 1.5.2 Non-Homogeneous System Solutions
PRACTICE
$\left[\begin{array}{ccc|c} 1 & 4 & -5 & 0 \\ 2 & -1 & 8 & 9 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 4 & -5 & 0 \\ 0 & -9 & 18 & 9 \end{array}\right] \sim$
$\sim \left[\begin{array}{ccc|c} 1 & 0 & 3 & 4 \\ 0 & 1 & -2 & -1 \end{array}\right]$
$x_1 + 3x_3 = 4$
$x_2 - 2x_3 = -1$
$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 - 3x_3 \\ -1 + 2x_3 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ -1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -3 \\ 2 \\ 1 \end{bmatrix}$
$p + t\vec{u}$
11:35 / 11:40

WHAT DOES IT MEAN?

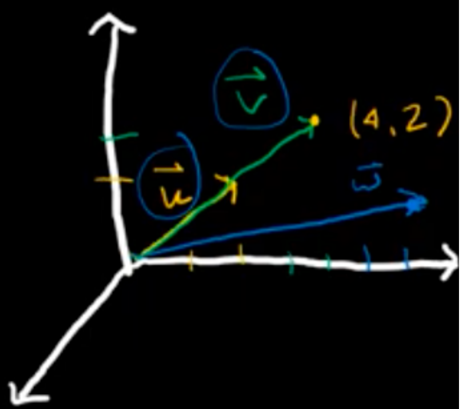
AN INDEXED SET OF VECTORS $\{v_1, v_2 \dots v_p\}$ in $\mathbb{R}^n$ is
INDEPENDENT IF THE VECTOR EQUATION $x_1v_1 + x_2v_2 + \dots +$
ONLY THE TRIVIAL SOLUTION.

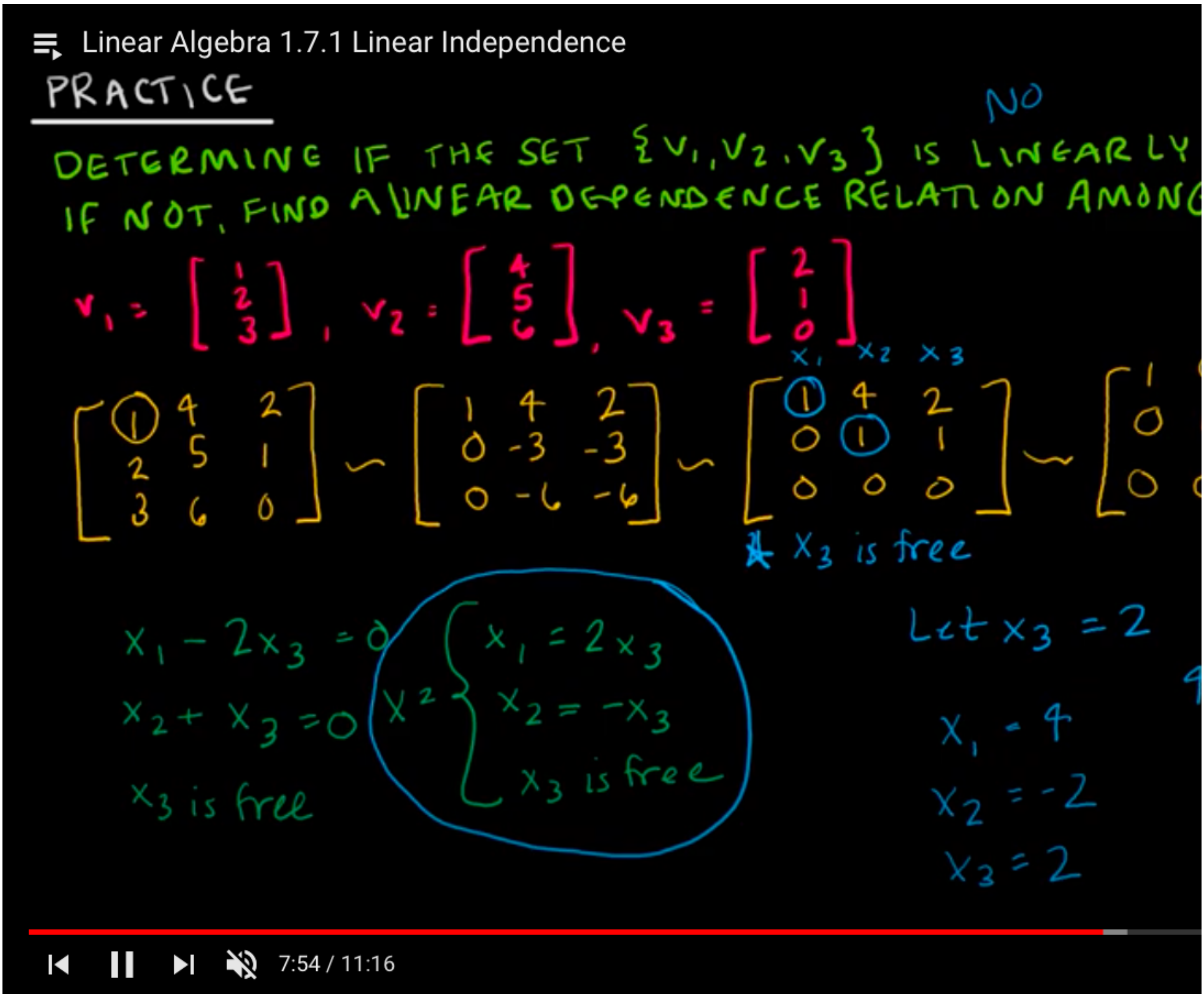
THEY ARE LINEARLY DEPENDENT IF A NON-TRIVIAL SOLUTIO

2

TRANSLATION?

free variable




Linear Algebra 1.7.1 Linear Independence
PRACTICE
NO
DETERMINE IF THE SET {v1, v2, v3} IS LINEARLY
IF NOT, FIND A LINEAR DEPENDENCE RELATION AMONG
v1 = [1 2 3], v2 = [4 5 6], v3 = [2 1 0]
x1 x2 x3
x3 is free
x1 - 2x3 = 0
x2 + x3 = 0
x3 is free
x1 = 2x3
x2 = -x3
x3 is free
Let x3 = 2
x1 = 4
x2 = -2
x3 = 2
7:54 / 11:16

PRACTICE

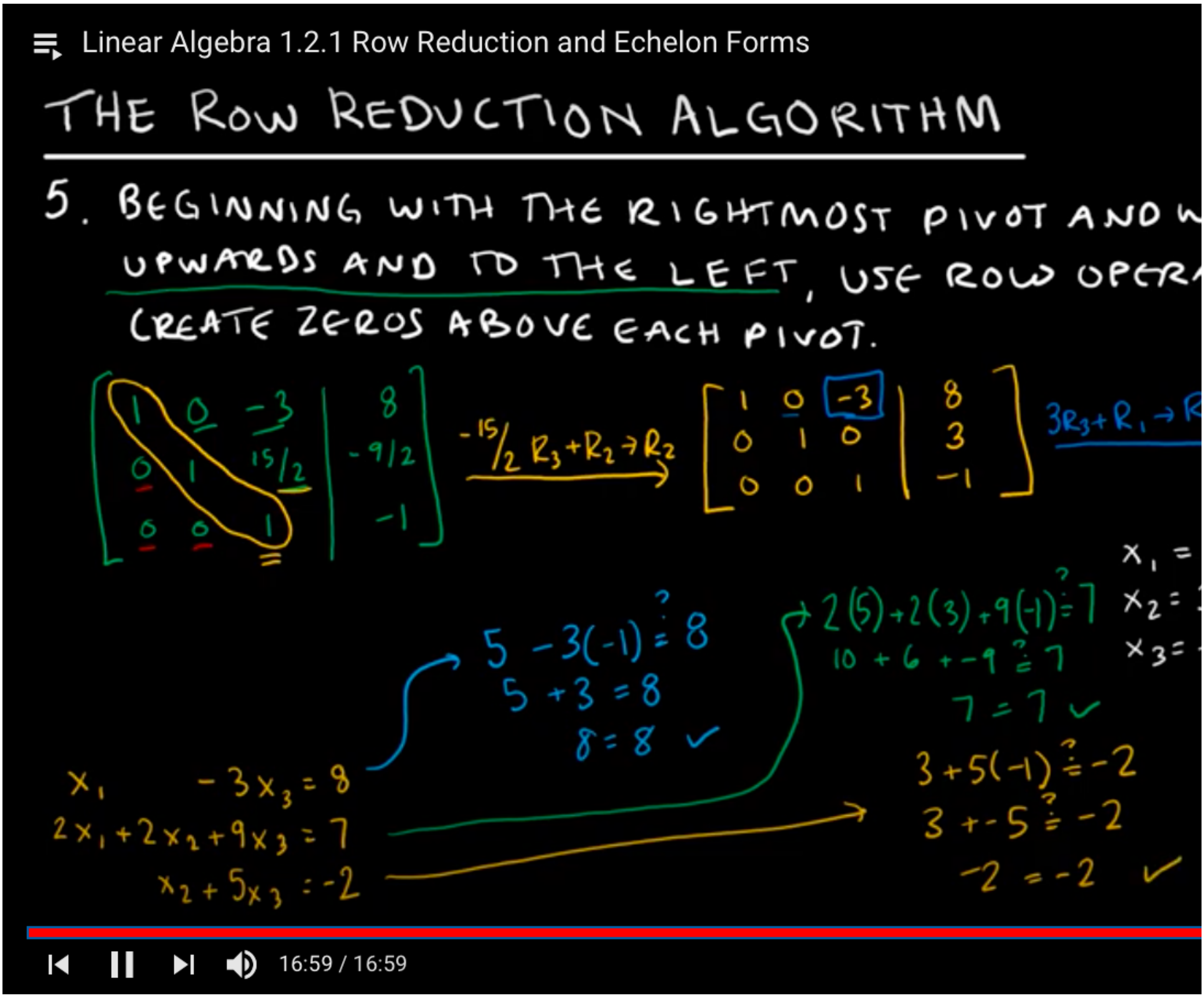
DETERMINE IF THE COLUMNS OF THE MATRIX $A = \begin{bmatrix} 0 & 1 & 4 \\ 1 & 2 & -1 \\ 5 & 8 & 0 \end{bmatrix}$

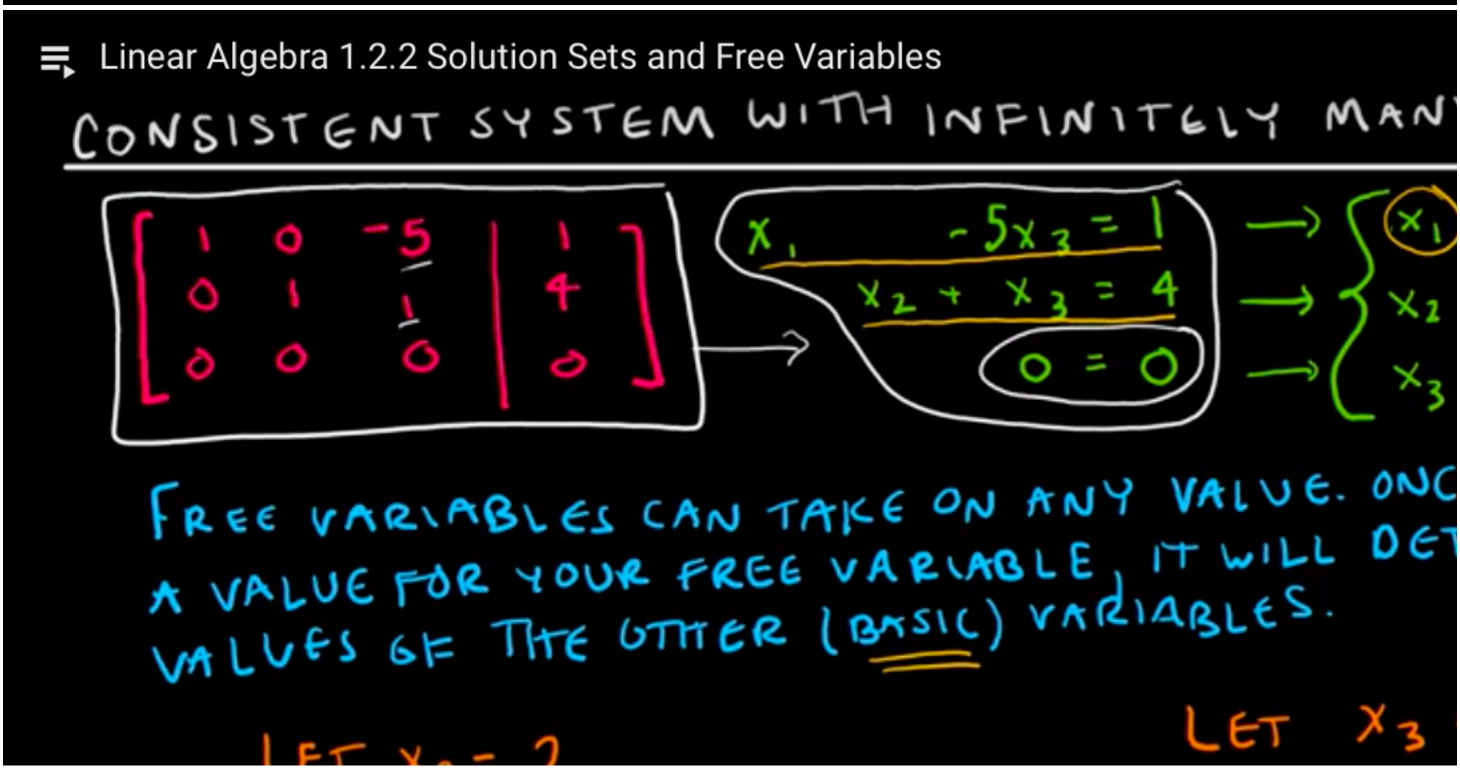
INDEPENDENT. IF NOT, FIND A LINEAR DEPENDENCE RELATION AM

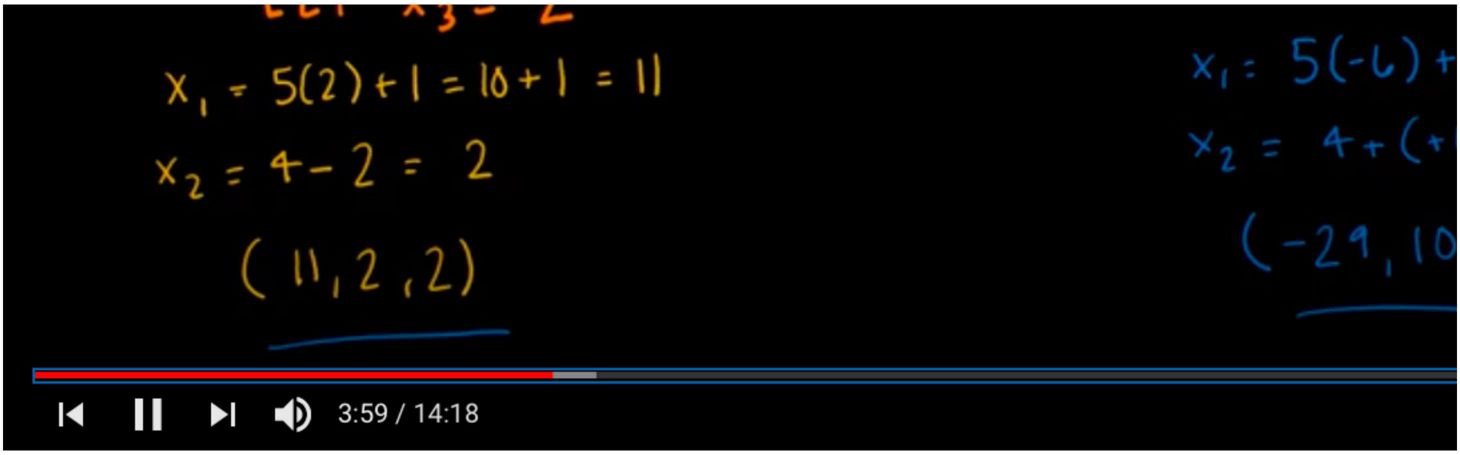
$$\left[\begin{array}{ccc|c} 0 & 1 & 4 & 0 \\ 1 & 2 & -1 & 0 \\ 5 & 8 & 0 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 2 & -1 & 0 \\ 0 & 1 & 4 & 0 \\ 0 & -2 & 5 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 2 & -1 & 0 \\ 0 & 1 & 4 & 0 \\ 0 & 0 & 13 & 0 \end{array}\right.$$

No free variables

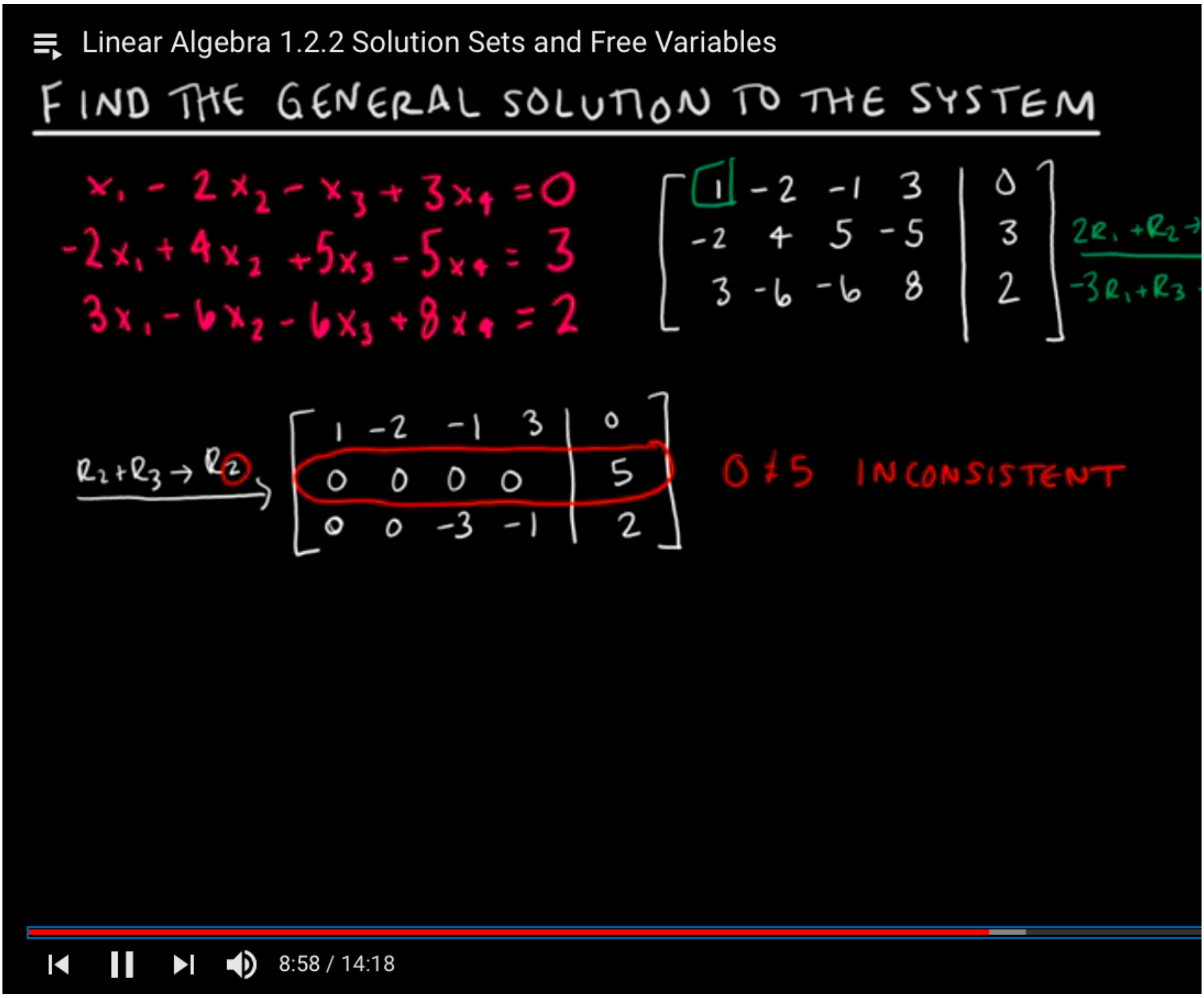
$Ax = 0$ has only the trivial solutio


Linear Algebra 1.2.1 Row Reduction and Echelon Forms
THE ROW REDUCTION ALGORITHM
5. BEGINNING WITH THE RIGHTMOST PIVOT AND W
UPWARDS AND TO THE LEFT, USE ROW OPERA
CREATE ZEROS ABOVE EACH PIVOT.
$-\frac{15}{2}R_3+R_2 \to R_2$
$3R_3+R_1 \to R$
$x_1 =$
$x_2 =$
$x_3 =$
$5-3(-1) \stackrel{?}{=} 8$
$5+3=8$
$8=8$ ✓
$2(5)+2(3)+9(-1) \stackrel{?}{=} 7$
$10+6+-9 \stackrel{?}{=} 7$
$7=7$ ✓
$3+5(-1) \stackrel{?}{=} -2$
$3+-5 \stackrel{?}{=} -2$
$-2=-2$ ✓
$x_1 - 3x_3 = 8$
$2x_1+2x_2+9x_3=7$
$x_2+5x_3=-2$
16:59 / 16:59


Linear Algebra 1.2.2 Solution Sets and Free Variables
CONSISTENT SYSTEM WITH INFINITELY MAN
$x_1 - 5x_3 = 1$
$x_2 + x_3 = 4$
$0 = 0$
FREE VARIABLES CAN TAKE ON ANY VALUE. ONC
A VALUE FOR YOUR FREE VARIABLE, IT WILL DET
VALUES OF THE OTHER (BASIC) VARIABLES.
LET x_3



x1 = 5(2) + 1 = 10 + 1 = 11
x2 = 4 − 2 = 2
(11, 2, 2)
x1 = 5(−6) +
x2 = 4 + (+
(−29, 10
3:59 / 14:18



Linear Algebra 1.2.2 Solution Sets and Free Variables
FIND THE GENERAL SOLUTION TO THE SYSTEM
x1 − 2x2 − x3 + 3x4 = 0
−2x1 + 4x2 + 5x3 − 5x4 = 3
3x1 − 6x2 − 6x3 + 8x4 = 2
[1 −2 −1 3 | 0; −2 4 5 −5 | 3; 3 −6 −6 8 | 2]
2R1 + R2 →
−3R1 + R3
R2 + R3 → R2
[1 −2 −1 3 | 0; 0 0 0 0 | 5; 0 0 −3 −1 | 2]
0 ≠ 5 INCONSISTENT
8:58 / 14:18

Linear Algebra 1.2.2 Solution Sets and Free Variables

PRACTICE

FIND THE $\boxed{\text{GENERAL}}$ SOLUTION OF THE AUGMENTED

$$\left[\begin{array}{ccc|c} \boxed{1} & 3 & 4 & 7 \\ \boxed{3} & 9 & 7 & 6 \end{array}\right] \xrightarrow{-3R_1 + R_2 \to R_2} \left[\begin{array}{ccc|c} \boxed{1} & 3 & 4 & 7 \\ 0 & 0 & \boxed{-5} & -15 \end{array}\right] \quad -\tfrac{1}{5}R_2 \to R$$

☆ $x_1 + 3x_2 + 4x_3 = 7$

$x_3 = 3$

$x_1 + 3x_2 + 4(3) = 7$

$x_1 + 3x_2 + 12 = 7$

$x_1 + 3x_2 = -5$

$x_1 = -5 - 3x_2$

$\left\{ \begin{array}{l} x \\ x \\ x \end{array} \right.$

14:16 / 14:18

VECTORS IN $\mathbb{R}^2$

$(1,3) \rightarrow \begin{bmatrix} 1 \\ 3 \end{bmatrix}$ 2×1

VECTOR – AN ORDERED LIST OF NUMBERS (MORE O

COLUMN VECTOR – A VECTOR WITH ONLY ONE COLUM

USE THESE FOR ORDERED PAIRS, TRIP

VECTORS IN $\mathbb{R}^2$ – THE SET OF ALL VECTORS WITH 2

$\mathbb{R} \rightarrow$ REAL NUMBERS $2 \rightarrow$ NUMB

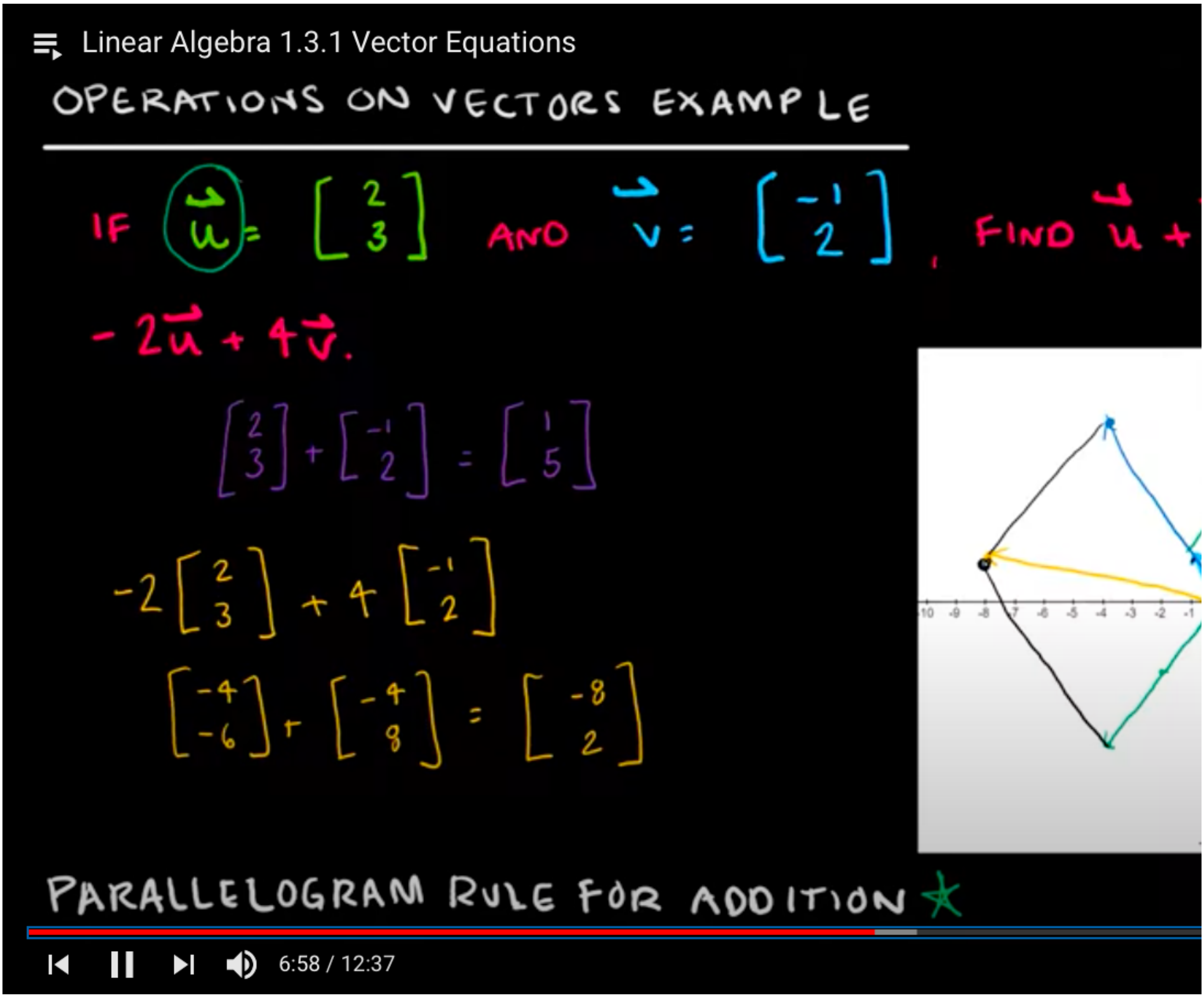
THIS IS THE SET OF ALL POINTS

OPERATIONS WITH VECTORS – SAME AS WITH OTHER

$2\begin{bmatrix} 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 2 \\ 6 \end{bmatrix}$ SCALAR – MULTIPLY VECTOR BY A CO

$\begin{bmatrix} 1 \\ 3 \end{bmatrix} + \begin{bmatrix} 4 \\ -1 \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$ ADDITION – ADD CORRESPONDING VA

MULTIPLICATION – $\boxed{\text{NOPE!}}$ DIMENSIO


Linear Algebra 1.3.1 Vector Equations
OPERATIONS ON VECTORS EXAMPLE
IF $\vec{u} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ AND $\vec{v} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$, FIND $\vec{u}$ +
$-2\vec{u} + 4\vec{v}$.
$\begin{bmatrix} 2 \\ 3 \end{bmatrix} + \begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 5 \end{bmatrix}$
$-2\begin{bmatrix} 2 \\ 3 \end{bmatrix} + 4\begin{bmatrix} -1 \\ 2 \end{bmatrix}$
$\begin{bmatrix} -4 \\ -6 \end{bmatrix} + \begin{bmatrix} -4 \\ 8 \end{bmatrix} = \begin{bmatrix} -8 \\ 2 \end{bmatrix}$
-10 -9 -8 -7 -6 -5 -4 -3 -2 -1
PARALLELOGRAM RULE FOR ADDITION
6:58 / 12:37

Linear Algebra 1.3.1 Vector Equations

VECTORS IN $\mathbb{R}^n$

IF $n \in \mathbb{R}$, THEN $\mathbb{R}^n$ IS THE COLLECTION OF AL
ORDERED N-TUPLES OF N REAL NUMBERS WR
N × 1 COLUMN MATRICES.

$$\vec{u} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ \vdots \\ u_n \end{bmatrix}$$

$$0 \quad \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

ZERO VECTOR - THE VECTOR WHOSE ENTRIES ARE ALL 0, DENOTED BY $\mathbf{0}$.

8:06 / 12:37

Linear Algebra 1.4.1 The Matrix Equation Ax=b

LINEAR COMBINATIONS AS THE PRODUCT OF A MATRIX

IF A IS AN $m \times n$ MATRIX WITH COLUMNS $a_1, a_2, \ldots,$
$x \in \mathbb{R}^n$, THEN Ax IS THE LINEAR COMBINATION
COLUMNS OF A USING THE CORRESPONDING
x AS WEIGHTS :

$n \times 1$

weights

$$A x = \overbrace{\begin{bmatrix} a_1 & a_2 & a_3 & \cdots & a_n \end{bmatrix}}^{A} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{bmatrix} = a_1 x_1 + \ldots$$

$1 \times n$

1:21 / 11:10

EXAMPLES

1. FIND Ax IF $A = \begin{bmatrix} 2 & -1 & 0 \\ 3 & 4 & 1 \end{bmatrix}$ AND $x = \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix}$.

2. WRITE THE LINEAR COMBINATION $2v_1 - 3v_2 + 4$
MATRIX TIMES A VECTOR.

PRACTICE 2

LET $a_1 = \begin{bmatrix}1\\4\\-2\end{bmatrix}$, $a_2 = \begin{bmatrix}-2\\-3\\7\end{bmatrix}$ AND $b = \begin{bmatrix}4\\1\\h\end{bmatrix}$

WHAT VALUE(S) OF h IS b IN THE SPAN OF a_1,

$$\left[\begin{array}{cc|c}1 & -2 & 4\\4 & -3 & 1\\-2 & 7 & h\end{array}\right] \sim \left[\begin{array}{cc|c}1 & -2 & 4\\0 & 5 & -15\\0 & 3 & 8+h\end{array}\right] \sim \left[\begin{array}{cc|c}1 & -2 & \\0 & 1 & \\0 & 3 & \end{array}\right]$$

$$\left[\begin{array}{cc|c}1 & -2 & 4\\0 & 1 & -3\\0 & 0 & 17+h\end{array}\right]$$

$0 = 0$

$17 + h = 0$

$h = -17$

h must be -1

b to be in t

PRACTICE

A MINING COMPANY HAS TWO MINES. ONE DAYS OP
MINE 1 PRODUCES ORE THAT CONTAINS 20 METRIC TO
AND 550 KG OF SILVER. MINE 2 PRODUCES 30 ME
COPPER AND 500 KG OF SILVER. HOW MANY DAYS S
MINE OPERATE TO PRODUCE 150 TONS COPPER AND 28

$$x_1\begin{bmatrix}20\\550\end{bmatrix} + x_2\begin{bmatrix}30\\500\end{bmatrix} = \begin{bmatrix}150\\2825\end{bmatrix}$$

$$\left[\begin{array}{cc|c}20 & 30 & 150\\550 & 500 & 2825\end{array}\right] \sim \left[\begin{array}{cc|c}1 & 1.5 & 7.5\\550 & 500 & 2825\end{array}\right] \sim \left[\begin{array}{cc}1 & 1.5\\0 & -325\end{array}\right.$$

$$\sim \left[\begin{array}{cc|c}1 & 1.5 & 7.5\\0 & 1 & 4\end{array}\right] \sim \left[\begin{array}{cc|c}1 & 0 & 1.5\\0 & 1 & 4\end{array}\right]$$

MINE 1 1.5

MINE 2 4

Linear Algebra 1.4.1 The Matrix Equation Ax=b

EXAMPLES

1. FIND Ax IF $A = \begin{bmatrix} 2 & -1 & 0 \\ 3 & 4 & 1 \end{bmatrix}$ AND $x = \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix}$.

$$Ax = 2\begin{bmatrix} 2 \\ 3 \end{bmatrix} + 3\begin{bmatrix} -1 \\ 4 \end{bmatrix} - 1\begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 4 \\ 6 \end{bmatrix} + \begin{bmatrix} -3 \\ 12 \end{bmatrix} + \begin{bmatrix} 0 \\ -1 \end{bmatrix}$$

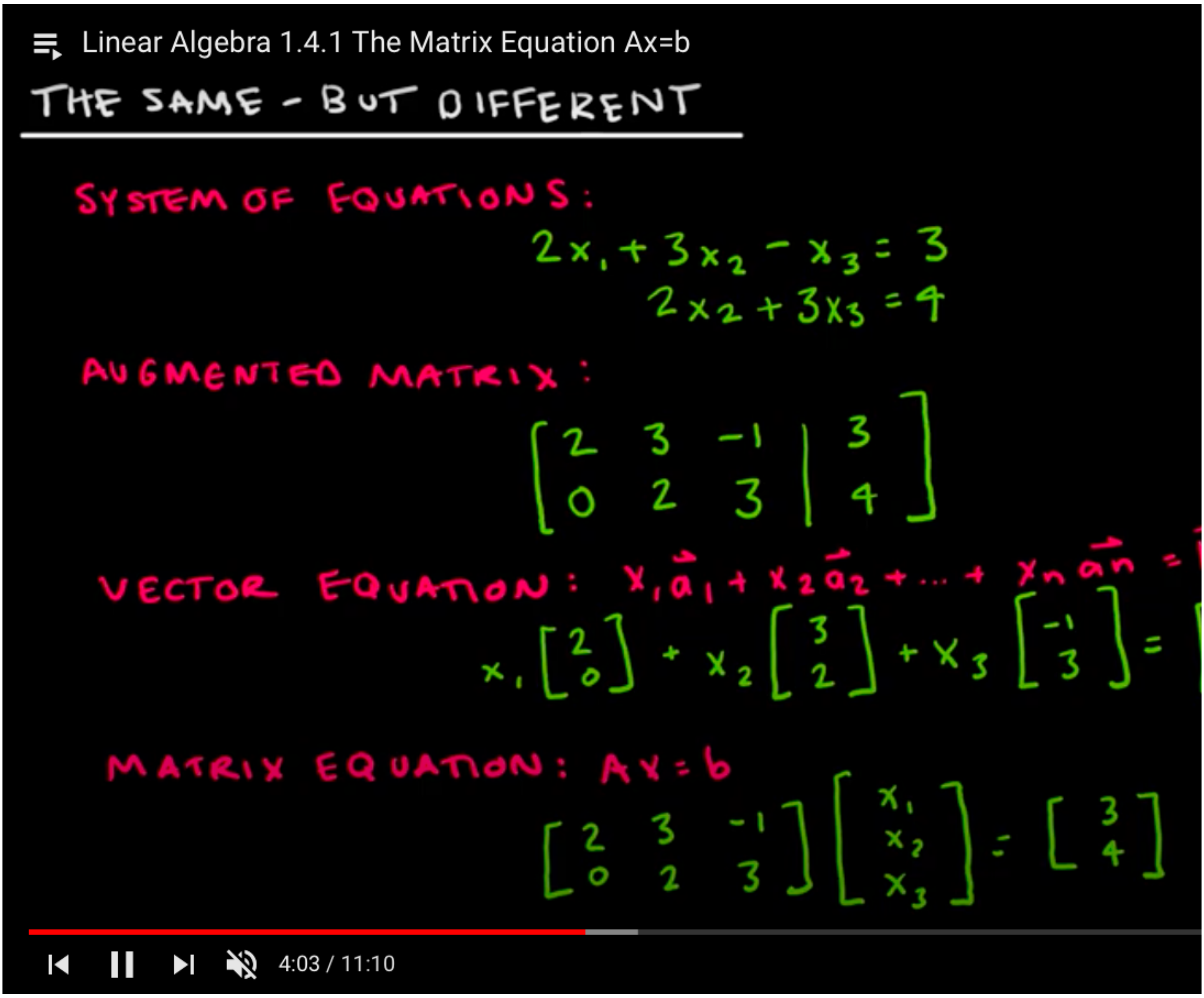
$$= \begin{bmatrix} 1 \\ 17 \end{bmatrix}$$

2. WRITE THE LINEAR COMBINATION $\underline{2}v_1 - \underline{3}v_2 + \underline{4}$

MATRIX TIMES A VECTOR. Ax

$$Ax = \begin{bmatrix} v_1 & v_2 & v_3 \end{bmatrix} \begin{bmatrix} 2 \\ -3 \\ 4 \end{bmatrix}$$

3:44 / 11:10


Linear Algebra 1.4.1 The Matrix Equation Ax=b
THE SAME - BUT DIFFERENT
SYSTEM OF EQUATIONS:
$2x_1 + 3x_2 - x_3 = 3$
$2x_2 + 3x_3 = 4$
AUGMENTED MATRIX:
$\left[\begin{array}{ccc|c} 2 & 3 & -1 & 3 \\ 0 & 2 & 3 & 4 \end{array}\right]$
VECTOR EQUATION: $x_1\vec{a}_1 + x_2\vec{a}_2 + \dots + x_n\vec{a}_n =$
$x_1\begin{bmatrix}2\\0\end{bmatrix} + x_2\begin{bmatrix}3\\2\end{bmatrix} + x_3\begin{bmatrix}-1\\3\end{bmatrix} =$
MATRIX EQUATION: $Ax = b$
$\begin{bmatrix}2 & 3 & -1\\0 & 2 & 3\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} = \begin{bmatrix}3\\4\end{bmatrix}$
4:03 / 11:10

Linear Algebra 1.4.1 The Matrix Equation Ax=b

EXISTENCE OF SOLUTIONS

LET A BE AN $m \times n$ MATRIX. THEN THESE STATE

ARE LOGICALLY EQUIVALENT :

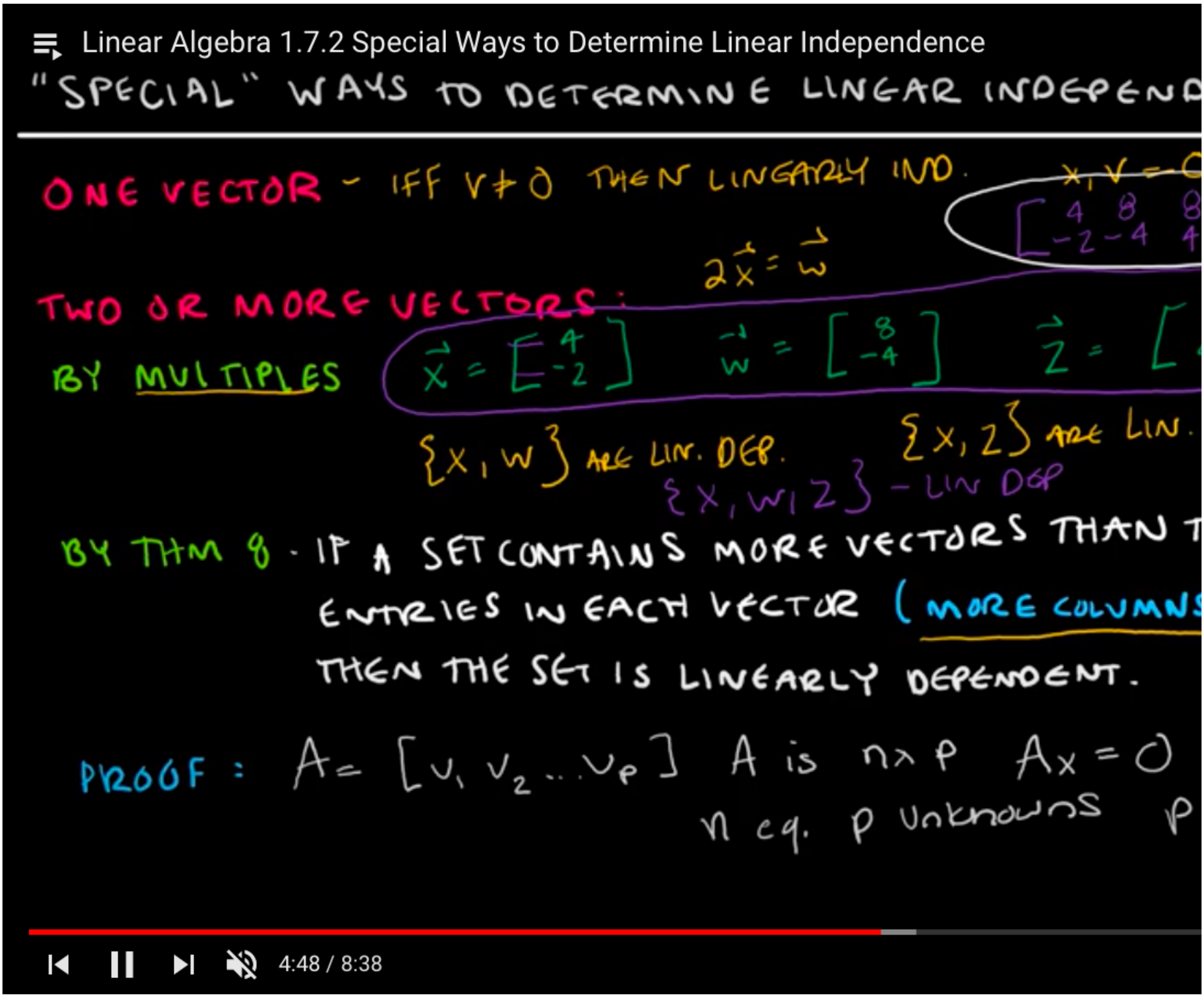
1) FOR EACH b IN $\mathbb{R}^m$, THE EQUATION $Ax = b$ HAS

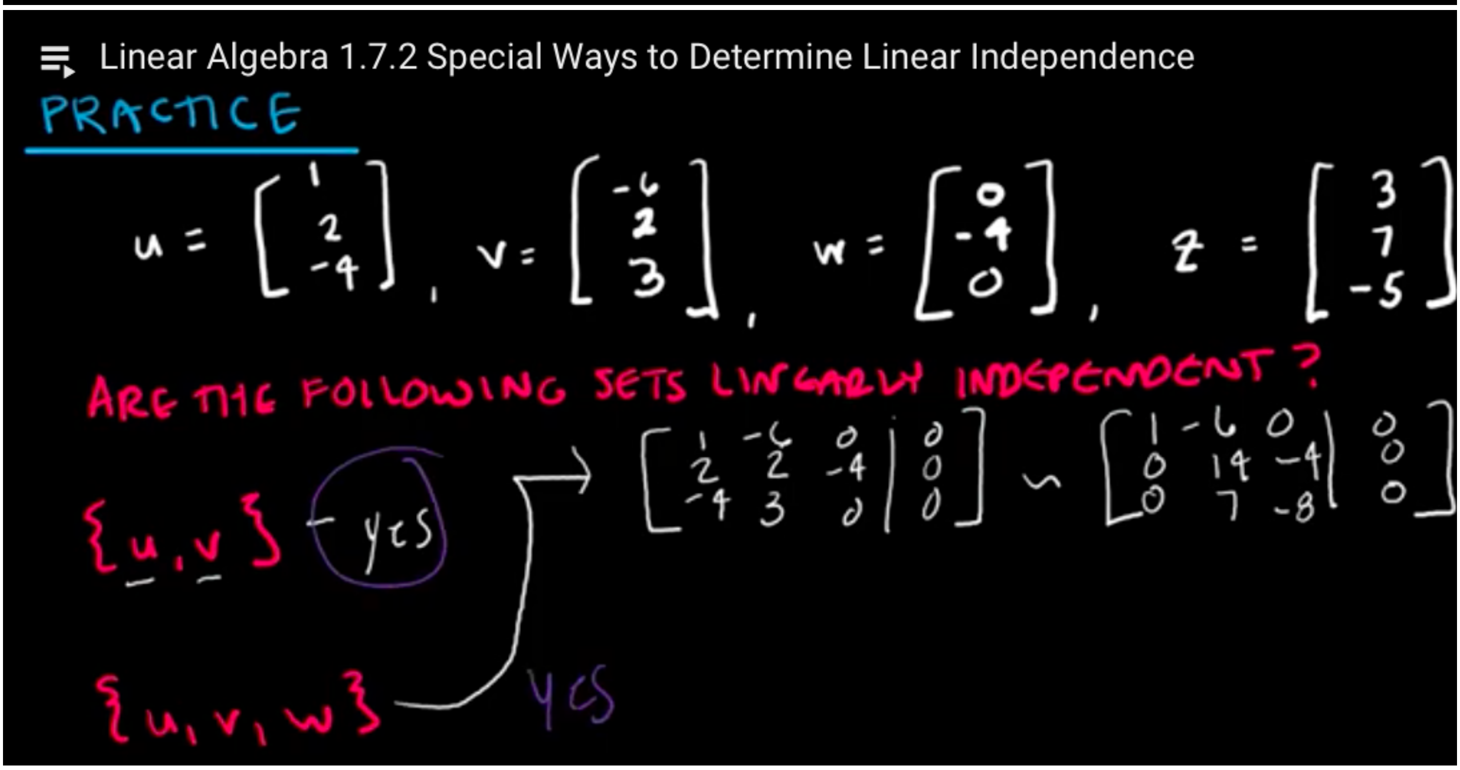
2) EACH b IN $\mathbb{R}^m$ IS A LINEAR COMBINATION O
COLUMNS OF A

3) THE COLUMNS OF A SPAN $\mathbb{R}^m$

4) A HAS A PIVOT POSITION IN EVERY ROW
(A MUST BE A COEFFICIENT MATRIX - NOT AUGM

6:08 / 11:10


Linear Algebra 1.7.2 Special Ways to Determine Linear Independence
"SPECIAL" WAYS TO DETERMINE LINEAR INDEPEND
ONE VECTOR - IFF V ≠ 0 THEN LINEARLY IND.
$2\vec{x} = \vec{w}$
TWO OR MORE VECTORS:
BY MULTIPLES
$\vec{x} = \begin{bmatrix} 4 \\ -2 \end{bmatrix}$ $\vec{w} = \begin{bmatrix} 8 \\ -4 \end{bmatrix}$ $\vec{z} =$
{x, w} ARE LIN. DEP.
{x, z} ARE LIN.
{x, w, z} - LIN DEP
BY THM 8 - IF A SET CONTAINS MORE VECTORS THAN T
ENTRIES IN EACH VECTOR (MORE COLUMNS
THEN THE SET IS LINEARLY DEPENDENT.
PROOF: $A = [v_1\ v_2 \dots v_p]$ A is $n \times p$ $Ax = 0$
n eq. p unknowns p
4:48 / 8:38


Linear Algebra 1.7.2 Special Ways to Determine Linear Independence
PRACTICE
$u = \begin{bmatrix} 1 \\ 2 \\ -4 \end{bmatrix}, v = \begin{bmatrix} -6 \\ 2 \\ 3 \end{bmatrix}, w = \begin{bmatrix} 0 \\ -4 \\ 0 \end{bmatrix}, z = \begin{bmatrix} 3 \\ 7 \\ -5 \end{bmatrix}$
ARE THE FOLLOWING SETS LINEARLY INDEPENDENT?
{u, v} - yes
{u, v, w} yes
$\left[\begin{array}{ccc|c} 1 & -6 & 0 & 0 \\ 2 & 2 & -4 & 0 \\ -4 & 3 & 0 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & -6 & 0 & 0 \\ 0 & 14 & -4 & 0 \\ 0 & 7 & -8 & 0 \end{array}\right]$

$\{u, v, w, z\}$ more columns than rows

linearly dependent

no

$$\begin{bmatrix} - & - & - & - & - \\ - & - & - & - & - \\ - & - & - & - & - \end{bmatrix}$$

8:35 / 8:38

Linear Algebra 1.8.1 Matrix Transformations

WHAT IS IT ?

$f(x) = x + 2$ Domain / Preimage $\{1, 2\}$ — $\{3, 4\}$ codomain / image

A MATRIX TRANSFORMATION IS A FUNCTION. ...
VECTOR x BY MULTIPLICATION BY A, AND MAP ... Ax

SO $T(x)$ FROM $\boxed{\mathbb{R}^n \text{ TO } \mathbb{R}^m}$ IS A RULE THAT ASSI...
VECTOR x IN $\mathbb{R}^n$ TO A VECTOR $T(x)$ IN $\mathbb{R}^m$.

$\mathbb{R}^n$ IS THE DOMAIN

$\mathbb{R}^m$ IS THE CODOMAIN / IMAGE

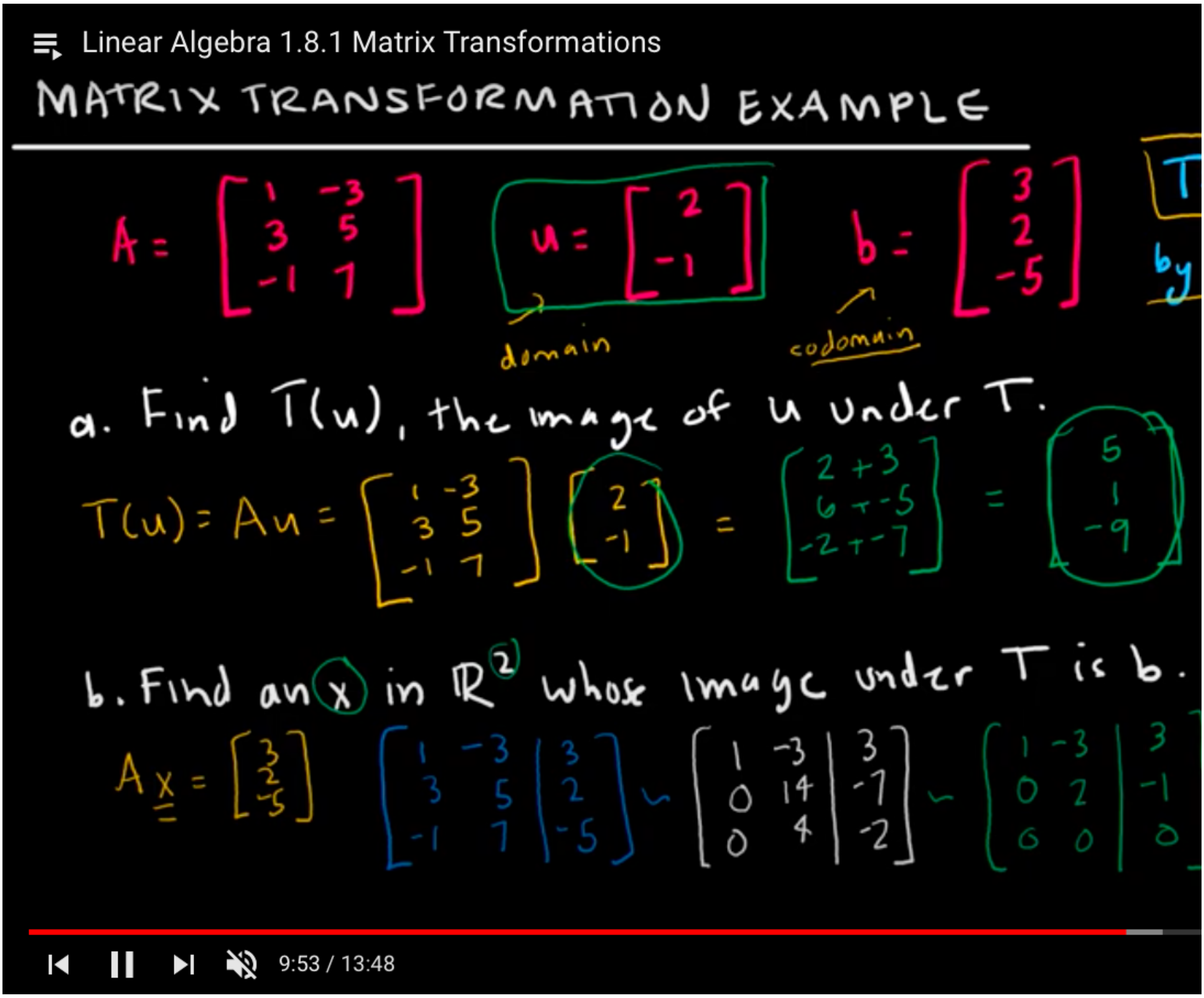
$$\boxed{T : \mathbb{R}^n \to \mathbb{R}^m}$$ or

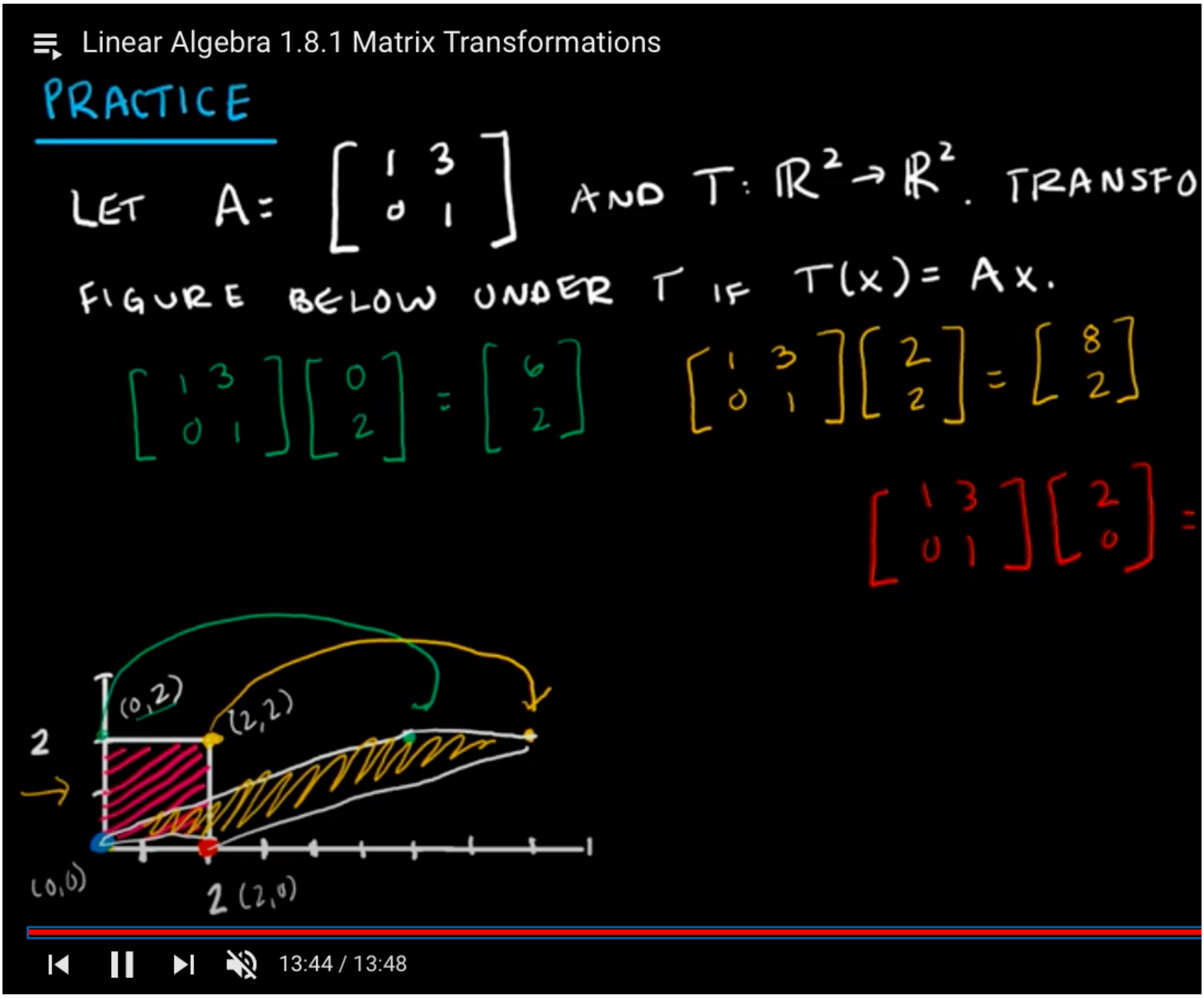
$\boxed{\text{Ex:}}$ $x = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ $\mathbb{R}^2$

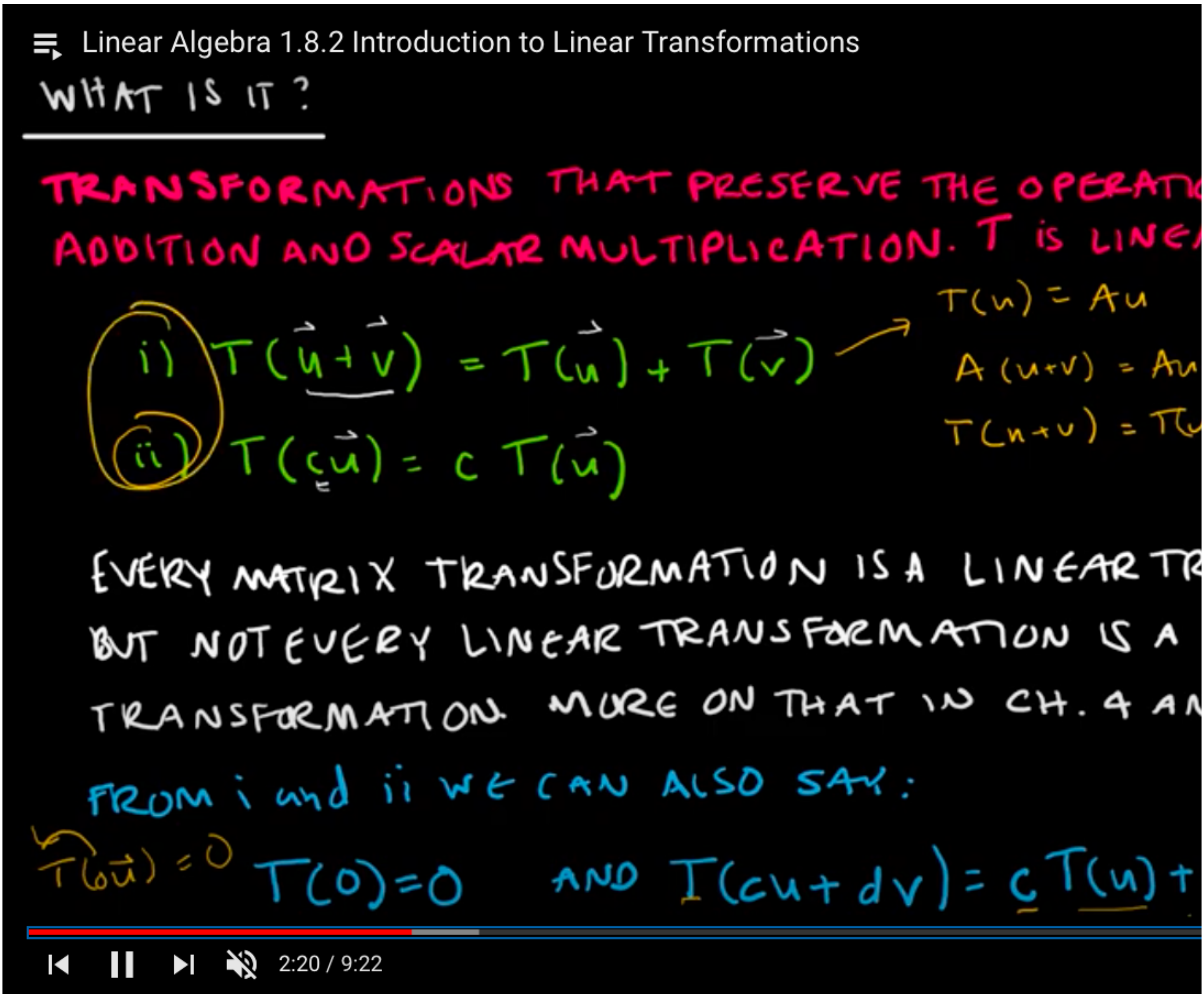
$A = \begin{bmatrix} 1 & 4 \\ -2 & 3 \\ 0 & 1 \end{bmatrix}$ $\mathbb{R}^3$

$$Ax = \begin{bmatrix} 2 + 4 \\ -4 + 3 \\ 0 + 1 \end{bmatrix} = \begin{bmatrix} 6 \\ -1 \\ 1 \end{bmatrix}$$

3:36 / 13:48


Linear Algebra 1.8.1 Matrix Transformations
MATRIX TRANSFORMATION EXAMPLE
$A = \begin{bmatrix} 1 & -3 \\ 3 & 5 \\ -1 & 7 \end{bmatrix}$
$u = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$
$b = \begin{bmatrix} 3 \\ 2 \\ -5 \end{bmatrix}$
domain
codomain
a. Find T(u), the image of u under T.
$T(u) = Au = \begin{bmatrix} 1 & -3 \\ 3 & 5 \\ -1 & 7 \end{bmatrix}\begin{bmatrix} 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 2+3 \\ 6+-5 \\ -2+-7 \end{bmatrix} = \begin{bmatrix} 5 \\ 1 \\ -9 \end{bmatrix}$
b. Find an x in $\mathbb{R}^2$ whose image under T is b.
$A\underline{x} = \begin{bmatrix} 3 \\ 2 \\ -5 \end{bmatrix}$
$\left[\begin{array}{cc|c} 1 & -3 & 3 \\ 3 & 5 & 2 \\ -1 & 7 & -5 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & -3 & 3 \\ 0 & 14 & -7 \\ 0 & 4 & -2 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & -3 & 3 \\ 0 & 2 & -1 \\ 0 & 0 & 0 \end{array}\right]$
9:53 / 13:48


Linear Algebra 1.8.1 Matrix Transformations
PRACTICE
LET A = [1 3; 0 1] AND T: R² → R². TRANSFO
FIGURE BELOW UNDER T IF T(x) = Ax.
[1 3; 0 1][0; 2] = [6; 2]
[1 3; 0 1][2; 2] = [8; 2]
[1 3; 0 1][2; 0] =
(0,2)
(2,2)
2
(0,0)
2 (2,0)
13:44 / 13:48


Linear Algebra 1.8.2 Introduction to Linear Transformations
WHAT IS IT ?
TRANSFORMATIONS THAT PRESERVE THE OPERATI
ADDITION AND SCALAR MULTIPLICATION. T is LINE
i) T(u + v) = T(u) + T(v)
ii) T(cu) = c T(u)
T(u) = Au
A(u+v) = Au
T(u+v) = T(u
EVERY MATRIX TRANSFORMATION IS A LINEAR TR
BUT NOT EVERY LINEAR TRANSFORMATION IS A
TRANSFORMATION. MORE ON THAT IN CH. 4 AN
FROM i and ii WE CAN ALSO SAY:
T(0u) = 0
T(0)=0 AND T(cu + dv) = cT(u) +
2:20 / 9:22

Linear Algebra 1.8.2 Introduction to Linear Transformations

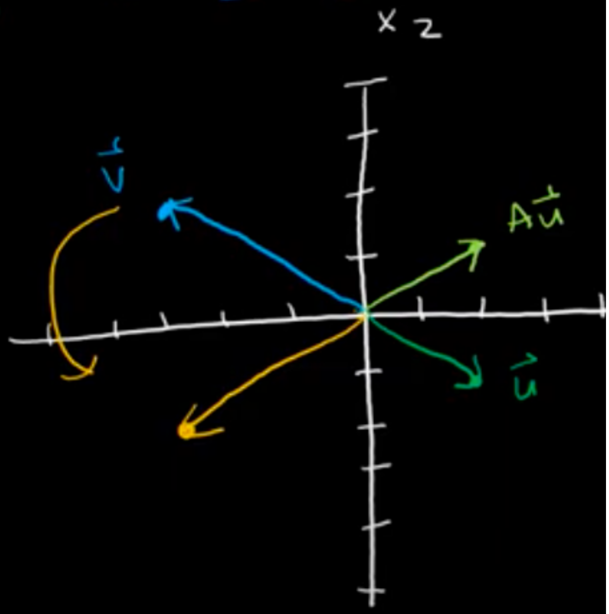
PRACTICE

GIVE A GEOMETRIC DESCRIPTION OF THE TRANSFORMA

IF $A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$. $\vec{u} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$ $\vec{v} = \begin{bmatrix} -3 \\ 2 \end{bmatrix}$

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} -3 \\ 2 \end{bmatrix} = \begin{bmatrix} -3 \\ -2 \end{bmatrix}$$



5:14 / 9:22

Linear Algebra 1.7.2 Special Ways to Determine Linear Independence

"SPECIAL" WAYS TO DETERMINE LINEAR INDEPEND

ONE VECTOR - IFF $v \neq 0$ THEN LINEARLY IND.

$x, v = 0$

$\begin{bmatrix} 4 & 8 & 8 \\ -2 & -4 & 4 \end{bmatrix}$

$2\vec{x} = \vec{w}$

TWO OR MORE VECTORS:

BY MULTIPLES $\vec{x} = \begin{bmatrix} 4 \\ -2 \end{bmatrix}$ $\vec{w} = \begin{bmatrix} 8 \\ -4 \end{bmatrix}$ $\vec{z} = [$

$\{x, w\}$ ARE LIN. DEP. $\{x, z\}$ ARE LIN.

$\{x, w, z\}$ - LIN DEP

BY THM 8. IF A SET CONTAINS MORE VECTORS THAN T

ENTRIES IN EACH VECTOR (MORE COLUMNS

THEN THE SET IS LINEARLY DEPENDENT.

PROOF: $A = [v_1\ v_2 \ldots v_p]$ A is $n \times p$ $Ax = 0$

n eq. p unknowns p

4:48 / 8:38

Linear Algebra 1.7.2 Special Ways to Determine Linear Independence

PRACTICE

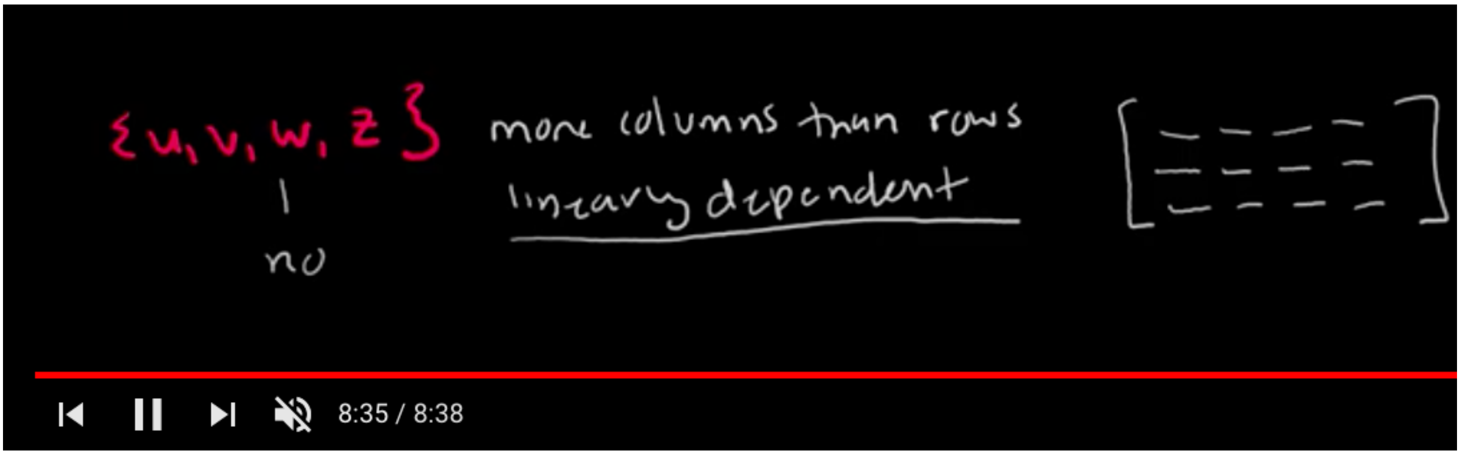
$u = \begin{bmatrix} 1 \\ 2 \\ -4 \end{bmatrix}, \quad v = \begin{bmatrix} -6 \\ 2 \\ 3 \end{bmatrix}, \quad w = \begin{bmatrix} 0 \\ -4 \\ 0 \end{bmatrix}, \quad z = \begin{bmatrix} 3 \\ 7 \\ -5 \end{bmatrix}$

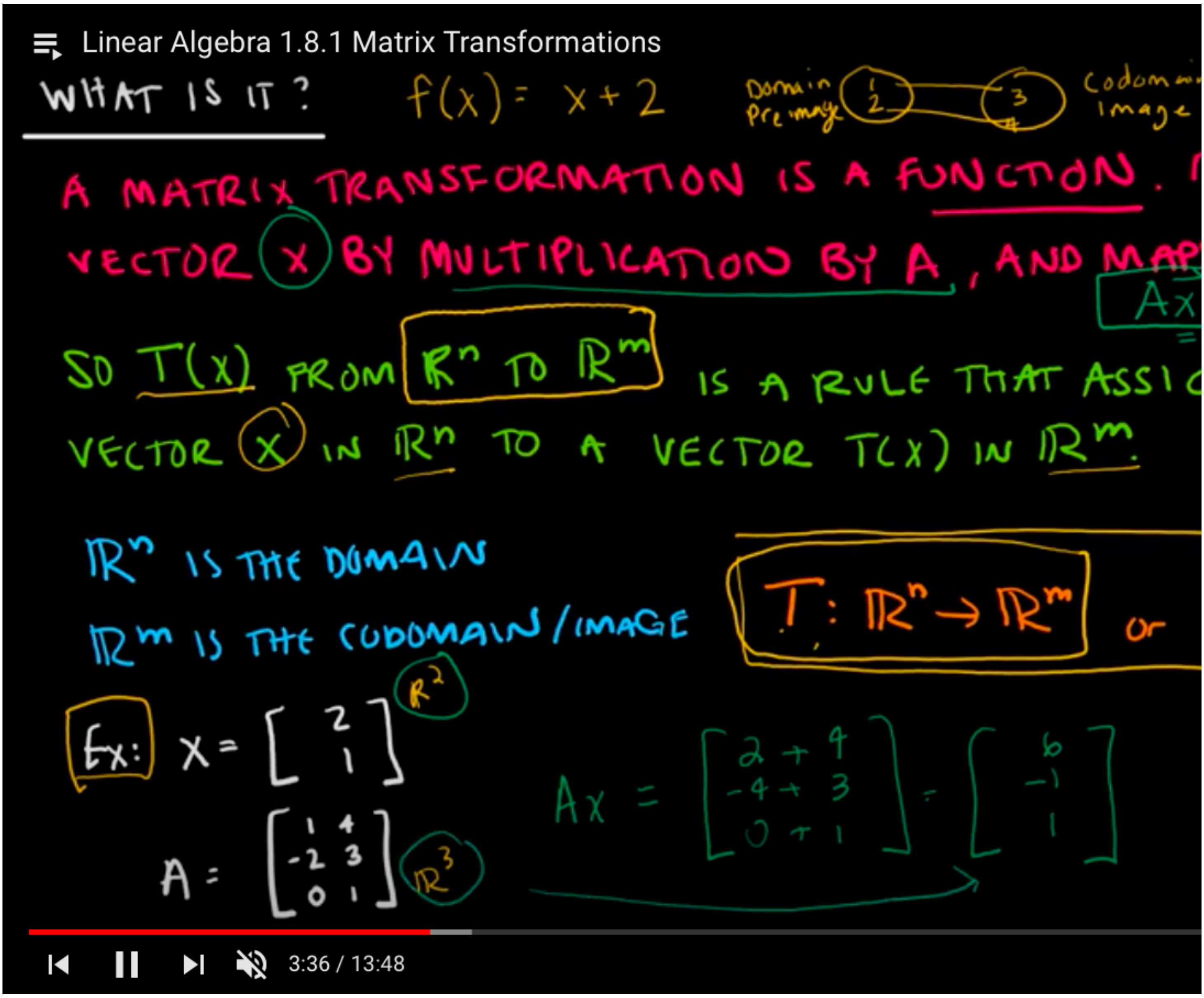
ARE THE FOLLOWING SETS LINEARLY INDEPENDENT?

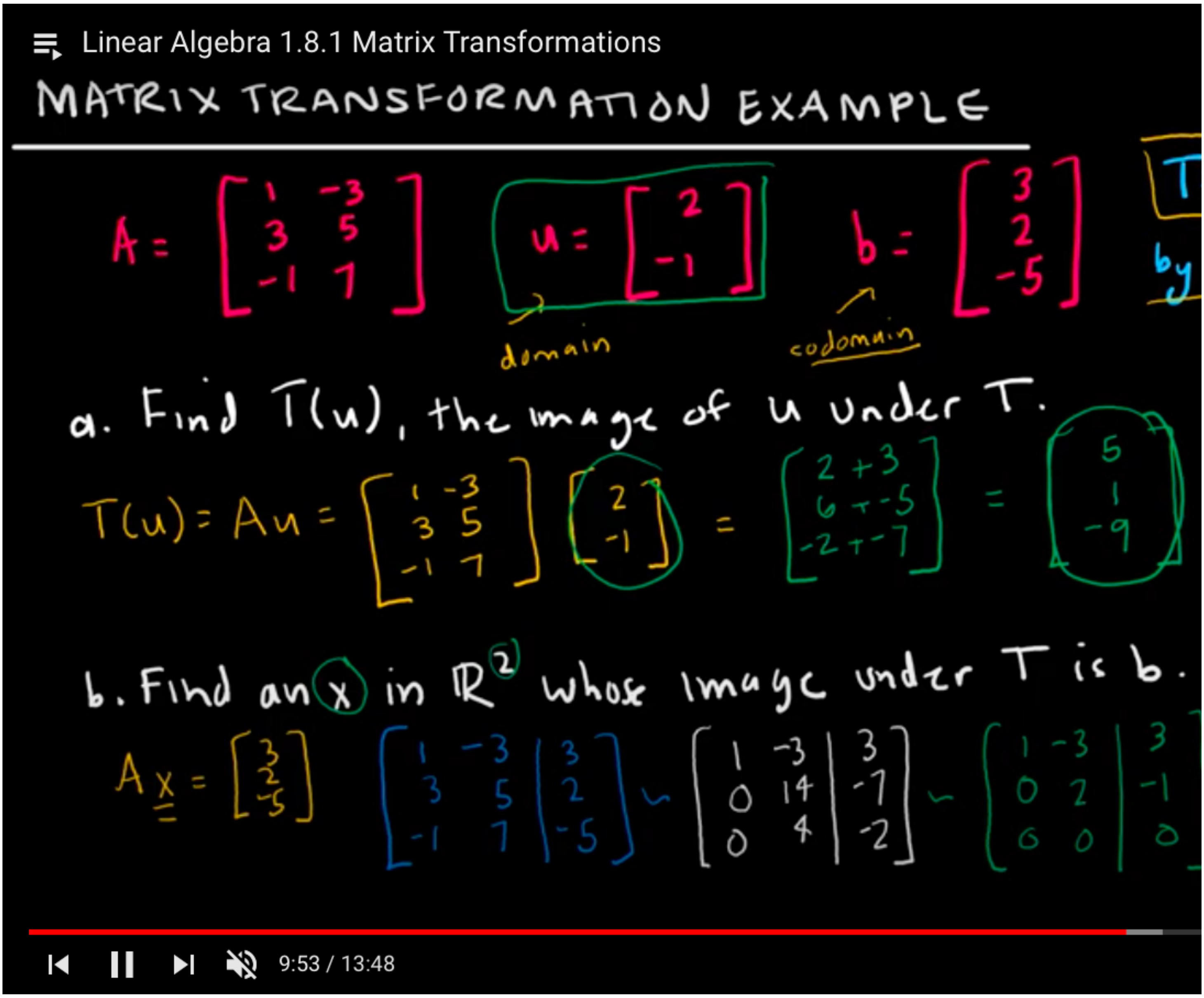
$\{u, v\}$ - yes

$\{u, v, w\}$ yes

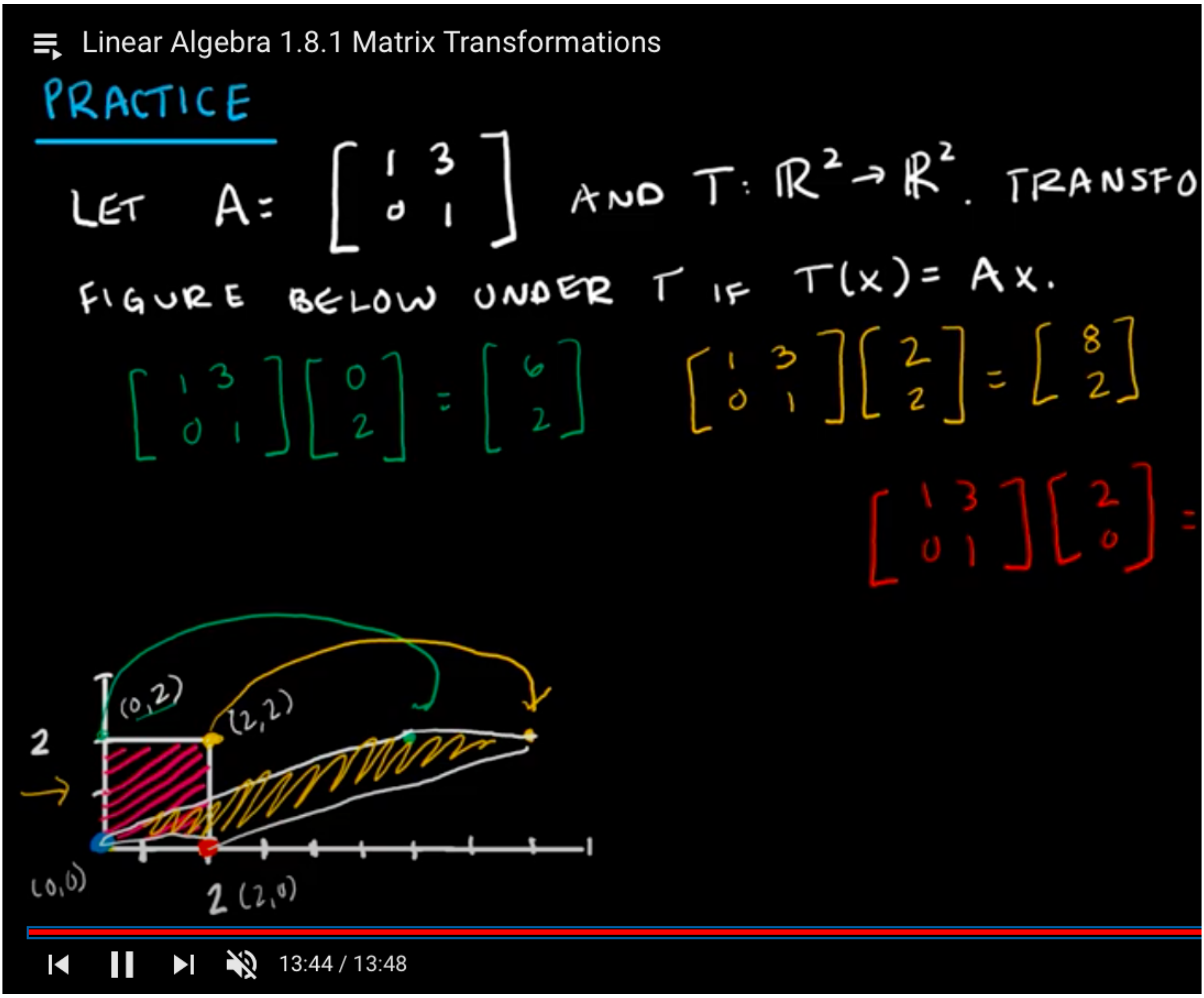
$\left[\begin{array}{ccc|c} 1 & -6 & 0 & 0 \\ 2 & 2 & -4 & 0 \\ -4 & 3 & 0 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & -6 & 0 & 0 \\ 0 & 14 & -4 & 0 \\ 0 & 7 & -8 & 0 \end{array}\right]$


{u, v, w, z}
no
more columns than rows
linearly dependent
8:35 / 8:38

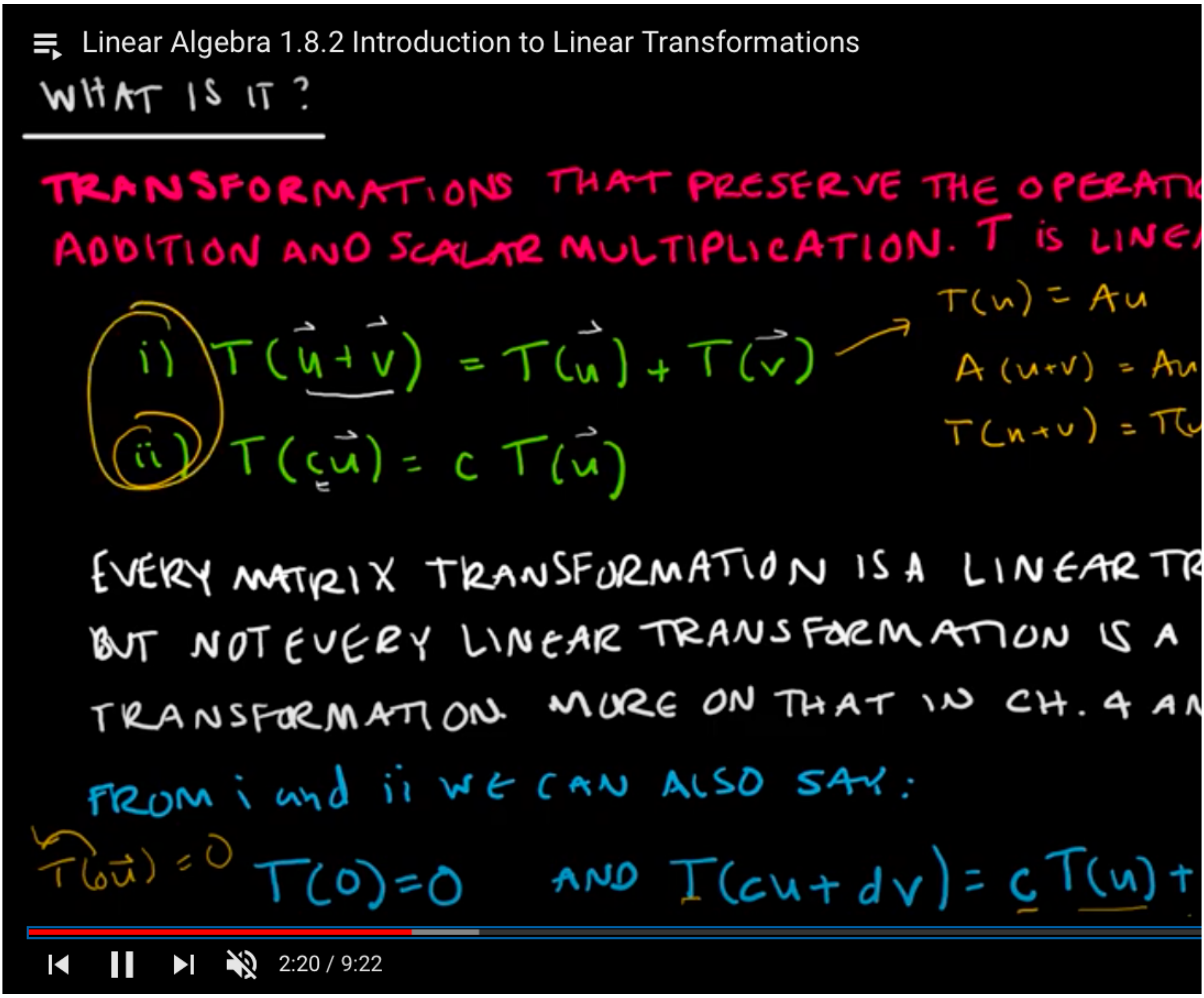

Linear Algebra 1.8.1 Matrix Transformations
WHAT IS IT ?
f(x) = x + 2
Domain
Preimage
Codomain
Image
A MATRIX TRANSFORMATION IS A FUNCTION.
VECTOR x BY MULTIPLICATION BY A, AND MAP
Ax
SO T(x) FROM R^n TO R^m IS A RULE THAT ASSI
VECTOR x IN R^n TO A VECTOR T(x) IN R^m.
R^n IS THE DOMAIN
R^m IS THE CODOMAIN / IMAGE
T : R^n → R^m
or
Ex: x = [2 1]
A = [1 4; -2 3; 0 1]
R^2
R^3
Ax = [2 + 4; -4 + 3; 0 + 1] = [6; -1; 1]
3:36 / 13:48

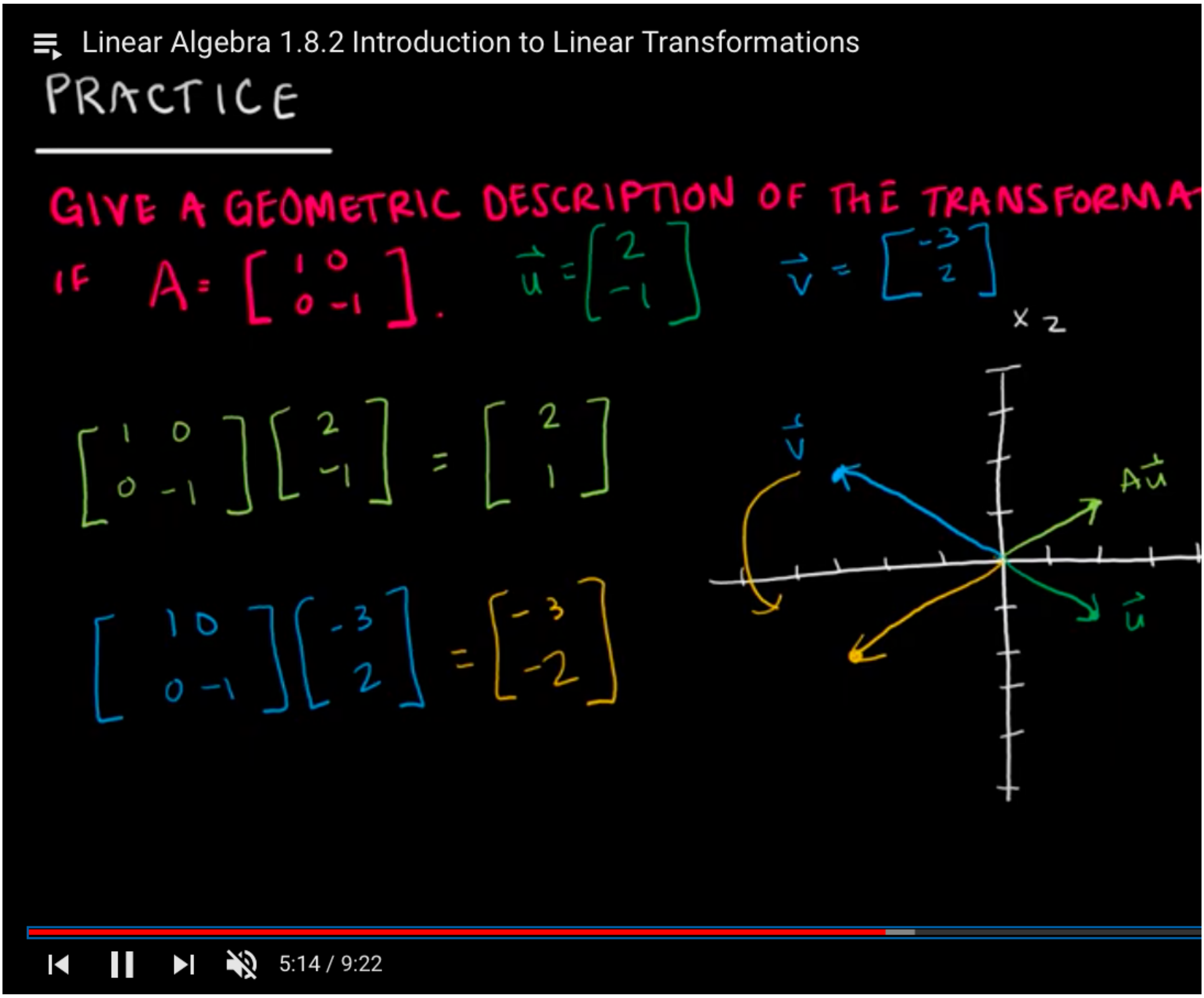


Linear Algebra 1.8.1 Matrix Transformations
MATRIX TRANSFORMATION EXAMPLE
$A = \begin{bmatrix} 1 & -3 \\ 3 & 5 \\ -1 & 7 \end{bmatrix}$ $u = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$ $b = \begin{bmatrix} 3 \\ 2 \\ -5 \end{bmatrix}$
domain
codomain
a. Find T(u), the image of u under T.
$T(u) = Au = \begin{bmatrix} 1 & -3 \\ 3 & 5 \\ -1 & 7 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 2+3 \\ 6+-5 \\ -2+-7 \end{bmatrix} = \begin{bmatrix} 5 \\ 1 \\ -9 \end{bmatrix}$
b. Find an x in $\mathbb{R}^2$ whose image under T is b.
$A\underline{x} = \begin{bmatrix} 3 \\ 2 \\ -5 \end{bmatrix}$
$\left[\begin{array}{cc|c} 1 & -3 & 3 \\ 3 & 5 & 2 \\ -1 & 7 & -5 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & -3 & 3 \\ 0 & 14 & -7 \\ 0 & 4 & -2 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & -3 & 3 \\ 0 & 2 & -1 \\ 0 & 0 & 0 \end{array}\right]$
9:53 / 13:48

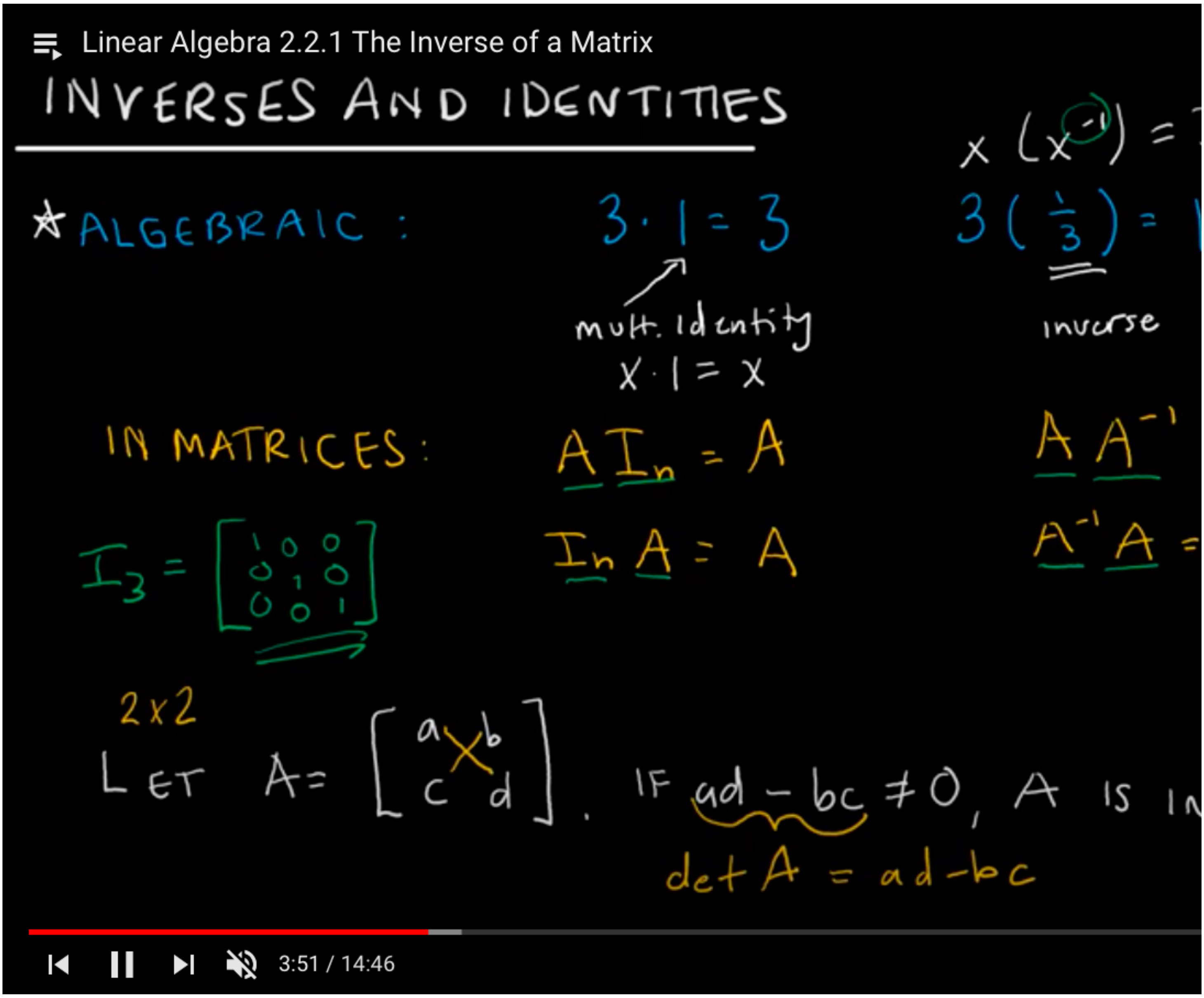


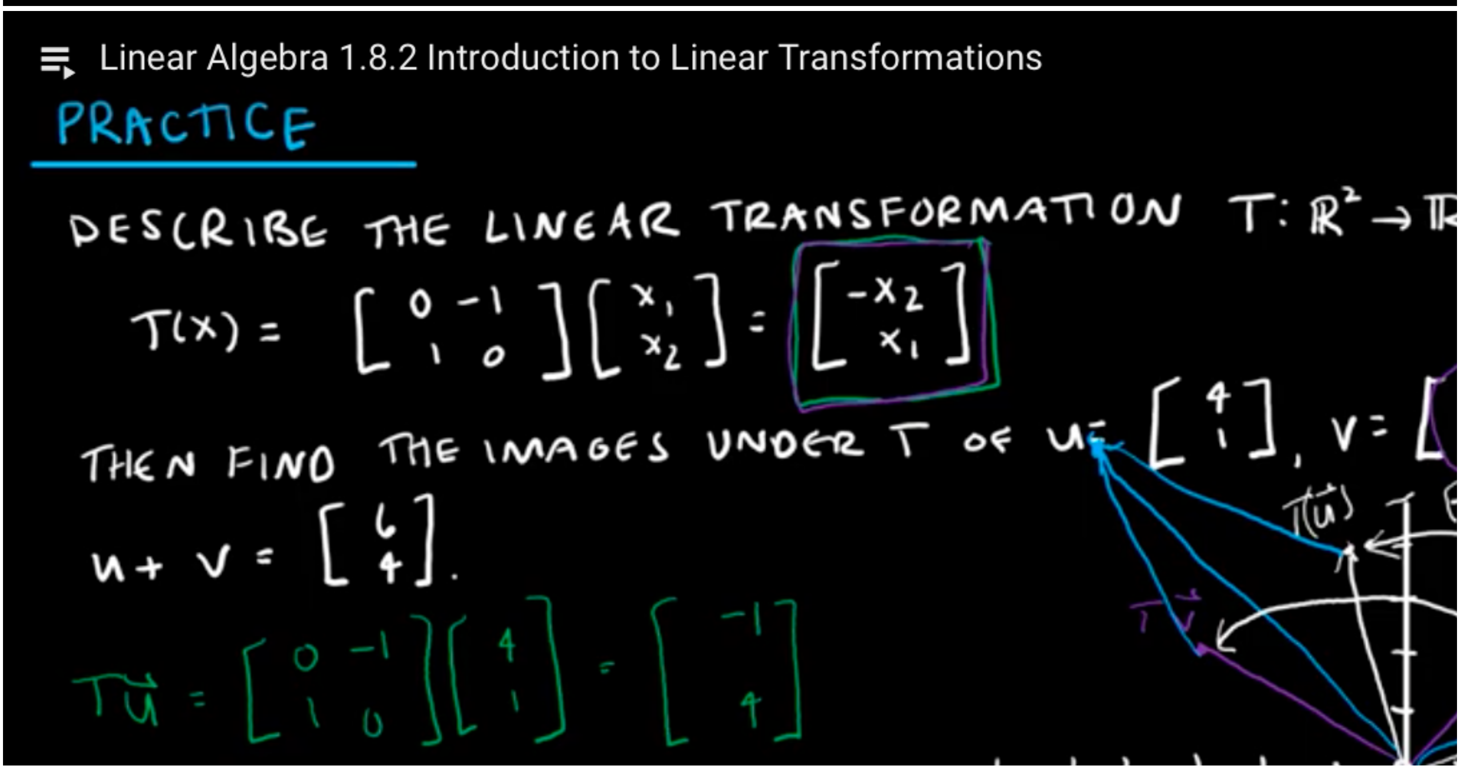
Linear Algebra 1.8.1 Matrix Transformations
PRACTICE
LET $A = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$ AND $T: \mathbb{R}^2 \to \mathbb{R}^2$. TRANSFO
FIGURE BELOW UNDER T IF $T(x) = Ax$.
$\begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 6 \\ 2 \end{bmatrix}$
$\begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 8 \\ 2 \end{bmatrix}$
$\begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 0 \end{bmatrix} =$
(0,2)
(2,2)
2
(0,0)
2 (2,0)
13:44 / 13:48


Linear Algebra 1.8.2 Introduction to Linear Transformations
WHAT IS IT ?
TRANSFORMATIONS THAT PRESERVE THE OPERATI
ADDITION AND SCALAR MULTIPLICATION. T is LINE
i) $T(\vec{u}+\vec{v}) = T(\vec{u}) + T(\vec{v})$
ii) $T(c\vec{u}) = c\,T(\vec{u})$
$T(u) = Au$
$A(u+v) = Au$
$T(u+v) = T(u$
EVERY MATRIX TRANSFORMATION IS A LINEAR TR
BUT NOT EVERY LINEAR TRANSFORMATION IS A
TRANSFORMATION. MORE ON THAT IN CH. 4 AN
FROM i and ii WE CAN ALSO SAY:
$T(0\vec{u}) = 0$
$T(0) = 0$ AND $T(cu + dv) = cT(u) +$
2:20 / 9:22

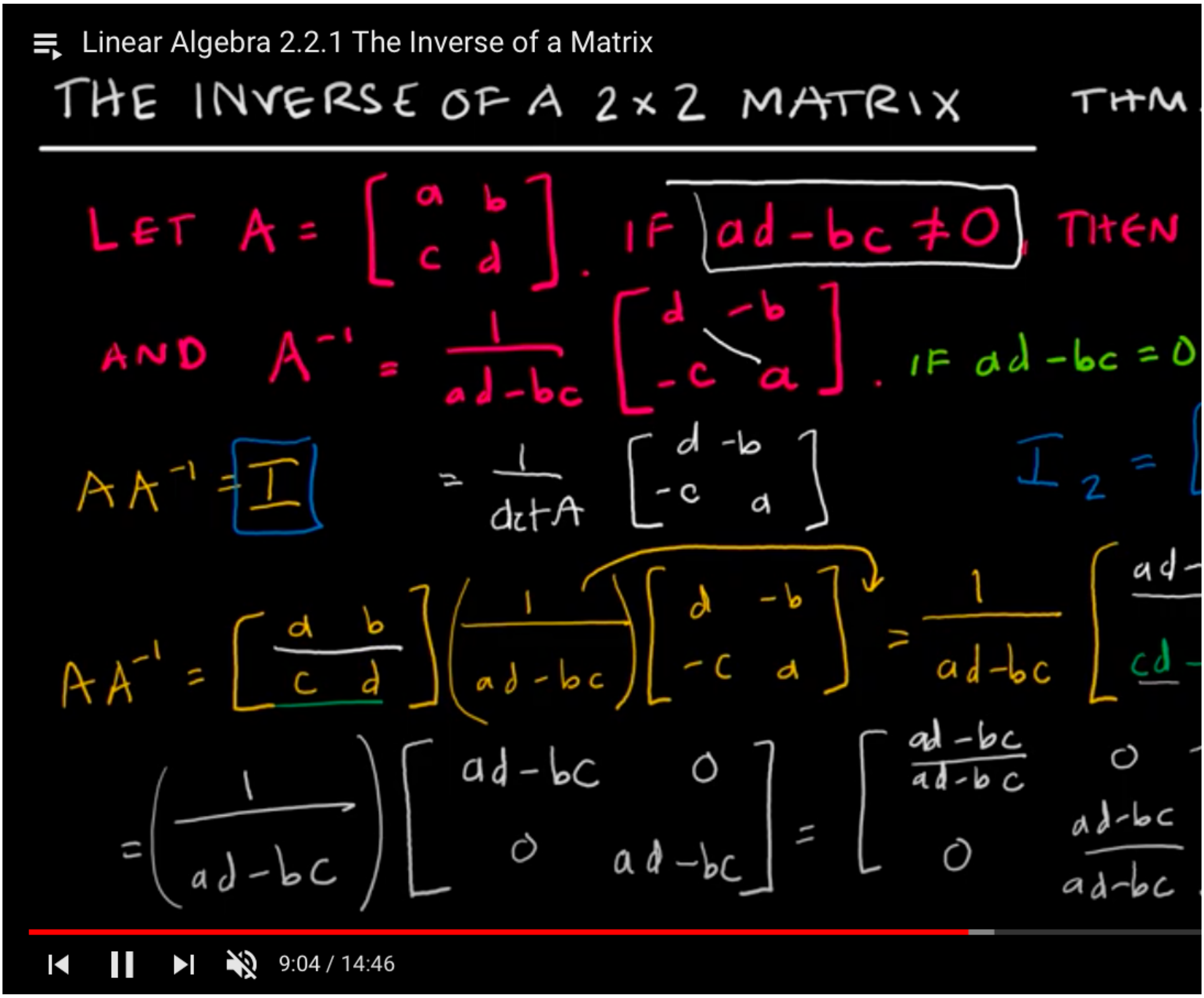


Linear Algebra 1.8.2 Introduction to Linear Transformations
PRACTICE
GIVE A GEOMETRIC DESCRIPTION OF THE TRANSFORMA
IF $A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$. $\vec{u} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$ $\vec{v} = \begin{bmatrix} -3 \\ 2 \end{bmatrix}$
x 2
$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$
$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} -3 \\ 2 \end{bmatrix} = \begin{bmatrix} -3 \\ -2 \end{bmatrix}$
$\vec{v}$
$A\vec{u}$
$\vec{u}$
5:14 / 9:22


Linear Algebra 2.2.1 The Inverse of a Matrix
INVERSES AND IDENTITIES
★ ALGEBRAIC :
3 · 1 = 3
mult. identity
x · 1 = x
x (x⁻¹) =
3 (1/3) =
inverse
IN MATRICES :
A Iₙ = A
Iₙ A = A
A A⁻¹
A⁻¹ A =
I₃ = [1 0 0; 0 1 0; 0 0 1]
2x2
LET A = [a b; c d].
IF ad − bc ≠ 0, A IS IN
det A = ad − bc
3:51 / 14:46


Linear Algebra 1.8.2 Introduction to Linear Transformations
PRACTICE
DESCRIBE THE LINEAR TRANSFORMATION T: ℝ² → ℝ
T(x) = [0 −1; 1 0][x₁; x₂] = [−x₂; x₁]
THEN FIND THE IMAGES UNDER T OF u = [4; 1], v =
u + v = [6; 4].
Tu = [0 −1; 1 0][4; 1] = [−1; 4]
T(u)

T v = [-3 2]
T(u+v) = [-4 6]
9:16 / 9:22



Linear Algebra 2.2.1 The Inverse of a Matrix
THE INVERSE OF A 2×2 MATRIX
THM.
LET A = [a b; c d]. IF ad−bc ≠ 0, THEN
AND A⁻¹ = 1/(ad−bc) [d −b; −c a].
IF ad−bc = 0
AA⁻¹ = I
= 1/detA [d −b; −c a]
I₂ =
AA⁻¹ = [a b; c d] (1/(ad−bc)) [d −b; −c a] = 1/(ad−bc)
= (1/(ad−bc)) [ad−bc 0; 0 ad−bc] =
ad−bc / ad−bc
9:04 / 14:46

PRACTICE

$A =$

FIND THE INVERSE OF $A = \begin{bmatrix} 2 & 1 \\ 0 & 4 \end{bmatrix}$ AN

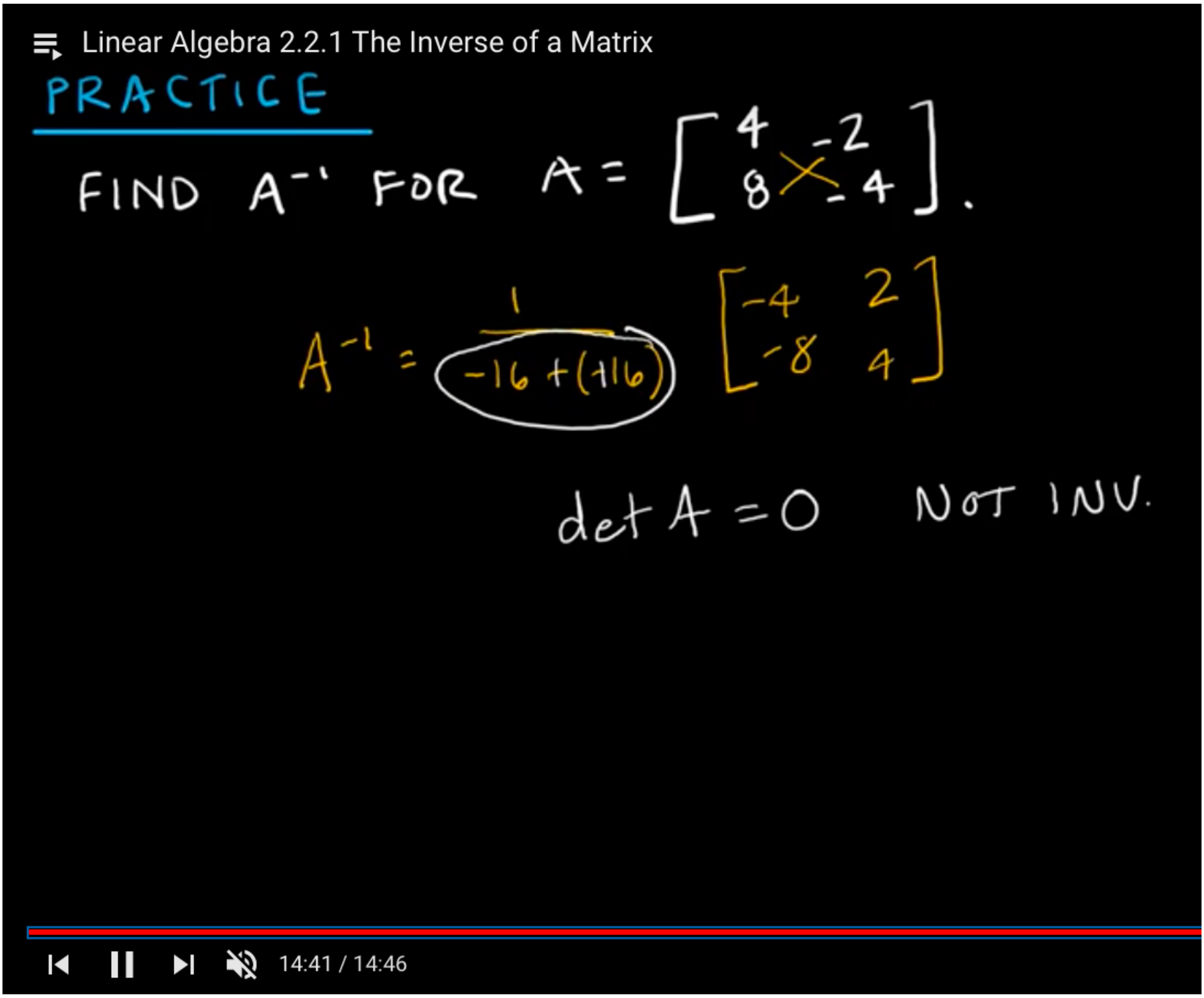
IS THE INVERSE.

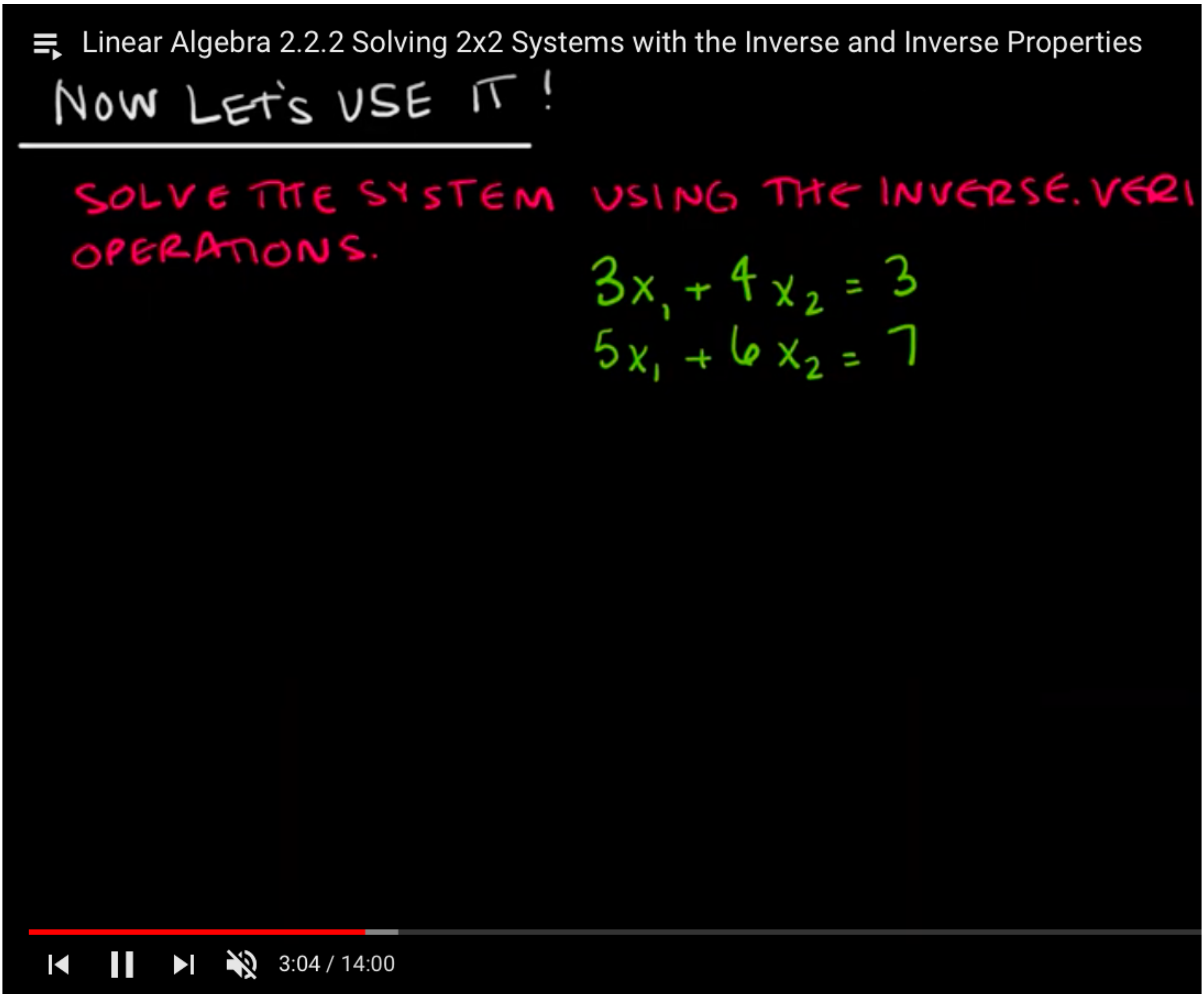
$$A^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$\boxed{A^{-1}} = \frac{1}{2(4) - 0(1)} \begin{bmatrix} 4 & -1 \\ 0 & 2 \end{bmatrix} = \frac{1}{8} \begin{bmatrix} 4 & -1 \\ 0 & 2 \end{bmatrix} =$$

$$AA^{-1} = \begin{bmatrix} 2 & 1 \\ 0 & 4 \end{bmatrix} \begin{bmatrix} \frac{1}{2} & -\frac{1}{8} \\ 0 & \frac{1}{4} \end{bmatrix} = \begin{bmatrix} 1+0 & -\frac{1}{4} + \frac{1}{4} \\ 0+0 & 0+1 \end{bmatrix}$$

$$A^{-1}A = \begin{bmatrix} \frac{1}{2} & -\frac{1}{8} \\ 0 & \frac{1}{4} \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} 1+0 & \frac{1}{2} + -\frac{1}{2} \\ 0+0 & 0+1 \end{bmatrix}$$


Linear Algebra 2.2.1 The Inverse of a Matrix
PRACTICE
FIND A^{-1} FOR $A = \begin{bmatrix} 4 & -2 \\ 8 & -4 \end{bmatrix}$.
$A^{-1} = \frac{1}{-16+(+16)} \begin{bmatrix} -4 & 2 \\ -8 & 4 \end{bmatrix}$
det $A = 0$ NOT INV.
14:41 / 14:46


Linear Algebra 2.2.2 Solving 2x2 Systems with the Inverse and Inverse Properties
NOW LET'S USE IT !
SOLVE THE SYSTEM USING THE INVERSE. VERI
OPERATIONS.
$3x_1 + 4x_2 = 3$
$5x_1 + 6x_2 = 7$
3:04 / 14:00