M 4

Deal, nicely

Descrete Structures (60%)

① Counting
② Set
③ Mathametical logic
④ Induction & Recursion
⑤ Relation (Matrix & Algebra) 40%
⑥ Solving linear eqn

① Counting :-

* Product & Sum rule

Product Rule :- if Procedue Can be broken into 2 Sequence of tasks

$A \longrightarrow B \longrightarrow C$

A to B = n_1 ways
B to C = n_2 ways

total Procedure Can be done by $n_1 n_2$ ways

Q.1) The Chairs of an auditorium are to be labelled with an uppercase Eng. letter followed by a +ve integer ≤ 100. what is the largest no. of Chairs that Can be labelled diff.

Ans:-

$26 \times 100 = 2600$

12 × 11 × 10 = 1320

44 × 60 = 2640

Q) 2 employees in an office, 12 flr building.

Ans

$11 \cdot 12 = 132$ or ${}^{12}C_2 \times 2!$

Th.

Q) if a task can be done either in one of n_1 ways or one of n_2 ways, where none of the n_1 ways is common with any of the n_2 ways, then the procedure can be done in $n_1 + n_2$ ways.

Q) A Student can choose a computer project from one of the 3 lists. The three lists contains 23, 15 & 19 projects respectively, how many possible projects are available.

Ans :- $23 + 15 + 19 = 57$

* Inclusion - Exclusion principle / Substraction Rule

Inclusion = Adding element

Subs. : Common element.

Th.

Eg:- if a task can be done in either n_1 ways or n_2 ways, then the procedure / task can be done in :-

$$(n_1 + n_2) - (\text{no. of common ways})$$

	CS	B	Both
Q 350 →	220	147	51

Pigeon hole principle

If we have 'k' boxes & 'k+1' objects, then we have atleast 1 box with atleast 2 objects

- eg:- ~~365~~ 367

1	2	3	4

1st P_1 P_2 P_3 P_4
P_5 P_6 P_7 P_8 atleast 1 hole with atleast 2 pegion
P_9 P_{10} P_{11} P_{12} " " 1 " " " " 3 "
P_{13} → " " 1 " " " " 4 " "

N = objects
k = boxes

general formulae = $\left\lceil \frac{N}{k} \right\rceil$

if 'n' objects are placed into 'k' boxes then there is atleast one box containing atleast $\left\lceil \frac{N}{k} \right\rceil$ objects.

Check
Eg:- Among 100 people, what is the atleast minimum no. of people born in same month.

Soln:- $N = 100$
$k = 12$

$\left\lceil \frac{100}{12} \right\rceil = 9$

Q) What is the min. no. of Students required for a Class such that atleast 6 will receive same grade A, B, C, D & F

Sol"

$\{$ K = no. of grade = 5

$\lceil \frac{N}{k} \rceil = 6 \Rightarrow \lceil \frac{N}{5} \rceil = 6$

$\frac{N}{5} + 1 = 6 \Rightarrow \frac{N}{5} = 5 \quad N = 25$

Ans $\Rightarrow 25 + 1 = 26$

Permutation

Ordered arrangements of object.

$^nP_r = \frac{n!}{(n-r)!} \rightarrow n(n-1)(n-2)\ldots(n-(r-1))(n-r)(n-$

eg) $^5C_3 \cdot 3! \Rightarrow \frac{5\times4\times3}{3\times2} \times 3 \times 2 \rightarrow 60$

$^{100}C_3 \cdot 3! \Rightarrow \frac{100\times99\times98}{3\times2} \times 3 \times 2$

eg:- How many ~~hands~~ of poker hands of five Cards Can be dealt from a deck of 52 Cards. Also how many ways are these to select 47 Cards of 52 Cards.

Sol"

$^{52}C_5 \quad \& \quad ^{52}C_{47}$

Q:- M, 9   CS 11   a Committee of 3 faculty from M dept & 4 from CS dept

Sol^n ${}^9C_3 \times {}^{11}C_4$

Binomial Coeff. & Identities

$$(x+y)^n = \sum_{r=0}^{n} \left({}^nC_r\right) x^{n-r} y^r$$

eg:- $(x+y)^4 = {}^4C_0 x^4 + {}^4C_1 x^3 y^1 + {}^4C_2 x^2 y^2 + {}^4C_3 x y^3 + {}^4C_4 x^0 y^4$

eg:- Coeff of $x^{12} y^{13}$ in $(2x-3y)^{25}$

Sol $\;{}^{25}C_{13}\,(2)^{12}\,(-3)^{13} = \text{ans}$

Pascal's Identity :-

Let n & k be any two integer where $n \geq k$ then :-

$$ {}^{n+1}C_k = {}^nC_{(k-1)} + {}^nC_k $$

Logic :-

$n+1$ items → ${}^{n+1}C_k$

'n' items

$(n+1)^{th}$ items let's 'a'

either ∈ S & ∉ S

∈ S → ${}^nC_{(k-1)}$ → a is already choosen so, $(k-1)$

∉ S → ${}^nC_k$ → $n+1$ → this 'a' is not included.

eg> ABCDEFGH : 8! (ABC DEFGH 6!

Vandermondes identity, let m, n and r be non-ve integer with $r < m, n$, then

$$\binom{m+n}{r} = \sum_{k=0}^{r} \binom{m}{r-k}\binom{n}{k}$$

Corollary, $$^{2n}C_n = \sum_{k=0}^{n} \left(^nC_k\right)^2$$ when, $m = r = n$

Sets & operations

- Sets is a Collection of well defined objects denoted by upper Case letter

$$A = \{a, b, c, d, e, f\} \qquad a \in A$$

Roster form :- all the elements are listed

o* Equality Sets :- each element of A Should be in A & vice Versa.

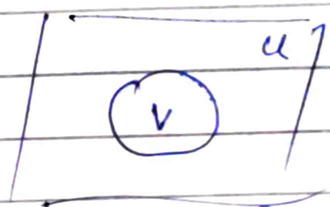
eg:- $A = \{a, b, c\}$
$B = \{a, b, c, a\}$ then, $A = B$
$\{a, b, c\}$

* Cardinality. no. of distinct elements in a set

$A = \{a, b, c, d, e, f\}$, $|A| = 6$
↓ Cardinality.

* Venn diagram :-

U = set of alphabets
V = vowels



* obvious
Trivial subsets :-

$A = \{a, b, c\}$, $B = \{a, b, c, d, e, f\}$, $D = \{a, b, c\}$

$A \subseteq A$
$\phi \subseteq A$
→ trivial subset

* Proper subset :- 'A' is said to be proper subset of 'B' if there is an element $a \in B$ & $a \notin A$.

$A \subseteq D$
$D \subseteq A$
$A \subset B$

$\forall x \in A$, $x \in B$
for all
$A \subseteq B$

* Power set :- $A = \{a, b, c\}$
Set of all subsets of a set is called power set denoted by $P(A)$

$P(A) = \{ \{a\}, \phi, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{c, a\}, \{a, b, c\} \}$

$|P(A)| = 2^{|A|}$ = $|A| = 3$
$= 2^3$.

Cartesian product :- let A & B be 2 sets. Cartesian product denoted by ~~A × B~~ = {

$$A \times B = \{(x,y) \mid x \in A, y \in B\}, \quad |A \times B| = |A| \cdot |B|$$

$A \times B = \{(a,1)\ (a,2), (b,1) \ldots \quad (c,2)\}$
$B \times A = \{(1,a)\ (1,b) \ldots \quad (2,c)\}$

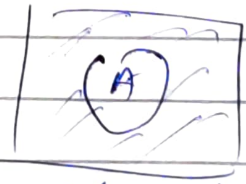
$|A \times B| = |B \times A|$ but $A \times B \neq B \times A$.

eg:- $A = \{0,1,2\}$, $B = \{a,b,c\}$, $C = \{3,4\}$

- $\neg$ = negation

Sets & operators . Membership table

3) Complement of a set A is the set of all element that are not present in A.
i.e U - A



4) Minus / Set diff : $(A - B)$ or ~~A/B~~ $A \setminus B$

5) Disjoint Sets.

Set identities.

1) Identity law

$A \cap U = A$
$A \cup \phi = A$

2) Domination law

$A \cap \phi = \phi$
$A \cup U = U$

3) Idempotent law

$A \cap A = A$
$A \cup A = A$

4) Complementation law

$(\overline{\bar{A}}) = A$

5) Commutative law

$A \cup B = B \cup A$
$A \cap B = B \cap A$

6) Associative law

$A \cup (B \cup C) = (A \cup B) \cup C$
$A \cap (B \cap C) = (A \cap B) \cap C$

7) Distributive law

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

8) De Morgan's law

$(\overline{A \cap B}) = \bar{A} \cup \bar{B}$
$(\overline{A \cup B}) = \bar{A} \cap \bar{B}$

9) Absorption law

$A \cap (A \cup B) = A$
$A \cup (A \cap B) = A$

10) Complement law

$A \cup \bar{A} = U$
$A \cap \bar{A} = \phi$

Q) eg) $A \cap B = B \cap C$

P.T. $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

A	B	C	$B \cup C$	$A \cap (B \cup C)$	$A \cap B$ (p)	$A \cap C$ (q)	$p \cup q$
0	0	0	0	0	0	0	0
0	0	1	1	0	0	0	0
0	1	0	1	0	0	0	0
0	1	1	1	0	0	0	0
1	0	0	0	0	0	0	0
1	0	1	1	1	0	1	1
1	1	0	1	1	1	0	1
1	1	1	1	1	1	1	1

Q) using set builder notation, prove Demorgan's law
$\overline{A \cap B} = \bar{A} \cup \bar{B}$

Sol: $\overline{A \cap B} = \{x / x \notin (A \cap B)\}$ set builder form.

$= \{x / \neg (x \in (A \cap B))\}$ negation definition.

$= \{x / \neg (x \in A \wedge x \in B)\}$ defination of and op.

$= \{x / \neg (x \in A) \vee \neg (x \in B)\}$ demorgan's law of logical.

$= \{x / x \in \bar{A} \vee x \in \bar{B}\}$ complement defination

$= \bar{A} \cup \bar{B}$

Sequence & Sums

Sequence :- Sequence is a ~~set~~ funct^n from set of the int. to a set S a_n, $\{a_n\}$

Reccurrence Relation :- the term a_n is ~~called~~ represented as funct^n of previous terms.

eg:- $a_n = a_{n-1} + a_{n-2}$ $a_{n_0} = 1$

eg:- $a_n = a_{n-1} + 3$ $n_1 = a_1 = a_0 + 3 = 4$

$n = 2$, $a_2 = 4 + 3 = 7$ ⇒ 1, 4, 7, 10

$a_n = a + (n-1)3$

$a_n = 3n - 2$

Now, $a_n = a_{n-1} + 3$

$= 3(n-1) - 2 + 3 = 3n - 3 - 2 + 3 = 3n - 2$

eg:- Determine whether $a_n = 3n$ is a solution of ~~$a_n = 2a_{n-1}$~~, $a_n = 2a_{n-1} - a_{n-2}$ for ~~what~~ $n = 2, 3, 4$. what if $a_n = 2^n$.

Sol: $a_n = 2 \cdot 3(n-1) - 3(n-2) = 6n - \cancel{6} - 3n + \cancel{6} = 3n$ yes

(ii) $a_n = 2(2^{n-1}) - 2^{n-2} = 2^n - \frac{2^n}{4} = \frac{3 \cdot 2^n}{4} = 3 \cdot 2^{n-2}$

eg:- $f_0 = 0$, $f_1 = 1$, $f_n = f_{n-1} + f_{n-2}$, $n = 2, 3, 4$

$f_2 = f_1 + f_0 = 1$

$f_3 = f_2 + f_1 = 1 + 1 = 2$

* ~~Sumati~~ Summations :-

$$\sum_{i=m}^{n} a_i = a_m + a_{m+1} + \_\_\_ a_n$$

index

eg: $\sum_{i=1}^{5} i^2 = 55$

eg: $\sum_{k=4}^{8} (-1)^k = 1$

** Theorem :- If a & r is real no.'s $r \neq 0$, then

$$\sum_{j=0}^{n} a \cdot r^j = \begin{cases} \frac{ar^{n+1} - 1}{r - 1} & , r \neq 1 \\ a(n+1) & , r = 1 \end{cases}$$

* $\sum_{r=1}^{n} r = \frac{n(n+1)}{2}$

* $\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$

* $\sum_{k=1}^{n} k^3 = \left[\frac{n^2(n+1)}{2}\right]^2$

* $\sum_{k=0}^{\infty} x^k, \ |x| < 1 = \frac{1}{1-x}$

$\sum_{k=1}^{\infty} k x^{k-1}, \ |x| < 1 = \frac{1}{(1-x)^2}$

Sum of ∞ terms

$x \in (-1, 1)$

eg: $(0.5)^0 + (0.5)^1 + (0.5)^2 + \_\_\_ \infty$

$= \frac{1}{(1-0.5)} = \frac{1}{0.5} = 2$ Ans

Logic

* Proposition :- Declaration sentences that is either true or false.

(1) Today is Sunday ✓ false

2> $x+1=2$ ✗

3> where is A now. ✗

4> Raju got 100 marks ✓

5> $0+1<2$ ✓ false

→ It is denoted by lower case letter.

* negation ($\neg$) :- opposing the statement.
(it is not the case that ...)

eg:- Memory of A's hardisk is atleast 16 GB $\geq 16GB$

$\neg p$:- Memory of A's hardisk is at most 16 GB $<16GB$

$\neg$:- it is not the case that ...

eg:- Geetha has red colour box.

$\neg p$:- It is not the case.

$\neg p$:- Geetha has green colour box ✗ not $\neg$

* Compound proposition :- is a proposition that has obtained from existing prop. & operators.

① Conjuction (∧) :- P & q denoted by $P \wedge q$ is only if both are true.

P	q	$P \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

eg:- ~~tou~~ today is Monday and there is a Class at 11:50

② ~~dis~~ dis junction :- ~~Por~~ P and q is denoted by $P \vee q$ is false only if all are false.

P	q	$P \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

③ Conditional Statement :- is denoted by $P \to q$
$P \to q$ is false only if P is true & q is false.

P	q	$P \to q$
T	T	T
F	F	T
F	T	T
T	F	F

$\neg(P \to q) = P \to \neg q$

p : A got 100 marks
q : A got a grade

$A \to q$: A got 99

eg:- if 2+1

* BiConditional Statement $p \leftrightarrow q$

$\downarrow$

$p \to q$ & $q \to p$

p	q	$p \to q$	$q \to p$	$p \leftrightarrow q$
T	F	F	T	F
F	T	T	F	F
T	T	T	T	T
F	F	T	T	T

Conditional :-

$p \to q$:- The home team wins whenever it is raining
if it is raining home team wins

* Converse :- $q \to p$:- if ~~home team wins~~ it is raining
* Contrapositive :- $\neg q \to \neg p$:- If the home team doesn't win
* Inverse :- $\neg p \to \neg q$

p	q	$p \to q$	$\neg p$	$\neg q$	$\neg p \to \neg q$
T	T	T	F	F	T
T	F	F	F	T	T
F	T	T	T	F	F
F	F	T	T	T	T

* Precedence of Operators

$(p \wedge q) \wedge (r \vee s) \vee (\neg s) \rightarrow (p \wedge s)$

Priority:-

1. $\neg$ or $\neg$
2. $\wedge$
3. $\vee$
4. $\rightarrow$
5. $\Leftrightarrow$

$(p \wedge q) \vee r \rightarrow r$

Translating eng. to logical.

→ you can access internet from campus only if you are a CS major or you are not fresher

a) you can access internet from campus

c : you are a CS major

f : you are a fresher

$a \rightarrow (c \vee \neg f)$

* A given compound proposition is said to be

* tautology :- if it true for all the set of possible cases.

* Contradiction :- if it is false for all set of possible cases.

* Contigency :-

p	$\neg p$	$p \vee \neg p$	$p \wedge \neg p$	$p \rightarrow q$
T	F	T	F	
F	T	T	F	

* logical equivalence :- 2 compound proposition are said to be logically eq^n if the truth values are same for all the possible cases.

Identities of Conditional Statements.	Laws of biconditional Statement
* $P \to q \equiv \neg P \vee q$	* $P \leftrightarrow q \equiv P \to q \wedge q \to P$
* $P \to q \equiv \neg q \to \neg P$	$P \leftrightarrow q \equiv \neg P \leftrightarrow \neg q$
$(P \to q) \wedge (P \to r) \equiv P \to (q \wedge r)$	$P \leftrightarrow q \equiv$ ~~$\neg P \leftrightarrow \neg q$~~
$(P \to r) \wedge (q \to r) \equiv (P \vee q) \to r$	$\equiv (P \wedge q) \vee (\neg P \wedge \neg q)$
$(P \to q) \vee (P \to r) \equiv P \to (q \vee r)$	$\neg(P \leftrightarrow q) \equiv P \leftrightarrow \neg q$
$(P \to r) \vee (q \to r) \equiv (P \wedge q) \to r$	

Q) using de-morgans law, write the negation of "A has taken M. course and A has scored 100 marks".

Sol:
$P \equiv$ A has taken M. cours
$q \equiv$ A has scored 100 marks.

$\therefore$ $\neg(P \wedge q) \equiv \neg P \vee \neg q \equiv$ A has not taken M. course or A has not scored 100 marks

Q) Show that $\neg(P \vee (\neg P \wedge q))$ and $\neg P \wedge \neg q$ are logically eq^n by developing a series of logical express.

Sol:

$$\begin{aligned}
\neg(P \vee (\neg P \wedge q)) &\equiv \neg[(P \vee \neg P) \wedge (P \vee q)] \\
&\equiv \neg[P \wedge (P \vee q)] \\
&\equiv \neg[(P \wedge P) \vee q] \\
&\equiv \neg(P \vee q) \\
&\equiv \neg P \wedge \neg q
\end{aligned}$$

m-2 $\sim(P \vee (\sim P \wedge q))$
$= \sim P \wedge \sim(\sim P \wedge q)$
$= \sim P \wedge (P \vee \sim q)$ (double negation law)
$= (\sim P \wedge P) \vee (\sim P \wedge \sim q)$
$= F \vee (\sim P \wedge \sim q)$
$= \sim P \wedge \sim q = \sim(P \vee q)$

* Satisfiability :-

we say that a compound proposition is satisfiable if there exist some set of truth values for which compound proposition is true

eg:- $(P \vee q) \wedge r = T$

Predicates & Quantifier (sec-1.4)

* Predicates :-

$P(x) : x + 3 \leq 5$ (value of x is not specified)

variable → x, Predicates → $x+3 \leq 5$

→ if x is specified it will become proposition.

eg:- $P(x, y) = x \cdot y = 10$

$x = 3, y = 2$. $P(3, 2) : 3 \times 2 = 6 \neq 10$ F → truth value of the Predicate.

eg:- $P(x)$: x has scored atleast 90 marks in M,

		truth value.
A = 93	P(A)	T
A = 89	P(A)	F

* **Quatifiers** :- Generalization of Predicates.

$\forall x \rightarrow$ for all x

$\exists x \rightarrow$ there exist 'x'

$\exists ! x \Rightarrow$ uniqueness quantifiers

(1) eg:- $\forall x\ P(x)$, $P(x) : x^2 > x$, ~~$x \in R$~~

eg: $\forall x\ (x^2 > x)$, $x \in R$, domain is $R \rightarrow$ false

domain :. $R^+ - \{1\} \rightarrow$ True

eg: $\forall x\ P(x)$, $P(x)$: x is wearing Red Colour Shirt.

domain $\in$ UGI

$\Rightarrow$ for false :- atleast one Student in UGI should not wear Red tshirt.

Note: Quantifiers becomes propositions once the domain is specified.

(2) eg:- $\exists x\ P(x)$, $P(x)$: x is wearing red colour shirt

domain UGI

$\rightarrow$ True :- if atleast one student of UGI wear Red tshirt.

false : if no student " " " " ".

eg:- $\exists x\ P(x)$ $P(x)$: $x^2 < 10$, domain is N

sol' True because 3 values of x is Satisfying the relation.

Quantifiers	True	False
$\forall x\ P(x)$	$P(x)$ is true for every 'x' in domain	$P(x)$ is false for some 'x' in domain.
$\exists x\ P(x)$	$P(x)$ is true for some 'x' or atleast one 'x'	$P(x)$ is false for every x in domain.

eg:- $\neg \forall x\ P(x)$ $\quad$ $\exists x\ \neg P(x)$
$\neg\ \exists x\ P(x)$ $\quad$ $\forall x\ \neg P(x)$ } Demorgan's law

* Uniqueness Quantifier.

$\exists !x\ P(x)$ (should have unique sol^n).

$P(x)$: $x+3=5$
$P(x)$: $x^2=x$, domain is N
$P(x)$: x is even prime.

* Quantifiers with restricted Domains.

$\forall (x \neq 0)\ x^2 > x$

⇒ for any real no. if it is non-zero, it's square is always greater than itself (don't see T/F)

⇒ or, Square of any non-zero real no. is greater than itself.

* Precedence :-

$\forall x$, & $\exists x$ are of higher importance compared to $\vee, \wedge, \rightarrow, \Leftrightarrow, \neg$

$(\forall x\, P(x)) \wedge Q(x)$

eg:- * There is a honest politician

Demorgan's law & negate.

1) There is a honest politician.

$\exists x\ h(x)$! x is honest.

$\neg(\exists x\ h(x))$ $\quad \forall x\ (\neg h(x))$. All politicians are dishonest.

2) $\forall x\ (x^2 > x)$ & $\exists x\ (x^2 = 2)$

$\neg(\forall x\ (x^2 > x))$ $\quad\quad \neg(\exists x\ (x^2 = 2))$

$\exists x\ \neg(x^2 > x)$ $\quad\quad \forall x\ \neg(x^2 = 2)$

$\exists x\ (x^2 \leq x)$ $\quad\quad \forall x\ (x^2 \neq 2)$

* Proposition $\quad P \quad$ True / false.

- Predicate $\quad P(x) \quad$ becomes proposition when 'x' is specified

$P(1), P(2), P(3)$

- Quantifiers, $\forall x (P(x))$ $\quad$ becomes proposition when domain is specified.

$\exists x$ there exist.

Eg:- Every Students in UMI ~~Clas~~ Class has Studied Calculus

Sol: $c(x)$: x has Studied ~~for~~ Calculus.

Domain : UMI

$\forall x \; (c(x))$

If Domain : IIITs Students

ans:- eqn logic Statement :-

we will introduce another predicate :-

$U(x)$: x is a student of UMI

$\forall x \; (U(x) \rightarrow c(x))$

sec-1.5

* Nested Quantifiers

One Variable is within the Scope of Other Variable

* Commutative law for real nos

Eg: for any two real no. x, y, $x+y = y+x$

Ans:- through logical Statement (using ~~given~~ predicates)

$P(x,y) = x+y = y+x$

$\forall x \; \forall y \; P(x,y)$

$\forall x \; (\forall y \; (x+y = y+x))$

$\forall y \; \forall x \; (x+y = y+x$

also, $\forall x \forall y (x^2+y^2 > xy) \equiv \forall y \forall x (x^2+y^2 > xy)$

Note: Order Can be Changed we we have Same quantifiers throughout the Statement.

* Additive Inverse for Real no.

eg:- for every real no. 'x', there exists a real no 'y' such that $x+y=0$

As:- $P(x,y) = x+y=0$

$\forall x \exists y \; x+y=0$

for all x there exist 'y'

but here $\exists y \forall x \; x+y=0 \neq \forall x \exists y \; x+y=0$

↳ there is not any single 'y' for which every 'x' $x+y=0$

Note: Chang of Order matters when ~~to~~ we have different Quantifiers.

eg:- $\forall x \forall y ((x>0) \wedge (y<0) \rightarrow (xy<0))$

Ans:- In Statement, :- for any ~~no~~ two real no. x & y ~~for all x~~ if x is +ve & y is negative the their Product is negative.

eg:- Every real no. except 0 has multiplicative inverse.

Sol:-

$$\forall x \; ((x \neq 0) \longrightarrow \exists y \; (xy = 1))$$

eg:- Translate the following:-

** ① $\forall x \; (C(x) \vee \exists y \; (C(y) \wedge F(x,y)))$, domain = all students in school.

$C(x) = x$ has a computer

$C(y) = y$ " "

$f(x,y) = x$ & y are friends.

Sol:- every student in this school has a computer or he has a friend ~~with~~ who has a computer.

** eg:- If a person is female and is a parent then this person is someone's mother. Domain (all people)

Sol:-
Predicates

$F(x) = x$ is female

$P(x) = x$ is a parent

$M(x,y) = x$ is mother of y

$$\forall x \; \cancel{\exists y} \; ((F(x) \wedge P(x)) \longrightarrow \exists y \, M(x,y))$$

for every person 'x' in the set of people if x is female, and x is a parent then there is a person 'y' such that x is the mother of 'y'

Note:- $\forall x \; (F(x) \wedge P(x) \rightarrow \exists y \; M(x,y))$

we can shift here bcz there is no 'y' between these

eg:- Negation of N Q

① $(\forall x \; \exists y \; (x^2y^2 < 100))$

Sol:

$\sim(\forall x \; \exists y \; (x^2y^2 < 100))$

$= \exists x . \sim(\exists y \; (x^2y^2 < 100))$

$= \exists x . \forall y \; \sim(x^2y^2 < 100)$

$= \exists x \; \forall y \; (x^2y^2 \geq 100)$

eg:- $\forall x \; (C(x) \vee \exists y \; (C(y) \wedge F(x,y)))$

$= \sim(\forall x \; (C(x) \vee \exists y \; (C(y) \wedge F(x,y))))$

$= \exists x \; (\sim C(x) \wedge \forall y \; (\sim C(y) \vee \sim F(x,y)))$

Sec - 1.6

Rules of Inference

Proposition

P1: You have password then you can log on to the network
q

P2: You have the password

$P \rightarrow q \therefore q$ = You have the password.

Conclusion

Premises

Argument.

→ Conclusion is valid if truth of all the premises imply ~~the~~ truth of conclusion.

Argument: Sequence of compound proposition

Rule 1) $p \rightarrow q$ Modus ponens

p

$\therefore q$

2) $\sim q$ ~~Modusponens~~ Modus tollens

$p \rightarrow q$

$\therefore \sim p$

3) $p \rightarrow q$ hypothetical Syllogism

$q \rightarrow r$

$p \rightarrow r$

$\boxed{P \to q \equiv \sim P \to \sim q \equiv P \vee q}$

4) $\dfrac{\begin{array}{c} P \vee q \\ \sim P \end{array}}{q}$ Disjunctive Syllogism

5) $\dfrac{\begin{array}{c} q \\ P \end{array}}{\therefore P \vee q}$ add^n

6) $\dfrac{\begin{array}{c} P \vee q \\ P \wedge q \end{array}}{\therefore P}$ Simplification

7) $\dfrac{\begin{array}{c} P \\ q \end{array}}{P \wedge q}$ Conjunction

8) $\dfrac{\begin{array}{c} P \vee q \\ \sim P \vee r \end{array}}{q \vee r}$

$\sim q \to P$

$\sim P \to \sim q$

$P \to r$ Resolution

$\Rightarrow \sim q \to r$

Prove that 'r' is the conclusion of $P, P \to q, q \to r$

Sol^n:-

P from ①

$P \to q$ from ②

q from ① & ②, Modus phonens ③

$q \to r$ ④

r Modus ponen ③ & (4)

$P \rightarrow q$

P only if q

$P \rightarrow q$

$\sim q \rightarrow \sim P$

eg:- S.T (show that) the premises "it is not sunny this ~~aft~~ afternoon and it is colder than yesterday", "we will go swimming only if it is sunny", "if we do not go swimming then we will take a ~~ch~~ canoe trip" and "if we canoe trip, then we will be home by sunset" lead to the conclusion "we will be home by sunset.

Sol^n

P: it is ~~not~~ sunny this afternoon

q: it is colder than yesterday

r:- we will go swimming

S:- we will take canoe trip

t:- we will be home by sunset.

$$\sim P \wedge q$$
$$r \rightarrow P$$
$$\sim r \rightarrow s$$
$$\underline{s \rightarrow t}$$
$$t$$

→ we need to show that this conclu. is valid.

1	$\sim P \wedge q$	Premise
2	$\sim P$	Simplification from 1
3	$r \rightarrow P$	Premise
4	$\sim r$	Modus tollens 2 & 3
5	$\sim r \rightarrow s$	Premise
6	s	Modus Ponens 4 & 5
7	$s \rightarrow t$	Premise
8	t ✓	Modus ponens 6 & 7

$p \to q \equiv \sim q \to \sim p$

Contra Positive

1.7 Proof Strategies

① Direct proof : $p \to q$. we assume that 'p' is true and P.T q is also true.

eg:- if n is odd integer, P.T n^2 is odd

Sol: let us suppose that n is odd, $n = 2k+1$, $k \in \mathbb{Z}$

we need to prove that n^2 is odd

Consider $n^2 = (2k+1)^2$

$= 4k^2 + 4k + 1$

$= 2(2k^2 + 2k) + 1$

$= 2p + 1$ $\quad p \in \mathbb{Z}$

$\therefore n^2$ is also odd

② Contrapositive proof :- $p \to q$ equivalent to $\sim q \to \sim p$

So, we assume $\sim q$ is true & P.T $\sim p$ is true

eg:- if (3n+2 is odd) P.T (n is odd), $n \in \mathbb{Z}$

is equivalent to if (n is even) then (3n+2 is even)

Proof :- Assume that 'n' is even, $n = 2k$, $k \in \mathbb{Z}$

we need to P.T $3n+2$ is even

$3n+2 = 3 \cdot 2k + 2$

$= 2(3k+2) = 2p$, thus

$3n+2$ is also even.

$$p \to q \equiv \sim q \to \sim p$$

Contrapositive

③ Proof by Contradiction :- $P \to q$

we assume that $\neg P$ is true, and arrive at some contradiction, we conclude that our assumption is wrong.

eg:- If $3n+2$ is odd, P.T n is odd
(P: $3n+2$ is odd; q: n is odd)

Proof: we assume that $\neg(P \to q)$ is true, i.e. $\neg(P \to q) \equiv P \wedge \neg q$
i.e. $3n+2$ is odd and n is even.

$\therefore$ 'n' is even, $n = 2k$, $k \in Z$
Consider $3n+2 = 3(2k)+2$
$= 2(3k+1)$, which S.T $3n+2$ is even

$\Rightarrow$ ~~Our~~ This is a contradiction
$\Rightarrow$ Our assumption is wrong, i.e. $3n+2$ is odd then n is odd.

④ Disprove Something :-

~~throu~~

Counter Examples

eg:- Every +ve integer can be expressed as sum of squares of two integers.

Sol Proof:
$6 = \_\_ + \_\_$
or $5 =$

} any no. which does not satisfy the statement wff = well formed formula

Normal forms :-

well formed formulla :- A String Consists of propositional variables connectives and parathesis in proper manner operations $\wedge \vee \sim \rightarrow \Leftrightarrow$

$(P \wedge q) \rightarrow (r \wedge \sim q)$ ✓, $((P \wedge) \rightarrow q) \vee \sim r$ ✗

* Literal :- Any variable or its negation. $P, \sim r, q, \sim q, \sim P$ but not $P \vee q$

* Sum

Disjunction, Elementry Sum :- disjunction of literals
eg:- $P \vee q$, $P \vee \sim r$, $P \vee \sim q \vee r$,

* Product

Conjuction, Elementry product :- Conjuction of literals
$P \wedge q$, $P \wedge r \wedge \sim q$, $q \wedge r \wedge \sim P$

* Disjuctive Normal forms :- DNF :- A formula which is (DNF) equivalent to given wff and consist of Sum of elementry products.

eg:- $(P \wedge q \wedge r) \vee (P \wedge \sim q \wedge r) \vee (P \wedge \sim r \wedge q)$

(CNF) :- $(P \vee q \vee r) \wedge (P \vee \sim q \vee r) \wedge (P \vee q \vee \sim r)$

DNF : $(P \rightarrow q) \wedge \sim q \equiv (\sim P \vee q) \wedge \sim q$ — CNF

$(\sim P \wedge \sim q) \vee (q \wedge \sim q)$ — DNF

CNF ≡ $\wedge$
DNF ≡ $\vee$

$P \leftrightarrow q \equiv \cancel{(P \wedge q)}$
$(P \wedge q) \vee (\sim P \wedge \sim q)$

DNF

$\sim(P \wedge q) \leftrightarrow (P \vee q)$

$((\sim P \vee \sim q) \wedge (P \vee q)) \vee ((P \wedge q) \wedge (\sim P \wedge \sim q))$

$(\sim P \wedge P) \vee (\sim P \wedge q) \vee (\sim q \wedge P) \vee (\sim q \wedge q)$
$\vee (P \wedge q \wedge \sim P \wedge \sim q)$

$\equiv (\sim P \wedge q) \vee (\sim q \wedge P)$

$P \leftrightarrow q \equiv (P \to q) \wedge (q \to P)$

$(\sim(P \wedge q) \to (P \vee q)) \wedge ((P \vee q) \to \sim(P \wedge q))$

$(\sim(\sim(P \wedge q)) \vee (P \vee q)) \wedge (\sim(P \vee q) \vee \sim(P \wedge q))$

$((P \wedge q) \vee (P \vee q)) \wedge ((\sim P \wedge \sim q) \vee (\sim P \vee \sim q))$
$(P \vee q \vee P) \wedge (P \vee q \vee q) \wedge (\sim P \vee \sim q \vee \sim P)$
$\wedge (\sim P \vee \sim q \vee \sim q)$

→ Every wff can be expressed as DNF & CNF
they are not unique

- finite, infinite, countable and uncountable ~~set with~~
→ A set with finite no. of elements is finite if it is not finite it is infinite.

* Countable set :- A set 's' countable if it is finite or there is a bijective mapping from N to S
uncountable set

→ finite, infinite, countable, and uncountable.

$N = \{1, 2, 3 \ldots\}$

Countable finites

$S = \{0, 2, 4, 6, 8 \ldots\}$
$S = \{0, 3, 6, 9 \ldots\}$
$S = \{\ldots, -4, -3, -2, -1\}$

$f(n) = 3n \quad , \quad n = 1, 2, 3 \ldots$

$S = \{3, 6, 9, 12, 15$

Domain N

one - one

$f(x_1) = f(x_2)$

$3x_1 = 3x_2$

$x_1 = x_2$

onto for every $y \in S \quad \exists x \in N$

$f(x) = y \quad\quad s.T \; f(x) = y$

$3x = y$

$x = y/3 \; , x \in N$

* Solving Reccurence Relation :-

A linear homogeneous R.R of degree with constant coeff. is of form

$$a_n = C_1 a_{n-1} + C_2 a_{n-2} + \ldots + C_k a_{n-k}$$

C_i is a real no. $\quad C_1 \neq 0$

eg:- if $i = 2$, $a_2 = 3a_1 + 4a_0$

$a_2 = a_1 a_0^2 \quad \times$ not linear

$a_3 = a_1 + n^2 a_0 \quad \times$ not homogeneous

Th:

Let C_1 & C_2 be real nos, Suppose that $r^2 - C_1 r - C_2 = 0$ has two distinct roots r_1 & r_2, then $a_n = \alpha_1 r_1^n + \alpha_2 r_2^n$ is a Sol^n of RR

$a_{n-2} = C_1 a_{n-1} = C_2 a_{n-2}$, where α_1 α_2 are constants i.e.

$$ax^2 + bx + c = 0$$

$$aS_n + bS_{n-1} + cS_{n-2} = 0 \quad \Big\} \; S_n = \alpha^n + \beta^n$$

***

eg:-

$$a_n = a_{n-1} + 2a_{n-2}$$

$$a_0 = 2, \quad a_1 = 7$$

$$a_0 = \alpha_1 (2)^0 + \alpha_2 (1)^0$$

$$r^2 - r - 2 = 0$$

$$r^2 + r - 2r - 2 = 0$$

$$(r+1)(r-2) = 0$$

$$r_1 = 2, r_2 = -1, \quad r = 2, -1$$

$$a_4 = \alpha_1 (r_1)^2 + \alpha_2 (r_2)^2$$

$\therefore$ $$a_n = \alpha_1 (2)^2 + \alpha_2 (-1)^n$$

$$a_0 = 2$$

$$a_0 = \alpha (2)^0 + \alpha_2 (-1)^0$$

$$2 = \alpha_1 + \alpha_2 \quad - (1)$$

$$a_1 = 7$$

$$\alpha_1 = \alpha_1 (2)^1 + \alpha_2 (-1)^1$$

$$7 = 2\alpha_1 - \alpha_2 \quad - (2)$$

$$\alpha_1 = 3, \quad \alpha_2 = -1$$

$$a_n = 3(2)^n + (-1)(-1)^n$$

$$= 3 2^n - (-1)^n$$

* for $(x-3)^3 = 0$

$$a_n = \alpha_1 r_0^n + \alpha_2 n r_0^n + \alpha_3 n^2 r_0^n$$

→ If $r^2 - C_1 r - C_2 = 0$, has same root r_1 then the

eg:- $a_n = 6a_{n-1} + 9a_{n-2}$, $a_0 = 1$, $a_1 = 6$

Sol^n $\Rightarrow r^2 - 6r + 9 = 0$

$(r-3)^2 = 0$

$r = 3, 3$

$a_n = \alpha_1 (3)^n + \alpha_2 n (3)^n$

for $a_0 = 1$, $n = 0$

$a_0 = \alpha_1 (3)^0 + \alpha_2 0 (3)^0$

$\alpha_1 = 1$

$a_1 = 6$
$n = 1$

$a_1 = \alpha_1 (3)^1 + \alpha_2 \cdot 1 \cdot 3$

$6 = 1 \times 3 + 3\alpha_2$

$\alpha_2 = \frac{6-3}{3} = 1$

* Non-homogeneous R.R with Const. Coeff. of the form

2, 2, 5

~~$\alpha_1 = \alpha_1 (2)^n + \alpha_2 n 2^n$~~ +

$a_n = C_1 a_{n-1} + C_2 a_{n-2} + \cdots + C_k a_{n-k} + F(n)$

$C_1 \neq 0$, $C_i \in R$

$a_n = a_n^{(h)} + a_n^{(p)}$, where $a_n^{(h)}$ is Sol^n of corres. h eq^n $a_n = C_1 a_{n-1} + \cdots C_k a_{n-k}$

& $a_n^{(P)}$ is particular Sol^n

a_n is depending one 1 term (degree = 1)

(P)

a_n	$f(n)$
$cn+d$	$2n+3$
cn^2+dn+e	n^2+3n+2

$\rightarrow$ a_n is a sol^n of given R.R

eg:- $a_n = 3a_{n-1} + \underbrace{2n}_{f(n)}$, $a_1 = 3$ Chapt. = 8.2

So $a_n = 3a_{n-1}$ (1st solve this)

$r - 3 = 0$

$r = 3$

$a_n^{(h)} = \alpha_1 3^n$

$a_n^{(p)} = cn + d$

Since $a_n^{(p)}$ is a sol^n of given R.R

$\therefore$ $cn + d = 3(c(n-1) + d) + 2n$

$\Rightarrow$ $cn + d = 3cn - 3c + 3d + 2n$

$0 = 2cn + 2d - 3c + 2n$

$\Rightarrow$ $(2c+2)n + (2d - 3c) = 0$

$\Rightarrow$ $2c + 2 = 0$ $\quad$ $2d - 3c = 0$

$c = -1$ $\quad$ $2d = -3$

$d = -3/2$

$a_n = -1n - 3/2$

$\rightarrow$ $a_n = a_n^{(h)} + a_n^{(p)}$

$= \alpha_1 3^n - n - 3/2$

$a_1 = 3$, $n = 1$

$3 = \alpha_1 3^1 - 1 - 3/2$

$\alpha_1 = 11/6$

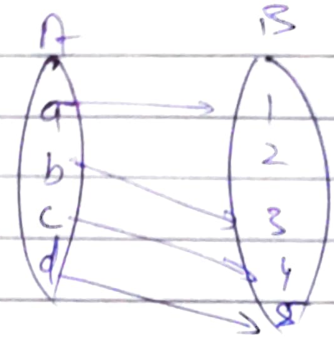
$\boxed{a_n = \frac{11}{6} 3^n - n - 3/2}$ gen sol^n

2.3 Mappings of finite & infinite sets. theorem.

→ function $f : A \to B$ is an assignment of each element of A to an element of B.

$A \to$ domain, B = Co-domain, $f(a) = b$, $a \in A$, $b \in B$

a → Preimage, b → image

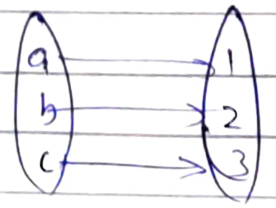


→ $f_1 + f_2 = (x^2+2x+3) + (3x^2+6x) = 4x^2+8x+3$

$f_1 \cdot f_2 = (x^2+1)(x+5) = x^2(x+5) + 1(x+5)$

$= x^3 + 5x^2 + x + 5$

* One-one / Injection



$f : R \to R$

$f(x) = 3x+2$

$f(x_1) = f(x_2)$

$x_1 = x_2$

* Onto / Surjection

for every $y \in B$, $\exists x \in A$

x^2

$f : R \to R^+$ such that $f(x) = y$

$f(x) = x^2$

$f(x) = y$

$x^2 = y$

$x = \sqrt{y}$

$x \in R$

* f is bijective if 'f' it is ~~one~~ one-one and onto

$R \to R$

$f(x) = 2x$

$f(x_1) = f(x_2)$

$2x_1 = 2x_2$

$x_1 = x_2$

for $y \in R \ \exists x \in R$

$f(x) = y$

$f(x) = 2x = y$

$x = y/2$

eg:- $R - \{0\} \to R$

$f(x) = 2/x$

* 'f' is bijective if f^{-1} exists

$f : R \to R \qquad f^{-1} : R \to R$

$f(x) = 2x+5$

$\boxed{\begin{array}{l} f(a) = b \\ f(b) = a \end{array}}$

$f^{-1}(f(x)) = f^{-1}(y) \Rightarrow f^{-1}(y) = x$

$f^{-1}(y) = x$

$f(x) = 2x+5 = y$

$2x = y-5$

$x = \frac{y-5}{2} = f^{-1}(y)$

* Compositions of functions

$f : A \to B, \ g : B \to A$

$g \circ f(x)$

$f(x) = 2x+5$

$g(x) = 3x+2$

$$f \circ g(x) = f(3x+2)$$
$$= 2(3x+2)+3$$
$$= 6x+4+3$$
$$= 6x+7$$

1) PT $u \to r$, $(r \wedge s) \to (p \vee t)$, $q \to (u \wedge s)$, $\neg t$, q lead to conclusion $\therefore p$

Sol:

①	q	Premise
②	$q \to (u \wedge s)$	Premise
③	$u \wedge s$	Modus Ponens
④	u	Simplification of ③
⑤	$u \to r$	Premise
⑥	r	Modus Pones.
⑦	s	Simpl. of ③
⑧	$r \wedge s$	Conjuction of ⑥ & ⑦
⑨	$(r \wedge s) \to (p \vee t)$	Premise
⑩	$p \vee t$	M.P of ⑧ & 9
⑪	$\neg t$	Premise
⑫	p	disjunct. syll. ⑩ & ⑪

2) $a_n = 2a_{n-1} - a_{n-2} + 2^n$, $a_0 = 1$, $a_1 = 2$

Sol: degree = 2 (a_n depends on two previous terms)

Quad eqn :- $r^2 - 2r + 1$
$r = 1, 1$

$$a_n^h = \alpha_1 (1)^n + \alpha_2 n (1)^n$$
$$a_n^p = C \cdot 2^n$$

$$\Rightarrow C \cdot 2^n = 2C2^{n-1} - C2^{n-2} + 2^n \qquad \boxed{C=4}$$
$$a_n^p = 4 2^n$$

$\frac{10}{5} = 5 | 10$

$n \nmid a$ → ~~and~~ n does not devide a / $n | a$ → n divides a with 0 remainder

$a_n = \alpha_1 (1)^n + \alpha_2 n (1)^n + 4 \cdot 2^n$

$\therefore a_0 = 1 = \alpha_1 + 0 + 4 \Rightarrow \alpha_1 = -3$

$a_1 = 2 = \alpha_1 + \alpha_2 + 8$

$-3 + \alpha_2 = 2 - 8$

$\alpha_2 = 3 - \cancel{6} = -3$, $\alpha_1 = \alpha_2 = -3$

$\therefore a_n = -3(1)^n + (-3) n (1)^n + 4 \, 2^n$

$= -3 - 3n + 4 \cdot 2^n$

3) Let, $a, b, c \in \mathbb{Z}$, P.T if $\underset{p}{n \nmid ab}$ then $\underset{q}{n \nmid a}$ and $n \nmid b$

Sol^n $\sim q \rightarrow \sim p$ Prov by Contrapositive method

$\sim (n \nmid a \wedge n \nmid b) \rightarrow \sim (n \nmid ab)$

$(n | a \vee n | b) \rightarrow n | ab$

let $n | a \Rightarrow a = n(c)$, $c \in \mathbb{Z}$

Consider $ab = (nc) b = n(cb)$, $c, b \in \mathbb{Z}$

$\Rightarrow n | ab$

let $n | b \Rightarrow b = n(d)$, $d \in \mathbb{Z}$

$ab = a(nd) = n(ad)$, $ad \in \mathbb{Z}$

$\Rightarrow n | ab$

for disprove only we give counter example

Q) There is a no. 'x' S.T if you add any no. to it then the result is that no. and if you multiply with any no. then the result is x.

Sol^n

y = any no. other than x

$\therefore \quad \forall y \, (\exists x \, (x+y=y \wedge xy=x))$

$\rightarrow \quad \exists x \, (\forall y \, (x+y=y \wedge x \cdot y = x))$

Q) Everyone has exactly one best friend.

Sol^n

with uniqueness :- $\forall x \, (\exists! y \;\; B(x,y))$

without uniqueness :- $\forall x \, \exists y \, (B(x,y) \wedge \forall z ((z \neq y) \rightarrow \neg B(x,z)))$

Q) P.T there are no integers a, b, S.T $2a+4b=1$

Sol^n

Let us assume that $\exists \, a, b \in \mathbb{Z}$
S.T $2a+4b=1$

$\therefore \; 2a+4b=1$

$2(a+2b)=1$

$a+2b = 1/2$

if $a, b \in \mathbb{Z}$, then $a+2b$ should also be integer
but $1/2 \in \mathbb{Q}$

which is a contradiction to our supposition.

Q) (i) Democracy survives if the leaders are not corrupt.
q p

Sol

$p \to q$

$\sim(p \to q) = \sim(\sim p \vee q)$

$= p \wedge \sim q$

→ Leaders are not corrupt and Democracy does not survive.

(ii) The necessary and sufficient condition for a person to be succesful is to be honest.
biconditional statement

S

Person is succesful if he is honest.
p q

$\sim(p \leftrightarrow q),\ \sim(p \to q \wedge q \to p),\ \sim((\sim p \vee q) \wedge (\sim q \vee p)),\ (\sim p \wedge q) \vee (p \wedge \sim q)$

Person is not successful and not honest and person is not succesful and honest.

Q) (i)
(iii) Some student in this class visited mexico.
S(x) M(x)

$\exists x\ (S(x) \wedge M(x))$

(iv) Every student in the class has visited Mexico or Canada.

$\forall x\ (S(x) \to (M(x) \vee C(x)))$

9) If n is prime then n is odd or n is 2

10) $\sim(P \vee q) \vee (\sim P \wedge q) \overset{?}{=} \sim P$ (without truth table.

#

Recursive Defination:-

$f(n) = a^n$, $n = 0, 1, 2, 4 \ldots$

$f(0) = 1$

$f(n+1) = a \cdot f(n)$, $n = 0, 1, 2, \ldots$

$f(0) = 1$, $f(1) = a$, $f(2) = a^2$

$\therefore$ $f(0) = 0$

$f(1) = 0 + 1$

$f(2) = 0 + 1 + 2$

$\vdots$

Recurr. Def^n :- $f(0) = 0$

$f(n+1) = f(n) + (n+1)$, .. $n = 0, 1, 2, \ldots$ $f(2) = f(1) + 2$

Fibonacci ~~numbers~~ numbers :- 0, 1, 1, 2, 3, 5, 8

$f(0) = 0$

$f(1) = 1$

$f(n) = f(n-1) + f(n-2)$, $n = 2, 3, 4$

$f(n+2) = f(n+1) + f(n)$, $n = 0, 1, 2, 3, 4, 5$

eg:- P.T for any +ve integer n, $6^n - 1$ is divisible by 5 by Mathematical Induction.

Sol $P(n) = 6^n - 1$ is divisible by 5 for $n = 1, 2 \ldots$

Basic Step:- P(1) is true

$6^1 - 1 = 5$ is divisible by 5

So, P(1) is true

Induction Step:- for arbitrary $k > 1$, P(k) is true

$P(k) = 6^k - 1$ is divisible by 5

$$\therefore \quad 6^{k+1} - 1 = 6 \times 6^k - 1$$

$$= 6(6^k - 1) + 6 - 1$$

$$= 6 \times 5m + 5$$

P(k+1) is true

$$= 30m + 5$$

$$= 5(6m + 1)$$

eg:- S.T $n! > 3^n$ for $n \geq 7$

Sol^n $P(n): n! > 3^n$ for $n = 7, 8, 9 \ldots$

Basic Step:- $P(7)$: $7! > 3^7$

$5040 > 2187$ it's true

Inductive Step:-

for aubitary cond. $k \geq 7$, $k(4) \ldots : k! > 3^k$

$$(k+1)! = (k+1)k! > (k+1)3^k \geq (7+1)3^k$$
$$= 8 \cdot 3^k > 3 \cdot 3^k$$
$$8 \cdot 3^k > 3^{k+1}$$

eg:- $a_0 = 1$

$a_1 = 1$

$a_{n+1} = a_n + a_{n-1}$, $n \geq 1$

Fibonaci sequence for $n \geq 1$ $a_n \geq \left(\frac{3}{2}\right)^{k-2}$

Sol^n BS:- $P(1)$: $a_1 \geq \left(\frac{3}{2}\right)^{-1}$

$1 \geq \frac{2}{3}$ it's true

Inductive:-

for $n = 1, 2, 3 \ldots k$ $a_n \geq \left(\frac{3}{2}\right)^{n-2}$

for $n = k+1$ $a_{k+1} \geq \left(\frac{3}{2}\right)^{k+1-2}$ — ①

it's true true [strong induction]

$\therefore a_{k+1} = a_k + a_{k-1}$

$\Rightarrow \geq \left(\frac{3}{2}\right)^{k-2} + \left(\frac{3}{2}\right)^{k-3} = \left(\frac{3}{2}+1\right) \times \left(\frac{3}{2}\right)^{k-3} = \frac{5}{2} \times \left(\frac{3}{2}\right)^{k-3}$

$\therefore \frac{5}{2}\left(\frac{3}{2}\right)^{k-3} \geq \frac{9}{4}\left(\frac{3}{2}\right)^{k-3}$

$\geq \left(\frac{3}{2}\right)^{k-1}$ [Proved eq ①]

Recursion def.

Define it recursively

eg:- $f(n) = 2n+1$, $n = 0, 1, 2 \ldots$ Define it recursively

Sol:- $f(0) = 1$ [B.S]
$f(1) = 3$ $f(n+1) = f(n)+2$ [R.S]

$f(0) = 1$
$f(1) = 3$
$f(2) = 5 = (3+2)$

$\therefore$ $f(0) = 1$
$f(n+1) = f(n)+2$, $n = 0, 1, 2 \ldots$

eg:- $a_n = n^2$, $n = 1, 2, 3$

$a_1 = 1$
$a(n+1) = a_n + 2n+1$, $n = 1, 2, 3 \ldots$

$a_1 = 1$
$a_2 = 4 = 1+3$
$a_3 = 9 = 4+5$
$a_4 = 16 = 9+7$

eg:- $f(n) = n!$, $n = 1, 2, 3 \ldots$

Sol:- $f(1) = 1!$
$f(2) = 2 \times 1$
$f(3) = 3 \times 2 \times 1 = 3 \cdot f(2)$

$f(1) = 1$
$f(2) = 2$
$f(3) = 3 \times 2 \times 1$
$f(4) = 4 \cdot f(3)$

$\therefore$ $f(n) = n!$
$f(n+1) = (n+1) f(n)$, $n = 1, 2, 3 \ldots$

eg:- S: $2 \in S$ B.S
if $x \in S$, then $x^2 \in S$ R.S