Boolean Algebra
Canonical and Standard forms

Dr. E. Paul Braineard

Canonical and Standard forms

- Canonical form
  - Boolean function expressed as sum of minterms
  - Boolean function expressed as product of maxterms
- For expressing a Boolean function algebraically from a truth table
- How?
  - By forming a minterm for each combination of the variable that produces a 1
  - Then take OR of all those terms

Canonical and Standard forms

- A binary variable can appear either in its normal form (x) or in its complement form (x')
- For two binary variables x and y, there are four possible combinations $x'y', x'y, xy'$ *and* xy
- These four **AND** terms are called as *standard Products* or *minterms*
- Similarly the other four possible OR combinations are $x + y, x + y', x' + y$ *and* $x' + y'$
- These four **OR** terms are called as *Standard Sums* or *maxterms*

Minterms and Maxterms for three Binary variables

			Minterms		Maxterms	
x	y	z	Term	Designation	Term	Designation
0	0	0	$x'y'z'$	m_0	$x+y+z$	M_0
0	0	1	$x'y'z$	m_1	$x+y+z'$	M_1
0	1	0	$x'yz'$	m_2	$x+y'+z$	M_2
0	1	1	$x'yz$	m_3	$x+y'+z'$	M_3
1	0	0	$xy'z'$	m_4	$x'+y+z$	M_4
1	0	1	$xy'z$	m_5	$x'+y+z'$	M_5
1	1	0	xyz'	m_6	$x'+y'+z$	M_6
1	1	1	xyz	m_7	$x'+y'+z$	M_7

Example: Express the functions f_1 and f_2 using minterms and maxterms

x	y	z	f_1	f_2
0	0	0	0	0
0	0	1	1	0
0	1	0	0	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

Function of three variables

$$f_1 = x'y'z + xy'z' + xyz = m_1 + m_4 + m_7$$

$$f_2 = x'yz + xy'z + xyz' + xyz = m_3 + m_5 + m_6 + m_7$$

SOP

$$f_1 = (x+y+z)(x+y'+z)(x+y'+z')(x'+y+z')(x'+y'+z)$$
$$= M_0M_2M_3M_5M_6$$

$$f_2 = (x+y+z)(x+y+z')(x+y'+z)(x'+y+z) \quad \text{POS}$$
$$= M_0M_1M_2M_4$$

Example: Find out odd and even numbers between 0 to 7

	x	y	z	O	E
0	0	0	0	0	1
1	0	0	1	1	0
2	0	1	0	0	1
3	0	1	1	1	0
4	1	0	0	0	1
5	1	0	1	1	0
6	1	1	0	0	1
7	1	1	1	1	0

Example: Express the Boolean function $F = A + B'C$ as **Sum of minterms**

- Identify the number of variables
- Add missing variables in each term
- $F = A + B'C$
  - Variables are three (A, B, C)
  - Terms are two (A, $B'C$)

Example: Express the Boolean function F = A + B′C as **Sum of minterms**

- F = A + B′C **The Function has three variables, Add missing terms**
- F = A(B+B′)(C+C′)+(A+A′) B′C
- F = ABC + ABC′ + AB′C + AB′C′ + AB′C + A′B′C
- F = ABC + ABC′ + AB′C + AB′C′ + A′B′C
- F = 111 + 110 + 101 + 100 + 001
- F = $m_7 + m_6 + m_5 + m_4 + m_1$ Rearranging terms
- F = $m_1 + m_4 + m_5 + m_6 + m_7$
- F(A, B, C) = Σ(1, 4, 5, 6, 7)

Example: Express the Boolean function $F = xy + x'z$ as **Product of maxterms**

- Convert the function to OR terms using distributive law $\left[(A+B)(A+C) = A+BC\right]$
- Identify the variables
- Add missing variables in each term

Example: Express the Boolean function

$F = xy + x'z$ as **Product of maxterms**

- $F = xy + x'z = (xy + x')(xy + z)$
- $F = (xy + x')(xy + z)$
- $F = (x + x')(y + x')(x + z)\,(y + z)$
- $F = (y + x')(x + z)\,(y + z)$
- $F = (x' + y + zz')(x + yy' + z)(x\,x' + y + z)$
- $F = (x' + y + z)(x' + y + z')(x + y + z)(x + y' + z)(x + y + z)(x' + y + z)$

$$\left[(A+B)(A+C) = A+BC\right]$$

Example: Express the Boolean function F = $xy + x'z$ as **Product of maxterms**

- F = $xy + x'z = (xy + x')(xy + z)$
- F = $(xy + x')(xy + z)$
- F = $(x + x')(y + x')(x + z)\ (y + z)$
- F = $(y + x')(x + z)\ (y + z)$
- F = $(x' + y + zz')(x + yy' + z)(x\,x' + y + z)$
- F = $(x' + y + z)(x' + y + z')(x + y + z)(x + y' + z)(x + y + z)(x' + y + z)$
- F = $(x' + y + z)(x' + y + z')(x + y + z)(x + y' + z)$
- F = (100)(101)(000)(010)
- F = $M_0M_2M_4M_5$
- F(x, y, z) = $\Pi(0, 2, 4, 5)$

$$[(A+B)(A+C) = A+BC]$$

Conversion between Canonical forms

- The maxterm with subscript j is a complement of the minterm with the same subscript j
- Ex: F(A, B, C) = Σ(1, 4, 5, 6, 7)
- Compliment is F′(A, B, C) = Π(0,2,3)

$$m_j' = M_j$$

Digital Logic Families

- The circuit technology is referred to as a *digital logic family*
  - TTL transistor–transistor logic;
  - ECL emitter-coupled logic;
  - MOS metal-oxide semiconductor;
  - CMOS complementary metal-oxide semiconductor

Definitions

- *Fan-out* specifies the number of standard loads that the output of a typical gate can drive without impairing its normal operation. A standard load is usually defined as the amount of current needed by an input of another similar gate in the same family.
- *Fan-in* is the number of inputs available in a gate.
- *Power dissipation* is the power consumed by the gate that must be available from the power supply.
- *Propagation delay* is the average transition delay time for a signal to propagate from input to output. For example, if the input of an inverter switches from 0 to 1, the output will switch from 1 to 0, but after a time determined by the propagation delay of the device. The operating speed is inversely proportional to the propagation delay.
- *Noise margin* is the maximum external noise voltage added to an input signal that does not cause an undesirable change in the circuit output.