"श्री गणेशाय नमः"

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Thursday

Pg 01

1) AR (Autoregressive) Method →

$$Y_t = c + \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \cdots + \phi_p Y_{t-p}$$

where − Y_t = current value
c = constant
ϕ_t = Co-efficient

Suppose we have the following time series data −

time	Y(t)
1	10
2	12
3	13
4	16
5	18
6	20
7	22

We will use the AR(2) model to forecast Y8 (the next value.

Given that. $c = 1.0$
$\phi_1 = 0.6$
$\phi_2 = 0.2$

forecast Y8 :

$$Y_8 = 1.0 + 0.6 \cdot Y_7 + 0.2 Y_6$$

$$= 1.0 + 0.6 * 22 + 0.2 * 20$$

$$\boxed{Y_8 = 18.2}$$

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New forecast $Y_9 = 1.0 + 0.6\, Y_8 + 0.2\, Y_7$

$$= 1.0 + 0.6 * 18.2 + 0.2 * 22$$

$$\boxed{Y_9 = 16.32}$$

MA (Moving Average) Method →

$$Y_t = \mu + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} + \cdots + \theta_q \varepsilon_{t-q}$$

Where

Y_t = is the current time series value

μ = mean of the series

ε_t = forecast error (white noise) at time t

θ_1, θ_2 = are the moving average coefficient

Given the time series data

time (t)	Y(t)
1	10
2	12
3	13
4	16
5	18
6	20
7	22

The mean (μ) = 15.86

MA(2) model

$$Y_t = \mu + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2}$$

Given $\theta_1 = 0.5$

$\theta_2 = 0.3$

Calculate Error

We recursively calculate ε_t as

$$\varepsilon_t = Y_t - \mu - \theta_1 \varepsilon_{t-1} - \theta_2 \varepsilon_{t-2}$$

Assume $\varepsilon_1 = \varepsilon_2 = 0$ (starting value)

Now compute error from $t=3$ to $t=7$

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t	Y(t)	ε_t (calculation)
3	13	$13 - 15.86 - 0.5(0) - 0.3(0) = -2.86$
4	16	$16 - 15.86 - 0.5(-2.86) - 0.3(0) = 1.57$
5	18	$18 - 15.86 - 0.5(1.57) - 0.3(-2.86) = 0.86$
6	20	$20 - 15.86 - 0.5(0.86) - 0.3(1.57) = 2.32$
7	22	$22 - 15.86 - 0.5(2.32) - 0.3(0.86) = 2.87$

$$\varepsilon_t = Y_t - \mu - \theta_1(\varepsilon_{t-1}) - \theta_2 \varepsilon_{t-2})$$

forecast Y_8

$$Y_8 = \mu + \theta_1 \varepsilon_7 + \theta_2 \varepsilon_6$$

$$Y_8 = \cancel{15.58}\ 15.86 + 0.5(2.87) + 0.3(2.32)$$

$$= 15.86 + 1.435 + 0.696 = 17.991$$

$$\boxed{Y_8 = 17.991}$$

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ARMA (AutoRegressive Moving Average) Method

Given the time series dataset

t	Y(t)
1	10
2	12
3	13
4	16
5	18
6	20
7	22

ARMA(2,2) Model Equation

$Y_t = C + \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \theta_1 \varepsilon_{t-1} + \ldots$

Assumed Estimated Parameters

$C = 1.0$

$\phi_1 = 0.6$

$\phi_2 = 0.2$

$\theta_1 = 0.5$

$\theta_2 = 0.3$

Assume initial Error $\varepsilon_1 = \varepsilon_2 = 0$

~~we compute~~

$$Y_t = \mu + \phi_1 (Y_{t-1} - \mu) + \phi_2 (Y_{t-2} - \mu) + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} + \varepsilon_t$$

where

$\mu = 15.86$

$\phi_1 = 0.6$

$\phi_2 = 0.2$

$\theta_1 = 0.5$

$\theta_2 = 0.3$

Calculate Residuals (forecast Error)

We start $\varepsilon_1 = \varepsilon_2 = 0$

At $t = 3$: $Y_3 = 13$, $Y_2 = 12$, $Y_1 = 10$

$\varepsilon_3 = X_3 - \mu - \phi_1 (Y_2 - \mu) - \phi_2 (Y_1 - \mu)$

$-\theta_1 \varepsilon_2 - \theta_2 \varepsilon_2$

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$= 13 - 15.86 - 0.6(12-15.86) - 0.2(10-15.86)$
$- 0 - 0$

$= 13 - 15.86 + 2.316 + 1.172$

$= 0.628$

At $t = 4$

$Y_4 = 16 \quad Y_3 = 13 \quad Y_2 = 12 \quad \varepsilon_3 = 0.628$

$\varepsilon_4 = 16 - 15.86 - 0.6(13 - 15.86) - 0.2(12 - 15.86)$
$- 0.5(0.628) - 0.3(0)$

$= 16 - 15.86 + 1.716 + 0.772 - 0.314$

$= 2.314$

At $t = 5$

$\boxed{\varepsilon_5 = 0.1382}$

At $t = 6$

$\boxed{\varepsilon_6 = 2.0647}$

At $t = 7$

$\boxed{\varepsilon_7 = 2.155}$

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forecast for Y_8

$$Y_8 = \mu + \phi_1 (Y_7 - \mu) + \phi_2 (Y_6 - \mu) + \theta_1 \varepsilon_7 + \theta_2 \varepsilon_6$$

$$= 15.86 + 0.6(6.14) + 0.2(4.14) + 0.5(2.150) + 0.3(2.0647)$$

$$= 15.86 + 3.884 + 0.828 + 1.0775 + 0.6194$$

$$\boxed{Y_8 = 22.07}$$

MA →

Given Time Series

t	Y(t)
1	10
2	12
3	13
4	16
5	18
6	20
7	22

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We will forecast Y_8 using ARIMA (p, d, q)

ARIMA (p, d, q) stands

P: Order of Autoregression

d: Degree of differencing (to make the series stationary)

q = Order of Moving average (MA)

Let's assume the best model for this data is ARIMA (2, 1, 2)

Let's Compute the first order difference

$$Y_t' = Y_t - Y_{t-1}$$

$$Y' = \{12-10, 13-12, 16-13, 18-16, 20-18, 22-20\}$$

$= \{2, 1, 3, 2, 2, 2\}$

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Now Y is our stationary since, we will

fit using ~~ARIMA~~. ARMA(2,2)

$$\mu_y = \frac{2+1+3+2+2+2}{6} = 2$$

Let's assume - $\phi_1 = 0.6$, $\phi_2 = 0.2$

$\theta_1 = 0.5$ $\theta_2 = 0.3$

We now model

$$Y_t = \mu_y + \phi_1(Y_{t-1} - \mu_y) + \phi_2(Y_{t-2} - \mu_y) + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2}$$

Let's initialize

$$\varepsilon_1 = \varepsilon_2 = 0$$

Let's Compute - ε_3 to ε_6

At $t=3$ $Y_3 = 3$, $Y_2 = 1$, $Y_1 = 2$

$$\varepsilon_3 = 3 - 2.0 - 0.6(1-2) - 0.2(2-2)$$

$$\boxed{\varepsilon_3 = 1.6}$$

Formula $\varepsilon =$

Compute ε_3 to ε_6

At $t = 3$ $\quad Y_3 = 3, \; Y_2 = 1 \quad Y_1 = 2$

$$\varepsilon_3 = Y_3 - \mu - \theta_1 (Y_{t-2} - \mu) - \theta_2 (Y_{t-1} - \mu)$$

$$= 3 - 2 - 0.5(1 - 2) - 0.3(2 - 2)$$

$$= 3 - 2 + 0.5 - 0$$

$$\boxed{\varepsilon_3 = 1.05}$$

Same ε_4

ε_5

$\& \; \varepsilon_6$

Forecast -

$$Y_7 = \mu_y + \phi_1 (Y_6 - \mu) + \phi_2 (Y_5 - \mu) + \theta_1 \varepsilon_6 + \theta_2 \varepsilon_5$$

$$= 2.0 + 0.6(2 - 2.0) + 0.2(2 - 2 - 2.0) + 0.5(0.4) + 0.3(-0.08)$$

$$= 2.0 + 0 + 0 + 0.2 - 0.24 = 2.176$$