INDIAN INSTITUTE OF TECHNOLOGY, KHARAGPUR

Computer Science and Engineering

Switching Circuits and Logic Design (CS21002)

Assignment – 1 (Spring)

Group: 9 *Marks:* 60

Answer ALL the questions using xournal or similar software to edit the PDF

1. Simplify the following boolean functions in sum of products form using the Karnaugh map.
   (a) $f_1(w,x,y,z) = \sum(0,2,4,5,6,7,8,10,13,15)$ 3
   (b) $f_2(w,x,y,z) = \sum(0,1,5,7,8,10,14,15)$ 3
   (c) $f_3(w,x,y,z) = \sum(2,3,12,13,14,15)$ 3
2. Simplify the following boolean functions in product of sum form using the Karnaugh map.
   (a) $g_1(w,x,y,z) = \prod(1,3,5,7,13,15)$ 3
   (b) $g_2(w,x,y,z) = \prod(1,3,6,9,11,12,14)$ 3
   (c) $g_3(w,x,y,z) = \prod(1,4,5,6,11,12,13,14,15)$ 3
3. Given the function: $h(w,x,y,z) = \sum(1,2,3,5,13) + \sum_{\phi}(6,7,8,9,11,15)$
   (a) Find the minimal sum of products expression 3
   (b) Minimal product of sum expression 3
4. A binary-coded-decimal (BCD) message appears in four input lines of a switching circuit. Design an AND, OR, NOT gate network that produces an output value 1 whenever the input combination is 0, 2, 3, 5, or 8. 4
5. Simplify the following functions and implement them using two level NAND gate circuits.
   (a) $F_1(A,B,C,D) = A' + B + D' + B'C$ 3
   (b) $F_2(A,B,C,D) = (A' + C' + D')(A' + C')(C' + D')$ 3
6. Using the Karnaugh map simplify the following function.
   $f(v,w,x,y,z) = \sum(3,6,7,8,10,12,14,17,19,20,21,24,25,27,28)$. 3
7. Given the function $T(w,x,y,z) = \sum(1,3,4,5,7,8,9,11,14,15)$:
   (a) Using the Karnaugh map find all the prime implicants and identify the essential ones 3
   (b) Find three distinct minimal expressions for T 3
8. Draw the Karnaugh map for a four variable function with even number of prime implicants of which exactly half are essential. 4
9. How many prime implicants are there for the function $f(x_1, x_2, \ldots, x_n)$ which assumes the value 1 if and only if k or more of the variables are equal to 1. 4
10. Use the Quine-McCluskey procedure to generate the set of prime implicants and obtain *all* minimal expressions for the following functions:
    (a) $h_1(w,x,y,z) = \sum(1,5,6,12,13,14) + \sum_{\phi}(2,4)$ 3
    (b) $h_2(w,x,y,z) = \sum(0,1,5,7,8,10,14,15)$ 3
    (c) $h_2(v,w,x,y,z) = \sum(1,5,6,7,9,13,14,15,17,18,19,21,22,23,25,29,30)$ 3

1A

The following is the Kaurnaugh map where the true minterms (0,2,4,5,6, 7,8,10,13,15) are marked in **bold**.

yz / **wx**	00	01	11	10
00	**0**	1	3	**2**
01	**4**	**5**	**7**	**6**
11	12	**13**	**15**	16
10	**8**	9	11	**10**

3 cubes of size 4 each can be formed to cover all the true minterms. These 3 cubes are distincly highlighted with **green background colour**, **blue background colour** and **yellow border colour**.

For each of the three cubes the corresponding prime implicants are written in the following table.

Cube	w	x	y	z	Prime Implicant
Green (0 2 4 6)	0	-	-	0	$\overline{w}\ \overline{z}$
Blue (5 7 13 15)	-	1	-	1	$x\ z$
Yellow (0 2 8 10)	-	0	-	0	$\overline{x}\ \overline{z}$

Note that all three of them are essential prime implicants. Since they also cover all the true minterms, the reduced sum of products expression can be written as

$$f(w, x, y, z) = \overline{w}\ \overline{z} + xz + \overline{x}\ \overline{z}$$

1B

The following is the Kaurnaugh map where the true minterms (0, 1, 5, 7, 8, 10, 14, 15) are marked in **bold**.

yz wx	00	01	11	10
00	**0**	**1**	3	2
01	4	**5**	**7**	6
11	12	13	**15**	**14**
10	**8**	9	11	**10**

4 cubes of size 2 each can be formed to cover all the true minterms. These 4 cubes are distincly highlighted with **blue background colour**, **yellow background colour**, **green background colour** and **red background colour**.

For each of the four cubes the corresponding prime implicants are written in the following table.

Cube	w	x	y	z	Prime Implicant
Blue (0 1)	0	0	0	-	$\overline{w}\ \overline{x}\ \overline{y}$
Yellow (5 7)	0	1	-	1	$\overline{w}\ x\ z$
Green (15 14)	1	1	1	-	$w\ x\ y$
Red (8 10)	1	0	-	0	$w\ \overline{x}\ \overline{z}$

Since these prime implicants cover all the true minterms, the reduced sum of products expression can be written as

$$f(w, x, y, z) = \overline{w}\,\overline{x}\,\overline{y} + \overline{w}\,x\,z + w\,x\,y + w\,\overline{x}\,\overline{z}$$

1C

The following is the Kaurnaugh map where the true minterms (2, 3, 12, 13, 14, 15) are marked in **bold**.

yz / **wx**	00	01	11	10
00	0	1	**3**	**2**
01	4	5	7	6
11	**12**	**13**	**15**	**14**
10	8	9	11	10

2 cubes, one of size 4 and other of size 2 can be formed to cover all the true minterms. These 2 cubes are distincly highlighted with **green background colour** and **blue background colour**.

For each of the two cubes the corresponding prime implicants are written in the following table.

Cube	w	x	y	z	Prime Implicant
Blue (2 3)	0	0	1	-	$\overline{w}\,\overline{x}\,y$
Green (12 13 14 15)	1	1	-	-	$w\,x$

Note that both of them are essential prime implicants. Since they also cover all the true minterms, the reduced sum of products expression can be written as

$$f(w, x, y, z) = w\,x + \overline{w}\,\overline{x}\,y$$

2A

The following is the Kaurnaugh map where the *true* minterms (0, 2, 4, 6, 8, 9, 10, 11, 12, 14 -- because *a minterm accepts if and only if the corresponding maxterm rejects*) are marked in **bold**.

yz / **wx**	00	01	11	10
00	**0**	1	3	**2**
01	**4**	5	7	**6**
11	**12**	13	15	**14**
10	**8**	**9**	**11**	**10**

2 cubes, of size 4 each can be formed to cover all the false minterms. These 2 cubes are distincly highlighted with **yellow background colour** and **red border colour**.

For each of the two cubes the corresponding products of literals are written in the following table.

Cube	w	x	y	z	POS
Yellow (1 3 5 7)	0	-	-	1	$\overline{w}\ z$
Red (5 7 13 15)	-	1	-	1	$x\ z$

Since both of them cover all the false minterms, they can be used to write the expression as a product of sums. Note that literals in a minterm and corresponding maxterm are complemented. Besides, a minterm accepts if and only if the corresponding maxterm rejects. Therefore, the product of sums can be written as

$$f(w,\ x,\ y,\ z) = (w + \bar{z}).(\bar{x} + \bar{z})$$

2B

The following is the Kaurnaugh map where the *true* minterms (0, 2, 4, 5, 7, 8, 10, 13, 15 -- because *a minterm accepts if and only if the corresponding maxterm rejects*) are marked in **bold**.

yz **wx**	00	01	11	10
00	**0**	1	3	**2**
01	**4**	**5**	**7**	6
11	12	**13**	**15**	14
10	**8**	9	11	**10**

3 cubes, one of size 4 and two of size 2 can be formed to cover all the false minterms. These 3 cubes are distincly highlighted with **blue background colour**, **green background colour** and **red border colour**.
For each of the three cubes the corresponding products of literals are written in the following table.

Cube	w	x	y	z	POS
Blue (1 3 9 11)	-	0	-	1	$\bar{x}\ z$
Green (6 14)	-	1	1	0	$x\ y\ \bar{z}$
Red (12 14)	1	1	-	0	$w\ x\ \bar{z}$

Since all three of them cover all the false minterms, they can be used to write the expression as a product of sums. Note that literals in a minterm and corresponding maxterm are complemented. Besides, a minterm accepts if and only if the corresponding maxterm rejects. Therefore, the product of sums can be written as

$$f\,(\,w,\ x,\ y,\ z\,) = (x + \overline{z})\ .\ (\overline{x} + \overline{y} + z)\ .\ (\overline{w} + \overline{x} + z)$$

2C

The following is the Kaurnaugh map where the *true* minterms (0, 2, 3, 7, 8, 9, 10, 13 -- because *a minterm accepts if and only if the corresponding maxterm rejects*) are marked in **bold**.

yz **wx**	00	01	11	10
00	**0**	1	**3**	**2**
01	4	5	**7**	6
11	12	**13**	15	14
10	**8**	**9**	11	**10**

4 cubes, two of size 4 and two of size 2 can be formed to cover all the false minterms. These 4 cubes are distincly highlighted with **blue border colour**, **green background colour**, **yellow background colour** and **red background colour**.

For each of the four cubes the corresponding products of literals are written in the following table.

Cube	w	x	y	z	POS
Blue (12 13 14 15)	1	1	-	-	$w\ x$

Red (4 6 12 14)	-	1	-	0	$x\,\overline{z}$
Green (11 15)	1	-	1	1	$w\,y\,z$
Yellow (1 5)	0	-	0	1	$\overline{w}\,\overline{y}\,z$

Since all four of them cover all the false minterms, they can be used to write the expression as a product of sums. Note that literals in a minterm and corresponding maxterm are complemented. Besides, a minterm accepts if and only if the corresponding maxterm rejects. Therefore, the product of sums can be written as

$$f\,(\,w,\;x,\;y,\;z\,) = (\overline{w}+\overline{x})\;.\;(\overline{x}+z)\;.\;(\overline{w}+\overline{y}+\overline{z})\;.\;(w+y+\overline{z})$$

3A

The following is the Kaurnaugh map where the *true* minterms (1, 2, 3, 5, 13) are marked in **bold**. The *dont-cares* are marked in **bold red**.

yz wx	00	01	11	10
00	0	**1**	**3**	**2**
01	4	**5**	**7**	**6**
11	12	**13**	**15**	14
10	**8**	**9**	**11**	10

2 cubes, one of size 8 and 1 of size 4 can be formed to cover all the true minterms. These 2 cubes are distincly highlighted with **blue border colour** and **green background colour**.

For both of the cubes the corresponding prime implicants are written in the following table.

Cube	w	x	y	z	Prime Implicant
Green (1 3 5 7 9 11 13 15)	-	-	-	1	z
Blue (2 3 6 7)	0	-	1	-	$\overline{w}\, y$

Since these prime implicants cover all the true minterms, the minimal sum of products expression can be written as

$$f\,(\,w,\ x,\ y,\ z\,) = z + \overline{w}\, y$$

3B

The following is the Kaurnaugh map where the *true* minterms (1, 2, 3, 5, 13) are marked in **bold**. The *dont-cares* are marked in **bold red**.

yz wx	00	01	11	10
00	0	**1**	**3**	**2**
01	4	**5**	**7**	**6**
11	12	**13**	**15**	14
10	**8**	**9**	**11**	10

2 cubes, of size 4 each can be formed to cover all the false minterms. These 2 cubes are distincly highlighted with **yellow background colour** and **blue background colour**.

For both the cubes the corresponding products of literals are written in the following table.

Cube	w	x	y	z	POS
Yellow (0 4 8 12)	-	-	0	0	$\overline{y}\ \overline{z}$
Blue (10 11 14 15)	1	-	1	-	$w\ y$

Since both the cubes cover all the false minterms, they can be used to write the expression as a product of sums. Note that literals in a minterm and corresponding maxterm are complemented. Besides, a minterm accepts if and only if the corresponding maxterm rejects. Therefore, the product of sums can be written as

$$f(w, x, y, z) = (y + z) \,.\, (\overline{w} + \overline{y})$$

4

Let the boolean function $f(w, x, y, z)$ be a function that produces output 1 if and only if the input 4-bit *binary coded decimal* ***wxyz*** represents 0, 2, 3, 5, or 8.

Therefore, $f(w, x, y, z) = \Sigma(0, 2, 3, 5, 8)$

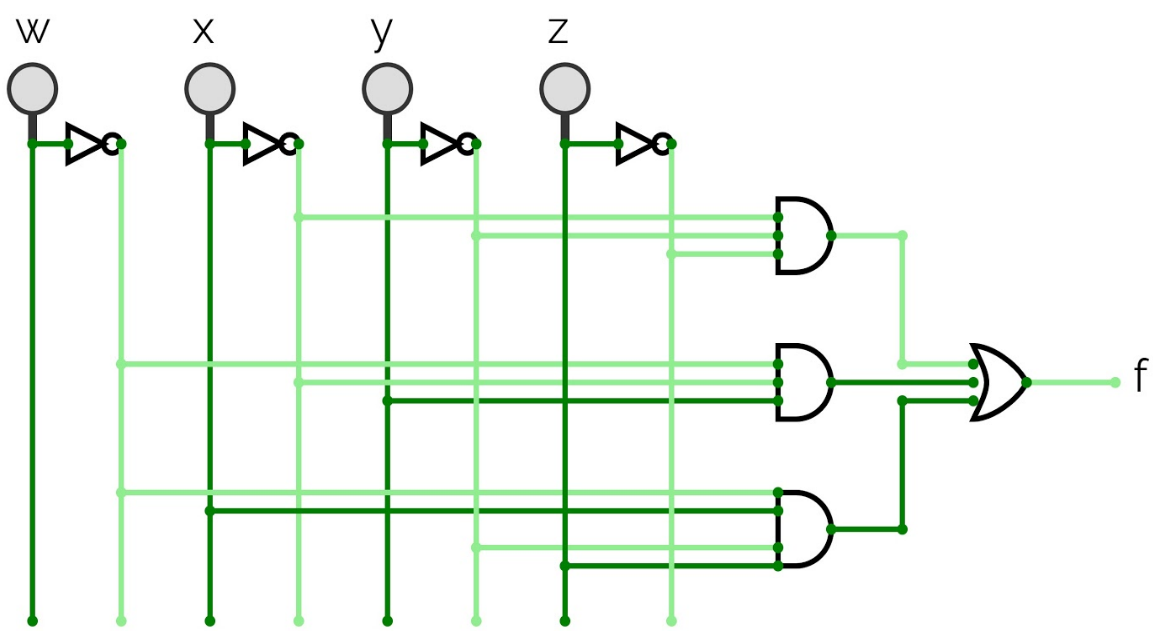
The following is the Kaurnaugh map where the *true* minterms (0, 2, 3, 5, 8) are marked in **bold**.

yz **wx**	00	01	11	10
00	**0**	1	**3**	**2**
01	4	**5**	7	6
11	12	13	15	14
10	**8**	9	11	10

For all of the 3 cubes the corresponding prime implicants are written in the following table.

Cube	w	x	y	z	Prime Implicant
Yellow (2 3)	0	0	1	-	$\overline{w}\,\overline{x}\,y$
Green (5)	0	1	0	1	$\overline{w}\,x\,\overline{y}\,z$
Red (0 8)	-	0	0	0	$\overline{x}\,\overline{y}\,\overline{z}$

$$f\,(\,w,\ x,\ y,\ z\,) = \overline{w}\,\overline{x}\,y + \overline{x}\,\overline{y}\,\overline{z} + \overline{w}\,x\,\overline{y}\,z$$



5A

$$F_1\,(\,A,\ B,\ C,\ D\,) = A' + B + D' + B'C$$

For each product term in the above *sum of products*, the corresponding minterms are stated in the following table.

Product	Minterms

A'	0 1 2 3 4 5 6 7
B	4 5 6 7 12 13 14 15
D'	0 2 4 6 8 10 12 14
$B'C$	2 3 10 11

Therefore,

$$F_1(A, B, C, D) = \Sigma(0, 1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 13, 14, 15)$$

The following is the Kaurnaugh map where the *true* minterms are marked in **bold**.

(Marked different prime implicants on different K-maps to avoid clutter)

Prime Implicant O1

CD AB	00	01	11	10
00	**0**	**1**	**3**	**2**
01	**4**	**5**	**7**	**6**
11	**12**	**13**	**15**	**14**
10	**8**	9	**11**	**10**

Prime Implicant O2

CD AB	00	01	11	10
00	**0**	**1**	**3**	**2**
01	**4**	**5**	**7**	**6**
11	**12**	**13**	**15**	**14**
10	**8**	9	**11**	**10**

Prime Implicant O3

CD AB	00	01	11	10
00	**0**	**1**	**3**	**2**
01	**4**	**5**	**7**	**6**
11	**12**	**13**	**15**	**14**

Prime Implicant O4

CD AB	00	01	11	10
00	**0**	**1**	**3**	**2**
01	**4**	**5**	**7**	**6**
11	**12**	**13**	**15**	**14**

10	**8**	9	**11**	**10**

10	**8**	9	**11**	**10**

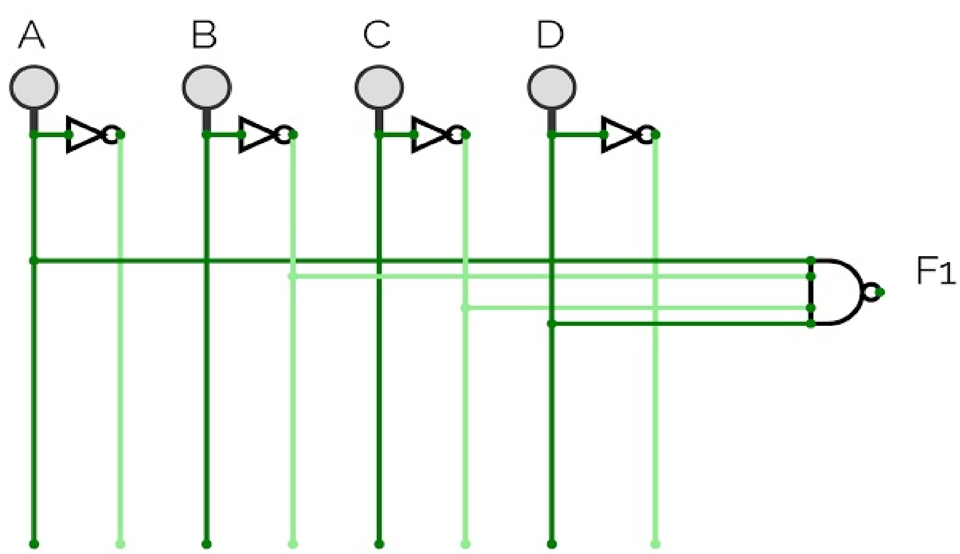
4 cubes, all of size 8 can be formed to cover all the true minterms. These 4 cubes are highlighted with **yellow background colour**, distinctly in different Karnaugh maps.

For all of the 4 cubes the corresponding prime implicants are written in the following table.

Cube	A	B	C	D	Prime Implicant
Prime Implicant O1	-	-	1	-	C
Prime Implicant O2	0	-	-	-	A'
Prime Implicant O3	-	1	-	-	B
Prime Implicant O4	-	-	-	0	D'

Since these prime implicants cover all the true minterms, the reduced expression can be written as

$$F_1(A, B, C, D) = A' + B + C + D'$$



5B

$$F_2(A, B, C, D) = (A' + C' + D')(A' + C')(C' + D')$$

For each sum term in the above *product of sums*, the corresponding maxterms are stated in the following table.

Sum	Maxterms
$A' + C' + D'$	11 15
$A' + C'$	10 11 14 15
$C' + D'$	3 7 11 15

Therefore, $F_2(A, B, C, D) = \Pi(3, 7, 10, 11, 14, 15)$

The following is the Kaurnaugh map where the *true* minterms (0, 1, 2, 4, 5, 6, 8, 9, 12, 13 -- because *a minterm accepts if and only if the corresponding maxterm rejects*) are marked in **bold**.

CD **AB**	00	01	11	10
00	**0**	**1**	3	**2**
01	**4**	**5**	7	**6**
11	**12**	**13**	15	14
10	**8**	**9**	11	10

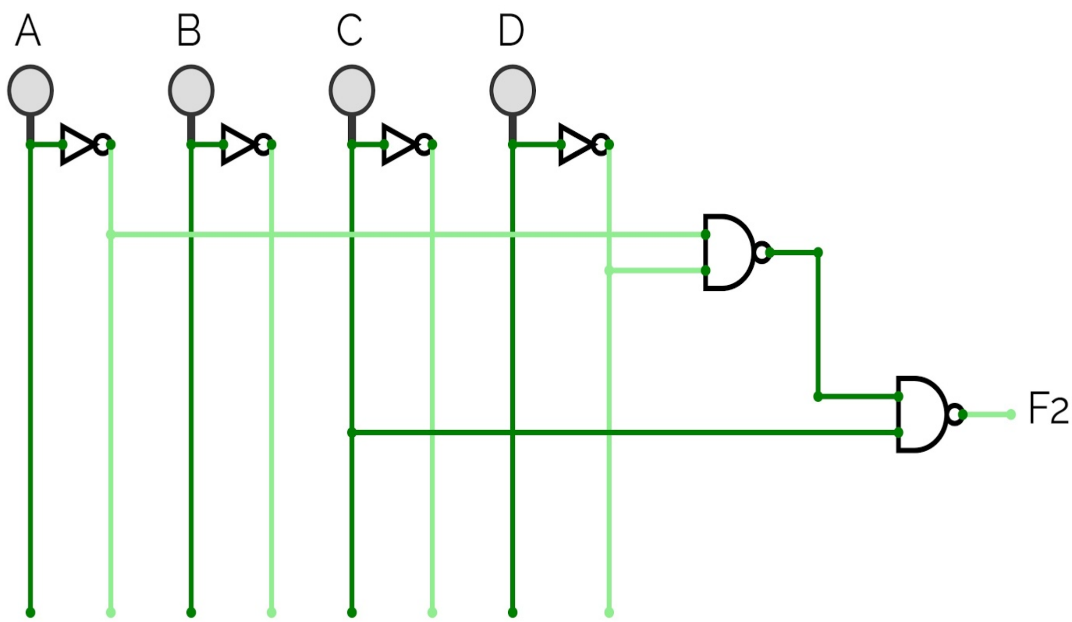
2 cubes, one of size 8 and one of size 4 can be formed to cover all the true minterms. These 2 cubes are distincly highlighted with **red border colour** and **green background colour**.

For both the cubes the corresponding prime implicants are written in the following table.

Cube	A	B	C	D	Prime Implicant
Green (0 1 4 5 8 9 12 13)	-	-	0	-	C'
Red (0 2 4 6)	0	-	-	0	$A'D'$

Since these prime implicants cover all the true minterms, the reduced expression can be written as

$$F_2(A, B, C, D) = C' + A'D'$$



6

The following is the Kaurnaugh map where the *true* minterms (3, 6, 7, 8, 10, 12, 14, 17, 19, 20, 21, 24, 25, 27, 28) are marked in **bold**.

xyz vw	000	001	011	010	110	111	101	100
00	0	1	**3**	2	**6**	**7**	5	4
01	**8**	9	11	**10**	**14**	15	13	**12**

11	**24**	**25**	**27**	26	30	31	29	**28**
10	16	**17**	**19**	18	22	23	**21**	**20**

6 cubes, three of size 4 and three of size 2 can be formed to cover all the true minterms. These 6 cubes are distincly highlighted with **blue background colour**, **purple background colour**, **pink background colour**, **red background colour**, **yellow background colour** and **green border colour**.
For all of the 6 cubes the corresponding prime implicants are written in the following table.

Cube	v	w	x	y	z	Prime Implicant
Blue (8 12 24 28)	-	1	-	0	0	$w\,\bar{y}\,\bar{z}$
Purple (17 19 25 27)	1	-	0	-	1	$v\,\bar{x}\,z$
Green (8 10 12 14)	0	1	-	-	0	$\bar{v}\,w\,\bar{z}$
Pink (21 20)	1	0	1	0	-	$v\,\bar{w}\,x\,\bar{y}$
Red (6 14)	0	-	1	1	0	$\bar{v}\,x\,y\,\bar{z}$
Yellow (3 7)	0	0	-	1	1	$\bar{v}\,\bar{w}\,y\,z$

Since these prime implicants cover all the true minterms, the reduced expression can be written as

$$f(v, w, x, y, z) =$$
$$w\,\bar{y}\,\bar{z} + v\,\bar{x}\,z + \bar{v}\,w\,\bar{z} + v\,\bar{w}\,x\,\bar{y} + \bar{v}\,x\,y\,\bar{z} + \bar{v}\,\bar{w}\,y\,z$$

7A

The following is the Kaurnaugh map where the *true* minterms (1, 3, 4, 5, 7, 8, 9, 11, 14, 15) are marked in **bold**.

yz wx	00	01	11	10
00	0	**1**	**3**	2
01	**4**	**5**	**7**	6
11	12	13	**15**	**14**
10	**8**	**9**	**11**	10

In the following set of Karnaugh maps, various prime implicants of the expression formed by the above true minterms are highlighted in **blue background colour**. Note that using the Karnaugh map, a prime implicant can be discovered by forming as large cubes as possible, which cannot be expanded (or combined with adjacent cells) any further.

Prime Implicant O1

yz wx	00	01	11	10
00	0	**1**	**3**	2
01	**4**	**5**	**7**	6
11	12	13	**15**	**14**
10	**8**	**9**	**11**	10

Prime Implicant O2

yz wx	00	01	11	10
00	0	**1**	**3**	2
01	**4**	**5**	**7**	6
11	12	13	**15**	**14**
10	**8**	**9**	**11**	10

Prime Implicant O3

yz wx	00	01	11	10
00	0	**1**	**3**	2
01	**4**	**5**	**7**	6
11	12	13	**15**	**14**
10	**8**	**9**	**11**	10

Prime Implicant O4

Prime Implicant O5

Prime Implicant O6

yz wx	00	01	11	10
00	0	**1**	**3**	2
01	**4**	**5**	**7**	6
11	12	13	**15**	**14**
10	**8**	**9**	**11**	10

yz wx	00	01	11	10
00	0	**1**	**3**	2
01	**4**	**5**	**7**	6
11	12	13	**15**	**14**
10	**8**	**9**	**11**	10

yz wx	00	01	11	10
00	0	**1**	**3**	2
01	**4**	**5**	**7**	6
11	12	13	**15**	**14**
10	**8**	**9**	**11**	10

In conclusion, there are 6 prime implicants of the expression formed by the above true minterms.

P.I.	1	3	4	5	7	8	9	11	14	15
3 7 11 15		X			X			X		X
1 3 5 7	X	X		X	X					
1 3 9 11	X	X					X	X		
4 5			X	X						
14 15									X	X
8 9						X	X			

From the above table, it is clear that *Prime Implicant O4, Prime Implicant O5* and *Prime Implicant O6* are essential prime implicants because they are the only implicants whose corresponding cubes cover the true minterms 4, 14, 8 respectively on the Karnaugh map.

Therefore the prime implicants are --

Prime Implicant O1 $= y\,z$

Prime Implicant O2 $= \overline{w}\,z$

Prime Implicant O3 $= \overline{x}\,z$

Prime Implicant O4 $= \overline{w}\, x\, \overline{y}$

Prime Implicant O5 $= w\, x\, y$

Prime Implicant O6 $= w\, \overline{x}\, \overline{y}$

And the essential prime implicants are --

Prime Implicant O4 $= \overline{w}\, x\, \overline{y}$

Prime Implicant O5 $= w\, x\, y$

Prime Implicant O6 $= w\, \overline{x}\, \overline{y}$

7B

yz wx	00	01	11	10
00	0	**1**	**3**	2
01	**4**	**5**	**7**	6
11	12	13	**15**	**14**
10	**8**	**9**	**11**	10

5 cubes, two of size 4 and three of size 2 can be formed to cover all the true minterms. These 5 cubes are distincly highlighted with **green background colour**, **purple background colour**, **yellow background colour**, **pink border colour** and **red border colour**.
For each of the three cubes the corresponding prime implicants are written in the following table.

Cube	w	x	y	z	Prime Implicant
Yellow (3 7 15 11)	-	-	1	1	$y\, z$

Red (1 3 9 11)	-	0	-	1	$\overline{x}\,z$
Purple (4 5)	0	1	0	-	$\overline{w}\,x\,\overline{y}$
Green (8 9)	1	0	0	-	$w\,\overline{x}\,\overline{y}$
Pink (14 15)	1	1	1	-	$w\,x\,y$

Since they all five of them cover all the true minterms, the minimal expression can be written as

$$f(w,\ x,\ y,\ z) = y\,z + \overline{x}\,z + \overline{w}\,x\,\overline{y} + w\,\overline{x}\,\overline{y} + w\,x\,y$$

yz wx	00	01	11	10
00	0	**1**	**3**	2
01	**4**	**5**	**7**	6
11	12	13	**15**	**14**
10	**8**	**9**	**11**	10

5 cubes, two of size 4 and three of size 2 can be formed to cover all the true minterms. These 5 cubes are distincly highlighted with **green background colour**, **purple background colour**, **yellow background colour**, **pink border colour** and **red border colour**.
For each of the three cubes the corresponding prime implicants are written in the following table.

Cube	w	x	y	z	Prime Implicant

Yellow (1 3 5 7)	0	-	-	1	$\overline{w}\, z$
Pink (1 3 9 11)	-	0	-	1	$\overline{x}\, z$
Red (4 5)	0	1	0	-	$\overline{w}\, x\, \overline{y}$
Green (8 9)	1	0	0	-	$w\, \overline{x}\, \overline{y}$
Purple (14 15)	1	1	1	-	$w\, x\, y$

Since they all five of them cover all the true minterms, the minimal expression can be written as

$$f\,(\,w,\ x,\ y,\ z\,) = \overline{w}\, z + \overline{x}\, z + \overline{w}\, x\, \overline{y} + w\, \overline{x}\, \overline{y} + w\, x\, y$$

yz **wx**	00	01	11	10
00	0	**1**	**3**	2
01	**4**	**5**	**7**	6
11	12	13	**15**	**14**
10	**8**	**9**	**11**	10

5 cubes, two of size 4 and three of size 2 can be formed to cover all the true minterms. These 5 cubes are distincly highlighted with **green background colour**, **purple background colour**, **yellow background colour**, **pink border colour** and **red border colour**.
For each of the three cubes the corresponding prime implicants are written in the following table.

Cube	w	x	y	z	Prime Implicant
Red (1 3 5 7)	0	-	-	1	$\overline{w}\,z$
Yellow (3 7 11 15)	-	-	1	1	$y\,z$
Green (4 5)	0	1	0	-	$\overline{w}\,x\,\overline{y}$
Purple (8 9)	1	0	0	-	$w\,\overline{x}\,\overline{y}$
Pink (14 15)	1	1	1	-	$w\,x\,y$

Since they all five of them cover all the true minterms, the minimal expression can be written as

$$f(w,\ x,\ y,\ z) = \overline{w}\,z + y\,z + \overline{w}\,x\,\overline{y} + w\,\overline{x}\,\overline{y} + w\,x\,y$$

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Let such function be

$$f(w,\ x,\ y,\ z) = \Sigma\,(3,\ 5,\ 7,\ 12,\ 13)$$

The following is the Kaurnaugh map where the *true* minterms (3, 5, 7, 12, 13) are marked in **bold**.

yz **wx**	00	01	11	10
00	0	1	**3**	2
01	4	**5**	**7**	6
11	**12**	**13**	15	14

10	8	9	11	10

In the following set of Karnaugh maps, various prime implicants of the expression formed by the above true minterms are highlighted in **blue background colour**. Note that using the Karnaugh map, a prime implicant can be discovered by forming as large cubes as possible, which cannot be expanded (or combined with adjacent cells) any further.

Prime Implicant O1

yz wx	00	01	11	10
00	0	1	**3**	2
01	4	**5**	**7**	6
11	**12**	**13**	15	14
10	8	9	11	10

Prime Implicant O2

yz wx	00	01	11	10
00	0	1	**3**	2
01	4	**5**	**7**	6
11	**12**	**13**	15	14
10	8	9	11	10

Prime Implicant O3

yz wx	00	01	11	10
00	0	1	**3**	2
01	4	**5**	**7**	6
11	**12**	**13**	15	14
10	8	9	11	10

Prime Implicant O4

yz wx	00	01	11	10
00	0	1	**3**	2
01	4	**5**	**7**	6
11	**12**	**13**	15	14
10	8	9	11	10

In conclusion, there are 4 prime implicants of the expression formed by the above true minterms.

P.I.	3	5	7	12	13

3 7	X		X		
5 7		X	X		
5 13		X			X
12 13				X	X

From the above table, it is clear that *Prime Implicant O1* and *Prime Implicant O4* are essential prime implicants because they are the only implicants whose corresponding cubes cover the true minterms 12, 3 respectively on the Karnaugh map.

Hence, the boolean function $f(w, x, y, z)$ has 4 prime implicants out of which 2are essential prime implicants. Therefore, this has the property that was desired in the problem statement.

9

$$f(x_1, x_2, x_3............, x_{n-1}, x_n) = x_1x_2...x_k + x_2x_3...x_{k+1} + \\ .... + x_{n-k+1}x_{n-k+2}...x_n + ... (nCk\ terms)$$

f is a sum of products where each product term is a *product of k distinct literals taken in non-complemented form*. Product of literals in every possible set of *k* variables out of the *n* variables must be a *product-term* of f.

Prove that f is equal to an n-variable boolean function g that assumes the value of 1 if and only if at least k variables are equal to 1 (true).

(1.) Prove that if value of f is 1 for some n-boolean inputs, then value of g must also be 1 for the same inputs.

Let value of f is 1 for some *n* boolean inputs. This implies there exists at least one product term *T* in the *sum of products* expression of f that is equal to 1. Otherwise, if all the terms are 0, then the boolean sum of them will also be 0 (leading to contradiction).

By definition of the function f the product term *T* must be a product of exactly *k* distinct variables in *non-complemented form*.

Value of *T* is 1. Therefore, each and every literal in the product must be equal to 1. Since all the *k* literals are distinct variables in non-complemented form, there must be exactly *k* distinct variables present in the term *T* that are true/1.

This implies, if the value of f is 1 then at least *k* distinct variables out of the *n* input variables must be equal to 1.

By definition of the function g , it gives a value 1 if and only if at least *k* of the *n* input variables are 1. Therefore, the inputs given to the function f must give a value of 1 when they are passed as input to the function g **.**

(2.) <u>Prove that if value of g is 1 for some n-boolean inputs, then value of f must also be 1 for the same inputs.</u>

Let value of g is 1 for some *n* boolean inputs. By definition of the function g , its value is 1 if and only if at least *k* of the n input variables are 1. Therefore, at least *k* of the *n* input variables (say x_1, x_2, x_3,....., x_{k-1}, x_k) must be 1. Consequently the product $P = x_1x_2x_3.....x_{k-1}x_k$ is also equal to 1.

Now pass the same inputs to the boolean function f .

By definition, f is a sum of products where each product term is a *product of k distinct literals taken in non-complemented form*. Product of literals in every possible set of *k* variables out of the *n* variables must be a *product-term* of f .

Consequently, *P* must be a product term in the *sum of products* expression of f . Since value of *P* is 1, the value of the function f must become 1.

From (1.) and (2.) it is clear that value of function f is 1 if and only if value of function g is 1.

Therefore, the two functions are equivalent.

Prove that all product terms in the function f are prime implicants of the function

A product terms in the *sum of products* expression of f are *implicants* (by definition). An *implicant* of a function is *prime implicant* if and only if it is not *covered/implied* by any other implicant.
Let any product term *T* in the expression be a non-prime implicant.
This implies that

$$\exists \ \ product\ term\ P \neq T\ in\ f \quad s.t. \quad P \rightarrow T$$

By definition of function f, both terms *P* and *T* have exactly *k* literals of *k* distinct variables, in non-complemented form. Since $P \neq T$ and both of them have exactly same number of literals, there must be at least one literal (say *p*) that is in *P* and is not in *T* and there must be at least one literal (say *t*) that is in *T* and is not in *P*.
If *P* is true, then all its literals must be true. Therefore, *p* is also true.
Choose *t = 0 independently* (not a literal of *P*). If *t* is 0 then the whole product term *T* must be 0.
Therefore here, $1 \rightarrow 0$ which is a contradiction.

Therefore, every term in the *sum of products* expression of f is a *prime implicant. Since there are nCk terms in* f *, it must have at least nCk prime implicants.*

Prove that the function f does not have any more prime implicants

Any product term with less than *k* distinct literals in non-complemented form cannot even be an *implicant* of the function f , let alone a *prime implicant*. This is because by definitional property of f , truth value of less

than *k* distinct variables does not ensure/imply a truth value of f ; it is only ensured when *k* or more distinct variables have truth value.
Secondly, any product term with more than *k* distinct literals in non-complemented form will definitely be an implicant of the function (as already stated), but not a *prime implicant*. This is because there must exist a product of any *k* of those non-complemented distinct literals in the *sum of products* expression of f (by definition of the function), which will *imply/cover* this product term with more than *k* distinct literals in non-complemented form (violation of the property of *prime implicants*).
In comclusion, other than the terms already present in the f , there are no more *prime implicants of this function.*

Hence, there are exactly *<u>nCk prime implicants</u> for the function* f ; that assumes a value of 1 if and only if at least *k* of the *n* input boolean variables are 1.

10A

NOTE *: All cubes highlighted with a* ***red background colour*** *in their corresponding groups are the prime implicants. These are the ones that do not further combine with any cube of any other group.*

Group 1.1 -- 1 on-set and 0 DC-set minterms

<1:1, 0:D>				
Minterm	**w**	**x**	**y**	**z**
1	0	0	0	1
2	0	0	1	0
4	0	1	0	0

Group 1.2 -- 2 on-set and 0 DC-set minterms

<2:1, 0:D>				
Minterm	**w**	**x**	**y**	**z**
5	0	1	0	1
6	0	1	1	0
12	1	1	0	0

Group 1.3 -- 3 on-set and 0 DC-set minterms

<3:1, 0:D>				
Minterm	**w**	**x**	**y**	**z**
13	1	1	0	1
14	1	1	1	0

Group 2.1 -- 1 on-set and 1 DC-set cubes
(combine Group 1.1 and Group 1.2)

<1:1, 1:D>				
Cube	**w**	**x**	**y**	**z**
1 5	0	-	0	1
2 6	0	-	1	0
4 5	0	1	0	-
4 6	0	1	-	0
4 12	-	1	0	0

Group 2.2 -- 2 on-set and 1 DC-set cubes
(combine Group 1.2 and Group 1.3)

<2:1, 1:D>				
Cube	**w**	**x**	**y**	**z**

5 13	-	1	0	1
6 14	-	1	1	0
12 13	1	1	0	-
12 14	1	1	-	0

Group 3.1 -- 1 on-set and 2 DC-set cubes
(combine Group 2.1 and Group 2.2)

<1:1, 2:D>				
Cube	**w**	**x**	**y**	**z**
4 5 12 13	-	1	0	-
4 6 12 14	-	1	-	0

Take all the cubes highlighted with red background colour (prime implicants) and construct the following matrix to illustrate the inclusion of various true minterms in each of the prime implicants.

REDUCTION STEP O1 : (1, 5) is an essential prime implicant because it is the only cube that covers the true minterm 5.
Remove the highlighted cells and include the prime implicant corresponding to the cube (1, 5) in the minimal expression.

P.I.	1	5	6	12	13	14
1 5	X	X				
2 6			X			
4 5 12 13		X		X	X	
4 6 12 14			X	X		X

REDUCTION STEP O2 : (4, 5, 12, 13) is an essential prime implicant because it is the only cube that covers the true minterm 13.

Remove the highlighted cells and include the prime implicant corresponding to the cube (4, 5, 12, 13) in the minimal expression.

P.I.	6	12	13	14
2 6	X			
4 5 12 13		X	X	
4 6 12 14	X	X		X

REDUCTION STEP O3 : (4, 6, 12, 14) is an essential prime implicant because it is the only cube that covers the true minterm 14.
Remove the highlighted cells and include the prime implicant corresponding to the cube (4, 6, 12, 14) in the minimal expression.

	6	14
2 6	X	
4 6 12 14	X	X

So there is only ***one*** minimal expression for the boolean function.

$$h_{1-min}(w,\ x,\ y,\ z) = \overline{w}\,\overline{y}\,z + x\,\overline{y} + x\,\overline{z}$$

10B

***NOTE** : All cubes highlighted with a **red background colour** in their corresponding groups are the prime implicants. These are the ones that do not further combine with any cube of any other group.*

Group 1.1 -- 0 on-set and 0 DC-set minterms

<0:1, 0:D>				
Minterm	**w**	**x**	**y**	**z**
0	0	0	0	0

Group 1.2 -- 1 on-set and 0 DC-set minterms

<1:1, 0:D>				
Minterm	**w**	**x**	**y**	**z**
1	0	0	0	1
8	1	0	0	0

Group 1.3 -- 2 on-set and 0 DC-set minterms

<2:1, 0:D>				
Minterm	**w**	**x**	**y**	**z**
5	0	1	0	1
10	1	0	1	0

Group 1.4 -- 3 on-set and 0 DC-set minterms

<3:1, 0:D>				
Minterm	**w**	**x**	**y**	**z**
7	0	1	1	1
14	1	1	1	0

Group 1.5 -- 4 on-set and 0 DC-set minterms

<4:1, 0:D>				
Minterm	**w**	**x**	**y**	**z**
15	1	1	1	1

Group 2.1 -- 0 on-set and 1 DC-set cubes
(combine Group 1.1 and Group 1.2)

<0:1, 1:D>

Cube	w	x	y	z
0 1	0	0	0	-
0 8	-	0	0	0

Group 2.2 -- 1 on-set and 1 DC-set cubes (combine Group 1.2 and Group 1.3)

<1:1, 1:D>				
Cube	**w**	**x**	**y**	**z**
1 5	0	-	0	1
8 10	1	0	-	0

Group 2.3 -- 2 on-set and 1 DC-set cubes (combine Group 1.3 and Group 1.4)

<2:1, 1:D>				
Cube	**w**	**x**	**y**	**z**
5 7	0	1	-	1
10 14	1	-	1	0

Group 2.4 -- 3 on-set and 1 DC-set cubes (combine Group 1.4 and Group 1.5)

<3:1, 1:D>				
Cube	**w**	**x**	**y**	**z**
7 15	-	1	1	1
14 15	1	1	1	-

Take all the cubes highlighted with red background colour (prime implicants) and construct the following matrix to illustrate the inclusion of various true minterms in each of the prime implicants.

REDUCTION PROCEDURE O1

REDUCTION STEP O1 : Remove the highlighted cells and include the prime implicant corresponding to the cube (0, 1) in the minimal expression.

P.I.	0	1	5	7	8	10	14	15
0 1	X	X						
0 8	X				X			
1 5		X	X					
8 10					X	X		
5 7			X	X				
10 14						X	X	
7 15				X				X
14 15							X	X

REDUCTION STEP O2 : Row for P.I. (5, 7) dominates the row for P.I. (1, 5). Remove the highlighted cells (the dominated row).

P.I.	5	7	8	10	14	15
0 8			X			
1 5	X					
8 10			X	X		
5 7	X	X				
10 14				X	X	
7 15		X				X

14 15					X	X

REDUCTION STEP O3 : (5, 7) is an essential prime implicant because it is the only cube that covers the true minterm 5.
Remove the highlighted cells and include the prime implicant corresponding to the cube (5, 7) in the minimal expression.

P.I.	5	7	8	10	14	15
0 8			X			
8 10			X	X		
5 7	X	X				
10 14				X	X	
7 15		X				X
14 15					X	X

REDUCTION STEP O4 : Row for P.I. (14, 15) dominates the row for P.I. (7, 15). Remove the highlighted cells (the dominated row).

P.I.	8	10	14	15
0 8	X			
8 10	X	X		
10 14		X	X	
7 15				X
14 15			X	X

REDUCTION STEP O5 : (14, 15) is an essential prime implicant because it is the only cube that covers the true minterm 15.
Remove the highlighted cells and include the prime implicant corresponding to the cube (14, 15) in the minimal expression.

P.I.	8	10	14	15
0 8	X			
8 10	X	X		
10 14		X	X	
14 15			X	X

REDUCTION STEP O6 : Row for P.I. (8, 10) dominates the rows for P.I. (0, 8) and (10, 14). Remove the highlighted cells (the dominated rows).

P.I.	8	10
0 8	X	
8 10	X	X
10 14		X

REDUCTION STEP O7 : (8, 10) is an essential prime implicant because it is the only cube that covers the true minterms 8 and 10.
Remove the highlighted cells and include the prime implicant corresponding to the cube (8, 10) in the minimal expression.

P.I.	8	10
8 10	X	X

So there is the ***first*** minimal expression for the boolean function.

$$h_{2-min-first}(w,\ x,\ y,\ z) = \overline{w}\ \overline{x}\ \overline{y} + w\ x\ y + \overline{w}\ x\ z + w\ \overline{x}\ \overline{z}$$

REDUCTION PROCEDURE O2

REDUCTION STEP O1 : Remove the highlighted cells and include the prime implicant corresponding to the cube (0, 8) in the minimal expression.

P.I.	0	1	5	7	8	10	14	15

0 1	X	X						
0 8	X				X			
1 5		X	X					
8 10					X	X		
5 7			X	X				
10 14						X	X	
7 15				X				X
14 15							X	X

REDUCTION STEP O2 : Row for P.I. (1, 5) dominates the row for P.I. (0, 1). Remove the highlighted cells (the dominated row).

P.I.	1	5	7	10	14	15
0 1	X					
1 5	X	X				
8 10				X		
5 7		X	X			
10 14				X	X	
7 15			X			X
14 15					X	X

REDUCTION STEP O3 : (1, 5) is an essential prime implicant because it is the only cube that covers the true minterm 1.

Remove the highlighted cells and include the prime implicant corresponding to the cube (1, 5) in the minimal expression.

P.I.	1	5	7	10	14	15
1 5	X	X				

8 10				X		
5 7		X	X			
10 14				X	X	
7 15			X			X
14 15					X	X

REDUCTION STEP O4 : Row for P.I. (10, 14) dominates the row for P.I. (8, 10). Remove the highlighted cells (the dominated row).

P.I.	7	10	14	15
8 10		X		
5 7	X			
10 14		X	X	
7 15	X			X
14 15			X	X

REDUCTION STEP O5 : (10, 14) is an essential prime implicant because it is the only cube that covers the true minterm 10.
Remove the highlighted cells and include the prime implicant corresponding to the cube (10, 14) in the minimal expression.

P.I.	7	10	14	15
5 7	X			
10 14		X	X	
7 15	X			X
14 15			X	X

REDUCTION STEP O6 : Row for P.I. (7, 15) dominates the rows for P.I. (5, 7) and (14, 15). Remove the highlighted cells (the dominated rows).

P.I.	7	15
5 7	X	
7 15	X	X
14 15		X

REDUCTION STEP O7 : (7, 15) is an essential prime implicant because it is the only cube that covers the true minterms 7 and 15.
Remove the highlighted cells and include the prime implicant corresponding to the cube (7, 15) in the minimal expression.

P.I.	7	15
7 15	X	X

So there is the ***second*** minimal expression for the boolean function.

$$h_{2-min-second}(w, x, y, z) = \overline{x}\,\overline{y}\,\overline{z} + \overline{w}\,\overline{y}\,z + w\,y\,\overline{z} + x\,y\,z$$

10C

***NOTE** : All cubes highlighted with a **red background colour** in their corresponding groups are the prime implicants. These are the ones that do not further combine with any cube of any other group.*

Group 1.1 -- 1 on-set and 0 DC-set minterms

<1:1, 0:D>					
Minterm	**v**	**w**	**x**	**y**	**z**
1	0	0	0	0	1

Group 1.2 -- 2 on-set and 0 DC-set minterms

<2:1, 0:D>					
Minterm	**v**	**w**	**x**	**y**	**z**

5	0	0	1	0	1
6	0	0	1	1	0
9	0	1	0	0	1
17	1	0	0	0	1
18	1	0	0	1	0

Group 1.3 -- 3 on-set and 0 DC-set minterms

<3:1, 0:D>					
Minterm	**v**	**w**	**x**	**y**	**z**
7	0	0	1	1	1
13	0	1	1	0	1
14	0	1	1	1	0
19	1	0	0	1	1
21	1	0	1	0	1
22	1	0	1	1	0
25	1	1	0	0	1

Group 1.4 -- 4 on-set and 0 DC-set minterms

<4:1, 0:D>					
Minterm	**v**	**w**	**x**	**y**	**z**
15	0	1	1	1	1
23	1	0	1	1	1
29	1	1	1	0	1
30	1	1	1	1	0

Group 2.1 -- 1 on-set and 1 DC-set cubes (combine Group 1.1 and Group 1.2)

<1:1, 1:D>					
Cube	**v**	**w**	**x**	**y**	**z**
1 5	0	0	-	0	1
1 9	0	-	0	0	1
1 17	-	0	0	0	1

Group 2.2 -- 2 on-set and 1 DC-set cubes (combine Group 1.2 and Group 1.3)

<2:1, 1:D>					
Cube	**v**	**w**	**x**	**y**	**z**
5 7	0	0	1	-	1
5 13	0	-	1	0	1
5 21	-	0	1	0	1
6 7	0	0	1	1	-
6 14	0	-	1	1	0
6 22	-	0	1	1	0
9 13	0	1	-	0	1
9 25	-	1	0	0	1
17 19	1	0	0	-	1
17 21	1	0	-	0	1
17 25	1	-	0	0	1
18 19	1	0	0	1	-
18 22	1	0	-	1	0

Group 2.3 -- 3 on-set and 1 DC-set cubes (combine Group 1.3 and Group 1.4)

<3:1, 1:D>					
Cube	**v**	**w**	**x**	**y**	**z**
7 15	0	-	1	1	1
7 23	-	0	1	1	1
13 15	0	1	1	-	1
13 29	-	1	1	0	1
14 15	0	1	1	1	-
14 30	-	1	1	1	0
19 23	1	0	-	1	1
21 23	1	0	1	-	1
21 29	1	-	1	0	1
22 23	1	0	1	1	-
22 30	1	-	1	1	0
25 29	1	1	-	0	1

Group 3.1 -- 1 on-set and 2 DC-set cubes (combine Group 2.1 and Group 2.2)

<1:1, 2:D>					
Cube	**v**	**w**	**x**	**y**	**z**
1 5 9 13	0	-	-	0	1
1 5 7 21	-	0	-	0	1
1 9 17 25	-	-	0	0	1

Group 3.2 -- 2 on-set and 2 DC-set cubes
(combine Group 2.2 and Group 2.3)

<2:1, 2:D>					
Cube	**v**	**w**	**x**	**y**	**z**
5 7 13 15	0	-	1	-	1
5 7 21 23	-	0	1	-	1
5 13 21 29	-	-	1	0	1
6 7 14 15	0	-	1	1	-
6 7 22 23	-	0	1	1	-
6 14 22 30	-	-	1	1	0
9 13 25 29	-	1	-	0	1
17 19 21 23	1	0	-	-	1
17 21 25 29	1	-	-	0	1
18 19 22 23	1	0	-	1	-

Group 4.1 -- 1 on-set and 3 DC-set cubes
(combine Group 3.1 and Group 3.2)

<1:1, 3:D>					
Cube	**v**	**w**	**x**	**y**	**z**
1 5 9 13 17 21 25 29	-	-	-	0	1

Take all the cubes highlighted with red background colour (prime implicants) and construct the following matrix to illustrate the inclusion of various true minterms in each of the prime implicants.

REDUCTION STEP O1 : (1, 5, 9, 13, 17, 21, 25, 29) is an essential prime implicant because it is the only cube that covers the true minterms 1, 9, 25 and 29.
Remove the highlighted cells and include the prime implicant corresponding to the cube (1, 5, 9, 13, 17, 21, 25, 29) in the minimal expression.

P.I.	1	5	6	7	9	13	14	15	17	18	19	21	22	23	25	29	30
5 7 13 15		X		X		X		X									
5 7 21 23		X		X								X		X			
6 7 14 15			X	X			X	X									
6 7 22 23			X	X									X	X			
6 14 22 30			X				X						X				X
17 19 21 23									X		X	X		X			
18 19 22 23										X	X		X	X			
1 5 9 13 17 21 25 29	X	X			X	X			X			X			X	X	

REDUCTION STEP O2 : (18, 19, 22, 23) is an essential prime implicant because it is the only cube that covers the true minterm 18.
Remove the highlighted cells and include the prime implicant corresponding to the cube (18, 19, 22, 23) in the minimal expression.

P.I.	6	7	14	15	18	19	22	23	30
5 7 13 15		X		X					
5 7 21 23		X						X	
6 7 14 15	X	X	X	X					
6 7 22 23	X	X					X	X	
6 14 22 30	X		X				X		X
17 19 21 23						X		X	

18 19 22 23					X	X	X	X	

REDUCTION STEP O3 : (6, 14, 22, 30) is an essential prime implicant because it is the only cube that covers the true minterm 30.
Remove the highlighted cells and include the prime implicant corresponding to the cube (6, 14, 22, 30) in the minimal expression.

P.I.	6	7	14	15	30
5 7 13 15		X		X	
5 7 21 23		X			
6 7 14 15	X	X	X	X	
6 7 22 23	X	X			
6 14 22 30	X		X		X
17 19 21 23					

REDUCTION STEP O4 : Row for P.I. (17, 19, 21, 23) is dominated by all other rows. Remove the highlighted cells (the empty row).

P.I.	7	15
5 7 13 15	X	X
5 7 21 23	X	
6 7 14 15	X	X
6 7 22 23	X	
17 19 21 23		

REDUCTION STEP O5 : Row for P.I. (5, 7, 13, 15) dominates all other rows. Remove the highlighted cells (the dominated rows).

REDUCTION STEP O5 : Row for P.I. (6, 7, 14, 15) dominates all other rows. Remove the highlighted cells (the dominated rows).

P.I.	7	15
5 7 13 15	X	X
5 7 21 23	X	
6 7 14 15	X	X
6 7 22 23	X	

REDUCTION STEP O6 : (5, 7, 13, 15) is an essential prime implicant because it is the only cube that covers the true minterms 7 and 15.
Remove the highlighted cells and include the prime implicant corresponding to the cube (5, 7, 13, 15) in the minimal expression.

P.I.	7	15
5 7 13 15	X	X

P.I.	7	15
5 7 13 15	X	X
5 7 21 23	X	
6 7 14 15	X	X
6 7 22 23	X	

REDUCTION STEP O6 : (6, 7, 14, 15) is an essential prime implicant because it is the only cube that covers the true minterm 7 and 15.
Remove the highlighted cells and include the prime implicant corresponding to the cube (6, 7, 14, 15) in the minimal expression.

P.I.	7	15
6 7 14 15	X	X

So there are ***two*** minimal expressions for the boolean function.

$$h_{3-min-first}(v, w, x, y, z) = \overline{y}\, z + v\, \overline{w}\, y + x\, y\, \overline{z} + \overline{v}\, x\, z$$

$$h_{3-min-second}(v, w, x, y, z) = \overline{y}\, z + v\, \overline{w}\, y + x\, y\, \overline{z} + \overline{v}\, x\, y$$