ANSWERS TO SELECTED PROBLEMS

Problems 1.3

1. (a) Yes; (b) yes; (c) no. 2. (a) Yes; (b) no; (c) no.
6. (a) 0.9; (b) 0.05; (c) 0.95. 7. 1/16. 8. $\frac{1}{3}+\frac{2}{9}\ell\text{n}2 = 0.487$.

Problems 1.4

3. $\binom{R}{n}\binom{W}{n-r}\Big/\binom{N}{n}$ 4. 352146 5. $(n-k+1)!/n!$

6. $1 - {}_7P_5/{}_75$ 8. $\binom{n+k-r}{n-r}\Big/\binom{n+k}{k}$ 9. $1-\sum_{i=1}^{n-k}\binom{2i}{i}\Big/\binom{2n}{n-k}$

12. (a) $4\Big/\binom{52}{5}$ (b) $9(4)\Big/\binom{52}{5}$ (c) $13\binom{48}{1}\Big/\binom{52}{5}$

(d) $13\binom{4}{3}12\binom{4}{2}\Big/\binom{52}{5}$ (e) $\left[4\binom{13}{5}-9(4)-4\right]\Big/\binom{52}{5}$

(f) $[10(4)^5-4-9(4)]\Big/\binom{52}{5}$ (g) $13\binom{12}{2}\binom{4}{3}4^2\Big/\binom{52}{5}$

(h) $\binom{13}{2}\binom{4}{2}\binom{4}{2}\binom{44}{1}\Big/\binom{52}{5}$ (i) $\binom{13}{1}\binom{4}{3}\binom{12}{3}4^3\Big/\binom{52}{5}$

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Problems 1.5

3. $\alpha(pb)^r \sum_{\ell=0}^{\infty} \binom{r+\ell}{\ell} [p(1-b)]^{\ell}$ 4. $p/(2-p)$

5. $\sum_{j=0}^{N} (j/N)^{n+1} \Big/ \sum_{j=0}^{N} (j/N)^n \simeq \dfrac{n+1}{n+2}$ for large N 6. $n = 4$

10. $r/(r+g)$ 11. (a) $1/4$; (b) $1/3$ 12. 0.08

13. (a) 173/480 (b) 108/173; 15/173 14. 0.0872

Problems 1.6

1. $1/(2-p)$; $(1-p)/(2-p)$ 4. $p^2(1-p)^2[3-7p(1-p)]$

12. For any two disjoint intervals $I_1, I_2 \subseteq (a,b)$, $\ell(I_1)\ell(I_2) = (b-a)\ell(I_1 \cap I_2)$, where $\ell(I)$ = length of interval I.

13. (a) $p_n = \begin{cases} 8/36 & \text{if } n = 1 \\ 2\left(\frac{27}{36}\right)^{n-2}\left(\frac{3}{6}\right)^2 + 2\left(\frac{26}{36}\right)^{n-2}\left(\frac{4}{36}\right)^2 + 2\left(\frac{25}{36}\right)^{n-2}\left(\frac{5}{36}\right)^2, n \geq 2 \end{cases}$

(b) $22/45$

(c) $12/36$; $2\left(\frac{27}{36}\right)^{n-2}\left(\frac{9}{36}\right)\left(\frac{3}{16}\right) + 2\left(\frac{26}{36}\right)^{n-2}\left(\frac{10}{36}\right)\left(\frac{4}{36}\right) + 2\left(\frac{25}{36}\right)^{n-2}\left(\frac{11}{36}\right)\left(\frac{5}{36}\right)$ for $n = 2, 3, \ldots$.

Problems 2.2

3. Yes; yes
4. ϕ; $\{(1,1,1,1,2),(1,1,1,2,1),(1,1,2,1,1),(1,2,1,1,1),(2,1,1,1,1)\}$; $\{(6,6,6,6,6)\}$; $\{(6,6,6,6,6),(6,6,6,6,5),(6,6,6,5,6),(6,6,5,6,6),(6,5,6,6,6),(5,6,6,6,6)\}$
5. Yes; $(1/4, 1/2) \cup (3/4, 1)$

Problems 2.3

1.

x	0	1	2	3
$P(X = x)$	1/8	3/8	3/8	1/8

$F(x) = 0,\ x < 0,\ = 1/8,\ 0 \leq x < 1;\ = 1/2,\ 1 \leq x < 2;\ = 5/8,\ 2 \leq x < 3;\ = 1,\ x \geq 3$

3. (a) Yes; (b) yes; (c) yes; yes

Problems 2.4

1. $(1-p)^{n+1} - (1-p)^{N+1}, N \geq n$
2. (b) $\frac{1}{\pi(1+x^2)}$; (c) $1/x^2$; (d) e^{-x}
3. Yes; $F_\theta(x) = 0\ \ x \leq 0,\ = 1 - e^{-\theta x} - \theta x e^{-\theta x}$ for $x > 0$; $P(X \geq 1) = 1 - F_\theta(1)$
4. Yes; $F(x) = 0, x \leq 0; = 1 - \left(1 + \frac{x}{\theta+1}\right)e^{-x/\theta}$ for $x > 0$
6. $F(x) = e^x/2$ for $x \leq 0$, $= 1 - e^{-x}/2$ for $x > 0$
8. (c), (d), and (f)
9. Yes; (a) $1/2, 0 < x < 1$, $1/4$ for $2 < x < 4$; (b) $1/(2\theta)$, $|x| \leq \theta$; (c) $xe^{-x}, x > 0$; (d) $(x-1)/4$ for $1 \leq x < 3$, and $P(X = 3) = 1/2$; (e) $2xe^{-x^2}, x > 0$
10. If $S(x) = 1 - F(x) = P(X > x)$, then $S'(x) = -f(x)$

Problems 2.5

2. $X \stackrel{d}{=} 1/X$
4. $\theta[1-\exp(-2\pi\theta)]\sqrt{1-y^2}\left[e^{-\theta \text{ arc cos } y}+e^{-2\pi\theta+\theta \text{ arc cos } y}\right]$, $|y| \le 1$;
$$\begin{cases} \theta\exp\{-\theta \text{ arctan } z\}[(1+z^2)(1-e^{-\theta\pi}]^{-1}, & z>0 \\ \theta\exp\{-\pi\theta - \text{arctan } z\}[(1+z^2)(1-e^{-\theta\pi})]^{-1}, & z<0 \end{cases}$$
10. $f_{|X|}(y) = 2/3$ for $0<y<1$, $=1/3$ for $1<y<2$
12. (a) $0, y<0; F(0)$ for $-1 \le y < 1$, and 1 for $y \ge 1$;
(b) $=0$ if $y<-b$, $=F(-b)$ if $y=-b$, $=F(y)$ if $-b \le y < b$, $=1$ if $y \ge b$;
(c) $=F(y)$ if $y<-b$, $=F(-b)$ if $-b \le y < 0$, $=F(b)$ if $0 \le y < b$, $=F(y)$. if $y \ge b$.

Problems 3.2

3. $EX^{2r} = 0$ if $2r < 2m-1$ is an odd integer,
$$= \frac{\Gamma\left(m-r+\frac{1}{2}\right)\Gamma\left(r+\frac{1}{2}\right)}{\Gamma\left(\frac{1}{2}\right)\Gamma\left(\frac{m-1}{2}\right)} \quad \text{if } 2r < 2m-1 \text{ is an even integer}$$
9. $\mathfrak{z}_p = a(1-v)/v$, where $v=(1-p)^{1/k}$
10. Binomial: $\alpha_3 = (q-p)/\sqrt{npq}$, $\alpha_4 = 3+(1-6pq)/3npq$
Poisson: $\alpha_3 = \lambda^{-1/2}$, $\alpha_4 = 3+1/\lambda$.

Problems 3.3

1. (b) $e^{-\lambda}(e^{\lambda s}-1)/(1-e^{-\lambda})$; (c) $p[1-(qs)^{N+1}]/[(1-qs)(1-q^{N+1})]$, $s<1/q$.
6. $f(\theta s)/f(\theta)$; $f(\theta e^t)/f(\theta)$.

Problems 3.4

3. For any $\sigma^2 > 0$ take $P(X=x) = \frac{\sigma^2}{\sigma^2+x^2}$, $P\left(X=-\frac{\sigma^2}{x}\right) = \frac{x^2}{\sigma^2+x^2}$, $x \ne 0$.
5. $P\left(X^2 = \frac{\sigma^4 K^2 - \mu_4}{K^2\sigma^2-\sigma^2}\right) = \frac{\sigma^4[K^2-1]^2}{\mu_4+K^4\sigma^4-2K^2\sigma^4}$ $1<K<\sqrt{2}$
$P(X^2 = K^2\sigma^2) = \frac{\mu_4-\sigma^4}{\mu_4+K^4\sigma^4-2K^2\sigma^4}$.

Problems 4.2

1. No 4. 1/6; 0. 7. Marginals negative binomial, so also conditionals.
8. $h(y|x) = \frac{1}{2}(c^2+x^2)/(c^2+x^2+y^2)^{3/2}$.
9. $X \sim B(p_1, p_2+p_3)$; $Y/(1-x) \sim B(p_2, p_3)$.
10. $X \sim G(\alpha, 1/\beta)$, $Y \sim G(\alpha+\gamma, 1/\beta)$, $X/y \sim B(\alpha, \gamma)$, $Y-x \sim G(\gamma, 1/\beta)$.
14. $P(X \le 7) = 1-e^{-7}$ 15. 1/24; 15/16. 17. 1/6.

Problems 4.3

3. No; Yes; No. 10. $=1-a/(2b)$ if $a<b$, $=b/(2a)$ if $a>b$.
11. $\lambda/(\lambda+\mu)$; 1/2.

Problems 4.4

2. (b) $f_{V|U}(v|u) = 1/(2u)$, $|v|<u$, $u>0$.
6. $P(X=x, M=m) = \pi(1-\pi)^m[1-(1-\pi)^{m+1}]$ if $x=m$, $=\pi^2(1-\pi)^{m+x}$ if $x<m$. $P(M=m) = 2\pi(1-\pi)^m - \pi(2-\pi)(1-\pi)^{2m}$, $m \ge 0$.

7. $f_X(x) = \lambda^k e^{-\lambda}/k!,\ k \le x < k+1,\ k = 0,1,2,\ldots$
11. $f_U(u) = 3u^2/(1+u)^4$, $u > 0$.
13. (a) $F_{U,V}(u,v) = \left[1 - \exp\left(-\frac{u^2}{2\sigma^2}\right)\right]\left(\frac{\pi+2v}{2\pi}\right)$ if $u > 0$, $|v| \le \pi/2$,
$= 1 - \exp[1 - u^2/(2\sigma^2)]$ if $u > 0,\ v > \pi/2$, $= 0$ elsewhere.
(b) $f(u,v) = \frac{1}{\sqrt{\pi}} e^{-u^2} \frac{v^{1/2-1}e^{-v/2}}{\Gamma(1/2)\sqrt{2}}$.

Problems 4.5

2. $EX^kY^\ell = \frac{2^{\ell+1}}{(k+3)(\ell+1)} + \frac{2^{\ell+2}}{3(k+2)(\ell+2)}$. 3. $\text{cov}(X,Y) = 0$; X, Y dependent.
15. $M_{U,V}(u,v) = (1-2v)^{-1}\exp\{u^2/(1-2v)\}$ for $v < 1/2$; $\rho(U,V) = 0$; no.
18. $\rho_{Z,W} = (\sigma_2^2 - \sigma_1^2)\sin\theta\cos\theta/\sqrt{\text{var}(Z)\,\text{var}(W)}$.
21. If U has PDF f, then $EX^m = EU^m/(m+1)$ for $m \ge 0$; $\rho = \frac{1}{2} - \frac{EU^2}{\frac{8}{3}\text{var}(U) + \frac{2}{3}(EU)^2}$.

Problems 4.6

1. $\mu + \sigma\left[f\left(\frac{a-\mu}{\sigma}\right) - f\left(\frac{b-\mu}{\sigma}\right)\right]/\Phi\left(\frac{b-\mu}{\sigma}\right) - \Phi\left(\frac{a-\mu}{\sigma}\right)\big]$ where Φ is the standard normal DF.
2. (a) $2(1+X)$. 3. $E\{X|y\} = \mu_1 + \rho\frac{\sigma_1}{\sigma_2}(y - \mu_2)$. 4. $E(\text{var}\{Y|X\})$.
6. 4/9. 7. (a) 1; (b) 1/4. 8. $x^k/(k+1)$; $1/(1+k)^2$.

Problems 4.7

5. (a) $\left(\sum_{j=1}^{n} 1/j\right)/\beta$; (b) $\frac{n}{n+1}$.

Problems 5.2

5. $F_Y(y) = \binom{y}{M}/\binom{N}{M}$, $P(Y = y) = \binom{y-1}{M-1}/\binom{N}{M}$, $y \ge M+1$, and
$P(Y = M) = 1/\binom{N}{M}$. $P(x_1,\ldots,x_m|Y = y) = \frac{(y-m)!}{(y-1)!M}$, $0 < x_i \le y$,
$i = 1,\ldots,j$, $x_i \ne x_j$ for $i \ne j$.
9. $P(Y_1 = x) = qp^x + pq^x$, $x \ge 1$. $P(Y_2 = x) = p^2q^{x-1} + q^2p^{x-1}$, $x \ge 1$
$P(Y_n = x) = P(Y_1 = x)$ for n odd; $= P(Y_2 = x)$ for n even.

Problems 5.3

2. (a) $P\left\{F(X) = \sum_{k=0}^{x}\binom{n}{k}p^k(1-p)^{n-k}\right\} = \binom{n}{x}p^x(1-p)^{n-x}$, $x = 0,1,\ldots,n$.
13. $\mathcal{C}\left(\sum_{i=1}^{n}\frac{|a_i|}{a_i^2+b_i^2}, \sum_{i=1}^{n}\frac{b_i}{a_i^2+b_i^2}\right)$
22. $X/|Y| \sim \mathcal{C}(1,0)$; $(2/\pi)(1+z^2)^{-1}$, $0 < z < \infty$.
27. (a) t/α^2; (c) $= 0$ if $t \le \theta$, $= \alpha/t$ if $t > \theta$; (d) $(\alpha/\beta)t^{\alpha-1}$.
29. (b) $1/(2\sqrt{\pi})$; $1/2$.

Problems 5.4

1. (a) $\mu_1 = 4$; $\mu_2 = 15/4$, $\rho = -3/4$; (b) $\mathcal{N}\left(6 - \frac{9}{16}x, \frac{63}{16}\right)$; (c) 0.3191.
4. $B\mathcal{N}(a\mu_1 + b, c\mu_2 + d, a^2\sigma_1^2, c^2\sigma_2^2, \rho)$. 6. $\tan^2\theta = EX^2/EY^2$. 7. $\sigma_1^2 = \sigma_2^2$.

Problems 6.2

1. $P(\overline{X} = 0) = P(\overline{X} = 1) = 1/8$, $P(\overline{X} = 1/3) = P(\overline{X} = 2/3) = 3/8$
   $P(S^2 = 0) = 1/4$, $P(S^2 = 1/3) = 3/4$.
2.

$\bar{x}$	1	1.5	2	2.5	3	3.5	4	4.5	5	5.5	6
$p(\bar{x})$	1/36	2/36	3/36	4/36	5/36	6/36	5/36	4/36	3/36	2/36	1/36

Problems 6.3

1. $\{F(\min(x,y)) - F(x)F(y)\}/n$.
6. $E(S^2)^k = \frac{\sigma^2}{(n-1)^k}(n-1)(n+2)\cdots(n+2k-3)$, $k \geq 1$.
9. (a) $P(\overline{X} = t) = e^{-n\lambda}(n\lambda)^{tn}/(tn)!$, $t = 0, 1/n, 2/n, \ldots$; (b) $\mathcal{C}(1,0)$;
   (c) $\Gamma(nm/2, 2/n)$. 10. (b) $2/\sqrt{\alpha n}$; $3 + 6/(\alpha n)$.
11. $0, 1, 0, E(\overline{X}_n - 0.5)^4/(144n^2)$. 12. $\text{var}(S^2) = \frac{1}{n}\left(\lambda + \frac{2n\lambda^2}{n-1}\right) > \text{var}(\overline{X})$.

Problems 6.4

2. $n(m+\delta)/[m(n-2)]$; $2n^2\{(m+\delta)^2 + (n-2)(m+2\delta)\}/[m^2(n-2)^2(n-4)]$.
3. $\delta\sqrt{\frac{n}{2}}\frac{\Gamma(\frac{n-1}{2})}{\Gamma(\frac{n}{2})}$, $n > 1$; $\frac{n}{n-2}(1+\delta^2) - \left(\delta\sqrt{\frac{n}{2}}\frac{\Gamma(\frac{n-1}{2})}{\Gamma(\frac{n}{2})}\right)^2$, $n > 2$.
11. $2m^{m/2}n^{n/2}(n + me^{2z})^{-(m+n)/2}e^{zm}/B\left(\frac{m}{2}, \frac{n}{2}\right)$, $-\infty < z < \infty$.

Problems 6.5

1. $t(n-1)$ 2. $t(m+n-2)$ 3. $\left(\frac{2\sigma^2}{n-1}\right)^k\Gamma\left(\frac{n-1}{2} + k\right)/\Gamma\left(\frac{n-1}{2}\right)$.

Problems 6.6

3. $[2\pi(1-\rho^2)]^{-1/2}\left[1 + \frac{y_1^2 + y_2^2 - 2\rho y_1 y_2}{n(1-\rho^2)}\right]^{-(\frac{n}{2}+1)}$; both $\sim t(n)$.
4. $\sqrt{n-1}T \sim t(n-1)$.

Problems 7.2

1. No. 2. Yes
3. $Y_n \to Y \sim F(y) = 0$ if $y < 0$, $= 1 - e^{-y/\theta}$ if $y \geq 0$.
4. $F(y) = 0$ if $y \leq 0$, $= 1 - e^{-y}$ if $y > 0$.
9. $\mathcal{C}(1,0)$ 12. No
13. (a) $\exp(-x^{-\alpha})$, $x > 0$; $EX^k = \Gamma(1 - k/\alpha)$, $k < \alpha$.
    (b) $\exp(-e^{-x})$, $-\infty < x < \infty$; $M(t) = \Gamma(1-t)$, $t < 1$.
    (c) $\exp\{-(-x)^\alpha\}$, $x < 0$; $EX^k = (-1)^k\Gamma(1 + k/\alpha)$, $k > -\alpha$.
20. (a) Yes; No (b) Yes; No.

Problems 7.3

3. Yes; $A_n = n(n+1)\mu/2$, $B_n = \sigma\sqrt{n(n+1)(2n+1)/6}$
5. (a) $M_n(t) \to 0$ as $n \to \infty$; no. (b) $M_n(t)$ diverges as $n \to \infty$
   (c) Yes (d) Yes (e) $M_n \to e^{t^2/4}$; no.

Problems 7.4

1. (a) No; (b) No. 2. No. 3. For $\alpha < 1/2$. 7. (a) Yes; (b) No.

Problems 7.5

4. Degenerate at β. 5. Degenerate at 0.
6. For $\rho \geq 0$, $\mathcal{N}(0, \sqrt{\rho})$, and for $\rho < 0$, $S_n/n \xrightarrow{L}$ degenerate.

Problems 7.6

1. (b) No; (c) Yes; (d) No.
2. $\mathcal{N}(0,1)$. 3. $\mathcal{N}(0, \sigma^2/\beta^2)$. 4. 163. 8. 0.0926; 1.92

Problems 7.7

1. (a) $A\mathcal{N}(\mu^2, 4\mu^2\sigma_n^2)$ for $\mu \neq 0$, $\overline{X}^2/\sigma_n^2 \xrightarrow{L} \chi^2(1)$ for $\mu = 0$, $\sigma_n^2 = \sigma^2/n$.
   (b) For $\mu \neq 0$, $1/\overline{X} \sim A\mathcal{N}(1/\mu, \sigma_n^2/\mu^4)$; for $\mu = 0$, $\sigma_n/\overline{X}_n \xrightarrow{L} 1/\mathcal{N}(0,1)$.
   (c) For $\mu \neq 0$, $\ell\text{n}|\overline{X}| \sim A\mathcal{N}(\ell\text{n}|\mu|, \sigma_n^2/\mu^2)$; for $\mu = 0$, $\ell\text{n}(|\overline{X}|/\sigma_n) \xrightarrow{L} \ell\text{n}|\mathcal{N}(0,1)|$.
   (d) $A\mathcal{N}(e^{\mu}, e^{2\mu}\sigma_n^2)$.
2. $c = 1/2$ and $\sqrt{X} \sim A\mathcal{N}(\sqrt{\lambda}, 1/4)$.

Problems 8.3

2. No. 7. $f_{\theta_2}(x)/f_{\theta_1}(x)$. 9. No. 10. No.
11. (b) $X_{(n)}$; (e) $(\overline{X}, S^2)$; (g) $\left(\prod_1^n X_i, \prod_1^n (1 - X_i)\right)$ (h) $X\left((1), X_{(2)}, \ldots, X_{(n)}\right)$.

Problems 8.4

2. $\left(\frac{n-1}{2}\right)^p \frac{\Gamma\left(\frac{n-1}{2}\right)}{\Gamma\left(\frac{n+p-1}{2}\right)} S^p$; $\left(\frac{n-1}{2}\right)^{p/2} \frac{\Gamma\left(\frac{n+p-1}{2}\right)}{\Gamma\left(\frac{n+2p-1}{2}\right)} S$.
3. $S_1^2 = \frac{n-1}{n+1} S^2$; $\text{var}(S_1^2) = \left(\frac{n-1}{n+1}\right)^2 \frac{2\sigma^4}{n-1} < \text{var}(S^2) = \frac{2\sigma^4}{n-1}$; 4. No; 5. No.
6. (a) $\binom{n-s}{t-s} \Big/ \binom{n}{t}$, $0 \leq s \leq t \leq n$, $t = \sum_1^n x_i$; (b) $= \binom{s}{t} \Big/ \binom{n}{t}$ if $0 \leq t < s$,
   $= 2 \Big/ \binom{n}{t}$ if $t = s$, and $\binom{n-s}{t-s} \Big/ \binom{n}{t}$ if $s + 1 \leq t \leq n$.
9. $\binom{t+n-2}{t} \Big/ \binom{t+n-1}{t}$, $t = \Sigma x_i$. 11. (a) NX/n; (b) No.
12. $t = \Sigma_1^n x_i$, $1 - \left(1 - \frac{t_0}{t}\right)^{n-1}$ if $t > t_0$, and 1 if $t \leq t_0$.
13. (a) With $t = \sum_1^n x_j$, $\sum_{j=0}^t \frac{t!}{j!} n^{j-t}$; (b) $\frac{t!}{(t-s)!} n^{-s}$, $t \geq s$ (c) $(1 - 1/n)^t$;
    (d) $(1 - 1/n)^{t-1}[1 + \frac{t-1}{n}]$.
14. With $t = x_{(n)}$, $[t^n \psi(t) - (t-1)^n \psi(t-1)]/[t^n - (t-1)^n]$, $t \geq 1$.
15. With $t = \sum_1^n x_j$, $\binom{t}{k} \left(\frac{1}{n}\right)^k \left(1 - \frac{1}{n}\right)^{t-k}$.

Problems 8.5

1. (a), (c), (d) Yes; (b) No. 2. $0.64761/n^2$.
3. $n^{-1}\sup\limits_{x\neq 0}\{x^2/[e^{x^2}-1]\}$. 5. $2\theta(1-\theta)/n$

Problems 8.6

2. $\hat{\beta}=(n-1)S^2/(n\overline{X})$, $\hat{\alpha}=\overline{X}/\hat{\beta}$ 3. $\hat{\mu}=\overline{X}$, $\hat{\sigma}^2=(n-1)S^2/n$.
4. $\hat{\alpha}=\overline{X}(\overline{X}-\overline{X^2})[\overline{X^2}-\overline{X}^2]^{-1}$, $\overline{X^2}=\sum_1^n X_i^2/n$ $\hat{\beta}=(1-\overline{X})(\overline{X}-\overline{X^2})[\overline{X^2}-\overline{X}^2]^{-1}$.
5. $\hat{\mu}=\ell\text{n}\{\overline{X}^2/[\overline{X^2}]^{1/2}\}$, $\hat{\sigma}^2=\ell\text{n}\{\overline{X^2}/\overline{X}^2\}$, $\overline{X^2}=\sum_1^n X_i^2/n$.

Problems 8.7

1. (a) $\text{med}(X_j)$; (b) $X_{(1)}$; (c) $n/\sum_1^n X_j^{\alpha}$; (d) $-n/\sum_1^n \ell\text{n}(1-X_j)$.
2. (a) X/n; (b) $\hat{\theta}_n=1/2$ if $\overline{X}\leq 1/2$, $=\overline{X}$ if $1/2\leq\overline{X}\leq 3/4$, $=3/4$ if $\overline{X}\geq 3/4$;
(c) $\hat{\theta}=\begin{cases}\hat{\theta}_0, & \text{if } \overline{X}\geq 0\\ \hat{\theta}_1, & \text{if } \overline{X}\leq 0\end{cases}$ where $\hat{\theta}_0=-\frac{\overline{X}}{2}+\sqrt{\overline{X^2}+(\frac{\overline{X}}{2})^2}$,
$\hat{\theta}_1=-\frac{\overline{X}}{2}-\sqrt{\overline{X^2}+(\frac{\overline{X}}{2})^2}$, $\overline{X^2}=\sum X_1^2/n$;
(d) $\hat{\theta}=\frac{n_3}{n_1+n_3}$ if $n_1,n_3>0$; = any value in (0,1) if $n_1=n_3=0$;
no mle if $n_1=0,n_3\neq 0$; no mle if $n_1\neq 0,n_3=0$;
(e) $\hat{\theta}=-\frac{1}{2}+\frac{1}{2}\sqrt{1+4\overline{X^2}}$; (f) $\hat{\theta}=X$.
3. $\hat{\mu}=-\Phi^{-1}(m/n)$.
4. (a) $\hat{\alpha}=X_{(1)}$, $\hat{\beta}=\sum_1^n(X_i-\hat{\alpha})/n$; (b) $\Delta=P_{\alpha,\beta}(X_1\geq 1)=e^{(\alpha-1)\beta}$ $\alpha\leq 1$,
$=1,\ \alpha\geq 1$. $\hat{\Delta}=1$ if $\hat{\alpha}\geq 1$, $=\exp\{(\hat{\alpha}-1)/\hat{\beta}\}$ if $\hat{\alpha}<1$.
5. $\hat{\theta}=1/\overline{X}$. 6. $\hat{\mu}=\Sigma\ell\text{n}X_i/n$, $\hat{\sigma}^2=\sum_1^n(\ell\text{n}X_i-\hat{\mu})^2/n$.
8. (a) $\hat{N}=\frac{M+1}{M}X_{(M)}-1$; (b) $X_{(M)}$.
9. $\hat{\mu}_i=\sum_{j=1}^n X_{ij}/n=\overline{X}_i$, $i=1,2,\ldots,s$ $\hat{\sigma}^2=\Sigma\Sigma(X_{ij}-\overline{X}_i)^2/(ns)$.
11. $\hat{\mu}=\overline{X}$, 13. $d(\hat{\theta})=(X/n)^2$. 15. $\hat{\mu}=\max(\overline{X},0)$.
16. $\hat{p}_j=X_j/n$, $j=1,2,\ldots,k-1$.

Problems 8.8

2. (a) $(\Sigma x_i+1)/(n+1)$; (b) $\left(\frac{n+1}{n+2}\right)^{\Sigma x_i+1}$. 3. $\overline{X}$. 5. X/n.
6. $(X+1)(X+n)/[(n+2)(n+3)]$. 8. $(\alpha+n)\max(a,X_{(n)})/(\alpha+n-1)$.

Problems 8.9

5. (c) $(n+2)\left[(X_{(n)}/2)^{-(n+1)}-(X_{(1)})^{-(n+1)}\right]/\{(n+1)[(X_{(n)}/2)^{-(n+2)}-(X_{(1)})^{-(n+2)}]\}$
10. $(\Sigma X_i)^k\Gamma(n+k)/\Gamma(n+2k)$

Problems 9.2

1. 0.019, 0.857. 2. $k=\mu_0+\sigma z_\alpha/\sqrt{n}$; $1-\Phi\left(z_\alpha-\frac{\mu_1-\mu_0}{\sigma}\sqrt{n}\right)$.
5. $\exp(-2)$; $\exp(-2/\theta)$, $\theta\geq 1$.

Problems 9.3

1. $\phi(x) = 1$ if $x < \theta_0(1 - \sqrt{1-\alpha}) = 0$ otherwise.
4. $\phi(x) = 1$ if $||x| - 1| > k$. 5. $\phi(\mathbf{x}) = 1$ if $x_{(1)} > c = \theta_0 - \ell n(\alpha^{1/n})$.
11. If $\theta_0 < \theta_1$, $\phi(\mathbf{x}) = 1$ if $x_{(1)} > \theta_0 \alpha^{-1/n}$, and if $\theta_1 < \theta_0$, then $\phi(\mathbf{x}) = 1$ if $x_{(1)} < \theta_0(1-\alpha^{1/n})^{-1}$.
12. $\phi(x) = 1$ if $x < \sqrt{\alpha}/2$ or $> 1 - \sqrt{\alpha}/2$.

Problems 9.4

1. (a), (b), (c), (d) have MLR in ΣX_j; (e) and (f) in $\prod_1^n X_j$
4. Yes. 5. Yes; yes.

Problems 9.5

1. $\phi(x_1, x_2) = 1$ if $|x_1 - x_2| > c, = 0$ otherwise, $c = \sqrt{2} z_{\alpha/2}$.
2. $\phi(\mathbf{x}) = 1$ if $\Sigma x_i > k$. Choose k from $\alpha = P_{\lambda_0}\left(\sum_1^n X_i > k\right)$.

Problems 9.6

3. $\phi(\mathbf{x}) = 1$ if (no. of x_i's $> 0 -$ no. of x_i's $< 0) > k$.

Problems 10.2

2. $Y = \#$ of x_1, x_2 in sample, $Y < c_1$ or $Y > c_2$. 3. $X < c_1$ or $> c_2$.
4. $S^2 > c_1$ or $< c_2$. 5. (a) $X_{(n)} > N_0$; (b) $X_{(n)} > N_0$ or $< c$.
6. $|X - \theta_0/2| > c$. 7. (a) $\overline{X} < c_1$ or $> c_2$; (b) $\overline{X} > c$.
11. $X_{(1)} > \theta_0 - \ell n(\alpha)^{1/n}$. 12. $X_{(1)} > \theta_0 \alpha^{-1/n}$.

Problems 10.3

1. Reject at $\alpha = 0.05$. 3. Do not reject $H_0 : p_1 = p_2 = p_3 = p_4$ at 0.05 level.
4. Reject H_0 at $\alpha = 0.05$. 5. Reject at 0.10 but not at 0.05 level.
7. Do not reject H_0 at $\alpha = 0.05$. 8. Do not reject H_0 at $\alpha = 0.05$.
10. $U = 15.41$. 12. P-value = 0.5447.

Problems 10.4

1. $t = -4.3$, reject H_0 at $\alpha = 0.02$. 2. $t = 1.64$, do not reject H_0.
5. $t = 5.05$. 6. Reject H_0 at $\alpha = 0.05$. 7. Reject H_0. 8. Reject H_0.

Problems 10.5

1. Do not reject $H_0 : \sigma_1 = \sigma_2$ at $\alpha = 0.10$.
3. Do not reject H_0 at $\alpha = 0.05$. 4. Do not reject H_0.

Problems 10.6

2. (a) $\phi(\mathbf{x}) = 1$ if $\Sigma x_i = 5, = 0.12$ if $\Sigma x_i = 4, = 0$ otherwise;
   (b) Minimax rule rejects H_0 if $\Sigma x_i = 4$ or 5, and with probability 1/16 if $\Sigma x_i = 3$;
   (c) Bayes rule rejects H_0 if $\Sigma x_i \geq 2$.

3. Reject H_0 if $\bar{x} \le (1-1/n)\ell\text{n}2$
$\beta(1) = P(Y \le (n-1)\ell\text{n}2)$, $\beta(2) = P(Z \le (n-1)\ell\text{n}2)$, where $Y \sim G(n,1)$ and $Z \sim G(n,1/2)$

Problems 11.3

1. (77.7, 84.7). 2. $n = 42$. 7. $\left(\frac{2\Sigma X_i}{\chi^2_{2n,\alpha/2}}, 2\Sigma X_i/\chi^2_{2n,1-\alpha/2}\right)$.
9. $(2X/(2-\lambda_1), 2X/(2-\lambda_2))$, $\lambda_2^2 - \lambda_1^2 = 4(1-\alpha)$. 10. $[\alpha^{1/n}N]$.
11. $n \ge \frac{\ell\text{n}(1/\alpha)}{[\ell\text{n}(1+d/X_{(n)})]}$.
12. Choose k from $\alpha = (k+1)e^{-k}$. 13. $\overline{X} + z_\alpha \sigma/\sqrt{n}$
14. $(\Sigma X_i^2/c_2, \Sigma X_i^2/c_1)$, where $\int_{c_1}^{c_2} \chi_n^2(y)dy = 1-\alpha$ and $\int_{c_1}^{c_2} y\chi_n^2(y)dy = n(1-\alpha)$.
15. Posterior $B(n+\alpha, \Sigma x_i + \beta - n)$.
16. $h(\mu|\mathbf{x}) = \sqrt{\frac{n}{2\pi}}\exp\{-\frac{n}{2}(\mu-\bar{x})^2\}[\Phi(\sqrt{n}(1-\bar{x})) - \Phi(-\sqrt{n}(1+\bar{x}))]$, where Φ is standard normal DF.

Problems 11.4

1. $(X_{(1)} - \chi^2_{2,\alpha}/(2n), X_{(1)})$.
2. $(2n\overline{X}/b, 2n\overline{X}/a)$, choose a,b from $\int_a^b \chi^2_{2n}(u)du = 1-\alpha$, and $a^2\chi^2_{2n}(a) = b^2\chi^2_{2n}(b)$, where $\chi_v^2(x)$ is the PDF of $\chi^2(v)$ RV.
3. $(X/(1-b), X/(1-a))$, choose a,b from $1-\alpha = b^2 - a^2$ and $a(1-a)^2 = b(1-b)^2$.
4. $n = [4z^2_{1-\alpha/2}/d^2] + 1$; $n > (1/\alpha)\ell\text{n}(1/\alpha)$.

Problems 11.5

1. $(X_{(n)}, \alpha^{-1/n}X_{(n)})$.
2. $(2\Sigma X_i/\lambda_2, 2\Sigma X_i/\lambda_1)$, where λ_1, λ_2 are solutions of $\lambda_1 f_{2n\alpha}(\lambda_1) = \lambda_2 f_{2n\alpha}(\lambda_2)$ and $P(1) = 1-\alpha$, f_v is $\chi^2(v)$ PDF.
3. $(X_{(1)} - \frac{\chi^2_{2,\alpha}}{2n}, X_{(1)})$. 5. $(\alpha^{1/n}X_{(1)}, X_{(1)})$. 8. Yes.

Problems 12.3

4. Reject $H_0: \alpha_0 = \alpha_0'$ if $\frac{|\hat{\alpha}_0 - \alpha_0'|\sqrt{n\Sigma(t_i-\bar{t})^2/\Sigma t_i^2}}{\sqrt{\Sigma(Y_i - \hat{\alpha}_0 - \hat{\alpha}_1 t_i)^2/(n-2)}} > c_0$.
8. Normal equations $\hat{\beta}_0\Sigma x_i^k + \hat{\beta}_1\Sigma x_i^{k+1} + \hat{\beta}_2\Sigma x_i^{k+2} = \Sigma Y_i x_i^k$, $k = 0,1,2$.
Reject $H_0: \beta_2 = 0$ if $\{|\hat{\beta}_2|/\sqrt{c_1^2}\}/\sqrt{\Sigma(Y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i - \hat{\beta}_2 x_i^2)\}} > c_0$, where $\hat{\beta}_2 = \Sigma c_i Y_i$ and $\hat{\beta}_0 = \overline{Y} - \hat{\beta}_1\bar{x}$, $\hat{\beta}_1 = \Sigma(x_i - \bar{x})(Y_i - \overline{Y})/\Sigma(x_i - \bar{x})^2$.
10. (a) $\hat{\beta}_0 = 0.28$, $\hat{\beta}_1 = 0.411$; (b) $t = 4.41$, reject H_0.

Problems 12.4

2. $F = 10.8$. 3. Reject at $\alpha = 0.05$ but not at $\alpha = 0.01$.
4. BSS = 28.57, WSS = 26, reject at $\alpha = 0.05$ but not at 0.01.
5. $F = 56.45$. 6. $F = 0.87$.

Problems 12.5

4. SS Methods = 50, SS Ability = 64.56, ESS = 25.44; reject H_0 at $\alpha = 0.05$, not at 0.01.
5. $F_{\text{variety}} = 24.00$.

Problems 12.6

2. Reject H_0 if $\frac{am\sum_1^b(\bar{y}_{.j.}-\bar{y})}{\Sigma\Sigma\Sigma(y_{ijs}-\bar{y}_{ij.})^2} > c$.

4. SS_1 (machines) = 2.786, d.f. = 3; SSI = 73.476, d.f. = 6;
   SS_2 (machines) = 27.054, d.f. = 2; SSE = 41.333, d.f. = 24.

5.

Cities	3	227.27	4.22
Auto	3	3695.94	68.66
Interactions	9	9.28	0.06
Error	16	287.08	

Problems 13.2

1. d is estimable of degree 1; (number of x_i's in $A)/n$.
2. (a) $(mn)^{-1}\Sigma X_i \Sigma Y_j$; (b) $S_1^2 + S_2^2$.
3. (a) $\Sigma X_i Y_i/n$; (b) $\Sigma(X_i + Y_i - \overline{X} - \overline{Y})^2/(n-1)$.

Problems 13.3

3. Do not reject H_0. 7. Reject H_0. 10. Do not reject H_0 at 0.05 level.
11. $T^+ = 133$, do not reject H_0.
12. (Second part) $T^+ = 9$, do not reject H_0 at $\alpha = 0.05$.

Problems 13.4

1. Do not reject H_0. 2. (a) Reject; (b) Reject.
3. $U = 29$, reject H_0. 5. $d = 1/4$, do not reject H_0.
7. $t = 313.5$, $z = 3.73$, reject; $r = 10$ or 12, do not reject at $\alpha = 0.05$.

Problems 13.5

1. Reject H_0 at $\alpha = 0.05$. 4. Do not reject H_0 at $\alpha = 0.05$.
9. (a) $t = 1.21$; (b) $r = 0.62$; (c) Reject H_0 in each case.

Problems 13.6

1. (a) 5; (b) 8. 3. $p^{n-2}(n+p-np) \leq 1$.
4. $n \geq (z_{1-\gamma}\sqrt{p_0(1-p_0)} - z_{1-\delta}\sqrt{p_1(1-p_1)})^2/(p_1-p_0)^2$.

Problems 13.7

1. (c) $E\{n(\overline{X}-\mu)^2\}/ES^2 = 1 + 2\rho(1-2\rho/n)^{-1}$; ratio = 1 if $\rho = 0$, > 1 for $\rho > 0$.
2. Chi-square test based on (c) is not robust for departures from normality.