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1a) $S \to a^b S c B \mid \varepsilon$
$B \to bB \mid b$

B generates b^+ [As there is no epsilon]

S → generates strings of the form.

$$\boxed{(ab)^n (c\, b^+)^n \quad \text{for some } n \geq 0}$$

Here b^+ is $b^* - \{\varepsilon\}$

In a more generalized form
$(ab)^n (c^{x_1} b^{y_1}) (c^{x_2} b^{y_2}) (c^{x_3} b^{y_3})$

$x_1 + x_2 \ldots x_i = n$
$y_i \geq 1$

The given grammar is not-ambiguous
because there is exactly one unique parse trees for a given left or right derivations string.

b) $L_1 = \{ a^l b^m c^n \mid l < m \text{ and } l < n \}$

We will prove that L_1 is not context-free
by using CONTRAPOSITIVE on pumping lemma.

Let the demon pick a given $p \geq 0$
We pick $w = a^p b^{p+1} c^{p+1}$

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→ Let the adversary decompose

$w = uvxyz$ $|vy| > 0$ $|vxy| \le p$

into distinct ways.

~~Case 1~~

~~When v and y both~~

Let us divide our string w

into three parts $\underbrace{a^p}_{I} \underbrace{b^{p+1}}_{II} \underbrace{c^{p+1}}_{III}$

Case 1

When v and y are both present in part I

then uv^2xy^2z will contain more a's

than b's. hence $w \notin L_1$ [pumping in]

Case 2

When v and y are both present in part II

then uv^0xy^0z will contain

less b's and more a's [pumping out]

Hence $w \notin L_2$. Similar argument

when v and y are both present

in part III

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Case 3

When u and v straddles across two adjacent parts

then uv^2xy^3z, will have more a's

then c's ~~Since it will be of the form~~

Hence $w_1 \notin L_1$

Case 4

u and v cannot be present in part I and simultaneously because then $|vxy| \nleq p$

Hence in all the cases there exists w_1 such that $w \notin L_1$. Hence not context-free.

[Hence Proved]

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3a) For all non-palindromes for $\{a, b\}$

we consider two cases

1) Odd length = (Not one)

2) Even length. – (Not 0)

It will be a non-palindrome if it differs in atleast one position in the second half of string [if even length]

So,

$S \to aSa \mid bSb \mid Y$ (this covers all non-palindromes)

$Y \to axb \mid axb$ [this generates strings of form a(x)b]

$X \to \varepsilon \mid aX \mid bX$ [this generates $(a+b)^*$]

(b) PDA for L2

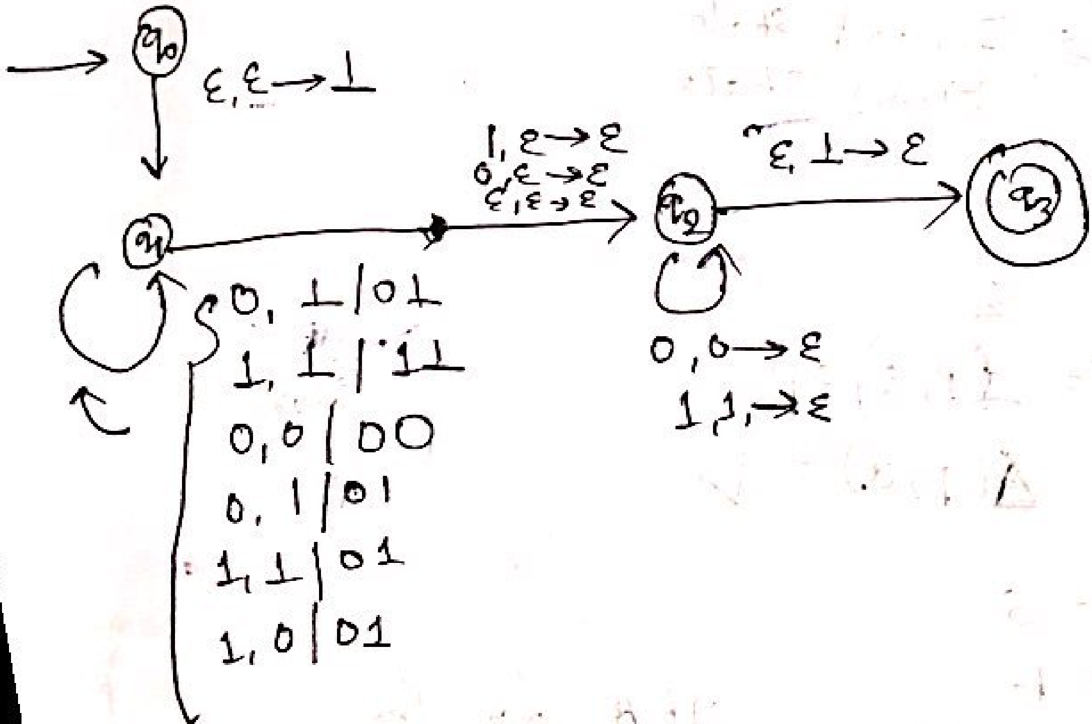
→ It accepts by empty stack

→ First we push '⊥' into the stack as bottom marker

→ Pushing 0's and 1's till the machine non-deterministically guesses that it has reached the middle of the string

→ Then poping the matched pairs

→ Finally popping the bottom marker as it accepts by empty stack



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8a) $P = (Q, \Sigma, \Gamma, \perp, \delta, s, F)$ is a

7-tuple PDA such that it

only has transitions $((p, a, A), (q, \gamma A))$

It will prove that $L(P)$ is regular

by constructing an equivalent NFA for the PDA.

NFA = $(Q', \Sigma', \Delta', s', F')$

Q' = ~~ID~~ ~~ID~~ finite States

Σ' = Input Alphabet

Δ' = ~~Trans~~ Transition

s' = Initial State

F' = Final State

Here $Q' = Q$

$\Sigma' = \Sigma$

$\Delta' = \Delta(p, \varepsilon) = p$

$\Delta(p, a) = q$

$S' = S$

$F' = F$

Hence there exists an NFA for the PDA. hence it is regular

3b) To Prove = Every CFL has a CFG of degree two

We can convert every CFL into a one or more CNF. A CNF produces the same language as generated by CFG. This can be easily proved by Induction on the length of derivation of the string $|w| = n$. ~~[illegible]~~

It requires '2n-1' production or steps in CNF.

$S \rightarrow AB$ $(n-1)$

$A \rightarrow a$ $[n]$

Hence 2n-1 steps

And snie Chromsky Normal Form is

$A \rightarrow BC$
for all $A, B, C \in N$
$a \in \Sigma - \{\varepsilon\}$
$A \rightarrow a$

There are exactly two non-terminals 'B' and 'C'

Hence it has a degree 2. [And the cases for epsilon in the grammer can be handled separately by defining the Production S->ε in the CFG]

i.e. $S \rightarrow \varepsilon$ in the

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